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Abstract

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ELECTRICAL ENGINEERING

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INVARIANCE UP TO ε IN LINEAR COMBINED AUTOMATICALLY CONTROLLED SYSTEMS

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The physical essence of the control problem consists in achieving, to one degree or another, independence of the controlled coordinate from the disturbance, i.e. its invariance with respect to the disturbance ([1–9]). The conditions for absolute invariance in combined automatically controlled systems and related questions have been elucidated in ([1–6,8,9]). The conditions found may prove to be stringent and, in practice, difficult to realize.

At the same time, invariance can be achieved to a sufficient extent when the indicated conditions are fulfilled partially or approximately. The realizable effect of invariance up to ε means independence of the controlled quantity from the disturbance with an accuracy up to a value determined by ε. In the present note we investigate the effect of invariance up to ε, caused by approximate fulfillment of the conditions of absolute invariance.

Fig. 1. Schematic diagram of a combined automatically controlled system; *A* —closed loop of the combined system, *B*—open loop of the combined system

Let us consider a combined automatically controlled system whose structural diagram is shown in Fig. 1.

Denote by $x(t)$ and $x_{\text{inv}}(t)$ the processes caused by the disturbance $f(t)$ in the automatically controlled system when control is carried out respectively according to the deviation principle (with the open loop of Fig. 1 disconnected from the system) and according to the deviation-and-disturbance principle:

$$x(t) = L^{-}\left\{\frac{N_1}{\Delta N} f(s)\right\}, \qquad x_{\text{inv}}(t) = L^{-}\left\{\frac{N_1 - N_2}{\Delta^{v}} f(s)\right\}, \tag{1}$$

where ΔN is the characteristic polynomial of the system (see Fig. 1), N_1 and N_2 are the corresponding polynomials of orders k and l, L^{-} is the inverse Laplace transform, and $f(s)$ is the Laplace image of the disturbance $f(t)$.

$$N_1 = b_0 + b_1 s + b_2 s^2 + \dots + b_k s^k, \qquad N_2 = \bar{a}_0 + \bar{a}_1 s + \bar{a}_2 s + \dots + \bar{a}_l s^l.$$

It is necessary, analogously to the conditions of absolute invariance, to establish the dependences for minimizing $x_{\text{inv}}(t)$ on the coefficients $\bar{a}_i$, or $a_i = b_i - \bar{a}_i$ (b_i given).

Using the convolution theorem for functions with respect to the expression $x_{\text{inv}}(t)$, we find the following relation between the processes under control according to the deviation principle and according to the deviation-and-disturbance principle:

$$x_{\text{inv}}(t) = \int_0^t \Phi(\tau)\, x(t-\tau)\, d\tau.$$

or

$$x_{\text{inv}}(t) = \int_0^t k(\tau)\Phi_1(t-\tau)\, d\tau, \tag{2}$$

where:

$$\Phi(s) = \frac{N_1 - N_2}{N_1} = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_k s^k}{b_0 + b_1 s + b_2 s^2 + \dots + b_k s^k},$$

$$\Phi_1(t) = \int_0^t \Phi(\tau)\, d\tau; \qquad k(t) = \dot{x}(t).$$

I. Assuming henceforth that the function $x(t)$ has all derivatives up to and including order n, and expanding $x(t-\tau)$ in a series in τ, the relation between the processes $x_{\text{inv}}(t)$ and $x(t)$ can be represented in the following form:

$$x_{\text{inv}}(t) \cong c_0 x(t) + c_1 \dot{x}(t) + \frac{c_2}{2!}\ddot{x}(t) + \dots + \frac{c_n}{n!}\frac{d^n x(t)}{dt^n} + R, \tag{3}$$

where $c_0, c_1, c_2, \dots, c_n$ are the corresponding moments of the function $\Phi(t)$, namely:

$$c_0 = \int_0^\infty \Phi(\tau)\, d\tau, \qquad c_1 = -\int_0^\infty \tau\Phi(\tau)\, d\tau, \dots, \quad c_r = (-1)^r \int_0^\infty \tau^r \Phi(r)\, d\tau. \tag{4}$$

The coefficients of the series (3) are computed directly from the parameters of the open and closed loops of the system (see Fig. 1):

$$c_0 = \Phi(0), \qquad c_1 = \left[\frac{d\Phi(s)}{ds}\right]_{s=0}, \dots, \quad c_r = \left[\frac{d^r\Phi(s)}{ds^r}\right]_{s=0}. \tag{5}$$

Then, for example, the first three coefficients c_i have the following form:

$$c_0 = \frac{a_0}{b_0}, \qquad c_1 = \frac{a_1 b_0 - a_0 b_1}{b_0^2}, \qquad c_2 = 2\left\{\frac{a_2 b_0 - a_0 b_2}{b_0^2} - \frac{b_1}{b_0}\frac{a_1 b_0 - a_0 b_1}{b_0^2}\right\}. \tag{6}$$

In the case of prescribed values of c_i, the parameters of the open loop of the system a_i are easily determined on the basis of the system of algebraic equations (5).

In the case of combined automatic-control systems, the coefficients $c_0, c_1, \dots, c_n$ characterize the degree of invariance of automatic-control systems. Indeed:

1) In the case of positional invariance up to ε: $c_0 = \varepsilon,\ a_0 = \varepsilon$; in the case of positional and velocity invariance $c_0 = 0,\ c_1 = \varepsilon,\ a_0 = 0,\ a_1 = \varepsilon$; in the case of positional, velocity, and acceleration invariance $c_0 = c_1 = 0,\ a_0 = a_1 = 0,\ c_2 = \varepsilon,\ a_2 = \varepsilon$. The indicated measure of invariance of automatic-control systems is approximate and applicable to slow processes $x(t)$.

