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Abstract

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DESCRIPTION OF TRANSIENT ELECTROMECHANICAL PROCESSES IN ASYNCHRONOUS MACHINES BY MEANS OF EQUATIONS WITHOUT PERIODIC COEFFICIENTS

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If the equations of transient electromechanical processes of an asynchronous machine are written with the aid of the phase currents, flux linkages, and voltages of the stator and rotor, i.e., in the so-called phase coordinates, then, as is known, the equations of the flux linkages and of the electromagnetic torque will contain periodic coefficients. Their presence is explained by the fact that, as a result of rotation of the machine rotor, the mutual inductances of the stator and rotor phase windings are periodic functions of the angle

$$\theta = \int_0^t \omega \, dt + \theta_0$$

between the magnetic axes of the stator phase *ca* and the rotor phase *pa*, and, respectively, between the axes *cb* and *pb*, or *cc* and *pc* (Fig. 1).

There exist a number of methods for replacing phase coordinates ([1–4]), which give expressions for the flux linkages and electromagnetic torque that do not contain periodic coefficients. This leads to a very substantial simplification of the entire system of equations of the transient electromechanical processes of an asynchronous machine. In deriving these equations, assumptions are made that are usual in investigations of this kind and that do not give rise to substantial discrepancies between the obtained results and experiment ([5,6]). We shall show that the method of the proposed transformations can be substantially simplified and carried out entirely without the use of complex numbers.

Fig. 1

Fig. 1

Figure 1: Fig. 1

We shall use, for the asynchronous machine, the Park transformation, or the coordinate system $d, q, 0$, which in what follows we shall denote by $d_k, q_k, 0$, since it rotates with an arbitrary angular velocity ω_k, whereas the rotor of the machine rotates with angular velocity ω.

Let the asynchronous machine under investigation be connected to a network whose phase voltages are u_{ca}, u_{cb}, u_{cc} (Fig. 2). The voltages at the rotor rings are u_{pa}, u_{pb}, u_{pc}. Figure 3 shows the corresponding stator currents (i_{ca}, i_{cb}, i_{cc}) and rotor currents (i_{pa}, i_{pb}, i_{pc}) and their positive directions relative to the like-named terminals of the windings.

We shall carry out the transformation of the equations of Ohm' s law and of the flux linkages separately for the stator and for the rotor, using the matrix form of their notation. We note that it would also have been possible to carry out this transformation

simultaneously for the stator and the rotor, but then one would have to use block matrices, or, if they were not introduced, the current, voltage, and flux-linkage matrices would have six elements each, which would greatly complicate their notation. On the other hand, block matrices will be expediently used for transforming the equation of the electromagnetic torque of the machine.

Fig. 2

Fig. 3

Let us write the equations of Ohm' s law and of the flux linkages of the stator and rotor phases:

$$[u_c] = r_c[i_c] + \frac{d[\psi_c]}{dt}, \qquad [u_p] = r_p[i_p] + \frac{d[\psi_p]}{dt},$$
$$[\psi_c] = [L_{cc}][i_c] + [M_{cp}][i_p], \qquad [\psi_p] = [M_{pc}][i_c] + [L_{pp}][i_p], \tag{1}$$

where

$$[u_c] = \begin{bmatrix} u_{ca} \\ u_{cb} \\ u_{cc} \end{bmatrix}, \qquad [u_p] = \begin{bmatrix} u_{pa} \\ u_{pb} \\ u_{pc} \end{bmatrix},$$

$$[L_{cc}] = \begin{bmatrix} L_c & M_c & M_c \\ M_c & L_c & M_c \\ M_c & M_c & L_c \end{bmatrix}, \qquad [L_{pp}] = \begin{bmatrix} L_p & M_p & M_p \\ M_p & L_p & M_p \\ M_p & M_p & L_p \end{bmatrix}; \tag{2}$$

$$[M_{cp}] = [M_{pc}^t] = M \begin{bmatrix} \cos\theta & \cos(\theta+120^\circ) & \cos(\theta-120^\circ) \\ \cos(\theta-120^\circ) & \cos\theta & \cos(\theta+120^\circ) \\ \cos(\theta+120^\circ) & \cos(\theta-120^\circ) & \cos\theta \end{bmatrix}. \quad (3)$$

Here L_c (L_p) is the inductance (constant) of one stator (rotor) phase; M_c (M_p) is the mutual inductance (constant) of two stator (rotor) phases; M is the maximum mutual inductance (constant) between one stator phase and one rotor phase.

