Assessment of the Grid-Forming Capability of Converters Based on Port Characteristics: Grid-Forming Index and Its Dynamic Strength Contribution

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Abstract

With the increasing penetration of renewable energy generation and power electronic equipment, the voltage and frequency support capability of power systems has been relatively weakened, and the secure and stable operation of the system imposes higher requirements on the grid-forming and support capabilities of power electronic equipment. Existing standards and studies regard autonomously establishing and maintaining internal potential and exhibiting voltage-source external characteristics as the core features of grid-forming converters. However, existing methods and indices still find it difficult to simultaneously account for the measurability of device port characteristics, the comparability among different devices, and the interpretability of contributions to system strength. To this end, starting from small-signal system strength, this paper proposes a method for evaluating the grid-forming capability of converters oriented toward engineering testing and control design. First, the dynamic strength of the system is characterized based on the norm of the sensitivity matrix from current disturbances in the closed-loop system to bus voltage responses, and the mechanism by which a grid-forming converter, as a "strength source," enhances the disturbance rejection capability of the system is clarified. Second, the Grid-Forming Index (FI) and Voltage Source Index (VI) are defined, and the voltage-source characteristics of the device are quantified by the peak sensitivity of port voltage to grid voltage disturbances. It is also proved that FI(jω)<1 is a sufficient condition for improving grid strength and the lower bound of system strength after device integration. Furthermore, an engineering calculation procedure based on port frequency-sweep/impedance measurement is provided, and the admittance shaping method is used to show that FI can be applied to grid-forming-oriented design of grid-following converters. Finally, simulations on the IEEE 39-bus system verify the consistency between the proposed indices and the dynamic support effect of devices.

Full Text

Assessment of the Grid-Forming Capability of Converters Based on Port Characteristics: Grid-Forming Index and Its Dynamic Strength Contribution

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Abstract: With the increasing penetration of renewable generation and power electronic equipment, the voltage and frequency support capability of power systems is relatively weakened, and the safe and stable operation of the system imposes higher requirements on the grid-forming and support capabilities of power electronic equipment. Existing standards and studies regard autonomously establishing and maintaining internal potential and presenting external voltage-source characteristics as the core features of grid-forming converters. However, existing methods and indices still have difficulty simultaneously considering the measurability of equipment port characteristics, comparability among different devices, and interpretability of contributions to system strength. Therefore, starting from the small-disturbance system strength, this paper proposes a converter grid-forming capability assessment method oriented toward engineering tests and control design. First, based on a sensitivity matrix paradigm from closed-loop system current disturbances to bus voltage responses, system dynamic strength is characterized, and the mechanism by which grid-forming converters, as "strength sources," improve system disturbance-withstanding capability is clarified. Next, the grid-forming index (Grid-Forming Index, FI) and the voltage source index (Voltage Source Index, VI) are defined, and the voltage-source characteristics of equipment are quantified by the peak value of the sensitivity of port voltage to grid voltage disturbances; it is also proved that $\mathrm{FI}(j\omega)<1$ is a sufficient condition for improving grid strength and the lower bound of system strength after the equipment is connected. Furthermore, an engineering calculation procedure based on port frequency-sweep/impedance measurement is given, and the use of the Nyquist method to show that FI can be used for grid-forming design of grid-following converters is explained. Finally, simulations on the IEEE 39-bus system verify the consistency between the proposed indices and the dynamic support effect of the equipment.

Keywords: grid-forming converter; system strength; grid-forming index; voltage source index; port impedance measurement; Nyquist

0 Introduction

Driven by the "dual-carbon" goals, China' s installed capacity of new energy and electricity generation continue to increase, and the power system is forming a "double-high" pattern in which a high proportion of renewable energy and a high proportion of power electronic equipment coexist[1]-[3]. Power electronic equipment has advantages such as fast response and flexible control, but it lacks the inherent rotational inertia of synchronous generators. Its terminal voltage and frequency support capability depends heavily on control strategies, making the system more prone to problems such as voltage oscillation and loss of synchronism under external disturbances. Therefore, how to evaluate and enhance the ability to maintain voltage and frequency under disturbances has become a key issue for the safe operation of new-type power systems.

