ON THE INFLUENCE OF GRAVITY ON THE PROPAGATION OF LIGHT*

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Abstract

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The equations of the electromagnetic field in vacuum in the presence of finite masses may be written in the form of Maxwell' s equations for a moving anisotropic medium. The character of the anisotropy in this case is determined by the properties of the metric tensor.

Since the spatial scales in problems of an astronomical character are enormous in comparison with the wavelengths of electromagnetic waves, even of radio frequency, the coefficients entering the equations turn out to be extremely slowly varying functions of the coordinates. This latter circumstance makes it possible in most cases to restrict oneself to the approximation of geometrical optics. Applying the method developed by S. M. Rytov, one can determine not only the path of the rays, but also the change in the character of the polarization of the wave along the ray.

It turns out, for example, that in the propagation of a plane electromagnetic wave near a rotating massive body the trajectory of the ray is, generally speaking, not a plane curve, but undergoes torsion in the direction of rotation of the body. At the same time there is a rotation of the plane of polarization proportional to the angular momentum of the rotating body. It is noteworthy that the plane of polarization of rays emerging from the poles of the rotating body and propagating along the axis of rotation is also rotated through a certain angle in the direction of rotation of the body.

The propagation of electromagnetic waves in vacuum in the presence of finite masses is described by the equations

$$\partial_\sigma f^{\rho\sigma} = 0; \qquad \partial_\lambda F_{\rho\sigma} + \partial_\rho F_{\sigma\lambda} + \partial_\sigma F_{\lambda\rho} = 0,$$

where

$$f^{\rho\sigma} = \varepsilon^{\rho\sigma\alpha\beta} F_{\alpha\beta},$$

$$\varepsilon^{\rho\sigma\alpha\beta} = \sqrt{-g}\, g^{\rho\alpha} g^{\sigma\beta}.$$

Introducing three-dimensional vectors according to the scheme

$$F_{\rho\sigma} \to (\mathbf{B}, \mathbf{E}); \qquad f^{\rho\sigma} \to (\mathbf{H}, -\mathbf{D})$$

and passing to vector notation, we obtain a system of equations formally coinciding with Maxwell' s equations in a medium:

$$\operatorname{rot} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \qquad \operatorname{div} \mathbf{B} = 0;$$

$$\operatorname{rot} \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}; \qquad \operatorname{div} \mathbf{D} = 0.$$

The tensor $\varepsilon^{\rho\sigma\alpha\beta}$, which relates the field vectors to one another, is analogous to the tensor encountered in the relativistic theory of moving anisotropic media—

* The formulas given in the paper for the torsion of the ray and the angle of rotation of the plane of polarization of an electromagnetic wave were obtained by A. R. Ptitsyn under the author' s supervision as early as 1951.

the tensor of electric and magnetic permeability ([1]). To determine the form of the tensor it is necessary to know the components of the metric tensor $g_{\alpha\beta}$.

Let us consider, as an example, a stationary gravitational field created by a uniformly rotating spherical mass m_0. Approximate solutions of the gravitational equations determining the field of gravity in space outside the body can, with high accuracy, be represented in the form ([2])

$$g^{00} = -1 - h; \qquad g^{11} = g^{22} = g^{33} = 1 - h; \qquad g^{0\alpha} = g_{0\alpha} = \frac{2\gamma}{c^3 r^3}[\mathbf{r}, \mathbf{M}]_\alpha,$$

where

$$h = \frac{2\gamma}{c^2}\frac{m_0}{r};$$

γ is the gravitational constant; $\mathbf{M}$ is the angular momentum of the rotating body.

Now, according to (2), we obtain

$$\mathbf{D} = \varepsilon\mathbf{E} - [\mathbf{G}, \mathbf{H}]; \qquad \mathbf{B} = \mu\mathbf{H} + [\mathbf{G}, \mathbf{E}],$$

where

$$\varepsilon = \sqrt{-g}\,(-g^{00})\,g^{11}; \qquad \mu = \frac{1}{\sqrt{-g}\,(g^{11})^2}; \qquad \mathbf{G} = \frac{2\gamma}{c^3 r^3}[\mathbf{r}, \mathbf{M}].$$

In the centrally symmetric field of a nonrotating mass ($\mathbf{G} = 0$) the relation between the field vectors becomes especially simple.