We introduce the matrices of the stator $[A_c]$ and rotor $[A_p]$ transformations[5]

$$[A_c] = \frac{2}{3} \begin{bmatrix} \cos\theta_k & \cos(\theta_k-120^\circ) & \cos(\theta_k+120^\circ) \\ -\sin\theta_k & -\sin(\theta_k-120^\circ) & -\sin(\theta_k+120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}; \quad (4)$$

$$[A_p] = \frac{2}{3} \begin{bmatrix} \cos(\theta_k-\theta) & \cos(\theta_k-\theta-120^\circ) & \cos(\theta_k-\theta+120^\circ) \\ -\sin(\theta_k-\theta) & -\sin(\theta_k-\theta-120^\circ) & -\sin(\theta_k-\theta+120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \quad (5)$$

Here

$$\theta_k = \int_0^t \omega_k\,dt + \theta_{k0}, \qquad \frac{d\theta_k}{dt} = \omega_k; \quad (6)$$

θ_k is the angle between the longitudinal axis d_k of a coordinate system rotating with arbitrary angular velocity ω_k and the magnetic axis of phase a of the machine stator. These matrices make it possible to relate the voltages, currents, and flux linkages of the stator and rotor before and after the transformation.

The voltage, current, and flux-linkage matrices after transformation will be denoted by a prime:

$$[u'_c] = \begin{bmatrix} u_{cd} \\ u_{cq} \\ u_{c0} \end{bmatrix} = [A_c][u_c], \qquad [u'_p] = \begin{bmatrix} u_{pd} \\ u_{pq} \\ u_{p0} \end{bmatrix} = [A_p][u_p]. \quad (7)$$

To transform Ohm' s-law equations and the stator flux linkages, we multiply both sides of (1) for $[u_c]$ and $[\psi_c]$ on the left by $[A_c]$. Taking (2), (3), and (7) into account, we obtain

$$[u'_c] = r_c[i'_c] + \frac{d[\psi'_c]}{dt} + \begin{bmatrix} -\psi_{cq} \\ \psi_{cd} \\ 0 \end{bmatrix} \frac{d\theta_k}{dt}, \qquad [\psi'_c] = [L'_{cc}][i'_c] + [M'_{cp}][i'_p], \quad (8)$$

where

$$[L'_{cc}] = [A_c][L_{cc}][A_c^{-1}] = \begin{bmatrix} L_{c1} & 0 & 0 \\ 0 & L_{c1} & 0 \\ 0 & 0 & L_{c0} \end{bmatrix}; \qquad (9)$$

$$[M'_{cp}] = [A_c][M_{cp}][A_p^{-1}] = M_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = [M'_{pc}]; \qquad (10)$$

$[A_c^{-1}]$ and $[A_p^{-1}]$ are the inverse matrices of the stator and rotor transformations; $L_{c1} = L_c - M_c$ and $L_{c0} = L_c + 2M_c$ are the inductances of the positive- and zero-sequence stator; $M_3 = {}^3/_2 M$ is the three-phase coefficient of mutual induction of the stator and rotor.

To transform Ohm' s-law equations and the rotor flux linkages, we multiply both sides of (1) for $[u_p]$ and $[\psi_p]$ on the left by $[A_p]$. Taking (2), (3), and (7) into account, we obtain

$$[u'_p] = r_p[i'_p] + \frac{d[\psi'_p]}{dt} + \begin{bmatrix} -\psi_{pq} \\ \psi_{pd} \\ 0 \end{bmatrix} \left(\frac{d\theta_k}{dt} - \frac{d\theta}{dt}\right), \qquad (11)$$

$$[\psi'_p] = [M'_{pc}][i'_c] + [L'_{pp}][i'_p],$$

where

$$[L'_{pp}] = [A_p][L_{pp}][A_p^{-1}] = \begin{bmatrix} L_{p1} & 0 & 0 \\ 0 & L_{p1} & 0 \\ 0 & 0 & L_{p0} \end{bmatrix}; \qquad (12)$$

$L_{p1} = L_p - M_p$ and $L_{p0} = L_p + 2M_p$ are the inductances of the positive- and zero-sequence rotor.

