Autonomous Orbit Determination Analysis for LEO–Lunar DRO Formation Non-Simultaneous Inter-Satellite Ranging*

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Abstract

To meet the requirements for autonomy and high precision in cislunar space navigation, this paper investigates a heterogeneous formation composed of Low Earth Orbit (LEO) satellites and cislunar Distant Retrograde Orbit (DRO) satellites. A theoretical method for differentially correcting DRO orbits under the circular restricted three-body problem is introduced. In the Barycentric Celestial Reference System (BCRS), a combined observation model for Dual One-Way Ranging (DOWR) is established, and the clock biases caused by light-time delay and general relativistic effects, as well as their magnitudes, are quantitatively analyzed. Based on numerical simulations, the influence of different uplink and downlink observation time intervals under non-instantaneous uplink and downlink observation conditions on the autonomous orbit determination accuracy of the LEO-cislunar DRO formation is examined. The simulation results show that: (1) in the cislunar space environment, the DOWR ranging error caused by general relativistic effects reaches the level of 5–30 ns, making it an error term that must be corrected in DOWR; (2) formation autonomous orbit determination can be achieved using only DOWR observations. Specifically, the three-dimensional positioning accuracy of the LEO satellites is better than 10 m, while that of the DRO satellites is better than 50 m, with errors mainly concentrated in the orbit-normal direction, and the radial and tangential accuracy in some arcs can reach the meter level; (3) within the uplink and downlink observation time interval range of 0–20 s, increasing the time interval can improve system observability, but the improvement is limited because the interval is relatively short, resulting in no significant enhancement of the overall orbit determination accuracy, and the orbit determination accuracy of all schemes is comparable. These results indicate that, under the premise of ensuring observation accuracy, the orbit determination system of the LEO-cislunar DRO formation can relax the requirements on uplink and downlink observation time intervals, which is conducive to improving the fault tolerance of system operation. This study verifies the feasibility of high-precision autonomous navigation in cislunar space using an LEO-cislunar DRO heterogeneous formation and provides a theoretical basis and technical reference for the design of future cislunar navigation constellation systems.

Full Text

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Autonomous Orbit Determination Analysis for LEO-Lunar DRO Formation Non-Simultaneous Inter-Satellite Ranging*

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Abstract To meet the requirements for autonomy and high precision in cislunar-space navigation, this paper focuses on a heterogeneous formation composed of a low Earth orbit (Low Earth Orbit, LEO) satellite and a cislunar distant retrograde orbit (Distant Retrograde Orbit, DRO) satellite. It introduces a theoretical method for differentially correcting DRO orbits under the circular restricted three-body problem; establishes, in the solar-system barycentric celestial reference system (Barycentric Celestial Reference System, BCRS), a combined observation model for dual one-way inter-satellite ranging (Dual One-Way Ranging, DOWR); and quantitatively analyzes the clock bias caused by light-time delay and general relativistic effects, as well as its magnitude. Based on numerical simulations, the influence of different uplink and downlink observation time intervals under non-simultaneous uplink and downlink observation conditions on the autonomous orbit-determination accuracy of an LEO–cislunar DRO formation is investigated. The simulation results show that: (1) in the cislunar-space environment, the DOWR ranging error caused by general relativistic effects reaches the level of 5–30 ns, and is an error term that must be corrected in DOWR; (2) autonomous formation orbit determination can be achieved using DOWR observations alone. Among them, the three-dimensional positioning accuracy of the LEO satellite is better than 10 m; that of the DRO satellite is better than 50 m; the errors are mainly concentrated in the along-track direction, while in some arcs the radial and tangential accuracies can reach the meter level; (3) when the uplink and downlink observation time interval is within 0–20 s, increasing the interval can improve system observability, but because the interval is relatively short the improvement is limited, the improvement in overall orbit-determination accuracy is not obvious, and the orbit-determination accuracies of the schemes are comparable. These results indicate that, while ensuring observation accuracy, the orbit-determination system of the LEO–cislunar DRO formation can relax the requirements on the uplink and downlink observation time interval, which is beneficial for improving the fault-tolerance capability of system operation. The study verifies the feasibility of high-precision autonomous navigation for a heterogeneous LEO–cislunar DRO formation in cislunar space, and provides a theoretical basis and technical reference for the design of future cislunar navigation constellations.

Keywords Spacecraft: cislunar DRO satellite, Spacecraft: LEO satellite, Celestial mechanics: orbit determination, Methods: inter-satellite links, Methods: autonomous navigation

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1 Introduction

Cislunar space is the natural extension of near-Earth orbital space and also the gateway to deep space, with enormous scientific, technological, and economic value[1]. At all stages of cislunar-space exploration, a large number of flight missions such as cislunar transfer, rendezvous and docking, and landing are involved, which has given rise to a strong demand for navigation and positioning. Therefore, achieving high-precision positioning, navigation, and timing (Position, Navigation and Timing, PNT) services in cislunar space is of great significance[2]. The cislunar distant retrograde orbit (Distant Retrograde Orbit, DRO) lies in the lunar revolution plane and is retrograde around the Moon. It has a wide range of motion, high orbital stability, and good coverage capability for both the Earth and the Moon; therefore, it has broad application prospects in exploring cislunar-space resources, establishing lunar space stations, asteroid warning and defense, and other areas, and is an ideal orbit for deploying future cislunar navigation systems[3–5].

At present, countries around the world mainly rely on ground-based monitoring and control networks for deep–

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spacecraft precision orbit determination (Precise Orbit Determination, POD). For example, Phase I of China' s lunar exploration project mainly used ground-based unified S-band (Unified S-Band, USB) measurement technology, with very-long-baseline interferometry (Very Long Base Interferometry, VLBI) measurements as an aid in key arc segments[^6]; starting with Chang' e-3, the measurement frequency band was upgraded to the S/X band[^7]. The United States and the European Space Agency (European Space Agency, ESA) mainly rely on the Deep Space Network (Deep Space Network, DSN)[^8]. The advantages of ground-based measurement lie in the strong ground-based computing and processing capabilities, and in the ability to employ multiple types of measurement methods to ensure accuracy and reliability; the disadvantage is the reliance on globally distributed observation stations. For China in particular, due to geographical limitations, it is difficult to achieve uniformly distributed stations worldwide, which limits tracking and control capability.

With the completion in 2020 of the global constellation of China' s independently developed BeiDou Navigation Satellite System (Beidou Navigation Satellite System, BDS)[^9], space-based tracking and control technology represented by inter-satellite links (Inter-Satellite Link, ISL), by virtue of its advantages of all-weather operation, high precision, and flexible geometric configuration, has become an important development trend for the future precise orbit determination of Global Navigation Satellite System (Global Navigation Satellite System, GNSS) spacecraft and cislunar spacecraft. The introduction of ISL technology can not only effectively compensate for blind zones in the geometric coverage of ground-based tracking and control networks, but can also, through autonomous processing of inter-satellite measurement data, greatly reduce spacecraft dependence on ground operation and control systems[^10].

However, directly transplanting existing near-Earth GNSS inter-satellite link technology to cislunar space still faces many challenges. In terms of observation-value modeling, BDS uses Ka-band phased-array antennas and realizes inter-satellite link establishment through a time-division, multi-address system, completing two-way one-way ranging[^11]. During data processing, BDS must rely on high-precision predicted ephemerides and clock offsets uploaded from the ground to first generate reduced two-way geometric range observations, and then perform precise orbit determination; the entire data-processing procedure is completed in the geocentric celestial reference system (Geocentric Celestial Reference System, GCRS)[^12]. By comparison, spacecraft located in cislunar space are in a weakly dynamically constrained environment and lack high-precision prediction information, making it difficult to guarantee the accuracy of the reduced two-way geometric range observations. Meanwhile, data processing for cislunar spacecraft should be performed in a spacetime coordinate system capable of simultaneously covering near-Earth space and near-Moon space, such as the barycentric celestial reference system (Barycentric Celestial Reference System, BCRS)[^2]. In addition, constrained by signal-propagation delays over the Earth–Moon distance and by the complex spatial environment, engineering practice mainly forms two-way one-way inter-satellite link measurements through mutual transmission between spacecraft[^13]. Therefore, establishing a two-way one-way combined observation model under the BCRS that is convenient for engineering implementation is a key prerequisite for realizing cislunar navigation.

In terms of orbital dynamics and observation geometry modeling, low-Earth-orbit (Low Earth Orbit, LEO) satellites, because of their short orbital periods, strong dynamical constraints, and low launch cost, are an ideal Earth–Moon navigation test platform[^14]. For example, in 2022 China initiated research on space-based autonomous navigation and time transfer by a LEO–cislunar DRO formation, and launched it in 2024; this successfully verified the feasibility of autonomous orbit determination in cislunar space[^1,^13]. However, because DRO satellites have long orbital periods and slow orbital evolution, for the heterogeneous constellation of LEO–cislunar DRO with "fast–slow coupling," uplink and downlink observations must be completed within a very short time (system requirements are usually at the millisecond level), whereas in actual observation the time interval inevitably reaches the second level; for example, BDS-3 completes uplink and downlink observations separately within 3 s[^11]. Nevertheless, research is still lacking on how different time intervals affect the orbit-determination accuracy of two types of satellites with distinctly different dynamical characteristics.