To characterize this capability, organizations such as international large power grid bodies have proposed the concept of system strength[12]. In engineering, the short-circuit ratio and its extended indices are commonly used for evaluation, including the short-circuit ratio (Short-Circuit Ratio, SCR), multi-infeed short-circuit ratio (multi-infeed SCR, MSCR), multiple renewable energy station short-circuit ratio (multiple renewable energy station SCR, MRSCR), and generalized short-circuit ratio (Generalized SCR, gSCR)[15], among others. These indices are based on AC network impedance, short-circuit capacity, or normalized network admittance matrices, and can reflect the electrical distance between the grid-connection point and the equivalent center of the AC grid. They are widely used in weak-grid identification, assessment of new energy hosting capacity, and planning and operation, but they have difficulty fully reflecting equipment control dynamics and their active support effect on system strength.

To improve system strength, grid-forming (Grid-Forming, GFM) converters have received extensive attention[4]-[6]. In recent years, domestic and international standards, technical specifications, and related studies have gradually regarded "autonomously establishing and maintaining the amplitude and frequency of the internal potential, and operating with voltage-source characteristics" as the core function of grid-forming control[21],[22]. Existing standards and technical specifications mainly define grid-forming equipment from the perspective of functional requirements, and still lack unified quantitative indices for comparing the grid-forming capability of different devices and their system support effects.

Existing studies mainly evaluate grid-forming capability from two aspects: equipment port characteristics and system support effect. Reference [7] quantifies the voltage-source characteristics of equipment based on the singular values of the impedance matrix and proposes the concepts of "one-dimensional voltage source" and "two-dimensional voltage source," but it is mainly oriented toward a single device and has difficulty reflecting its overall influence after being connected to a multi-node system. Reference [8] defines frequency-segment voltage and phase-angle support capability from the perspective of system strength and quantifies them using closed-loop sensitivity singular values, but it depends on global information such as network topology, operating state, and other equipment dynamics. Reference [9] evaluates the voltage-source characteristics of equipment and nodes by comparing dynamic similarity with an ideal voltage-source series impedance model, but the results are affected by the selection of the reference model. Reference [10] evaluates voltage stiffness and passivity based on black-box impedance scanning, but still focuses on equipment port performance. Overall, existing methods still have difficulty balancing the accuracy of system-level assessment with the implementability of equipment-level testing, and there remains a lack of a clear quantitative relationship between port characteristics and the improvement of system strength.

Therefore, this paper proposes a distributed assessment method that uses only converter port dynamic characteristics to analyze its grid-forming capability. First, [[unclear: text continues on next page]] embedded in the multi-infeed system small-disturbance closed-loop strength model, revealing the mechanism by which grid-forming equipment improves system strength; next, grid-forming index FI and voltage-source index VI are proposed, and their relationships with grid strength and system strength are established; finally, a calculation procedure based on port-frequency scanning is given, and the proposed method is verified using the IEEE 39-bus system.

1 Prior Knowledge and Problem Description

1.1 Prior Knowledge

1) Matrix D-norm

Let D be a symmetric positive-definite matrix. For any nonsingular complex matrix $A \in \mathbb{C}^{n\times n}$, the induced matrix D-norm is defined as follows:

$$\begin{aligned}\|A\|_D &= \left\|D^{\frac{1}{2}} A D^{-\frac{1}{2}}\right\|_2 \\ &= \sigma_{\max}\left(D^{\frac{1}{2}} A D^{-\frac{1}{2}}\right) = 1/\sigma_{\min}\left(D^{\frac{1}{2}} A^{-1} D^{-\frac{1}{2}}\right)\end{aligned} \tag{1}$$

where $\sigma_{\max}(\cdot)$ and $\sigma_{\min}(\cdot)$ denote the maximum singular value and minimum singular value of the matrix to be evaluated, respectively. Correspondingly, the induced matrix D-norm of the transfer-function matrix $G(s)$ at frequency ω is defined as:

$$\|G(j\omega)\|_D = \left\|D^{\frac{1}{2}} G(j\omega) D^{-\frac{1}{2}}\right\|_2 \tag{2}$$

Therefore, $\|G(j\omega)\|_D$ characterizes the maximum gain of the transfer-function matrix at frequency ω under the weighting matrix D.