2) Systems with positional, velocity, acceleration, etc. invariance will possess the properties of an astatical system of the corresponding order as $\varepsilon \to 0$ ([3]) and at the same time will substantially minimize deviations of the system in the transient process (for a slow process $x(t)$).

3) The limiting transition to the conditions of absolute invariance is effected, obviously, by the requirement

$$x_{\text{inv}}(t) \equiv 0 \quad (x(t) \neq 0). \tag{7}$$

The latter means that $c_0 = c_1 = c_2 = \dots = c_n = 0$, or $a_0 = a_1 = a_2 = \dots = a_n = 0$, which corresponds to the well-known condition of absolute invariance $N_1 - N_2 = 0$.

II. Let us consider invariance up to ε of the linear systems under consideration on a finite time interval t $(0 < t < t_1)$. Suppose that the "impulse" function of the system controlled according to the deviation principle, $k(t)$, on a bounded time interval t can be represented in the form of a polynomial in t that has no constant term:

$$k(t) = \sum_{n=1}^{\infty} \alpha_n t^n. \tag{8}$$

The function $\Phi_1(t)$ in the general case has a constant term:

$$\Phi_1(t) = \sum_{m=1}^{\infty} m\beta_m t^{m-1}. \tag{9}$$

Then, according to (2),

$$x_{\text{inv}}(t) = \int_0^t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} m\alpha_n \beta_m \tau^n (t-\tau)^{m-1}\, d\tau. \tag{10}$$

Introducing the substitution $V = \dfrac{\tau}{t}$, we transform (10):

$$x_{\text{inv}}(t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} m\alpha_n \beta_m t^{m+n} \int_0^1 V^n (1-V)^{m-1}\, dV.$$

Let us note that the integrand is the beta function ([10]) of the real values n, m:

$$B_{n+1,m} = \int_0^1 V^n (1-V)^{m-1}\, dV. \tag{11}$$

$x_{\text{inv}}(t)$ will finally take the following form:

$$x_{\text{inv}}(t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \alpha_n \beta_m t^{n+m} \frac{n!\, m!}{(n+m)!}. \tag{12}$$

Expanding $x_{\text{inv}}(t)$ in a series in t, we obtain

$$x_{\text{inv}}(t) = \frac{1}{2}\alpha_1\beta_1 t^2 + \left(\frac{1}{3}\alpha_1\beta_2 + \frac{1}{3}\alpha_2\beta_1\right)t^3 + \left(\frac{1}{4}\alpha_1\beta_3 + \frac{1}{6}\alpha_2\beta_2 + \frac{1}{4}\alpha_3\beta_1\right)t^4$$
$$+ \left(\frac{1}{5}\alpha_1\beta_4 + \frac{1}{10}\alpha_2\beta_3 + \frac{1}{10}\alpha_3\beta_2 + \frac{1}{5}\alpha_4\beta_1\right)t^5$$
$$+ \left(\frac{1}{6}\alpha_1\beta_5 + \frac{1}{15}\alpha_2\beta_4 + \frac{1}{20}\alpha_3\beta_3 + \frac{1}{15}\alpha_4\beta_2 + \frac{1}{6}\alpha_5\beta_1\right)t^6 + \ldots \tag{13}$$

It is necessary to find the conditions imposed on the parameters of the open-loop cycle of the system β (see Fig. 1), which determine the requirements for minimizing $x(t)$ and the continuous transition to $x_{\text{inv}}(t)$, and in the case of absolute invariance—to zero on a finite time interval t.

The condition of invariance of the system with accuracy up to $\varepsilon(t)$, determining the required parameters*, may be specified in the form

$$x_{\text{inv}}(t) = \varepsilon(t) \cdot x(t) \qquad (0 < t < t_1).$$

In particular, if the measure of invariance is $\varepsilon(t) = \varepsilon \cdot t$, the required parameters β_i are determined on the basis of (13) from the following system of algebraic equations:

$$\begin{aligned} \varepsilon &= \beta_1, \\ \varepsilon\alpha_2 &= \alpha_1\beta_2 + \alpha_2\beta_1, \\ \frac{1}{4}\varepsilon\alpha_3 &= \frac{1}{4}\alpha_1\beta_3 + \frac{1}{6}\alpha_2\beta_2 + \frac{1}{4}\alpha_3\beta_1, \\ \frac{1}{5}\varepsilon\alpha_4 &= \frac{1}{5}\alpha_1\beta_4 + \frac{1}{10}\alpha_2\beta_3 + \frac{1}{10}\alpha_3\beta_2 + \frac{1}{10}\alpha_4\beta_1 \end{aligned} \tag{14}$$

and so on.

* For a number of processes $x(t)$.

The limiting transition to absolute invariance on a finite time interval t is determined, obviously, by the condition $\varepsilon \to 0$. In the indicated case, according to (14), all coefficients $\beta_i \to 0$ and, consequently, $N_1 - N_2 \equiv 0$, which corresponds to the known conditions of absolute invariance over the entire time interval t $(0 \leqslant t \leqslant \infty)$.

The foregoing may be summarized as follows:

The conditions of invariance up to ε indicate the measure of the system' s nondisturbability, depending on the effectiveness of the disturbance links and on a sufficiently small number of the required disturbance actions for ensuring, with accuracy up to ε, the independence from it of the regulated coordinate.

The conditions found broaden the domain of realization of the ideas of invariance in combined automatic-control systems, making it possible to create high-quality automatically operating systems by means of a very limited—often only static—action with respect to the disturbance.

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Note: Figure translations are in progress. See original paper for figures.