In view of the deficiencies of the above studies, this paper carries out the following work. In terms of observation-value modeling, a two-way one-way inter-satellite ranging observation model under the BCRS is established. By recording the total reception time, the interval between uplink and downlink observation times, and the sum of the uplink and downlink observations, efficient observation-value recording is realized. On this basis, for cislunar DRO satellites of different periods and for different uplink-downlink observation time intervals, DRO orbit search and autonomous orbit-determination simulation analysis of LEO–cislunar DRO formations are conducted. Through numerical computation, their autonomous orbit-determination performance is evaluated, with the aim of providing a reference for the design of subsequent cislunar space navigation systems.

2 Earth-Moon DRO Spacecraft Orbits

2.1 DRO Periodic Orbit Family

The Hill restricted three-body problem is a special form of the circular restricted three-body problem (Circular Restricted Three-Body Problem, CR3BP). Let $r = [x, y, z]^{\mathrm{T}}$ be the coordinates of a mass point P relative to the common center of mass of two primaries (P_1 and P_2) in the barycentric rotating coordinate system. Then the dynamical equations of the normalized CR3BP can be expressed as[^15]

$$\begin{cases} \ddot{x} - 2\dot{y} = \dfrac{\partial \Omega}{\partial x} \\ \ddot{y} + 2\dot{x} = \dfrac{\partial \Omega}{\partial y}, \\ \ddot{z} = \dfrac{\partial \Omega}{\partial z} \end{cases} \qquad (1)$$

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}, \qquad (2)$$

where $r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2}$ is the distance of mass point P relative to P_1; $r_2 = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$ is the distance of mass point P relative to P_2; Ω is the potential function; and μ is the mass parameter, written as

$$\mu = \frac{m_2}{m_1 + m_2}, \quad m_1 > m_2, \qquad (3)$$

where m_1 and m_2 are the masses of the two primaries, respectively. The only integral constant in the CR3BP, namely the Jacobi constant C, is expressed as

$$C = 2\Omega - v^2, \qquad (4)$$

where $v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$.

If the following transformation is applied to Eq. (1):

$$\begin{cases} x = 1 - \mu + \mu^{1/3}\xi \\ y = \mu^{1/3}\eta \\ z = \mu^{1/3}\zeta \end{cases}, \qquad (5)$$

and when $m_2 \ll m_1$, the Hill equations can be obtained[^16]:

$$\begin{cases} \ddot{\xi} - 2\dot{\eta} - 3\xi = -\dfrac{\xi}{\rho^3} \\ \ddot{\eta} + 2\dot{\xi} = -\dfrac{\eta}{\rho^3} \\ \ddot{\zeta} + \zeta = -\dfrac{\zeta}{\rho^3} \end{cases}, \qquad (6)$$

where $\rho = \sqrt{\xi^2 + \eta^2 + \zeta^2}$. Similarly, the Jacobi constant C can be transformed as

$$C = 3 - \mu^{2/3}\Gamma, \qquad (7)$$

where Γ is an integral constant equivalent to the Jacobi constant, and its expression is[^16]:

$$\Gamma = 3\xi^2 - \zeta^2 + \frac{2}{\rho} - \left(\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2\right). \qquad (8)$$

The Hill equations can be used to study more general families of CR3BP periodic orbits. Among them, the f-family orbits correspond to DRO orbits. In the Earth-Moon DRO orbit, a satellite moves around the Moon in a direction (clockwise) opposite to the overall rotation direction of the Earth-Moon system (counterclockwise), and has relatively high orbital stability.

Reference [16] provides a series of initial values for f-family orbits, as shown in Table 1. These initial values can be used for subsequent differential correction searches of DRO orbits.

Table 1 Initial values of f-family orbit for Hill equation[^16]

DRO	Γ	ξ_0
DRO-0	4.0	-0.20421
DRO-1	2.0	-0.32163
DRO-2	1.0	-0.43991
DRO-3	0.5	-0.53182

2.2 Differential-correction DRO orbit search

The Hill equation is a special form when $\mu \to 0$. Therefore, the initial values in Table 1 do not completely satisfy the orbital dynamical characteristics of DROs under the CR3BP dynamical equations. To obtain orbits that satisfy the DRO orbital dynamical characteristics under the CR3BP dynamical equations, this paper performs a re-search of the orbital initial values using the differential correction method on the basis of Table 1.

Since DROs belong to the planar circular restricted three-body problem, only the two-dimensional case need be considered[^3]. Let $X = [x, y, \dot{x}, \dot{y}]^{\mathrm{T}}$ be the state vector, and write its initial value in the synodic frame as X_0. Then the flow of the CR3BP dynamical equations can be written as $\varphi(t, X_0)$. To find the retrograde orbit of this system, that is, to search for the solution of the following nonlinear equation,

$$\varphi(T, X_0) = X_0, \qquad (9)$$

where T is the orbital period of the retrograde orbit. Since a DRO orbit has the property of symmetry about the x-axis, its x-axis must satisfy the perpendicular-intersection condition; at the same time, the velocity in the x-direction is zero at the intersection. The initial state can be taken as

$$X_0 = [x_0, 0, 0, \dot{y}_0]^{\mathrm{T}}, \qquad (10)$$

where x_0 can be obtained by inverse transformation according to Eq. (5) based on the initial values in Table 1; $\dot{y}_0$ can be obtained by combining Eqs. (4) and (8).

To simplify the problem, x_0 may be fixed as known, and the DRO orbital dynamical characteristics may be taken as constraint conditions to search for a suitable $\dot{y}_0$ and orbital period T. Define the mapping $f : R^2 \to R^3$, and

$$f(\dot{y}_0, T) = \begin{bmatrix} \varphi_x(T, X_0) - x_0 \\ \varphi_y(T, X_0) \\ \varphi_{\dot{x}}(T, X_0) \end{bmatrix}. \qquad (11)$$

To find the initial state of the DRO orbit, it is necessary to satisfy the equation

$$f\left(\dot{y}_0^*, T^*\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (12)$$

Use Newton' s method for correction. Let $\Xi = \begin{bmatrix} \dot{y}_0 \\ T \end{bmatrix}$; then

$$-f = \frac{\partial f}{\partial \Xi} \Delta \Xi. \qquad (13)$$

Thus

$$\left[\frac{\partial f}{\partial \Xi}\right]^{\mathrm{T}} \left[\frac{\partial f}{\partial \Xi}\right] \Delta \Xi = -\left[\frac{\partial f}{\partial \Xi}\right]^{\mathrm{T}} f. \qquad (14)$$

After calculating $\Delta\Xi$, perform the correction iteratively, where

$$\frac{\partial f}{\partial \Xi} = \begin{bmatrix} \dfrac{\partial \varphi_x}{\partial \dot{y}_0} & \dfrac{\partial (\varphi_x - x_0)}{\partial T} \\ \dfrac{\partial \varphi_y}{\partial \dot{y}_0} & \dfrac{\partial \varphi_y}{\partial T} \\ \dfrac{\partial \varphi_{\dot{x}}}{\partial \dot{y}_0} & \dfrac{\partial \varphi_{\dot{x}}}{\partial T} \end{bmatrix}, \qquad (15)$$

Compared with the CR3BP dynamical equations, we have

$$\begin{cases} \dfrac{\partial \varphi_x}{\partial T} = \dot{x}(T), \\ \dfrac{\partial \varphi_y}{\partial T} = \dot{y}(T), \\ \dfrac{\partial \varphi_{\dot{x}}}{\partial T} = \left(2\dot{y} + \dfrac{\partial \Omega}{\partial x}\right)\Bigg|_{t=T}, \end{cases} \qquad (16)$$

and the other elements satisfy the solution of the variational equations of the original dynamical system. That is, let

$$\frac{\partial \varphi(t, X_0)}{\partial X_0} = \Phi(t, t_0), \qquad (17)$$

then

$$\begin{cases} \dot{\Phi} = A\Phi, \quad \Phi(t_0, t_0) = I \\ A = \dfrac{\partial F}{\partial X}, \quad F = \dot{X} \end{cases}. \qquad (18)$$

Fig. 1 Trajectory of searched DROs in the Earth-Moon synodic coordinate system

Figure 1: Fig. 1 Trajectory of searched DROs in the Earth-Moon synodic coordinate system

Fig. 2 Trajectory of DROs in the Earth-Moon synodic coordinate system from 1st January 2020 to 1st April 2020

Figure 2: Fig. 2 Trajectory of DROs in the Earth-Moon synodic coordinate system from 1st January 2020 to 1st April 2020

Through iteration by the above process, the initial orbital roots of the DRO under the CR3BP dynamical equations can be obtained. Because Newton' s method is used for correction, the convergence rate is relatively fast. Meanwhile, in the search process, not only is the initial state of the orbit improved, but the orbital period can also be obtained simultaneously. Since it is known that the DRO orbit is symmetric about the x-plane, when improving the orbit, the two sign changes of y can also be used as the integration termination time. In this way, no orbital-period search is required, because the two sign changes indicate that the orbit integration has covered one complete period.

2.3 Evolution of DRO Orbits under the Complete Dynamical Model

Figure 1 shows the trajectories, in the Earth–Moon synodic coordinate system, of the DRO orbits obtained through a new search based on Table 1. To illustrate the geometric features of each DRO orbit, the Earth–Moon Lagrange points L1 and L2 are also marked in the figure, corresponding respectively to "L1 approx" and "L2 approx" in the figure.