2) Passivity and rotational passivity

For a transfer-function matrix $G(s)$ with no unstable poles, its passivity index at frequency ω is defined as:

$$\rho\left(G(j\omega)\right) = \lambda_{\min}\left(\mathrm{He}\left(G(j\omega)\right)\right) \tag{3}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue to be evaluated, and $\mathrm{He}(\cdot)$ denotes the Hermitian part of the matrix to be evaluated, i.e., $\mathrm{He}(A) = 0.5 \cdot (A + A^H)$, with superscript H denoting the conjugate transpose. When $\rho(G(j\omega)) \geq 0$ holds for all frequencies ω, $G(s)$ is said to satisfy passivity.

For an open-loop stable transfer-function matrix $G(s)$, its rotational passivity index at frequency ω is defined as:

Schematic diagram of a renewable-energy-dominated multi-infeed system.

Figure 1: Schematic diagram of a renewable-energy-dominated multi-infeed system.

$$\rho_F\left(G(j\omega)\right) = \lambda_{\min}\left(\mathrm{He}\left(F^{-1}(j\omega)G(j\omega)\right)\right) \qquad (4)$$

where $F(s)$ is the admittance dynamics of a unit-inductance branch in the global xy coordinate system, and its detailed expression is given in Eq. (7). When $\rho_F(G(j\omega)) \geq 0$ holds for all frequencies ω, $G(s)$ is said to satisfy rotational passivity.

Equation (4) introduces a multiplier into the conventional definition of passivity, and uses the positive-realness of the transformed transfer-function matrix to characterize the dynamic properties of $G(s)$. Existing studies usually choose a constant rotation matrix as the multiplier, but its applicability is limited by frequency and line parameters. In this paper, the inverse dynamics $F^{-1}(s)$ of a unit-inductance branch is selected as a frequency-dependent multiplier. At low frequencies it is mainly manifested as a 90° cross rotation between the xy axes, and at high frequencies it gradually transitions to $j\omega I_2$. This multiplier can continuously extract common inductive dynamics with frequency, so that the rotational passivity index can measure the inductive characteristics of $G(s)$ at different frequencies [13], [14].

4) Basic inequalities for singular values and passivity indices

Let complex matrices $A, B \in \mathbb{C}^{n\times n}$. The following inequalities hold [20]:

1) $\sigma_{\min}(A) \geq \rho(A)$;
2) $\rho(A+B) \geq \rho(A) + \rho(B)$.

1.2 Problem Description

图 1 新能源多馈入系统示意图

Fig. 1. Schematic diagram of a renewable-energy-dominated multi-infeed system.

Consider a power system with multiple renewable-energy sources and power-electronic equipment infeed, as shown in Fig. 1 (hereinafter referred to as a multi-infeed system). This paper focuses on the small-disturbance response of bus voltage near a given operating point under the joint action of equipment control and network dynamics, and uses system strength to describe the system's ability to suppress voltage deviation and maintain stability.

Definition 1 (System strength) [16]
System strength refers to the ability of a power system, under a given oper-

ating mode and network structure, to keep bus voltage and frequency (phase angle) stable and resist deviations; it reflects the gap between the actual voltage response and the ideal voltage source, as well as the system' s disturbance resistance. This ability can be quantitatively evaluated by the degree to which the post-disturbance deviation departs from the nominal value and approaches the stability boundary.

To emphasize the combined effects and influence of equipment and network broadband dynamic characteristics on system strength, the concept of system strength can be further extended to the concept of dynamic strength (abbreviated as "dynamic strength").

Definition 2 (Dynamic strength)
Dynamic strength concerns the disturbance-response process under the joint action of equipment control and network dynamics, and describes the ability of a system to maintain stability and limit deviations of voltage and frequency (phase angle) during dynamic evolution.

Furthermore, depending on whether a linearized analysis method is adopted, dynamic strength can also be divided into small-disturbance strength and large-disturbance strength. Unless otherwise specified, "system strength" hereinafter refers to small-disturbance strength. Transient large disturbances, black-start, and islanded operation, which involve nonlinear scenarios such as current limiting, saturation, protection actions, and control-mode switching, are not within the scope of this paper. The dynamic strength under small disturbances can be described by the magnitude of the broadband sensitivity matrix in the frequency domain.

