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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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CONSTITUENTS SUFFICIENT TO ENSURE THE OPERATION OF RELAY-CONTACT CIRCUITS

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As is known from the theory of relay-contact circuits ([1]), the maximum number of unit constituents determining the switched-on (switched-off) state of a circuit is equal to $2^n - 1$, where n is the number of elements of the circuit making up the chain of some actuating element X operating once per cycle.

If the constituents are arranged as indicated in ([2,3]), where they correspond to sequentially arranged instants differing in the state of only one element, then in this case as well (and it gives a smaller number of repeated actuations of the elements compared with the usual order of writing the constituents), already for $n \geq 3$ there is multiple operation of the circuit elements. We shall show that the operating switched-on (switched-off) state of the circuit is ensured by fewer than $2^n - 1$ constituents, and that in the analytical expression of the circuit only single operation of its elements can be taken into account.

We shall call **superfluous** those constituents that are equal to zero in a definite combination of circuit chains, do not affect the analytical expression of the entire chain, and arise as a result of an unnecessary switching-on or switching-ons (switching-off or switching-offs) of an element or elements of the circuit.

The noted feature is very important, since it makes it possible to be limited to a smaller—and in some cases a considerably smaller—number of constituents. The use of a smaller number of constituents leads to the creation of a circuit with a smaller number of actuations of its elements, if repeated actuations of these elements are not caused by the necessity of acting upon some other elements or of receiving such actions, and to the achievement of single operation of each element while observing all the operating conditions of the circuit.

Fig. 1

The operation of any circuit whose chains consist of $2^n - 1$ constituents is shown

Fig. 2

Figure 2: Fig. 2

in Fig. 1a. As is seen from the figure, individual elements of the circuit are switched on (switched off) more than once per cycle, i.e., repeatedly.

If the number of elements of the circuit is equal to 3, then one of the possible operating schedules of such a circuit, under the above-mentioned condition that constituents of adjacent instants differ in the state of only one element, will be as shown in Fig. 2a,

$$f = x_1\overset{1}{\bar{x}_2}\bar{x}_3 + x_1\overset{2}{\bar{x}_2}x_3 + x_1\overset{3}{x_2}x_3 + x_1\overset{4}{x_2}\bar{x}_3 + \bar{x}_1\overset{5}{x_2}\bar{x}_3 + \bar{x}_1\overset{6}{x_2}x_3 + \bar{x}_1\overset{7}{\bar{x}_2}x_3. \quad (1)$$

Here 7 constituents are written out that determine the operating state of the circuit. In this case the element X_3 is switched on twice. It is taken into account that one constituent $x_1x_2x_3$ corresponds to the cycle preceding the release cycle. The sum of expression (1) is equal to $x_1 + x_2 + x_3$.

The sum of the 5 constituents with serial numbers 1, 4, 5, 6, and 7 is also equal to $x_1 + x_2 + x_3$. In this combination the remaining constituents with serial numbers 2 and 3 do not change the obtained sum and are superfluous. The graph taking into account only 5 constituents is given in Fig. 2 .

Fig. 2

Consideration of the remaining graphs of the operation of a three-element circuit gives analogous results. Thus, for a three-element circuit, with any arrangement of its elements in the switching table, the analytic expression will be a record of the successive overlap of the operating state of these elements. Suppose that this holds for the number of elements equal to $n - 1$, and show that it will be valid for the number of elements equal to n.

Let us consider the operation of the elements of the circuit from X_1 to X_n. If the operations of the elements from X_1 to X_{n-1} are represented graphically in the form Z (Fig. 1), then the analytic expression of the circuit paths will be

$$f = x_n\bar{z} + x_{nz} + \bar{x}_n\bar{z} + x_n\bar{z} + \bar{x}_n z + x_{nz} + \bar{x}_n z = x_n + z. \quad (2)$$

Graphically this is represented in Fig. 1 .

On the basis of expression (2) one may conclude that in the analytic expression the operation of the element X_n is recorded only once.

The complete expression of the circuit will be

$$f = x_1 + x_2 + \cdots + x_{n-1} + x_n. \quad (3)$$

Graphically this is represented in Fig. 1a.

Consequently, the notation of a single action of the elements of a circuit, independently of the number of their actuations, characterizes its actual operation. The operation of the circuit under the conditions noted above can be carried out by a single action of its elements.

Let us determine the number of constituents necessary for the operation of the circuit. If the number of elements composing the circuit of actions on some element X is equal to 3, then the number of constituents determining the operating state of the circuit is equal to 5 and can be written in the form $2n-1$.

We shall prove that the consideration stated above—that the number of constituents determining the operating state of the circuit is equal to $2n-1$—is valid also in the general case for a circuit consisting of any number of elements.

Suppose that, if a circuit consists of n elements, then the constituents of the circuit determining its operation, excluding the constituent of the cycle preceding the release cycle, will also be determined by the number $2n-1$, and the circuit itself by the sum of $2n-1$ elements,

$$\sum_{2n-1} K^{(1)}(x_1, x_2, \dots, x_n).$$

In Fig. 3a one of the possible combinations of the switched-on and switched-off states of the n elements of the circuit is shown. It should be noted that in Figs. 3a and b only the cycles of the operating state of the circuit are shown, and the cycles preceding the release cycles are not shown.

If the number of elements increases from n to $n+1$, then the number of cycles of the operating state of the circuit also increases by 2 (Fig. 3b). Its analytic expression will be:

$$\begin{aligned}\tilde{x}_{n+1} \sum_{2n-1} K^{(1)}(x_1, x_2, \dots, x_n) + \tilde{x}_{n+1} K^{(1)}_{2n-1}(x_1, x_2, \dots, x_n) \\ + \tilde{x}_{n+1} K^{(1)}_{2n}(x_1, x_2, \dots, x_n) = \sum_{2n-1} K^{(1)}(x_1, x_2, \dots, x_n, x_{n+1}) \\ + K^{(1)}_{2n}(x_1, x_2, \dots, x_n, x_{n+1}) + K^{(1)}_{2n+1}(x_1, x_2, \dots, x_n, x_{n+1}) \\ = \sum_{2(n+1)-1} K^{(1)}(x_1, x_2, \dots, x_n, x_{n+1}).\end{aligned} \tag{4}$$

Consequently, with an increase in the number of elements from n to $n+1$, the number of constituents of the switched-on (switched-off) state of the circuit has increased from $2n-1$ to $2(n+1)-1$.

Fig. 3

Figure 3: Fig. 3

Thus, the operating state of circuits is ensured by $2n-1$ unit constituents. This circumstance makes it possible to reduce the number of necessary circuits from $2^n - 1$ to $2n - 1$, i.e., by $2^n - 2n$ circuits.

Fig. 3

Thus, a circuit constituting the circuit of some element X, operating once per cycle, is distinguished by the fact that: 1) the analytic expression of the operating state of the circuits of the system will be determined by one zero constituent of the elements that change their state during operation, and, if repeated operations of the elements are not caused by the need to act upon some other elements outside the circuit or to receive such actions from outside, then they will be switched on (switched off) only once; 2) the operating state of the circuit is ensured by $2n - 1$ constituents.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.