Fig. 1 Trajectory of searched DROs in the Earth-Moon synodic coordinate system

This paper is based on the complete dynamical model (including the nonspherical gravity of the central bodies, the three-body gravitational perturbations of the planets in the solar system, solar radiation pressure, and relativistic effects, etc.). The above periodic orbits were integrated for 3 Earth–Moon periods, and the trajectories obtained in the Earth–Moon synodic coordinate system are shown in Figure 2. It can be seen from Figure 2 that, within 3 Earth–Moon periods, all DRO orbits can maintain their established orbital configurations, indicating that this type of orbit has good applicability in long-period missions [17].

Fig. 2 Trajectory of DROs in the Earth-Moon synodic coordinate system from 1st January 2020 to 1st April 2020

3 LEO-Earth-Moon DRO Formation Measurement and Orbit Determination

This section focuses on introducing the one-way inter-satellite ranging and dual one-way inter-satellite ranging models in the BCRS, the solution and order-of-magnitude analysis of their important influencing factors, as well as the theoretical method and feasibility analysis of autonomous orbit determination using inter-satellite ranging for LEO-Earth-Moon DRO formations in Earth-Moon space.

3.1 One-Way Inter-Satellite Ranging

As shown in Fig. 3, satellite A transmits a signal to satellite B at time τ_1, and satellite B receives the signal at time τ_2. The one-way ranging value is obtained by differencing the times; this is one-way inter-satellite ranging (One-Way Ranging, OWR).

Fig. 3 Inter-satellite one-way ranging mode

OWR observed values in the BCRS can be written as follows[^18]:

$$
\begin{aligned}
\rho_{12} &= (\tau_2 - \tau_1) \cdot c \\
&= \|r_{\mathrm{B}}(t_2) - r_{\mathrm{A}}(t_1)\| + \\
&\quad (\delta t_{\mathrm{A1}} - \delta t_{\mathrm{B2}}) \cdot c + (\Delta t_{\mathrm{B2}} - \Delta t_{\mathrm{A1}}) \cdot c + \\
&\quad (D_{\mathrm{B2}}^{\mathrm{recv}} - D_{\mathrm{A1}}^{\mathrm{tran}}) \cdot c + \Delta R_{12} + \varepsilon_{12},
\end{aligned}
\tag{19}
$$

where subscripts 1 and 2 denote the signal transmission time and reception time, respectively; subscripts A and B denote satellite A and satellite B, respectively; the combined subscript A1 (or B2) denotes satellite A at the transmission time (or satellite B at the reception time); τ_1 and τ_2 are the proper times of satellite A and satellite B, respectively, and their corresponding Barycentric Dynamical Time (TDB) are t_1 and t_2, respectively; $\|r_{\mathrm{B}}(t_2) - r_{\mathrm{A}}(t_1)\|$ is the geometric distance between satellite A and satellite B in the BCRS; c is the speed of light in vacuum; $D_{\mathrm{A1}}^{\mathrm{tran}}$ and $D_{\mathrm{B2}}^{\mathrm{recv}}$ are the hardware delays of satellite A and satellite B, respectively; superscripts tran and recv denote transmission and reception, respectively; ΔR_{12} is the correction term for the observation data, including phase-center correction, ionospheric correction, and Shapiro delay, etc.; ε_{12} is the measurement error; δt_{A1} and δt_{B2} are the satellite clock offsets caused by the generalized relativistic effect for satellite A at time t_1 and satellite B at time t_2, respectively (hereinafter referred to as the relativistic clock offset), also known as the difference between satellite proper time and coordinate time; Δt_{A1} and Δt_{B2} are the physical clock offsets of satellite A at time t_1 and satellite B at time t_2, respectively. For any satellite i, the difference between its satellite proper time and coordinate time at time t_j can be written as:

$$
\delta t_{ij} = t_{ij} - \tau_{ij}, \tag{20}
$$

and Δt_{A1} and Δt_{B2} can be expressed by a quadratic polynomial. For any satellite i, its physical clock offset at time t_j can be written as[^2]:

$$\Delta t_{ij} = a_{i0} + a_{i1}(t_j - t_{i0}) + a_{i2}(t_j - t_{i0})^2, \qquad (21)$$

where a_{i0} is the clock offset, a_{i1} is the clock drift, a_{i2} is the clock drift rate, and t_{i0} is the reference epoch of the physical clock offset of satellite i.

3.2 LEO-Earth-Moon DRO Formation Dual One-Way Inter-Satellite Ranging

For LEO-Earth-Moon DRO satellite formations, the dual one-way ranging mode is often used in engineering missions; that is, the LEO and the DRO mutually transmit and receive each other' s signals to obtain OWR observations, and then the dual one-way ranging observations are summed to obtain dual one-way inter-satellite ranging (Dual One-Way Ranging, DOWR), as shown in Fig. 4, where dT is the interval between the uplink and downlink observation times.

According to Fig. 4, the process can be described as follows: LEO satellite A transmits a signal at time τ_1, which is received by DRO satellite B at time τ_2, recorded as the uplink observation; subsequently, after a time interval dT, DRO satellite B transmits a signal at time $\tau_3 = \tau_2 + \mathrm{dT}$, which is received by LEO satellite A at time τ_4, recorded as the downlink observation. At this time, the LEO-Earth-Moon DRO formation DOWR in the BCRS can be expressed as[^13]:

$$\begin{aligned}
\rho_{12} + \rho_{34} &= (\tau_2 - \tau_1 + \tau_4 - \tau_3) \cdot c \\
&= (\tau_4 - \tau_1 - \mathrm{dT}) \cdot c \\
&= \|r_{\mathrm{B}}(t_2) - r_{\mathrm{A}}(t_1)\| + \|r_{\mathrm{A}}(t_4) - r_{\mathrm{B}}(t_3)\| + \\
&\quad (\delta t_{\mathrm{B3}} - \delta t_{\mathrm{B2}}) \cdot c + (\delta t_{\mathrm{A1}} - \delta t_{\mathrm{A4}}) \cdot c + \\
&\quad (\Delta t_{\mathrm{B2}} - \Delta t_{\mathrm{B3}}) \cdot c + (\Delta t_{\mathrm{A4}} - \Delta t_{\mathrm{A1}}) \cdot c + \\
&\quad (D_{\mathrm{B2}}^{\mathrm{recv}} - D_{\mathrm{A1}}^{\mathrm{tran}}) \cdot c + (D_{\mathrm{A4}}^{\mathrm{recv}} - D_{\mathrm{B3}}^{\mathrm{tran}}) \cdot c + \\
&\quad \Delta R_{12} + \Delta R_{34} + \varepsilon_{12} + \varepsilon_{34},
\end{aligned} \qquad (22)$$

In the formula, the definitions of the superscripts and subscripts are the same as in Eq. (19); τ_1 and τ_4 are the proper times of the LEO satellite, whose corresponding TDB epochs are t_1 and t_4, respectively; τ_2 and τ_3 are the proper times of the DRO satellite, whose corresponding TDB epochs are t_2 and t_3, respectively.

The above formula has strong universality in engineering applications. When $\mathrm{dT} = 0$, it is instantaneous transponder dual one-way inter-satellite ranging; when $\mathrm{dT} \neq 0$, it is the DOWR observation mode. In this mode, the order of magnitude of dT can range from milliseconds to seconds. Generally, DOWR ranging values can be obtained by simply summing and combining the uplink

Fig. 4 Inter-satellite dual one-way ranging mode

Figure 3: Fig. 4 Inter-satellite dual one-way ranging mode

and downlink OWR observations between the LEO satellite and the DRO satellite.

For the geometric distance (light time) and satellite clock offset related to Eq. (22), the specific computation and processing methods are described below.

Figure 4 Dual one-way ranging mode

Fig. 4 Inter-satellite dual one-way ranging mode

3.3 Light-Time Calculation

In the DOWR observation mode described above, among the observation data, only the total receiving epoch τ_4 and the uplink/downlink observation time interval dT are known. The other epochs, such as t_1, t_2, and t_3, are unknown in the precise orbit-determination process and must be obtained through iterative light-time computation.

The light time of the downlink signal can be iterated using the following fixed-point format:

$$\Delta t_{34}^{i+1} = \rho_{34}\left(\Delta t_{34}^{i}\right) = \frac{\left\| r_{\mathrm{A}}(t_4) - r_{\mathrm{B}}(t_4 - \Delta t_{34}^{i}) \right\| + \Delta R_{34}}{c}. \tag{23}$$

where the superscript $i = 1, 2, 3, \cdots$ denotes the iteration number; the initial value of Δt_{34} can be set to 0. After Δt_{34} is obtained, t_3 and t_2 can be computed by

$$\begin{cases} t_3 = t_4 - \Delta t_{34} \\ t_2 = t_3 - \mathrm{dT} \end{cases} . \tag{24}$$

Then, the light time of the uplink signal can likewise be obtained by iterating the following fixed-point format:

$$\Delta t_{12}^{i+1} = \rho_{12}\left(t_2, \Delta t_{12}^{i}\right) = \frac{\left\| r_{\mathrm{B}}(t_2) - r_{\mathrm{A}}(t_2 - \Delta t_{12}^{i}) \right\| + \Delta R_{12}}{c}. \tag{25}$$

The initial value of Δt_{12} can likewise be set to 0. It is worth noting that the above computational procedure must be carried out in the BCRS.

