Optimizing the Coordinates of NGLR-1 Using Lunar Laser Ranging Data*

Wang Zhenzhen, Tang Rufeng, Yang Yongzhang, Li Zhiying, Li Yuqiang

2026-08-18

ChinaRxiv

Abstract

Accurately determining the lunar surface coordinates of newly deployed retroreflectors is important not only for improving the prediction accuracy of laser ranging, but also for enhancing telescope tracking and pointing capabilities, thereby increasing the success rate and observational efficiency of lunar laser ranging. To verify the initial coordinates of the Next Generation Lunar Retroreflector (NGLR-1) recommended by the International Laser Ranging Service (ILRS), relevant observational data obtained in the first half of 2025 were processed and analyzed based on the lunar laser ranging observation model independently developed by Yunnan Observatories. The results show that, when the ILRS-recommended coordinates are used, the two-way range residuals (O-C) between the observed (Observation, O) and theoretical (Calculation, C) ranges of NGLR-1 have both mean and root mean square values at the hundred-meter level; in contrast, the corresponding results for the Apollo and Luna series retroreflectors during the same period remain at the centimeter level. Furthermore, the three-dimensional coordinates of NGLR-1 were estimated by the least-squares method, and the observational data were reprocessed using the optimized coordinate solution. The mean and root mean square values of the residuals were significantly reduced to the centimeter level, comparable to the results for other retroreflectors. In addition, this study verifies the feasibility of rapidly determining retroreflector coordinates under the condition of limited observational data, providing a reference for the rapid positioning of NGLR-1, precise laser ranging observations, and related scientific research.

Full Text

doi: 10.15940/j.cnki.0001-5245.2026.04.004

Optimizing the Coordinates of NGLR-1 Using Lunar Laser Ranging Data*

Wang Zhenzhen[1,2] Tang Rufeng[1] Yang Yongzhang[1†] Li Zhiying[1,3] Li Yuqiang[1,3]

(1 Yunnan Observatories, Chinese Academy of Sciences, Kunming 650216)
(2 School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049)
(3 Key Laboratory of Space Object and Debris Observation, Chinese Academy of Sciences, Nanjing 210023)

Abstract Accurately determining the lunar-surface coordinates of newly deployed retroreflectors not only helps improve the prediction accuracy of laser ranging, but also enhances the tracking and pointing capabilities of telescopes, and is of great significance for increasing the success rate and observational efficiency of lunar laser ranging. To verify the initial coordinates of the next-generation single large-aperture lunar retroreflector (Next Generation Lunar Retroreflector, NGLR-1) recommended by the International Laser Ranging Service (ILRS), this study, based on a lunar laser ranging observation model independently constructed by Yunnan Observatories, processed and analyzed relevant observational data obtained in the first half of 2025. The results show that, using the ILRS-recommended coordinates, the two-way range residuals between the measured values (Observation, O) and theoretical values (Calculation, C) for NGLR-1, $(O - C)$, have both mean value and root-mean-square value reaching the hundred-meter level; by contrast, the computed results for the Apollo and Luna series retroreflectors during the same period remain at the centimeter level. Furthermore, least-squares parameter estimation was used for the three-dimensional coordinates of NGLR-1, and the observational data were reprocessed based on the optimized coordinates. The mean and root-mean-square values of the residuals were significantly reduced to the centimeter level, comparable to the results for other retroreflectors. In addition, this study also verifies the feasibility of rapidly determining retroreflector coordinates under conditions of a small amount of observational data, providing a reference for rapid positioning of NGLR-1, precise laser ranging observations, and related scientific research.

Keywords Astrometry: lunar laser ranging; instruments: NGLR-1; methods: least-squares fitting

Chinese Library Classification No.: P129; **Document code:** A

1 Introduction

Lunar Laser Ranging (LLR) originated from a gravitational research project at Princeton University in the late 1950s. Robert Henry Dicke and his collaborators considered installing retroreflectors on satellites and using pulsed lasers from the ground to conduct satellite laser ranging, thereby obtaining tiny variations in the gravitational constant G by precisely tracking artificial satellites[1]. In July 1969, American astronauts Neil Armstrong and Buzz Aldrin, during the Apollo 11 mission, placed the Apollo 11 retroreflector array in the lunar Mare Tranquillitatis. In the following four years, the United States successively deployed the Apollo 14 and Apollo 15 retroreflector arrays on the lunar surface during the Apollo 14 and Apollo 15 missions; at the same time, the former Soviet Union also deployed the Luna 17 and Luna 21 retroreflector arrays on the lunar surface through its lunar missions. On January 15, 2025, the "Blue Ghost" lunar lander manufactured by the U.S. company Firefly Aerospace, carried by Falcon 9