When system strength is insufficient, grid-forming equipment can provide voltage and phase-angle dynamic support, reduce bus-voltage deviations caused by disturbances, and improve the system stability margin. Therefore, grid-forming equipment can be regarded as a dynamic-strength source of the system. Let $\kappa_0(j\omega)$ and $\kappa(j\omega)$ denote the system strength at frequency ω before and after equipment access, respectively. If, after the equipment is connected, the system strength increases, i.e.,

$$\Delta\kappa(j\omega) = \kappa(j\omega) - \kappa_0(j\omega) > 0 \qquad (5)$$

then the equipment is considered to have grid-forming capability at frequency ω.

However, although the above criterion is well defined and has clear physical meaning, the analysis requires knowledge of the network topology, operating state, and other equipment dynamics; it relies on global information and is not conducive to evaluating the grid-forming capability of equipment.

Fig. 2. Closed-loop device-network interconnection model with the device under evaluation

Fig. 2. Closed-loop device-network interconnection model with the device under evaluation

Figure 2: Fig. 2. Closed-loop device-network interconnection model with the device under evaluation

Therefore, this paper incorporates the port dynamics of the equipment under evaluation into the equivalent network, as shown in Fig. 2, and transforms the evaluation of equipment grid-forming capability into an analysis of the influence of its dynamics on the strength of the equivalent network. This further establishes the relationship between equipment port characteristics and the improvement of system strength. This leads to the problem to be solved in this paper: **"How to construct a grid-forming capability evaluation index with a clear relationship to system dynamic strength based on equipment port information."**

2 Characterization and Quantification of Multi-Infeed System Strength

2.1 Dynamic Strength Analysis Model for Multi-Infeed Systems

In the global xy coordinate system, the linearized closed-loop model of the system consists of the device-side model and the network-side model, described as follows:

1) Device-side model. The converter-side linearized model is

$$\Delta I_g = -(S_{\mathrm{B}} \otimes I_2) Y_{\mathrm{VSC}}^{N}(s) \Delta U_g \qquad (6)$$

where $S_{\mathrm{B}} = \mathrm{diag}\{S_1, \cdots, S_n\}$ is a diagonal matrix composed of the ratios of the rated capacities of the converters to the global base capacity; $Y_{\mathrm{VSC}}^{N}(s) = \mathrm{diag}\{Y_{\mathrm{VSC},1}(s), \cdots, Y_{\mathrm{VSC},n}(s)\}$; $\Delta I_g = [\Delta I_{xy,1}, \cdots, \Delta I_{xy,n}]^{\mathrm{T}}$ is the disturbance vector of converter port currents in the global synchronous xy coordinate system, $\Delta I_{xy,i} = [\Delta I_{x,i} \ \ \Delta I_{y,i}]^{\mathrm{T}}$ is the injected current disturbance vector at node i in the global coordinate system; $\Delta U_g = [\Delta U_{xy,1}, \cdots, \Delta U_{xy,n}]^{\mathrm{T}}$ is the disturbance vector of converter port voltages in the global xy coordinate system, and $\Delta U_{xy,i} = [\Delta U_{x,i} \ \ \Delta U_{y,i}]^{\mathrm{T}}$ is the voltage disturbance vector at node i in the global xy coordinate system.

2) Network-side model. According to Ref. [19], when network resistance is neglected, the network-side dynamic model can be expressed as

$$\Delta I_g = B_{\mathrm{red}} \otimes F(s) \Delta U_g = Y_{\mathrm{Grid}}(s) \Delta U_g$$

$$F(s) = \begin{bmatrix} s & -\omega_0 \\ \omega_0 & s \end{bmatrix}^{-1} \tag{7}$$

where $F(s)$ is the admittance dynamics of a unit inductor in the global xy coordinate system, ω_0 is the rated angular frequency of the AC grid; B_{red} is the network admittance matrix that retains only the converter port nodes after eliminating intermediate nodes.

Combining the dynamic models of the equipment and the grid, the system strength reflects the response performance of node voltages after disturbances, and can be characterized by the maximum displacement of the voltage vector under a unit current disturbance. Considering a current input disturbance proportional to capacity, $(S_{\text{B}} \otimes I_2)\Delta I_r(s)$, and combining (6) and (7), the current disturbance vector to output voltage $\Delta U_g(s)$ satisfies the following relationship:

$$\Delta U_g(s) = \underbrace{\left[S_{\text{B}}^{-1} B_{\text{red}} \otimes F(s) + Y_{\text{VSC}}^{N}(s)\right]^{-1}}_{T(s)} \Delta I_r(s) \tag{8}$$

where $T(s)$ is the broadband sensitivity transfer-function matrix from current disturbance to voltage response.