3.4 Satellite Clock-Offset Processing

The satellite clock offset consists of two parts. One part is the inherent time-measurement error of the satellite clock, also called the physical clock offset; the other part is the generalized relativistic effect caused by the gravitational attraction of massive stars/planets, producing the difference between the satellite proper time and coordinate time, also called the relativistic clock offset. If the influence of satellite clock offsets is neglected, on the one hand it will cause biases in the records of the signal transmission/reception epochs, thereby directly affecting the value of the dual one-way ranging measurement; on the other hand, it will also affect the computation of satellite positions during the light-time calculation. With the development and progress of time-synchronization technology and clock-manufacturing technology, the satellite clock offsets of different central celestial bodies and even different types of satellites will exhibit different characteristics and processing methods. Therefore, it is necessary to discuss satellite clock-offset corrections in detail.

3.4.1 Physical Clock Offset

From Eqs. (21) and (22), the influence of the satellite physical clock offset on the DOWR observable can be expressed as

$$\begin{cases} \Delta t_{\mathrm{B2}} - \Delta t_{\mathrm{B3}} = a_{\mathrm{B1}} \cdot (t_2 - t_3) + a_{\mathrm{B2}} \cdot (t_2 - t_3) \cdot (t_2 + t_3 - 2t_{\mathrm{B0}}), \\ \Delta t_{\mathrm{A4}} - \Delta t_{\mathrm{A1}} = a_{\mathrm{A1}} \cdot (t_4 - t_1) + a_{\mathrm{A2}} \cdot (t_4 - t_1) \cdot (t_4 + t_1 - 2t_{\mathrm{A0}}), \end{cases} \tag{26}$$

In LEO-Earth-Moon DRO formation DOWR observations, the choice of the uplink/downlink time interval dT will be affected simultaneously by multiple factors, such as the measurement system, clock stability, and nonlinear orbital dynamics.

This paper mainly focuses on the influence of clock and dynamical characteristics and does not discuss the limitations of the measurement system for the time being. Regarding clock stability, the frequency stability of current onboard atomic clocks (such as rubidium clocks and hydrogen clocks) or ultra-stable oscillators is better than 1×10^{-12} [12,19], and their linear frequency bias can be calibrated and compensated with high precision through ground monitoring and control or inter-satellite links. After compensation, the ranging residual introduced by clock random noise within a short time ($\mathrm{dT} < 20$ s) is better than 1 cm, and its contribution to the conventional meter-level distance measurement accuracy in the Earth–Moon space is negligible. In view of this, in order to remove the interference of clock hardware performance, this paper assumes in the subsequent simulations that the satellite clock rates have been ideally calibrated, with emphasis placed on the dynamical and geometric characteristics.

3.4.2 Difference Between Satellite Proper Time and Coordinate Time

For a LEO-Earth-Moon DRO satellite formation, because the gravitational and velocity potentials of the LEO satellite and the DRO satellite differ significantly [13], the two types of satellites need to be discussed separately.

For a LEO satellite, according to the resolutions of the International Astronomical Union (IAU), the conversion between its onboard proper time τ_{leo} and the coordinate time TDB must strictly follow the following time-transformation chain [20–21]:

$$\tau_{\mathrm{leo}} \to \mathrm{TAI} \overset{(A)}{\longleftrightarrow} \mathrm{TT} \overset{(B)}{\longleftrightarrow} \mathrm{TCG} \overset{(C)}{\rightleftarrows} \mathrm{TCB} \overset{(D)}{\longleftrightarrow} \mathrm{TDB}. \tag{27}$$

The transformation relationships among the above times are:

$$\text{(A)} \quad \mathrm{TT} = \mathrm{TAI} + 32.184\ \mathrm{s};$$

$$\text{(B)} \quad \left\langle \frac{\mathrm{dTT}}{\mathrm{dTCG}} \right\rangle = 1 - L_G;$$

$$\text{(C)} \quad \mathrm{TCB} - \mathrm{TCG} = \frac{1}{c^2} \int \left(\frac{1}{2} v_E^2 + \sum_{B \neq E} U_B \right) \mathrm{dTCG} + \frac{1}{c^2} \left(v_E \cdot X \right) + O(c^{-4});$$

$$\text{(D)} \quad \left\langle \frac{\mathrm{dTDB}}{\mathrm{dTCB}} \right\rangle = 1 - L_B;$$

where TAI is International Atomic Time; TT is Terrestrial Time; TCG is Geocentric Coordinate Time; TCB is Barycentric Coordinate Time; $\langle \cdots \rangle$ denotes a long-time average; $L_G = 6.969290134 \times 10^{-10}$ is the constant mean rate difference between TT and TCG; $L_B = 1.550519768 \times 10^{-8}$ is the constant mean rate difference between TDB and TCB; v_E is the orbital velocity of the Earth; and U_B is the gravitational potential at the geocenter due to each solar-system body except the Earth. The integral expression for the calculation of TDB–TT is therefore[^21]:

$$\begin{aligned} \mathrm{TDB} - \mathrm{TT} = \mathrm{TDB}_0 - L_C(\mathrm{TT} - T_0) + \frac{1}{c^2} \int_{T_0}^{\mathrm{TT}} \left(\frac{1}{2} v_E^2 + \sum_{B \neq E} U_B \right) \mathrm{d}t \\ + \frac{1}{c^2} \left(v_E \cdot X_{\mathrm{TT}} \right) + O(c^{-4}), \end{aligned} \tag{28}$$

Fig. 5 Comparison of series analytical solutions with numerical integration results for stations 7821, 7090, and 7105 from March 1, 2012 to March 1, 2013

Figure 4: Fig. 5 Comparison of series analytical solutions with numerical integration results for stations 7821, 7090, and 7105 from March 1, 2012 to March 1, 2013

where $L_C = 1 - \left\langle \frac{\mathrm{dTCG}}{\mathrm{dTCB}} \right\rangle$ is the constant mean rate difference between TCB and TCG, with the value $1.48082686741 \times 10^{-8}$; $T_0 = 2443144.5003725$; $\mathrm{TDB}_0 = -65.5\ \mu$s; and X_{TT} is the position vector of the LEO satellite clock relative to the geocenter at the TT epoch. For convenient computation, the IAU provides a series analytical solution in its published SOFA (the Standards of Fundamental Astronomy Service) software, whose expression is[^22]:

$$\mathrm{TDB} - \mathrm{TT} = \mathrm{TDB}_0 + P(\mathrm{TT}) - P(T_0) + \frac{1}{c^2}\left(v_E \cdot X_{\mathrm{TT}}\right) + O(c^{-4}). \qquad (29)$$

At present, in SOFA this analytical solution contains 787 terms. In this paper, computational comparisons between the analytical solution and the integral-form solution were carried out for satellite laser ranging (Satellite Laser Ranging, SLR) stations located in Shanghai, China (7821), Yarragadee, Australia (7090), and Washington, USA (7105). The results show that the difference between the series analytical solution and the integral-form solution is better than 5 ns, as shown in Fig. 5 (in the figure, the initial epoch has been unified to March 1, 2012 UTC).

Fig. 5 Comparison of series analytical solutions with numerical integration results for stations 7821, 7090, and 7105 from March 1, 2012 to March 1, 2013

If the onboard proper time of a LEO satellite has been corrected to GNSS time (GNSS Time, GNSST), then the transformation between its onboard proper time and coordinate time only requires adding the time-system offset between GNSST and TT on the basis of Eq. (28) or Eq. (29):

$$\mathrm{TDB} - \tau_{\mathrm{leo}} = \mathrm{TDB} - \mathrm{TT} + [\mathrm{TT} - \mathrm{GNSST}]. \qquad (30)$$

For a low-Earth-orbit navigation satellite, or for a LEO satellite that has been corrected for the GNSS agreement relativistic correction during time correction, an additional GNSS agreement relativistic correction term must be added on the basis of Eq. (30), so as to ensure strict synchronization between the satellite time and GNSS time[^23], namely:

$$\mathrm{TDB} - \tau_{\mathrm{leo}} = \mathrm{TDB} - \mathrm{GNSST} + \frac{1}{c^2}\left(X_{\mathrm{TT}} \cdot \dot{X}_{\mathrm{TT}}\right). \qquad (31)$$

For a LEO satellite for which no time correction has been performed, its onboard time may be regarded as proper time. The transformation between LEO onboard proper time and TDB also needs to comply with the time-transformation chain in Eq. (27). That is, the onboard proper time is first transformed to TT, and then TT is transformed to TDB using Eq. (28) or Eq. (29). According to the GCRS metric, the transformation between LEO onboard proper time and TT can be calculated using the following equation[^24]:

$$\mathrm{TT}-\tau_{\mathrm{leo}}=(\mathrm{TT}_0-\tau_{\mathrm{leo}}(\mathrm{TT}_0))+\int_{\mathrm{TT0}}^{\mathrm{TT}}\left[\frac{1}{c^2}\left(U_E+U_{\mathrm{tid}}+\frac{v_{\mathrm{leo}}^2}{2}\right)-L_G\right]\mathrm{d}t, \qquad (32)$$

where U_E is the value of the Earth' s gravitational potential at the position of the LEO satellite clock; U_{tid} is the tidal potential generated by celestial bodies such as the Sun and the Moon; and v_{leo} is the velocity of motion of the LEO satellite clock in the GCRS.