Received original manuscript: 2025-09-15; revised manuscript received: 2025-10-27

* Supported by the Strategic Priority Research Program of the Chinese Academy

Fig. 1 Distribution map of the six corner retroreflectors on the lunar near side

Figure 1: Fig. 1 Distribution map of the six corner retroreflectors on the lunar near side

of Sciences (XDA0350300), the National Natural Science Foundation of China (12033009), the National Key Research and Development Program of China (2021YFA0715101), the International Partnership Program of the Chinese Academy of Sciences (020GJHZ2022034FN), and the Yunnan Provincial Basic Research Programs (202301AT070328, 202401AT070141, 202501AT070208, 202501AT070026).

† yang.yongzhang@ynao.ac.cn

The (Falcon-9) rocket was successfully launched from the Kennedy Space Center. This mission is part of the Commercial Lunar Payload Services (CLPS) program carried out by Firefly Aerospace for the U.S. National Aeronautics and Space Administration; the purpose of this program is to achieve lunar resupply through commercial companies and to provide important support for the United States' return-to-the-Moon "Artemis" program. On March 2, 2025, the "Blue Ghost" lunar lander descended in the Mare Crisium region of the Moon, at coordinates 17.0 °N, 59.1 °E. The "Blue Ghost" lunar lander is about 2 m high and about 3.5 m wide, and carries a total of 10 scientific payloads, including a next-generation monolithic large-aperture laser corner retroreflector (Next Generation Lunar Retroreflector, NGLR-1). NGLR-1 adopts a single-reflector structure, with a diameter of 100 mm and a height of 72 mm. It was designed and manufactured by the University of Maryland (UMD). Compared with the retroreflector arrays carried by the Apollo and Luna series missions, the single-prism structure can effectively reduce the random errors caused by the inclination of retroreflector arrays[^2], thereby supporting millimeter-level laser-ranging precision and helping to further improve the accuracy of observational data. The distribution of the NGLR-1 corner retroreflector and the other five corner retroreflectors on the lunar near side is shown in Fig. 1.

图 1 月球正面 6 个角反射器分布图

Fig. 1 Distribution map of the six corner retroreflectors on the lunar near side

The successful deployment of corner retroreflectors on the lunar surface has significantly promoted the construction and development of global lunar laser ranging stations, and has facilitated the long-term accumulation of LLR observational data. Since August 1969, the Lick Observatory, University of California, United States, used a telescope with an aperture of 3 m to successfully detect laser return signals from the Apollo 11 mission' s retroreflector array, obtaining the first LLR observational data. As of December 2024, the Paris Observatory Lunar Analysis Center (POLAC) has cumulatively collected 35,069 LLR data records from major lunar laser ranging stations worldwide since 1969. Among

Fig. 2 Contribution proportion of global LLR stations (1969—2024)

Figure 2: Fig. 2 Contribution proportion of global LLR stations (1969—2024)

them, the Grasse station in France contributed 21,433 data records; the McDonald station and Apache Point station in the United States contributed 7,905 and 4,196 records, respectively; the Haleakala station in the United States provided 770 records; the Matera station in Italy contributed 439 records; and the Wettzell station in Germany contributed 326 records. The contribution proportions of LLR data from the various stations are shown in Fig. 2. It can be seen from the figure that the amount of observational data from the Grasse station in France accounts for more than half of the total global LLR data, making it the main source of global LLR observational data. On March 3, 2025, the scientific research team of the Geoazur laboratory under the Côte d' Azur Observatory (OCA) in France announced that it had successfully obtained the first batch of laser-ranging data from the NGLR-1 retroreflector. At 11:38:26 on March 3, 2025, Coordinated Universal Time (UTC), the French lunar laser ranging station (Metrology and Optics, MEO) measured the distance to the NGLR-1 retroreflector as 358758.807 km, with a corresponding laser round-trip light time of 2.393381 s. Subsequently, telescope stations in Germany, the United States, and China successively announced that they had successfully obtained valid return-wave data from NGLR-1.