For periodic processes $\omega = ck$, equations (4) in the case under consideration take the form

$$\operatorname{rot}\mathbf{E} = -ik(\mu\mathbf{H} + [\mathbf{G}, \mathbf{E}]); \qquad \operatorname{rot}\mathbf{H} = ik(\varepsilon\mathbf{E} - [\mathbf{G}, \mathbf{H}]).$$

Using the method developed by S. M. Rytov ([3,4]), we shall seek solutions of these equations in the form

$$\mathbf{E} = \frac{\vec{\mathcal{E}}}{\sqrt{\varepsilon}}\,e^{ik\Phi}; \qquad \mathbf{H} = \frac{\vec{\mathcal{H}}}{\sqrt{\mu}}\,e^{-ik\Phi},$$

where $\vec{\mathcal{E}}$, $\vec{\mathcal{H}}$, and Φ are functions of the coordinates. Representing, further, $\vec{\mathcal{E}}$ and $\vec{\mathcal{H}}$ in the form of series

$$\vec{\mathcal{E}} = \vec{\mathcal{E}}_0 + \frac{i}{k}\vec{\mathcal{E}}_1 + \ldots; \qquad \vec{\mathcal{H}} = \vec{\mathcal{H}}_0 + \frac{i}{k}\vec{\mathcal{H}}_1 + \ldots,$$

we obtain in the zeroth approximation a system of equations for determining $\vec{\mathcal{E}}_0$ and $\vec{\mathcal{H}}_0$:

$$[\operatorname{grad}\Phi - \mathbf{G}, \vec{\mathcal{H}}_0] + \sqrt{\varepsilon\mu}\,\vec{\mathcal{E}}_0 = 0; \qquad -\sqrt{\varepsilon\mu}\,\vec{\mathcal{H}}_0 + [\operatorname{grad}\Phi - \mathbf{G}, \vec{\mathcal{E}}_0] = 0.$$

In the first approximation

$$[\operatorname{grad}\Phi - \mathbf{G}, \vec{\mathcal{H}}_1] + \sqrt{\varepsilon\mu}\,\vec{\mathcal{E}}_1 = -\operatorname{rot}\vec{\mathcal{H}}_0 + \frac{1}{2\mu}[\operatorname{grad}\mu, \vec{\mathcal{H}}_0];$$

$$-\sqrt{\varepsilon\mu}\,\vec{\mathcal{H}}_1 + [\operatorname{grad}\Phi - \mathbf{G}, \vec{\mathcal{E}}_1] = -\operatorname{rot}\vec{\mathcal{E}}_0 + \frac{1}{2\varepsilon}[\operatorname{grad}\varepsilon, \vec{\mathcal{E}}_0].$$

Equating to zero the determinant of the system of homogeneous linear equations (11), we find

$$[(\operatorname{grad}\Phi - \mathbf{G})^2 - \varepsilon\mu]^2 = 0.$$

Thus, in the zeroth approximation we obtain the generalized eikonal equation:

$$\operatorname{grad}\Phi - \mathbf{G} = \sqrt{\varepsilon\mu}\,\mathbf{e},$$

where $\mathbf{e}$ is the unit vector in the direction of propagation of the wave at the given point. The field lines of the vector $\mathbf{e}$ coincide with the lines of energy flow of the electromagnetic field.

The general solution of the equations of the zeroth approximation (11), which in this case take the form

$$[\mathbf{e}, \vec{\mathcal{H}}_0] + \vec{\mathcal{E}}_0 = 0; \qquad -\vec{\mathcal{H}}_0 + [\mathbf{e}, \vec{\mathcal{E}}_0] = 0,$$

can be written in the form

$$\vec{\mathcal{E}}_0 = f_1\mathbf{n} + f_2\mathbf{b}; \qquad \vec{\mathcal{H}}_0 = f_1\mathbf{b} - f_2\mathbf{n},$$

where f_1 and f_2 are arbitrary functions of the coordinates, which cannot be determined from the zeroth approximation; $\mathbf{n}$ and $\mathbf{b}$ are the unit vectors of the principal normal and binormal to the ray. The vectors $\mathbf{n}$, $\mathbf{b}$, $\mathbf{e}$ form an orthogonal trihedron.