For Earth-Moon-space DRO satellites, it may be assumed that their onboard time

if the initial epoch is always taken as the original epoch, then the conversion between the onboard proper time τ_{dro} of the lunar DRO satellite and TDB can be realized through the following time-transformation chain:

$$\tau_{\mathrm{dro}}\rightarrow \mathrm{TCB}\overset{(\mathrm{D})}{\longleftrightarrow}\mathrm{TDB}. \qquad (33)$$

According to the BCRS metric, the time transformation between τ_{dro} and TDB can be calculated by the following equation [25]:

$$\mathrm{TDB}-\tau_{\mathrm{dro}}=(\mathrm{TDB}_0-\tau_{\mathrm{dro}}(\mathrm{TDB}_0))+\int_{\mathrm{TDB0}}^{\mathrm{TDB}}\left[\frac{1}{c^2}\left(U_{\mathrm{total}}+\frac{1}{2}v_{\mathrm{dro}}^2(t)\right)-L_{\mathrm{B}}\right]\mathrm{d}t, \qquad (34)$$

where U_{total} is the gravitational potential of all Solar-System bodies at the DRO satellite clock, and $v_{\mathrm{dro}}(t)$ is the velocity of the DRO satellite clock in the BCRS. Combining Eq. (22), the influence of the difference between the satellite proper time and coordinate time on the DOWR observation value can be expressed as:

$$\delta t_{\mathrm{B3}}-\delta t_{\mathrm{B2}}=\int_{t_2}^{t_3}\left[\frac{U_{\mathrm{total}}}{c^2}+\frac{1}{2}\frac{v^2(t)}{c^2}-L_{\mathrm{B}}\right]\mathrm{d}t, \qquad (35)$$

$$\delta t_{\mathrm{A1}}-\delta t_{\mathrm{A4}}=(\mathrm{TDB}-\mathrm{TT})_{t_1}-(\mathrm{TDB}-\mathrm{TT})_{t_4}. \qquad (36)$$

It can be seen that Eq. (35) is mainly determined by the DRO orbit and t_3-t_2. To facilitate analysis of the magnitude of Eq. (35), in this paper the four DRO

Fig. 6 General relativistic clock correction for Earth-Moon DRO orbits with different orbital periods over a 90-day period from January 1, 2020, to April 1, 2020

Figure 5: Fig. 6 General relativistic clock correction for Earth-Moon DRO orbits with different orbital periods over a 90-day period from January 1, 2020, to April 1, 2020

orbits in Table 1 are analyzed and calculated for this term over a total duration of 3 months; the results are shown in Fig. 6.

From Fig. 6 it can be seen that, as $t_3 - t_2$ continuously increases, the magnitude of Eq. (35) also gradually increases from < 1 ns to < 20 ns. This magnitude cannot be ignored for DOWR; therefore, this term must be corrected in the observation values.

Considering that it is now very common for LEO satellites to carry onboard GNSS receivers to realize high-precision time synchronization between satellite time and GNSS time [26−27], Eq. (36) and the subsequent content of this paper use Eq. (31) to calculate the generalized relativistic clock correction term for LEO satellites.

Fig. 7 shows the analytical calculation results of Eq. (36) for different LEO satellites (Sentinel-1A, Sentinel-3B, and Swarm-B). From Fig. 7 it can be seen that the influence of the generalized relativistic clock correction term of LEO satellites on DOWR is more significant than that of DRO satellites. As $t_4 - t_1$ continuously increases, the amplitude of this term also gradually increases from 5 ns to 60 ns; therefore, this term must also be corrected in the observation values.

3.4.3 Influence of the uplink and downlink observation time interval on DOWR

From Eq. (22), the uplink and downlink observation time interval $\mathrm{d}T$ determines the magnitude of the influence of the satellite clock difference on the DOWR observation value; this part has already been discussed in the preceding subsection. Meanwhile, the time-tag error generated by $\mathrm{d}T$ will bias the interpolation result for the satellite position, thereby adversely affecting the calculation of the light time.

图 6　对不同轨道周期的地月 DRO 轨道，在 2020 年 1 月 1 日至 2020 年 4 月 1 日共 90 d 的相对论效应时延量级分析

Fig. 6　General relativistic clock correction for Earth-Moon DRO orbits with different orbital periods over a 90-day period from January 1, 2020, to April 1, 2020

图 7　对不同 LEO 卫星，在 2020 年 1 月 1 日至 2020 年 4 月 1 日共 90 d 的相对论效应时延量级分析

Fig. 7 General relativistic clock correction for LEO satellites over a 90-day period from January 1, 2020, to April 1, 2020

Figure 6: Fig. 7 General relativistic clock correction for LEO satellites over a 90-day period from January 1, 2020, to April 1, 2020

Fig. 7 General relativistic clock correction for LEO satellites over a 90-day period from January 1, 2020, to April 1, 2020

For ease of analysis, the observation time interval between the uplink and downlink can be further written as:

$$\mathrm{dT} = \tau_3 - \tau_2 = t_3 - t_2 + \delta t_{\mathrm{B2}} - \delta t_{\mathrm{B3}} + a_{\mathrm{B1}} \cdot (t_3 - t_2). \tag{37}$$

From Eq. (37), dT likewise contains a relativistic clock-error correction term and a physical clock-error correction term. Among them, the relativistic clock-error correction term of dT can be corrected according to Eq. (35); the physical clock-error correction term of dT depends on the correction accuracy of the satellite clock drift. If the LEO satellite velocity relative to the Earth is about 7.2 km$\cdot$s^{-1}, and the DRO satellite velocity relative to the Earth is about 1.5 km $\cdot$ s^{-1}, then under the assumptions in Section 3.4.1, the LEO–DRO relative position error introduced by clock random noise over a short time (dT $\leq$ 20 s) is better than 2×10^{-4} mm, far smaller than the measurement error of DOWR. Therefore, the physical clock-error correction term of dT can be neglected.

3.5 Orbit Determination of Earth–Moon Spacecraft

In Earth–Moon space, a spacecraft is affected by multiple forces during its motion. Its equation of motion in an inertial coordinate system can be described by the following differential equation:

$$\ddot{r} = f_{\mathrm{TB}} + f_{\mathrm{NB}} + f_{\mathrm{NS}} + f_{\mathrm{TD}} + f_{\mathrm{RL}} + f_{\mathrm{SR}} + f_{\mathrm{DG}} + f_{\mathrm{TH}}, \tag{38}$$

where r, $\dot{r}$, and $\ddot{r}$ are respectively the position, velocity, and acceleration vectors of the spacecraft mass center; f_{TB} is the two-body force; f_{NB} is the N-body gravitational perturbation; f_{NS} is the nonspherical gravitational perturbation of the central body; f_{TD} is the tidal perturbation of the central body; f_{RL} is the influence of relativistic effects on the spacecraft motion; f_{SR} is solar radiation pressure; f_{DG} is the drag of the Earth' s upper atmosphere on the spacecraft, which is 0 for spacecraft near the Moon; and f_{TH} denotes other forces acting on the spacecraft. For LEO satellites, Eq. (38) is described in GCRS; for Earth–Moon DRO satellites, Eq. (38) is described in the Lunar Celestial Reference System (LCRS). The transformation among the Earth, Moon local systems and BCRS is given in Ref. [28].

Let $X = (r, \dot{r}, p)^{\mathrm{T}}$, where p is the vector of dynamical parameters. Then the spacecraft state equation and initial state can be written as:

$$\begin{cases} \dot{X} = F(X, t) \\ X(t_0) = X_0 \end{cases} . \qquad (39)$$

If the orbital parameters X_0 at the initial epoch are known, then by integrating Eq. (38), the satellite orbital reference value $X(t)$ at any epoch can be obtained. Linearizing Eq. (39) gives:

$$\dot{X} = \dot{X}^* + \left.\frac{\partial F}{\partial X}\right|_{X^*} (X - X^*). \qquad (40)$$

Let $A = \left.\frac{\partial F}{\partial X}\right|_{X^*}$, $x = X - X^*$. Then Eq. (40) can be written as:

$$\dot{x} = Ax, \qquad (41)$$

whose solution has the form:

$$x = \Psi(t, t_0) x_0, \qquad (42)$$

where $\Psi(t, t_0)$ is the state transition matrix, and $x_0 = x(t_0)$ is the correction to X_0. If there is a linearized observation equation at epoch t:

$$y = Hx + \varepsilon, \qquad (43)$$

where $H = \frac{\partial y}{\partial x}$ is the partial derivative of the observations with respect to the parameters to be estimated; ε is the observation noise, and the variance-covariance matrix of ε is D. If the prior covariance of X_0 is known as D_X, then combining Eqs. (42) and (43), according to the least-squares batch-processing principle, the normal-equation iteration form for x_0 can be written as:

$$\begin{cases} Nx(t_0)_n = \Psi^{\mathrm{T}} H^{\mathrm{T}} P \cdot y - D_X^{-1} x(t_0)_{n-1} \\ N = \Psi^{\mathrm{T}} H^{\mathrm{T}} P H \Psi + D_X^{-1} \\ P = D^{-1} \end{cases} . \qquad (44)$$

where the subscript $n = 1, 2, \cdots$ denotes the iteration number. Solving the above equations yields the improved initial orbit value:

$$\hat{X}_0 = X^* + \hat{x}_0. \qquad (45)$$

Usually, the above process needs to be solved iteratively to obtain the final solution.

