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1957

SovietRxiv

Abstract

Full Text

GEOPHYSICS

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FURTHER ON THE KINEMATICS OF SEA WAVES

In a number of previous papers ([1–4]) we established that the profile of sea waves, especially wind waves, differs from trochoids, derived equations for the profile in parametric form

$$x = R\theta + a\sin\theta, \quad y = b\cos\theta \tag{1}$$

and discovered the physical causes of the strong sharpening of wind waves. In all the cited articles it was assumed that a and b are the semi-axes of a certain auxiliary ellipse, making it possible, with sufficient approximation, to construct the wave profile point by point. Such a representation was very convenient, since the ratio of the semi-axes a/b could be connected with the characteristics of new phenomena discovered by us in the kinematics of sea waves. Namely, from the continuity equation it was found that, in the absence of wind or with a very weak wind, there must exist pulsations of the velocity of the "wave current" discovered by Stokes; these pulsations of the current (previously regarded as constant in velocity) give rise to the difference between a and b, with

$$\frac{a}{b} = 1 + \frac{r}{R}. \tag{2}$$

The same continuity equation enabled us to establish that, with a strong wind, sharp oscillations of the velocity of the drift current must arise, with the same period, equal to the period T of the waves. The generalized expression for a/b is written as follows:

$$\frac{a}{b} = 1 + \frac{r}{R}\left(1 + \frac{\overline{u}}{v}\right). \tag{3}$$

In equations (2) and (3), r is the radius of the orbit of a surface particle; R is the so-called radius of the rolling circle ($R = \lambda/2\pi$, where λ is the wavelength); $\overline{u}$ is the period-mean velocity of the drift current (previously regarded as constant); v is the velocity of the orbital motion of particles on the sea surface.

In the present article we shall show that the ellipse with semi-axes a and b, to which we previously assigned only the modest role of an auxiliary curve, acquires

Fig. 1

Figure 1: Fig. 1

a quite definite physical meaning, while the equations in parametric form (1) are not a simple "template," more or less successfully devised for tracing the profile of sea waves, but are more valid equations of kinematics than the equations of the trochoid, which served as a first approximation.

First of all, using formula (89) of paper ([4]), let us express the instantaneous value of the velocity of the wave current w; using formula (95) of paper ([4]), let us express the instantaneous value of the velocity of the drift current u; let us also take into account the instantaneous value of the horizontal component of the orbital velocity $v\cos\theta$ of the motion of a surface particle:

$$w = \omega\frac{r^2}{R} + \omega\frac{r^2}{R}\cos\theta, \quad u = \overline{u} + \overline{u}\frac{r}{R}\cos\theta, \quad v\cos\theta = \omega r\cos\theta. \tag{4}$$

Adding the left-hand sides of the three equations (4), we obviously obtain the instantaneous value of the full velocity of the horizontal motion of the surface particles. Adding all the right-hand sides, we obtain an expression that will make it possible to understand the physical meaning of the phenomena:

$$w + u + v\cos\theta = \left(\omega\frac{r^2}{R} + \bar{u}\right) + \left(\omega r + \omega\frac{r^2}{R} + \bar{u}\frac{r}{R}\right)\cos\theta. \tag{5}$$

Equation (5) shows that the full velocity of the horizontal motion of the surface particles splits into two parts: one of them is the "constant" current with which the kinematics of sea currents operated until quite recently; this "constant" current consists of a "wave" current, possessing, according to Stokes, the velocity $\omega\frac{r^2}{R}$, and of a drift current, previously known by its mean velocity $\bar{u}$.

Fig. 1

Of greatest interest is the other—the oscillating—part of the current; we shall write its velocity, first taking the angular velocity ω of the orbital motion outside the brackets:

$$U_x = \omega\left(r + \frac{r^2}{R} + \frac{\bar{u}}{\omega}\frac{r}{R}\right)\cos\theta. \tag{6}$$

It is easy to show that the expression in parentheses in (6) is nothing other than the expression for the semiaxis a of our "auxiliary" ellipse. Indeed, multiply the left-hand side of equation (3) by b, and the right-hand side by the equal quantity r (since the minor semiaxis is equal to half the wave height, i.e. r):

$$a = r + \frac{r^2}{R} + \frac{\bar{u}}{v}\frac{r^2}{R}. \tag{7}$$

Multiply the numerator and denominator of the third term in parentheses in formula (6) by r. Then the expression (7) will appear in the parentheses, since $\omega r = v$.

Thus, the velocity of the horizontal oscillations of the surface particles is

$$U_x = \omega a \cos\theta. \tag{8}$$

The velocity of the vertical oscillations of the same particles is found simply—by differentiating the second equation of system (1):

$$U_y = -\omega b \sin\theta. \tag{9}$$

The equations derived reveal the physical meaning of the ellipse with semiaxes a and b. Namely, in a coordinate system moving with velocity

$\omega\frac{r^2}{R} + \bar{u}$ in the direction of the waves, an ellipse with semiaxes a and b is the trajectory of a water particle.

Thus, in the most general case, the complex motion of a surface water particle, in the indicated moving coordinate system, can be described by the following components:

1. As in the classical schemes, the particle rotates with angular velocity ω about a certain center, remaining constantly at a distance r from it.
2. The center of rotation itself executes horizontal oscillations about its mean position with velocity $\omega\frac{r^2}{R}\left(1 + \frac{\bar{u}}{v}\right)\cos\theta$.

Figure 1 graphically depicts three cases of such motions of particles with the corresponding phase shift as they advance in the direction of wave propagation. Case 1*a* corresponds to a classical trochoidal wave; here the centers of rotation are immobile. Case 1*b* corresponds to a somewhat sharpened form of the wave profile; the centers of rotation execute oscillations in horizontal directions. Finally, in Fig. 1*c* a very strongly sharpened wave is presented; the centers of rotation execute horizontal oscillations of large amplitude.

Fig. 2

In Fig. 2, attention is concentrated on one surface water particle for all three variants mentioned. Fig. 2*a* represents the different phases of the orbital motion of the particle about an immobile center. In Fig. 2*b*, the particle moves with the same angular velocity about the center of rotation, remaining at the same distance r from it, but the center of rotation itself oscillates in horizontal directions about its mean position. In Fig. 2*c*, such horizontal oscillations of the center of rotation attain the greatest magnitude possible for a stable wave profile.

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Received
24 XII 1956

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Note: Figure translations are in progress. See original paper for figures.