图 2 1969—2024 年全球 LLR 台站贡献数据比例

Fig. 2 Contribution proportion of global LLR stations (1969—2024)

In recent years, with the continuous development of observation technology and observing equipment, the observational accuracy of LLR has steadily improved. Battat et al.

3

systematically analyzed nearly 50 yr of LLR data and summarized the evolutionary trend of observational accuracy. In the 1970s, the accuracy of LLR observations was about 10 cm; by the beginning of the 21st century it had improved to 1 cm, and the present observational accuracy has approached the 1 mm level. In China, on January 22, 2018, the Yunnan Observatories of the Chinese Academy of Sciences obtained returns from the Apollo 15 corner-cube reflector array using a 1.2 m telescope, becoming the first station in China to realize lunar laser ranging

4

, with the accuracy of two-way range residuals at the meter level

5

. On November 7, 2019, the Tianqin station of Sun Yat-sen University detected signals from five corner-cube reflector arrays of the Apollo series and Lunokhod series on the lunar surface, with measurement accuracy at the decimeter level

6

. The lunar laser ranging observation model is continuously optimized through historical LLR data, and can simulate and calculate the theoretical value of the light-time at a given epoch. Owing to the complexity of Earth–Moon motion and the fact that the laser’ s passage through the Earth–Moon space is affected by many complex factors such as atmospheric turbulence, Earth–Moon tides, and spacetime transformations in general relativity, the accuracy of the observation model lags behind the observational accuracy of the equipment. As time goes on, the accuracy of lunar laser ranging observation models continues to improve, and high-precision models are of important guiding significance for improving LLR observing efficiency and data-analysis accuracy.

During March to May 2025, the Grasse station, Apache Point station, and Wettzell station submitted nearly one hundred valid ranging data points for NGLR-1 to the European Laser Ranging Stations Data Center (EDC). However, the first measurement results for NGLR-1 from the Côte d’ Azur Observatory in France showed that the coordinates of the “Blue Ghost”lunar lander differed from the predetermined coordinates by about 100 m; the Wettzell station in Germany found that the return time of NGLR-1 was usually later than the set range-gate epoch, and inferred that this might be caused by insufficient accuracy of the lunar-surface coordinates of the new corner-cube reflector. Therefore, based on NGLR-1 observation data and the lunar laser ranging observation model, this paper adopts the Gauss–Newton nonlinear least-squares method to optimize the coordinates of NGLR-1, thereby providing a new coordinate reference for NGLR-1.

2 Lunar laser ranging observation model

The development of the lunar laser ranging observation model of the Yunnan Observatories is based on the Standards of Fundamental Astronomy (SOFA) of the International Astronomical Union (IAU), the IERS Conventions 2010 document of the International Earth Rotation and Reference Systems Service (IERS), and the ephemeris of France’ s INPOP21a (Intégrateur Numérique Planétaire de l’ Observatoire de Paris 21a). This model covers key processes including time correction, coordinate transformation, tidal correction, atmospheric-delay correction, relativistic correction, and light-time iteration calculation. By processing LLR data with the lunar laser ranging observation model, one can obtain the difference between the observed light-time and the theoretical light-time. According to the relation that distance equals the product of light speed and time, the mean and root-mean-square values of the two-way range residuals between observation and theory can be further obtained.

Because the laser emission time t_1 adopts UTC, whereas the time t_2 when the laser reaches the lunar corner-cube reflector and the time t_3 when the laser returns after reflection and is received by the station involve celestial motion, Barycentric Dynamical Time (TDB) is used. Therefore, the emission time t_1 must be corrected from UTC to Terrestrial Time (TT), and then to TDB. The time-correction relationship is as follows:

$$\begin{cases} (t_3 - t_1)^{(\mathrm{UTC})} = \dfrac{(t_3 - t_1)^{(\mathrm{TT})}}{1+\zeta}, \\ \mathrm{TDB} - \mathrm{TT} = \dfrac{L_G - L_B}{1 - L_B}(\mathrm{TDB} - T_0) + \dfrac{1 - L_G}{1 - L_B}\mathrm{TDB}_0 + \dfrac{1 - L_G}{1 - L_B}\displaystyle\int_{T_0+\mathrm{TDB0}}^{\mathrm{TDB}} \frac{1}{c^2}\left(\frac{\dot{r}_E^2}{2} + W_{0E} + W_{LE}\right)\mathrm{d}t + \frac{1}{c^2} \end{cases}$$

where TT and UTC denote terrestrial time and UTC time, respectively; ζ had a value of 3×10^{-8} before 1972 and a value of 0 after 1972; $\mathrm{TDB} - \mathrm{TT}$ denotes the difference between TDB and TT; $T_0 = 2443144.5003725$, $\mathrm{TDB}_0 = 6.55 \times 10^{-5}/86400$; $L_G = 6.969290134 \times 10^{-10}$ is the rate of TT relative to Geocentric Coordinate Time (TCG); $L_B = 1.550519768 \times 10^{-8}$ is the rate of TDB relative to Barycentric Coordinate Time (TCB); c denotes the speed of light; W_{0E} is the Earth' s rotational gravitational potential; W_{LE} is the external gravitational potential of the Earth; r_E is the position vector of the geocenter relative to the solar-system barycenter; and r_S is the position vector of the station relative to the solar-system barycenter.

In the selection of spatial coordinate systems, station coordinates are usually expressed in the International Terrestrial Reference System (ITRS), while retroreflector coordinates are based on the lunar principal-axes coordinate system (Principal Axes, PA). To achieve accurate distance computation between the station and the retroreflector, it is necessary to further transform both coordinate systems uniformly into the Barycentric Celestial Reference System (BCRS). The coordinate transformation methods for the station and the retroreflector are as follows:

$$\begin{cases} s_{\mathrm{BCRS}} = r_{\mathrm{E}} + s_{\mathrm{GCRS}}\left(1 - \dfrac{U_{\mathrm{E}}}{c^2} - L_{\mathrm{C}}\right) - \dfrac{1}{2}\left(\dfrac{\dot{r}_{\mathrm{E}} \cdot s_{\mathrm{GCRS}}}{c^2}\right)\dot{r}_{\mathrm{E}}, \\ l_{\mathrm{BCRS}} = r_{\mathrm{M}} + l_{\mathrm{LCRS}}\left(1 - \dfrac{U_{\mathrm{M}}}{c^2}\right) - \dfrac{1}{2}\left(\dfrac{\dot{r}_{\mathrm{M}} \cdot l_{\mathrm{LCRS}}}{c^2}\right)\dot{r}_{\mathrm{M}}, \end{cases} \tag{2}$$

where s_{BCRS} and s_{GCRS} are respectively the position vectors of the station in the BCRS and in the Geocentric Celestial Reference System (GCRS); U_{E} is the gravitational potential of other celestial bodies at the Earth; L_{C} is a constant, with value $1.48082686741 \times 10^{-8}$; l_{BCRS} and l_{LCRS} are respectively the position vectors of the retroreflector in the BCRS and in the Lunar-Centered Reference System (LCRS); r_{M} is the position vector of the lunar barycenter relative to the solar-system barycenter; and U_{M} is the gravitational potential of other celestial bodies at the Moon.

In addition, the positions of Earth stations and lunar retroreflectors are not fixed, but continuously change under the influence of pole tides, solid tides, and ocean tides. The correction relations are as follows:

$$\begin{cases} s_{\mathrm{GCRS}} = R_{\mathrm{T2C}}\left(s_{\mathrm{TRS}} + \Delta_{\mathrm{pole}} + \Delta_{\mathrm{solid}} + \Delta_{\mathrm{ocean}}\right), \\ l_{\mathrm{LCRS}} = R_{\mathrm{L2C}}\left(l_{\mathrm{PA}} + \Delta^{(\mathrm{E})}_{\mathrm{solidmoon}} + \Delta^{(\mathrm{S})}_{\mathrm{solidmoon}}\right), \end{cases} \tag{3}$$

where s_{TRS} is the position vector of the station in the ITRS coordinate system; R_{T2C} is the transformation matrix from the station coordinates in the ITRS coordinate system to the GCRS coordinate system; Δ_{pole}, Δ_{solid}, and Δ_{ocean} are respectively the displacement vectors of the station caused by pole tide, solid tide, and ocean tide; l_{PA} is the position vector of the retroreflector in the PA coordinate system; R_{L2C} is the transformation matrix from the retroreflector coordinates in the PA coordinate system to the LCRS coordinate system; and $\Delta^{(\mathrm{E})}_{\mathrm{solidmoon}}$ and $\Delta^{(\mathrm{S})}_{\mathrm{solidmoon}}$ are respectively the lunar solid-tide displacement vectors caused by the Earth and the Sun.