Fig. 8 Strength of the asymmetry in cislunar space

Figure 7: Fig. 8 Strength of the asymmetry in cislunar space

3.6 Feasibility of Autonomous Orbit Determination for an LEO-Earth-Moon DRO Formation

According to Ref. [29], without the participation of ground stations, autonomous navigation of spacecraft formations cannot be achieved in near-Earth space by relying only on inter-satellite ranging. This is related to the high symmetry of the gravitational field in near-Earth space [14].

Ref. [30] proposed that autonomous navigation of spacecraft formations can be achieved by establishing inter-satellite measurements with spacecraft in regions of high gravitational-field asymmetry in Earth–Moon space. This is the LiAISON (Linked Autonomous Interplanetary Satellite Orbit Navigation) autonomous navigation principle. To analyze the degree of asymmetry of the gravitational field in Earth–Moon space, Ref. [30] used the asymmetry index α to characterize the strength of the asymmetry of the Earth–Moon three-body gravitational field; that is, the spacecraft

the ratio of the magnitude of the third-body gravitational acceleration received to the sum of the magnitudes of all accelerations:

$$\alpha(x, y, z) = \frac{|a_{3\mathrm{rd}}(x, y, z)|}{\sum_{i=1}^{n} |a_i(x, y, z)|}, \qquad (46)$$

where $a_i(x, y, z)$ represents the acceleration of dynamical model i at position (x, y, z), $i = 1, 2, \cdots, n$. $a_{3\mathrm{rd}}(x, y, z)$ is the third-body gravitational acceleration. In cislunar space, if the spacecraft position (x, y, z) is close to Earth, the Earth' s gravity is higher than the Moon' s gravity, and the lunar gravity is regarded as the third body; conversely, if the position is close to the Moon, the Earth' s gravity is regarded as the third body. Figure 8 shows the asymmetry in various regions of cislunar space.

Figure 8 Analysis of asymmetry in cislunar space

Fig. 8 Strength of the asymmetry in cislunar space

As can be seen from Figure 8, from Earth to the Moon, the asymmetry first increases and then decreases. For the LEO-Earth-Moon DRO constellation studied in this paper, Table 2 summarizes the asymmetry corresponding to each region:

Table 2 Strength of the asymmetry in LEO and Earth-Moon DRO regions

Fig. 9 LEO-Earth-Moon DRO constellation autonomous orbit determination simulation process

Figure 8: Fig. 9 LEO-Earth-Moon DRO constellation autonomous orbit determination simulation process

Orbit type	Orbital period	Strength of the asymmetry
LEO	1.68 h	0.00000547%
DRO-0	3.5 d	16.4%
DRO-1	6.0 d	34.1%
DRO-2	9.0 d	49.3%
DRO-3	12.0 d	38.8%

From Table 2, it can be seen that although the asymmetry of the LEO orbit is close to 0, the DRO orbits that make up the constellation have relatively high asymmetry. According to the LiAISON autonomous navigation theory proposed in Ref. [30], the LEO–Earth–Moon DRO constellation has the conditions for implementing autonomous navigation.

4 Design of simulation cases

The entire simulation process follows the procedure shown in Figure 9. That is, first, precise DRO ephemerides are generated based on dynamical orbit integration, and OWR observations are then generated; subsequently, according to different time intervals between uplink and downlink observations, the OWR observations are combined to obtain DOWR observations; then these observations are used to carry out autonomous orbit determination for the LEO–Earth–Moon DRO constellation; finally, the orbit determination results are analyzed.

Figure contents:

Simulation for OWR observations
↓
Generating DOWR observations
↓
Autonomous orbit determination for LEO-DRO constellation
↓
Accuracy analyze of orbit determination

Figure 9 Simulation process for autonomous orbit determination of the LEO–Earth–Moon DRO constellation

Fig. 9 LEO-Earth-Moon DRO constellation autonomous orbit determination simulation process

4.1 Constellation and payload settings

This paper uses a LEO satellite constellation composed of three satellites launched by the European Space Agency: Sentinel-1A (hereinafter abbreviated as SE1A), Sentinel-3B (hereinafter abbreviated as SE3B), and Swarm-B (hereinafter abbreviated as SWMB), to conduct simulation experiments. The distribution of the LEO satellite constellation in near-Earth space is shown in Figure 10.

For the DRO satellites, the aforementioned four DRO orbits with different orbital periods are used. Tables 3 and 4 show the initial orbital values of each DRO satellite in the LCRS at 00:00:00 (UTC) on January 2, 2020.

4.2 Observation simulation settings

The models and calculation strategies used in the OWR observation simulation are shown in Table 5 [31-34].

Fig. 10 LEO satellites orbit distribution

In the simulation, the Earth orientation parameter (EOP) Bulletin A product released by the International Earth Rotation and Reference Systems Service (IERS) was used; the solar-system planetary positions and lunar orientation parameters were taken from the Development Ephemerides (DE440) released by the Jet Propulsion Laboratory (JPL)[31]. The lunar gravity-field model adopts the Grail Gravity Model (GRGM900C)[33], with the gravity-field degree and order truncated to 100×100.

Table 3 Initial position of Earth-Moon DRO satellites (unit: km)

Satellite	x	y	z
DRO-0	−18900.960	−1.720	1875.502
DRO-1	−29768.918	−2.708	2953.906
DRO-2	−40716.531	−3.705	4040.215
DRO-3	−49223.375	−4.479	4884.331

Table 4 Initial velocity of Earth-Moon DRO satellites (unit: km $\cdot$ s^{-1})

Satellite	v_x	v_y	v_z
DRO-0	0.0178280	0.4575956	0.1895429
DRO-1	0.0143245	0.3721242	0.1541565
DRO-2	0.0125884	0.3297702	0.1366215
DRO-3	0.0118551	0.3118817	0.1292154

Table 5 Settings in observation simulation

Items	Models & Description
Earth orientation parameters	EOP IERS Bulletin A
Lunar orientation parameters	DE440[31]
LEO reference orbit	Precise orbit provided by TU Graz[32]
Spacecraft clocks	Consider the general relativity corrections only
Gravity model	DRO: GRGM900C (100×100)[33]
N body	DE440[31]
General relativity	IERS 2010 Conventions
Solar radiation pressure	Cannon-ball model
Empirical accelerations	Consider the parameters of Ca, Sa, Cc and Sc for the along- and cross-track directions (cosine/sine)

Table 5 Continued

Items	Models & Description
Integrator	Krogh-Shampine-Gordon integrator[34]
Integral step	60 s
Antenna half-cone angle	30°
Sampling for observation	120 s
Random Noise	OWR: 1 m

To achieve fast and high-precision simulation of observations, this paper uses the centimeter-level dynamic orbits obtained from precise orbit determination based on onboard GNSS by Technische Universität Graz (TU Graz) as the true orbits of the LEO satellites, and directly participates in the simulation of observations. Considering that the orbit-determination accuracy of conventional cislunar spacecraft is at the meter level, directly using centimeter-level precise orbit products as the true values will not affect the final orbit-determination accuracy.

In the simulation of observations, the antennas of both the LEO satellites and the DRO satellites are simplified as conical pointing models, and the antenna phase center is coincident with the satellite center of mass. The LEO antenna points to the zenith, with its antenna half-aperture angle set to 30°; the DRO antenna always points toward the Earth, and its antenna half-aperture angle can cover the entire near-Earth space. When the angle between the LEO–Earth–Moon DRO vector and the antenna principal axis lies within the field of view, the link is considered available.

Because the orbital period of LEO satellites is short, the orbit-determination arc should not be set too long. Combining the visibility of the LEO–Earth–Moon DRO, six orbit-determination arcs can be divided, and each orbit-determination

Fig. 11 Visibility durations of Earth–Moon DRO satellites from each LEO satellite

Figure 9: Fig. 11 Visibility durations of Earth–Moon DRO satellites from each LEO satellite

arc requires only one LEO satellite to participate in the intersatellite link, so that different LEO satellites can take turns in observations and thereby achieve relay observation of the DRO satellites. Figure 11 shows, under consideration of Earth–Moon occultation, the distribution of visibility durations of four DRO satellites by LEO satellites and the division of orbit-determination arcs in this paper. Table 6 shows the start and end day of year (Day Of Year, DOY) for each orbit-determination arc and the participating LEO satellite.

To analyze the accuracy of autonomous orbit determination results under different uplink and downlink observation time intervals dT, when generating the DOWR observations in combination, this paper selects different dT values (including 0 s, 5 s, 10 s, and 20 s) for comparative analysis. Considering the current ranging level of lunar spacecraft in various countries[7,35], the observation noise for one-way intersatellite ranging is set to 1 m. The sampling rate is set to 120 s.

4.3 Autonomous Orbit-Determination Strategy

According to Table 6, different LEO satellites are used for different orbit-determination arcs to perform autonomous orbit determination. Therefore, the dynamic information of each LEO satellite is difficult to achieve temporal continuity. Considering the wide application of onboard GNSS technology, before performing autonomous orbit determination for each arc, precise orbit determination is carried out using onboard GNSS observation data of the LEO satellites together with GNSS broadcast ephemerides, thereby obtaining the initial orbit values of the LEO satellites. In the autonomous orbit-determination stage, only DOWR observations are used.