The equations of the first approximation (12) can now be rewritten as follows:

$$[\mathbf{e}, \vec{\mathcal{H}}_1] + \vec{\mathcal{E}}_1 = -\frac{1}{\sqrt{\varepsilon\mu}}\left\{\operatorname{rot}\vec{\mathcal{H}}_0 - \frac{1}{2\mu}[\operatorname{grad}\mu, \vec{\mathcal{H}}_0]\right\},$$

$$-\vec{\mathcal{H}}_1 + [\mathbf{e}, \vec{\mathcal{E}}_1] = -\frac{1}{\sqrt{\varepsilon\mu}}\left\{\operatorname{rot}\vec{\mathcal{E}}_0 - \frac{1}{2\varepsilon}[\operatorname{grad}\varepsilon, \vec{\mathcal{E}}_0]\right\}.$$

The solvability condition for this system of equations consists in requiring the orthogonality of their right-hand sides to each of the linearly independent solutions of the transposed homogeneous system. Combining the two conditions thus obtained, we obtain from this the law of variation of the intensity along the ray and the important relation

$$\frac{1}{2}(\mathbf{n}\operatorname{rot}\mathbf{n} + \mathbf{b}\operatorname{rot}\mathbf{b}) = (\mathbf{e}\operatorname{grad})\Psi,$$

where

$$\Psi = \operatorname{arc\,tg}\frac{f_2}{f_1},$$

which determines the angle Ψ between the principal normal to the ray and the vector $\vec{\mathcal{E}}_0$. Making use further of the identity known from differential geometry

$$\mathbf{n}\ \mathrm{rot}\,\mathbf{n} + \mathbf{b}\ \mathrm{rot}\,\mathbf{b} = \frac{2}{\rho} + \mathbf{e}\ \mathrm{rot}\,\mathbf{e},$$

where ρ is the radius of curvature of the ray (the vector line $\mathbf{e}$), we obtain, taking into account that $\mathbf{e}$ grad $= d/ds$, where ds is the element of arc length measured along the curve, the equation

$$\frac{d\Psi}{ds} = \frac{1}{\rho} + \frac{1}{2}\mathbf{e}\ \mathrm{rot}\,\mathbf{e},$$

which determines the law of rotation of the plane of polarization of an electromagnetic wave during its propagation in the gravitational field of a spherically symmetric rotating mass.

If the angular velocity of rotation of the star is zero, then $\mathbf{e}\ \mathrm{rot}\,\mathbf{e} = 0$ and $1/\rho = 0$. In this case the trajectory of the ray proves to be plane, and the plane of polarization does not rotate during its propagation.

The right-hand side of equation (18) is ultimately determined by the geometry of space and does not depend on the frequency of the wave. Thus, in a gravitational field there is no rotatory dispersion.

When a wave propagates in a direction parallel to the axis of rotation of the body, the light ray experiences not only the Einsteinian curvature, but is also twisted in the direction of rotation of the body through an angle

$$\varphi = \frac{4\gamma M}{c^3 r^2},$$

remaining all the time on the surface of revolution of the Einstein curve. In this case, taking into account the twisting of the ray, there is a rotation of the plane of polarization through the angle

$$\Delta\Psi_{\mp\infty} = \frac{8\gamma M}{c^3 r^2}.$$

It is remarkable that the plane of polarization of light rays coming from the pole of a rotating body along the axis of rotation also undergoes a rotation through the angle

$$\Delta\Psi_0 = 3\frac{\gamma M}{c^3 R^2},$$

where R is the radius of the body. The latter circumstance once again testifies to the absolute character of rotation in the theory of relativity.

For rays perpendicular to the axis of rotation of the body z and parallel, for example, to the axis y, the rotation of the plane of polarization on different sides of the plane (x, z) occurs in different directions, so that if the wave goes from $-\infty$ to $+\infty$, the total rotation is equal to zero. For waves emitted by the surface of the body, in this case

$$\Delta\Psi_{\perp} = \frac{4\gamma M}{c^3 R^2}\, z.$$

The factor $\gamma/c^3 \sim 10^{-39}$, and therefore the effects of twisting and of rotation of the plane of polarization can be noticeable only for very large values of M/R^2. For ordinary stars the rotation effect is very small. Thus, for the Sun $\Delta\Psi \sim 6 \cdot 10^{-6}$ sec. A noticeable effect is obtained for stars of the white-dwarf type. Thus, for the dwarf [5] L 745–46, $\Delta\Psi = 1.65$ sec. For light coming from an external source, the effect is doubled.

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([5]) S. M. Petrov, *Priroda*, No. 6 (1945).

Note: Figure translations are in progress. See original paper for figures.