When the laser propagates through the Earth' s atmosphere, atmospheric refraction causes the optical path to bend. It is necessary to consider the effects of atmospheric pressure, temperature, and humidity at the station location on the laser path. The correction in the station zenith direction is calculated from the atmospheric refractive index, and then the atmospheric delay is obtained through a mapping function. The specific formula is as follows:

$$\begin{cases} d_{\mathrm{atm}} = 10^{-6}\displaystyle\int_{r_s}^{r_d} N\,\mathrm{d}z = \int_{r_s}^{r_d} (n-1)\mathrm{d}z, \\ \Delta_{\mathrm{atm}} = d_{\mathrm{atm}} m(e), \end{cases} \tag{4}$$

where d_{atm} is the atmospheric delay in the zenith direction, N is the total refractivity of moist air, n is the refractive index of moist air, Δ_{atm} is the delay introduced by the atmosphere during laser propagation, r_s and r_d are respectively the geocentric radial distances of the ground station and the top of the atmosphere, and $m(e)$ is the mapping function, normalized at the zenith, where its value is 1 at the zenith.

During Earth–Moon propagation, the laser is also affected by the gravitational fields of solar-system bodies, producing path bending. Therefore, the influence of different celestial bodies on the light path must be considered. Specifically, the effects of bodies such as the Earth, Jupiter, Saturn, the Moon, and the Sun are considered. The relativistic correction relation is as follows:

$$\Delta_{\mathrm{grav}} = \sum_{J} \frac{2GM_j}{c^3} \ln\left|\frac{r_{j1} + r_{j2} + \rho}{r_{j1} + r_{j2} - \rho}\right|, \tag{5}$$

Fig. 3 Relativistic and atmospheric corrections of Grasse station LLR data from January to May 2025

Figure 3: Fig. 3 Relativistic and atmospheric corrections of Grasse station LLR data from January to May 2025

where Δ_{grav} is the delay caused by relativistic gravitation during laser propagation, J is the number of celestial bodies, G is the universal gravitational constant, M_j is the mass of the j-th celestial body, r_{j1} is the distance from the celestial body to the station, r_{j2} is the distance from the celestial body to the retroreflector, and ρ is the distance along the straight-line propagation of the laser.

After processing the LLR data from the Grasse station from January to May 2025, the two-way atmospheric delay correction is approximately 3–12 m, and the two-way relativistic correction is approximately 14–17 m. The specific results are shown in Fig. 3.

The laser is emitted at time t_1, reaches the lunar retroreflector at time t_2, and is finally received by the station at time t_3. Since the Earth and the Moon are continuously rotating and moving during these two time intervals, the relative positions of the station and the retroreflector change with time. Therefore, the times t_2 and t_3 must be dynamically computed through iteration of the light-time. The iterative relations are respectively:

$$\begin{cases} t_2 - t_1 = \dfrac{|s_{\text{BCRS}}(t_1) + l_{\text{BCRS}}(t_2)|}{c} + \Delta_{\text{atm}}(t_1, t_2) + \Delta_{\text{grav}}(t_1, t_2), \\ t_3 - t_2 = \dfrac{|s_{\text{BCRS}}(t_2) + l_{\text{BCRS}}(t_3)|}{c} + \Delta_{\text{atm}}(t_3, t_2) + \Delta_{\text{grav}}(t_3, t_2). \end{cases} \tag{6}$$

3 Coordinate Optimization of NGLR-1

3.1 Validation of the Lunar Laser Ranging Observation Model

First, lunar laser ranging data obtained from different stations during January to May 2025 and covering multiple types of retroreflectors were selected to test the accuracy of the lunar laser ranging observation model. The three-dimensional coordinates of the Apollo and Luna series retroreflector arrays in the PA coordinate system are shown in Table 1; the data source is INPOP21a. After processing the lunar laser ranging data from January to May 2025, the mean value of the two-way range residuals between the observed range values (Observation, O) and the theoretical computed values (Calculation, C), i.e., $O-C$, was -2.2 cm, and the root-mean-square value was 6.17 cm. The results are shown in Fig. 4.