Fig. 11 Visibility durations of Earth–Moon DRO satellites from each LEO satellite

Table 6 Table of start and end times for each POD arcs

ARC	LEO	Start DOY	End DOY
1	SE1A	002.0	006.0
2	SE3B	006.0	011.0
3	SWMB	011.0	016.0
4	SE1A	016.0	020.0
5	SE3B	020.0	024.0
6	SWMB	024.0	028.0

Because the orbital period of the DRO satellites is relatively long (greater than the orbit-determination arc length), after each orbit determination is completed, it is necessary to output the results for the next orbit determination.

The initial orbit information of the arc segment and its uncertainty are used to maintain the continuity of its dynamical information; it is even expected that, through the accumulation of multiple arc segments, the orbit determination accuracy of DRO satellites can be improved.

In terms of dynamical parameter settings, considering that in practical engineering missions lunar ephemerides are often used to obtain lunar position information, taking the DE430 and DE440 ephemerides released by JPL as examples, the difference in the Earth–Moon distance computed by the two is at the decimeter-to-meter level[^36]. This difference affects the light-time calculation. Therefore, to be as consistent as possible with engineering practice, this paper uses the DE430[^37] ephemeris to simulate the influence of lunar ephemeris error on autonomous orbit determination. The Earth's nonspherical gravitational field adopts the EIGEN-GL04C (GRACE- and LAGEOS-based high resolution combination European Improved Gravity model of the Earth by New techniques) model[^38], truncated to degree and order 120×120. The lunar nonspherical gravitational field adopts the GRGM900C model[^33], truncated to degree and order 50×50. Other dynamical models and parameter-estimation strategies are listed in Table 7[^33–34,^37–43].

5 Orbit Determination Results Analysis

5.1 Analysis of Orbit Determination Results for DRO Satellites and LEO Satellites

Figures 12–15 show the autonomous orbit determination accuracy of each DRO satellite under different ascending/descending observation time intervals, namely the three-dimensional root mean square (3D-Root-Mean-Square, 3D-RMS). It can be seen from the figures that, after considering lunar ephemeris prediction errors, the orbit determination performance of DOWR observations under different ascending/descending observation time intervals is comparable, and all can achieve orbit determination accuracy better than 50 m.

Table 7 Settings in autonomous orbit determination

Items	Models & Description
Measurement Model	Measurement Model
LEO satellite attitude	Nominal attitude[^39–41]
Outlier observation process	3σ criterion
Parameter estimation	Batch least-squares estimation
Dynamic Model	Dynamic Model
Earth orientation parameters	EOP IERS Bulletin A
Lunar orientation parameters	DE430[^37]

Items	Models & Description
Gravity model	LEO: EIGEN GL04C (120×120)[^38], DRO: GRGM900C (50×50)[^33]
N body	DE430[^37]
Solid earth tide	IERS 2010 Conventions
Ocean tide	FES2004[^42]
Pole tides	IERS 2010 Conventions
General relativity	IERS 2010 Conventions
Solar radiation pressure	Cannon-ball model
Atmospheric drag	NRLMSIS-00[^43]
Empirical accelerations	Consider the parameters of Ca, Sa, Cc and Sc for the along- and cross-track directions (cosine/sine)
Integrator	Krogh-Shampine-Gordon integrator[^34]
Integral step	LEO: 15 s; DRO: 60 s

Table 7 Continued

Items	Models & Description
Solved Parameters	
Orbit parameters	Position and velocity at initial epoch for all spacecrafts
Solar radiation factor	LEO: 1 factor for each arc, DRO: 1 factor for each arc
Atmospheric drag factor	LEO: 1 factor for every 24 hours
Empirical accelerations	LEO: 1 factor for every 24 hours, DRO: 1 factor for each arc
Parameters Prior Information	
Orbit parameters	LEO: Calculated by spaceborne GNSS data and GNSS broadcast ephemeris, DRO: Integrated from last POD arc
Solar radiation factor	Init value: 2; Standard deviation: 2
Atmospheric drag factor	Init value: 2; Standard deviation: 2
Empirical accelerations	Init value: $0\ \mathrm{m} \cdot \mathrm{s}^{-2}$; Standard deviation: $1 \times 10^{-8}\ \mathrm{m} \cdot \mathrm{s}^{-2}$

Fig. 12 labels: y-axis, 3D-RMS/m; x-axis, Arc 1, Arc 2, Arc 3, Arc 4, Arc 5, Arc 6; legend, 0 s, 5 s, 10 s, 20 s.

Fig. 12 Autonomous orbit determination accuracy of DRO-0 under different uplink-downlink observation intervals

Fig. 14 labels: y-axis, 3D-RMS/m; x-axis, Arc 1, Arc 2, Arc 3, Arc 4, Arc 5, Arc 6; legend, 0 s, 5 s, 10 s, 20 s.

Fig. 14 Autonomous orbit determination accuracy of DRO-2 under different uplink-downlink observation intervals

Fig. 13 labels: y-axis, 3D-RMS/m; x-axis, Arc 1, Arc 2, Arc 3, Arc 4, Arc 5, Arc 6; legend, 0 s, 5 s, 10 s, 20 s.

Fig. 13 Autonomous orbit determination accuracy of DRO-1 under different uplink-downlink observation intervals

Fig. 15 labels: y-axis, 3D-RMS/m; x-axis, Arc 1, Arc 2, Arc 3, Arc 4, Arc 5, Arc 6; legend, 0 s, 5 s, 10 s, 20 s.

Fig. 15 Autonomous orbit determination accuracy of DRO-3 under different uplink-downlink observation intervals

Table 8 summarizes, when dT = 5 s, the orbital accuracies ΔR, ΔT, and ΔN of each DRO satellite in the radial (Radial, R), tangential (Tangential, T), and normal (Normal, N) directions.

As can be seen from Table 8, when dT = 5 s, the orbit-determination accuracy of all DRO satellites is better than 50 m; in particular, for DRO-0, which has the shortest orbital period, the orbit-determination accuracy is better than 20 m. Meanwhile, as the operating period of the DRO increases, the overall orbit-determination accuracy also continuously decreases, and the orbital errors of all DRO satellites are mainly concentrated in the orbit-normal direction. On the one hand, this is related to the relatively short orbit-determination arc length. The orbit-determination arc length selected in this paper is shorter than the orbital periods of the DRO-1, DRO-2, and DRO-3 satellites, causing their orbit-determination accuracy to decrease as the orbital period increases, and also making the error in the orbit-normal direction more significant. Such a phenomenon also exists in orbit determination for Earth GEO satellites [44].

On the other hand, this is also affected by the observation geometry of LEO–cislunar DRO. Because the orbits of DRO satellites lie close to the ecliptic plane, while the LEO–cislunar DRO line-of-sight direction is almost parallel to the ecliptic plane, the observations are insensitive to changes in the orbital plane. For example, for the DRO-0 satellite in Table 8, although the orbit-determination arc length is greater than the satellite' s orbital period, owing to the influence of the observation geometry, its orbital error in the N direction (mostly 10–20 m) is more significant than the orbital errors in the R and T directions (meter level or even sub-meter level).

To further analyze the LEO orbit-determination results, Table 9 presents the orbit-determination accuracies of each LEO satellite in the three RTN directions under different dT values. It can be seen from Table 9 that the orbital errors of the LEO satellites are mainly distributed in the along-track T direction, which is related to the low frequency and precision of the atmospheric drag coefficient estimated during orbit determination. The accuracy in the orbit-normal N direction is the highest, with an average at the decimeter level.

Table 8 Autonomous orbit determination accuracy of all DROs in RTN-directions when dT = 5 s (unit: m)

Arc	DRO-0 ΔR	DRO-0 ΔT	DRO-0 ΔN	DRO-0 3D	DRO-1 ΔR	DRO-1 ΔT	DRO-1 ΔN	DRO-1 3D	DRO-2 ΔR	DRO-2 ΔT	DRO-2 ΔN	DRO-2 3D	DRO-3 ΔR	DRO-3 ΔT	DRO-3 ΔN	DRO-3 3D
1	0.33	0.57	10.20	10.22	0.48	0.81	24.76	24.78	1.19	3.29	8.23	8.94	34.57	27.82	22.36	49.69
2	0.32	0.70	11.02	11.05	0.43	0.87	3.83	3.95	4.80	1.55	45.69	45.97	3.47	10.69	15.06	18.79
3	0.64	1.38	17.50	17.57	0.23	0.27	14.23	14.23	0.92	1.10	42.36	42.38	10.94	13.59	42.71	46.14
4	1.09	0.95	21.12	21.17	0.71	0.49	27.20	27.21	4.98	3.16	24.08	24.79	8.20	4.78	21.07	23.10
5	0.17	0.34	14.82	14.82	1.18	0.83	20.70	20.75	0.89	0.99	21.63	21.67	3.31	3.47	24.15	24.63
6	0.32	0.81	10.87	10.90	2.18	1.48	16.41	16.62	4.24	8.67	15.14	17.95	10.17	12.24	28.98	33.06

Compared with DRO satellites, LEO satellites, as Earth satellites, have stronger orbital-dynamics constraints. At the same time, in autonomous orbit determination, LEO satellites also use the variance-covariance matrix of broadcast ephemeris orbit-determination results as prior constraints. Therefore, the orbit-determination accuracy of LEO satellites is relatively high, at the meter level, as shown in Fig. 16.