2.2 Characterization of System Strength and Grid Strength

At $s = j\omega$, the system strength of the closed-loop system shown in Fig. 1 can be quantified by the reciprocal of the norm of the closed-loop transfer function. This paper adopts the reciprocal $\|T(j\omega)\|_D^{-1}$ of the norm of the closed-loop sensitivity matrix D to quantify system strength (where $D = S_B \otimes I_2$), expressed as

$$\kappa(j\omega) := \sigma_{\min}\left[Y_{\text{VSC}}^{N}(j\omega) + S_{\text{B}}^{-1/2} B_{\text{red}} S_{\text{B}}^{-1/2} \otimes F(j\omega)\right] \tag{9}$$

The larger this index is, the smaller the maximum bus-voltage displacement caused by the same current disturbance, and the stronger the system disturbance-rejection capability and system strength.

Note: Near the rated voltage, the closed-loop admittance matrix and the power–voltage Jacobian matrix can be transformed into each other through a similarity transformation. Therefore, (9) can also characterize strength analysis in the power–voltage angle coordinates. Due to space limitations, this is not discussed further here; details can be found in Ref. [8].

If the dynamics of grid-connected equipment are ignored and only the support effect of the network side on bus voltages is retained, then this support capability is defined as the grid strength index, expressed as

$$\alpha(j\omega) := \sigma_{\min}\left[S_{\mathrm{B}}^{-1/2} B_{\mathrm{red}} S_{\mathrm{B}}^{-1/2} \otimes F(s)\right] \tag{10}$$

The indices shown in (9) and (10) have clear input–output physical meanings, but both retain the common line dynamics $F(s)$. To eliminate the nonuniform scaling of strength indices at different frequencies caused by common line dynamics, define the transformed device admittance and transformed grid admittance as

$$\tilde{Y}_{\mathrm{VSC}}^{N}(s) = (I_n \otimes F^{-1}(s)) Y_{\mathrm{VSC}}^{N}(s)$$

$$\tilde{Y}_{\mathrm{Grid}}(s) = (S_{\mathrm{B}}^{1/2} \otimes F^{-1}(s)) Y_{\mathrm{Grid}}(s)(S_{\mathrm{B}}^{-1/2} \otimes I_2) \tag{11}$$

where the transformed admittance matrix $\tilde{Y}_{\mathrm{VSC}}^{N}(s) = \mathrm{diag}\{\tilde{Y}_{\mathrm{VSC},i}^{N}(s)\}$, and $\tilde{Y}_{\mathrm{VSC},i}(s) = F^{-1}(s) Y_{\mathrm{VSC},i}(s)$. For an ideal AC grid composed only of inductive lines, after extracting the common line dynamics, the transformed grid admittance degenerates into a constant matrix related only to the network topology, line parameters, and capacity distribution. Therefore, the grid strength index is independent of frequency and is equal to the generalized short-circuit ratio [15].

Correspondingly, the transformed system strength index and grid strength index after extracting the common line dynamics are defined as

$$\kappa_{\mathrm{F}}(j\omega) := \sigma_{\min}\left[\tilde{Y}_{\mathrm{VSC}}^{N}(j\omega) + S_{\mathrm{B}}^{-1/2} B_{\mathrm{red}} S_{\mathrm{B}}^{-1/2} \otimes I_2\right]$$

$$\alpha_{\mathrm{F}}(j\omega) := \sigma_{\min}\left[S_{\mathrm{B}}^{-1/2} B_{\mathrm{red}} S_{\mathrm{B}}^{-1/2} \otimes I_2\right] = gSCR \tag{12}$$

where the subscript F denotes the system strength and grid strength after extracting the common line dynamics. The transformed system strength after extracting the common line dynamics is consistent and positively correlated with the original system strength, satisfying

$$\kappa(j\omega) \cdot \sigma_{\min}(F(j\omega)) \leq \kappa_{\mathrm{F}}(j\omega) \leq \kappa(j\omega) \cdot \sigma_{\max}(F(j\omega)),$$

The grid-strength index also satisfies the above relationship.