Fig. 3 Relativistic and atmospheric corrections of Grasse station LLR data from January to May 2025

Fig. 4 Two-way residuals of Apollo and Luna series LLR data from January to May 2025

Figure 4: Fig. 4 Two-way residuals of Apollo and Luna series LLR data from January to May 2025

Fig. 5 Two-way residuals of LLR data for NGLR-1 corner reflector before coordinate optimization from March to May 2025

Figure 5: Fig. 5 Two-way residuals of LLR data for NGLR-1 corner reflector before coordinate optimization from March to May 2025

Table 1 Three coordinates of the Apollo and Luna retroreflectors in the Principal Axes (PA) coordinate system

Reflector	x/km	y/km	z/km
Apollo 11	1591.9666407	690.6994669	21.0037578
Apollo 14	1652.6895625	−520.9975929	−109.7305181
Apollo 15	1554.6783071	98.0955262	765.0052077
Lunokhod 1	1114.2922303	−781.2984355	1076.0586227
Lunokhod 2	1339.3633937	801.8719437	756.3586633

https://ilrs.gsfc.nasa.gov/missions/satellite-missions/current-missions/ngl1-general.html

The coordinates of NGLR-1 in the PA coordinate system recommended by the International Laser Ranging Service (ILRS) are $x = 776.7643$ km, $y = 1448.3320$ km, $z = 552.1420$ km[^1]. Taking the above coordinates as the position input for NGLR-1 in the lunar laser ranging observation model, the NGLR-1 ranging data collected during March–May 2025 were processed. The mean value of $O-C$ obtained was 18481.31 cm, and the root-mean-square value was 18498.10 cm. The results are shown in Fig. 5, with a comparatively large positive bias distribution in $O-C$.

Fig. 4 Two-way residuals of Apollo and Luna series LLR data from January to May 2025

Fig. 5 Two-way residuals of LLR data for NGLR-1 corner reflector before coordinate optimization from March to May 2025

3.2 Coordinate Estimation of NGLR-1

Using the lunar laser ranging observation model described in the previous section to process contemporaneous LLR data, compared with the Apollo and Luna series reflector data over the same period, the mean and root-mean-square values of $O-C$ were at the centimeter level, The data processing results for NGLR-1 at the hundred-meter level show relatively large deviations. This may be due to the influence of positional deviations during the initial placement of the new corner reflector. Therefore, the Gauss-Newton nonlinear least-squares method was adopted to estimate the parameters of the NGLR-1 coordinates, and the fitted NGLR-1 coordinates were then used to reprocess the NGLR-1 LLR data through the lunar laser ranging model.

Specifically, the NGLR-1 coordinates recommended by ILRS were used as the initial input coordinates in the lunar laser ranging observation model. By processing the NGLR-1 data, the two-way range residuals $O-C$ between the observed and theoretically computed station–corner-reflector ranges were obtained as follows:

$$\begin{cases} \rho_{\mathrm{calc}} = \left| r + l_{\mathrm{LCRS}} - s_{\mathrm{GCRS}} \right|, \\ x = \rho_{\mathrm{obs}} - \rho_{\mathrm{calc}}, \end{cases} \tag{7}$$

where r is the distance vector from the geocenter to the lunar center, ρ_{calc} is the theoretical two-way range between the station and the corner reflector, ρ_{obs} is the observed two-way range between the station and the corner reflector, and x is the two-way range residual between the observed and theoretically computed station–corner-reflector ranges.