5.2 Influence of increasing the uplink/downlink observation time interval

The condition number of the normal equation (the ratio of the maximum eigenvalue to the minimum eigenvalue of the normal equation) can characterize the observability of the estimation system [14]. Meanwhile, to more clearly express the trend of condition-number variation, the condition number can be normalized to form the normalized condition number as follows:

$$\mathrm{Cond}_{\mathrm{norm}} = \frac{\mathrm{Cond} - \min(\mathrm{Cond})}{\max(\mathrm{Cond}) - \min(\mathrm{Cond})}, \tag{47}$$

where Cond is the condition number, and $\mathrm{Cond}_{\mathrm{norm}}$ is the normalized condition number. Figure 17 shows the condition number, normalized condition number, and corresponding average orbit-determination accuracy of each arc segment for LEO satellites under different uplink/downlink observation time intervals dT.

As can be seen from Fig. 17, the condition numbers for different uplink/downlink observation time intervals dT in each arc segment are of the same order of magnitude. However, as dT increases, the normalized condition number exhibits an obvious decreasing trend, indicating that the system observability improves as dT increases.

Fig. 16

Figure 10: Fig. 16

The increase in the sensitivity of DOWR to satellite orbital shape is strictly···

···is the fundamental reason for this phenomenon. As shown in Fig. 18, the LEO satellite transmits a signal at time τ_1, and the DRO satellite receives it at time τ_2; after dT, the DRO satellite transmits a signal at time $\tau_3 = \tau_2 + \mathrm{dT}$, and the LEO satellite receives it at time τ_4. If dT is increased to dT′, then the DRO satellite transmits the signal at time τ_3', and the LEO satellite receives it at time τ_4'. Since $\tau_4' - \tau_1 > \tau_4 - \tau_1$, and $\tau_3' - \tau_2 > \tau_3 - \tau_2$, the "transmit-receive baseline" of the LEO/DRO under dT′ is longer than that under dT. Therefore, the DOWR under a longer uplink-downlink observation interval contains more information on the orbital variations of the LEO/DRO, thereby improving the observation geometry of the observation system.

Table 9 Autonomous orbit determination accuracy of LEO satellites in RTN-directions under different uplink-downlink observation intervals (unit: m)

Arc	Radial 0 s	Radial 5 s	Radial 10 s	Radial 20 s	Tangential 0 s	Tangential 5 s	Tangential 10 s	Tangential 20 s	Normal 0 s	Normal 5 s	Normal 10 s	Normal 20 s
1	0.89	0.58	0.38	0.52	2.17	1.32	1.05	1.14	0.5	0.2	0.54	0.83
2	1.5	2.62	1.15	2.26	3.55	6.44	3.11	5.39	0.35	0.51	0.42	0.62
3	0.19	0.54	0.53	0.29	0.83	1.98	1.96	1.54	0.43	0.46	0.62	0.57
4	0.44	0.63	0.39	0.38	1.44	1.78	0.96	1.41	0.82	0.95	0.96	0.55
5	2.04	2.43	0.91	1.5	4.76	5.77	2.22	3.75	0.69	0.85	0.47	0.3
6	2.27	1.91	1.84	2.47	5.39	4.65	4.5	5.91	0.38	0.33	0.43	0.48

Fig. 16 Autonomous orbit determination accuracy for all LEO satellites under different uplink-downlink observation intervals

It should be pointed out that, under the current simulation conditions, a longer uplink-downlink observation time interval does not make an obvious contribution to improving the autonomous orbit determination accuracy of the LEO-lunar DRO formation. The main reason is that the uplink-downlink observation time interval set in the simulation is relatively short, so the improvement in observation geometry is limited, causing it to be obscured by observation noise and other factors such as the relatively long DRO orbital period. However, it imposes lower requirements on the scheduling of the entire observation system and improves the fault-tolerance capability of the observation system. For example, as can be seen from Figs. 12-16, whether dT = 0 s or dT = 20 s, the orbit determination accuracies of LEO satellites and DRO satellites under different

Fig. 17

Figure 11: Fig. 17

Fig. 18 Transmission and reception position changes of LEO/DRO satellite caused by different uplink-downlink observation intervals

Figure 12: Fig. 18 Transmission and reception position changes of LEO/DRO satellite caused by different uplink-downlink observation intervals

schemes are comparable. This indicates that, in the autonomous orbit determination system of the LEO-lunar DRO formation, provided that the precision of the observations is ensured, it is not necessary to impose excessive requirements on the length of the uplink-downlink observation time interval. This has certain reference significance for improving the redundancy of the observation system in China' s future implementation of Earth-Moon space-related missions.

Fig. 17 Normalized condition numbers for each orbit determination arc and average autonomous orbit determination accuracy of LEO satellites at different uplink-downlink observation intervals

Figure 18 Position changes of LEO/DRO transmission-reception signal epochs caused by different uplink-downlink observation intervals

Fig. 18 Transmission and reception position changes of LEO/DRO satellite caused by different uplink-downlink observation intervals

6 Conclusion

As the frontier of human deep-space exploration, cislunar space is currently receiving extensive attention. Research on autonomous navigation of cislunar spacecraft based on inter-satellite links is of great significance for the future development of China' s cislunar-space PNT enterprise.

This paper discusses in detail the theoretical methods and related technologies for autonomous navigation of a LEO-cislunar DRO formation based on DOWR observations with different uplink-downlink time intervals, including the DRO orbit-search method, DOWR observation modeling in the BCRS and its satellite clock-error correction, and precision orbit determination for cislunar spacecraft. Simulation verification and analysis were carried out based on multiple periods of DRO orbits obtained through autonomous search. The simulation results show that, after accounting for lunar ephemeris prediction errors, DOWR observations with uplink-downlink observation time intervals at the second level or even the tens-of-seconds level can all achieve LEO-cislunar DRO autonomous orbit-determination accuracy better than 50 m, and this accuracy tends to decrease as the DRO orbital period increases. Meanwhile, appropriately increasing the uplink-downlink observation time interval can improve the system' s observability, but the extent of improvement is limited; therefore, the improvement in orbit-determination accuracy is not obvious, and the autonomous orbit-determination accuracy of the LEO–cislunar DRO formation is comparable under different uplink–downlink observation time intervals.

It should be pointed out that this paper aims to isolate clock-error interference and focuses on analyzing the influence of observation geometry and dynamical characteristics on orbit-determination accuracy; therefore, certain simplifications were made to the clock-error terms in the simulation strategy. In engineering practice, long-interval observations impose higher requirements on the short-term stability of spaceborne atomic clocks. Therefore, when China implements cislunar-space related missions in the future, it will be necessary to make a comprehensive trade-off in combination with the actual performance of the onboard time-frequency system. On the premise of satisfying accuracy requirements, the requirement on the time interval between uplink and downlink observations may be appropriately relaxed, thereby effectively improving the redundancy of the observation system.

Through the above research, this paper has accumulated relevant theory and methodological experience for high-precision orbit determination in cislunar inter-satellite autonomous navigation. At the same time, oriented toward the major future demand for cislunar-space PNT services, the conclusions of this study also have certain reference significance for China' s future implementation of cislunar-space related missions.

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Analysis of Autonomous Orbit Determination for LEO-Earth-Moon DRO Constellation Using Time-Delayed Inter-satellite Ranging Observation

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Abstract To address the demand for autonomy and high precision in cislunar space navigation, this study focuses on a heterogeneous constellation composed of Low Earth Orbit (LEO) satellites and Earth-Moon Distant Retrograde Orbit (DRO) satellites. The theoretical method for differential correction of DRO orbits under the Circular Restricted Three-Body Problem (CR3BP) is introduced. In the Barycentric Celestial Reference System (BCRS), a combined observation model for Dual One-Way Ranging (DOWR) is established, and the clock biases caused by light-time delay and general relativistic effects are quantitatively analyzed. Based on numerical simulations, the impact of different uplink-downlink observation intervals on the autonomous orbit determination accuracy of the LEO-Earth-Moon DRO constellation is investigated under non-instantaneous observation conditions. The simulation results show that: (1) In the cislunar space environment, the ranging error in DOWR caused by general relativistic effects reaches the order of 5–30 ns, which is an error term that must be corrected. (2) Autonomous orbit determination for the LEO-Earth-Moon DRO constellation can be achieved using only DOWR observations. Specifically, the three-dimensional positioning accuracy of LEO satellites is better than 10 m. For DRO satellites, the accuracy is better than 50 m, with errors primarily concentrated in the orbital normal direction, while radial and tangential accuracies can reach the meter level in some arc segments. (3) Within the range of 0–20 seconds for uplink-downlink observation time intervals, extending the interval can improve system observability. However, due to the short duration of the interval, the improvement is limited, and the enhancement of overall autonomous orbit determination accuracy is not significant. The accuracy of different schemes is comparable. These results indicate that, subject to maintaining measurement accuracy, the temporal constraints on uplink-downlink observation intervals for the LEO-Earth-Moon DRO constellation orbit determination system can be relaxed. This relaxation contributes to enhanced system robustness and operational redundancy. This study validates the feasibility of high-precision autonomous navigation for LEO-Earth-Moon DRO heterogeneous constellation in cislunar space, providing a theoretical basis and technical reference for the design of future cislunar navigation constellations.

Key words spacecraft: earth-moon distant retrograde orbit (DRO) satellite, spacecraft: low earth orbit (LEO) satellite, celestial mechanics: orbit determination, methods: inter-satellite link, methods: autonomous orbit determination