Starting from the known expression for the electromagnetic torque

$$T = \frac{1}{2}[i_t]\frac{d[L]}{d\theta}[i], \qquad \text{where} \quad [i] = \begin{bmatrix} i_c \\ i_p \end{bmatrix}, \quad [L] = \begin{bmatrix} L_{cc} & M_{cp} \\ M_{pc} & L_{pp} \end{bmatrix}, \qquad (13)$$

after transformation we obtain

$$T = \frac{1}{2}\begin{bmatrix} i'_{ct} & i'_{pt} \end{bmatrix} \begin{bmatrix} [A_{ct}^{-1}]\dfrac{d[M_{cp}]}{d\theta}[A_p^{-1}][i'_p] \\ [A_{pt}^{-1}]\dfrac{d[M_{cpt}]}{d\theta}[A_c^{-1}][i'_c] \end{bmatrix}, \qquad (14)$$

or, finally:

$$T = 2T_{cp} = \frac{3}{2}M_3 \begin{bmatrix} i_{cd} & i_{cq} & i_{c0} \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{pd} \\ i_{pq} \\ i_{p0} \end{bmatrix} = \frac{3}{2}M_3(i_{cq}i_{pd} - i_{cd}i_{pq}). \tag{15}$$

Now to the voltage and flux-linkage equations of the stator (8) and rotor (11) obtained above we add the equation of motion of the rotor

$$T - T_c = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}, \tag{16}$$

where T_c is the resisting torque on the motor shaft, and J is the moment of inertia of the rotor of the induction motor and of the parts rotating together with it.

Let us eliminate the flux linkages from (8) and (11), omit the equations for the zero components, which are solved separately from the others, and rewrite (8) and (11), as well as (15) and (16), in expanded form:

$$u_{cd} = \left(r_c + L_{c1}\frac{d}{dt}\right) i_{cd} + M_3\frac{di_{pd}}{dt} - \omega_k \left(L_{c1}i_{cq} + M_3 i_{pq}\right),$$

$$u_{cq} = \left(r_c + L_{c1}\frac{d}{dt}\right) i_{cq} + M_3\frac{di_{pq}}{dt} + \omega_k \left(L_{c1}i_{cd} + M_3 i_{pd}\right); \tag{17}$$

$$u_{pd} = \left(r_p + L_{p1}\frac{d}{dt}\right) i_{pd} + M_3\frac{di_{cd}}{dt} - (\omega_k - \omega)\left(M_3 i_{cq} + L_{p1}i_{pq}\right),$$

$$u_{pq} = \left(r_p + L_{p1}\frac{d}{dt}\right) i_{pq} + M_3\frac{di_{cq}}{dt} + (\omega_k - \omega)\left(M_3 i_{cd} + L_{p1}i_{pd}\right); \tag{18}$$

$$\frac{3}{2}M_3\left(i_{cq}i_{pd} - i_{cd}i_{pq}\right) - T_c = J\frac{d\omega}{dt}. \tag{19}$$

We have thus obtained 5 equations with 5 unknowns i_{cd}, i_{cq}, i_{pd}, i_{pq}, and ω, the solution of which makes it possible to investigate and calculate any electromechanical transient processes in an induction machine. In this case the voltages at the terminals of the stator (rotor), u_{cd}, u_{cq}, u_{c0} (u_{pd}, u_{pq}, u_{p0}), must either be specified or related by equations to the currents and parameters of the stator (rotor) circuit.

The angular velocity ω_k of rotation of the coordinate axes and the resisting torque T_c on the motor shaft must also be specified. Since the rotor of an induction machine is electrically and magnetically symmetrical, the equations for u_{cd} and u_{cq} (and also for u_{pd} and u_{pq}) contain identical inductances. As is known ([1–3]), the following values are taken for the angular velocity ω_k of the coordinate axes.

1) $\omega_k = \omega_0$, where ω_0 is the synchronous speed.
2) $\omega_k = \omega$, i.e., the coordinate axes will be stationary relative to the rotor. In this case equations (17) and (18) coincide with Stanley' s equations ([1]). This case is of interest because in the rotor equations the rotational emf' s disappear, while in the stator equations the rotational emf' s are the product of the current components and the angular velocity of the rotor $\omega = \omega_k$.
3) $\omega_k = 0$, i.e., the coordinate axes are stationary relative to the stator. In this case the rotational emf' s in the stator equations disappear.

It should be noted, however, that the case $\omega_k = 0$ is not equivalent to writing the equations in phase coordinates, from which we proceeded at the very beginning of the investigation. The point is that, as follows from equalities (7), even when $\omega_k = 0$ the values of the currents and other quantities transformed to the axes d_k, q_k, 0 are resultant waves produced by the windings of all three phases, and as a result the periodic coefficients are eliminated from the equations of flux linkages and electromagnetic torque.

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Note: Figure translations are in progress. See original paper for figures.