When numerically calculating the Jacobian matrix J, since the coordinates of the corner reflector are given in the PA coordinate system, and the PA coordinate system is a non-inertial system, the coordinates of the corner reflector must be transformed into the inertial coordinate system LCRS when differentiating the coordinates. The Jacobian matrix J is calculated as follows:

$$J = \frac{\partial x}{\partial l_{\mathrm{PA}}} = \frac{\partial \rho_{\mathrm{calc}}}{\partial l_{\mathrm{PA}}} = \frac{\partial \rho_{\mathrm{calc}}}{\partial l_{\mathrm{LCRS}}} \frac{\partial l_{\mathrm{LCRS}}}{\partial l_{\mathrm{PA}}} = \frac{r + l_{\mathrm{LCRS}} - s_{\mathrm{GCRS}}}{\rho_{\mathrm{calc}}} R_{\mathrm{L2C}}. \tag{8}$$

Through the Jacobian matrix J, the coordinates of the corner reflector are iteratively optimized. The value of the new coordinates is

$$\xi_{n+1} = \xi_n - \left(J^T J \right)^{-1} J^T \left[\rho_{\mathrm{obs}} - \rho_{\mathrm{calc}} \left(\xi_n \right) \right], \tag{9}$$

where ξ_n denotes the coordinates of the corner reflector in the PA coordinate system at the n-th optimization.

The termination condition for the optimization was set as a coordinate update step norm $|\xi_{n+1} - \xi_n|$ less than 1×10^{-8} cm. Finally, after 5–6 iterations, the optimized coordinates of NGLR-1 in the PA coordinate system were obtained as (x = 776.672841 km, y = 1448.359286 km, z = 552.204702 km). Taking these coordinates as the new position of NGLR-1 and inputting them into the lunar laser ranging observation model, the ranging data from March to May

Fig. 6

Figure 6: Fig. 6

Fig. 7

Figure 7: Fig. 7

2025 were reprocessed. The results show that the mean value of $O - C$ is 0.013 cm and the root-mean-square value is 6.96 cm, both at the centimeter level, and $O - C$ follows a good normal distribution. The processing results are shown in Fig. 6.

Fig. 6 Two-way residuals of LLR data for NGLR-1 corner reflector after coordinate optimization from March to May 2025

3.3 Rapid Optimization of NGLR-1 Coordinates

As a passive device, the laser corner reflector must be positioned after placement on the lunar surface by means of ground-based laser ranging or remote-sensing image data from lunar-orbiting satellites. To determine the precise position of the corner reflector more rapidly within a short time, this paper attempts to use a small amount of observation data from a few stations to achieve rapid positioning of the NGLR-1 coordinates. Two observations from the Wettzell station in Germany on March 4, 2025, and two observations each from the Grasse station in France on March 3 and March 4, 2025, were selected, giving a total of 6 observations. After optimization using the observation model combined with the Gauss–Newton nonlinear least-squares method, the optimized coordinates of NGLR-1 in the PA coordinate system were obtained as ($x = 776.672816$ km, $y = 1448.360301$ km, $z = 552.2049326$ km). Using the optimized coordinates as the new position of NGLR-1 and inputting them into the lunar laser ranging observation processing model, the six ranging observations from the two stations on the 3rd and 4th were reprocessed. The mean value of $O - C$ was 0.88 cm, and the root-mean-square value was 1.82 cm. The results are shown in Fig. 7. Furthermore, the optimized NGLR-1 coordinates were used to process the NGLR-1 lunar laser ranging data from March to May 2025, yielding a mean $O - C$ of -5.07 cm and a root-mean-square value of 13.67 cm. The results are shown in Fig. 8. Both of the above processing results are at the centimeter level, and are at the same order of magnitude as the results obtained in Section 3.2 using the March–May 2025 data for optimization. This result indicates that, even with a small amount of observational data, effective positioning of the lunar-surface corner reflector can be achieved, providing a feasible scheme for rapid initial positioning.

Fig. 7 Two-way range residuals of six LLR data after rapid optimization of NGLR-1 coordinates

Fig. 8 Two-way residuals of LLR data for NGLR-1 after rapid coordinate opti-

Fig. 8

Figure 8: Fig. 8

mization from March to May 2025

4 Conclusion

By inputting lunar laser ranging data obtained in 2025 from different stations and different corner reflectors into the lunar laser ranging model, it was found that, when the ILRS-recommended NGLR-1 coordinates were used as input, the mean and root-mean-square values of $O - C$ were relatively large. Taking into account the errors in remote-sensing image positioning, the Gauss–Newton nonlinear least-squares method was used to optimize the NGLR-1 coordinates, and the optimized coordinates were used as input to reprocess the data. The results show that the mean and root-mean-square values of the $O - C$ two-way residuals of NGLR-1 reached the same order of magnitude as the ranging results of the Apollo and Luna series. In response to the need for rapid positioning after the deployment of corner reflectors on the lunar surface in China' s future lunar exploration program, this paper shows that, even when only two stations and two nights of limited lunar laser ranging data are used, rapid optimization and effective determination of the NGLR-1 coordinates can still be achieved. It should be noted that the mean and root-mean-square values of $O - C$ are affected by many factors, and the coordinates of the corner reflector are only one of them. This study uses the Gauss–Newton nonlinear least-squares method to optimize the NGLR-1 coordinates, mainly considering the practical situation that the deployment time of NGLR-1 was relatively short and the available lunar laser ranging data were limited. The feasibility of rapid positioning in the future is verified, providing a better coordinate reference for future observations and scientific applications.

Acknowledgments The authors thank the Paris Observatory Lunar Analysis Center (POLAC) for its work in collecting lunar laser ranging data from global stations; through these measured data, the lunar laser ranging observation model of Yunnan Observatories has been continuously optimized. We thank the Grasse station in France, the Wettzell station in Germany, the Apache Point station in the United States, and the Matera station in Italy for providing lunar laser ranging data. We thank the International Laser Ranging Service (ILRS) for collecting NGLR-1 laser ranging data and providing the initial coordinates. Special thanks are due to the Clément Courde team at the Grasse station in France for providing a large amount of NGLR-1 observation data.

References

[1] Bender P L, Currie D G, Poultney S K, et al. *Science*, 1973, 182: 229

[2] Cao J, Tang R F, Huang K, et al. *Remote Sensing*, 2024, 16: 3030

[3] Battat J B R, Adelberger E, Colmenares N R, et al. *PASP*, 2023, 135: 104504

[4] Li Y Q, Fu H L, Li R W, et al. *Chinese Journal of Lasers*, 2019, 46: 188

[5] Huang K, Sun S B, Yang Y Z, et al. *Laser & Optoelectronics Progress*, 2022, 59: 1006

[6] Gao T Q, Zhang C S, Li M, et al. *Journal of Sun Yat-sen University (Natural Science Edition)*, 2021, 60: 24

Optimizing the Coordinates of NGLR-1 Using Lunar Laser Ranging Data

WANG Zhen-zhen[1,2] TANG Ru-feng[1] YANG Yong-zhang[1] LI Zhu-lian[1,3] LI Yu-qiang[1,3]

[1] Yunnan Observatories, Chinese Academy of Sciences, Kunming 650216
[2] School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049
[3] Key Laboratory of Space Object and Debris Observation, Chinese Academy of Sciences, Nanjing 210023

Abstract Accurately determining the lunar coordinates of newly deployed corner-cube retroreflectors is not only beneficial for improving the predictive accuracy of laser ranging but also enhances the tracking and targeting capabilities of telescopes. This has significant implications for increasing the success rate and observational efficiency of lunar laser ranging. To verify the initial coordinates of the next-generation monolithic large-aperture lunar retroreflector (Next Generation Lunar Retroreflector, NGLR-1) recommended by the International Laser Ranging Service (ILRS), this study processes and analyzes relevant observational data obtained in the first half of 2025 based on a lunar laser ranging observation model independently developed by Yunnan Observatories. The results show that when using the ILRS-recommended coordinates, the two-way $O-C$ residuals between the observed (O) and calculated (C) distances of NGLR-1 reach the order of hundreds of meters in both mean and root-mean-square values, whereas the corresponding results for the Apollo and Luna series retroreflectors remain at the centimeter level. Furthermore, by performing parameter estimation of the NGLR-1 three-dimensional coordinates using the least-squares method and reprocessing the observational data with the optimized coordinates, the mean and root-mean-square residuals are significantly reduced to the centimeter level, comparable to those of other retroreflectors. In addition, this study demonstrates the feasibility of rapidly determining retroreflector coordinates with a limited number of observations, providing a reference for the quick localization, precise laser ranging observations, and related scientific research of NGLR-1.

Key words astrometry: lunar laser ranging; instrumentation: NGLR-1; methods: least squares fitting

Note: Figure translations are in progress. See original paper for figures.