For a power network containing complex loads, SVGs, or other complex equipment, after the corresponding ports are reduced to the network side, $\tilde{Y}_{\mathrm{Grid}}(s)$ is no longer expressed as a constant matrix. Therefore, $\alpha_{\mathrm{F}}(j\omega)$ constitutes a frequency-dependent dynamic grid strength, which can be regarded as an extension of the traditional gSCR in non-ideal grids. According to the evaluation objective, the following complementary indices can be adopted to define grid strength:

$$\begin{aligned}\alpha_{\mathrm{F}}^{\sigma}(j\omega) &:= \sigma_{\min}(\tilde{Y}_{\mathrm{Grid}}(j\omega)) \\ \alpha_{\mathrm{F}}^{p}(j\omega) &:= \rho(\tilde{Y}_{\mathrm{Grid}}(j\omega))\end{aligned} \tag{13}$$

where $\alpha_{\mathrm{F}}^{\sigma}(j\omega)$ characterizes the voltage support capability of the grid under the most unfavorable disturbance, while $\alpha_{\mathrm{F}}^{p}(j\omega)$ characterizes the dissipative property of the grid and its support capability for equipment. Unless otherwise specified, the $\alpha_{\mathrm{F}}(j\omega)$ used below is the grid-strength index defined based on the minimum singular value.

In addition, according to the singular-value triangle inequality, the mathematical relationship between system strength and grid strength can be described by (14). The lower bound of system strength is jointly determined by grid strength and the characteristics of grid-connected equipment. By increasing $\alpha_{\mathrm{F}}(j\omega)$, the lower bound of system strength can be improved, thereby ensuring that the system has sufficient strength margin.

$$\kappa_{\mathrm{F}}(j\omega) \geq \alpha_{\mathrm{F}}(j\omega) - \sigma_{\max}(\tilde{Y}_{\mathrm{VSC}}^{N}(j\omega)) \tag{14}$$

It should be particularly noted that, for an ideal network containing only inductive lines, by extracting the common line dynamics $F(s)$, the frequency-dependent factors caused by line inductances are separated from the network topology characteristics. This processing can simplify the analysis and calculation of grid strength. For system strength, this processing can eliminate the uniform scale variation caused by line common dynamics, so that its frequency characteristics mainly reflect the dynamics of grid-connected equipment and their coupling with the network.

2.3 Analysis of the Influence of an Attached Converter on System Strength

Consider connecting a device to be evaluated to a certain bus of the system. Let the port admittance matrix of this device be $Y_{\mathrm{de}}(s)$. Then, after the device is connected, the system closed-loop admittance matrix can be expressed as:

$$\begin{bmatrix} Y_{\mathrm{VSC}}^{N}(s) & 0 \\ 0 & Y_{\mathrm{de}}(s) \end{bmatrix} + B'_{\mathrm{red}} \otimes F(s) \tag{15}$$

where $B'_{\mathrm{red}} \in \mathbb{R}^{(n+1)\times(n+1)}$ is the extended admittance matrix after the attached converter is connected, and can be expressed as:

$$B'_{\mathrm{red}} = \begin{bmatrix} B_1 \in \mathbb{R}^{n\times n} & B_2 \in \mathbb{R}^{n\times 1} \\ B_3 \in \mathbb{R}^{1\times n} & B_4 \in \mathbb{R}^{1\times 1} \end{bmatrix} \tag{16}$$

$$S_{\mathrm{B}}^{-1/2} B_{\mathrm{red}} S_{\mathrm{B}}^{-1/2} = B_1 - B_2 B_4^{-1} B_3$$

To explicitly characterize the dynamic support effect of the device to be evaluated on other grid-following equipment, the port variables of this device are eliminated using Schur complementation. The dynamic contribution of the device to be evaluated is strictly equivalently transferred to the network side, thereby obtaining an equivalent network model that accounts for its support effect. The partitioning method is shown in Fig. 2. At this time, the closed-loop model of the system can be rewritten as:

$$Y_{\mathrm{VSC}}^{N}(s) + Y_{\mathrm{GridC}}(s) \qquad (17)$$

where the equivalent network dynamics $Y_{\mathrm{GridC}}(s)$ is expressed as:

$$Y_{\mathrm{GridC}}(s) = (I_n \otimes F(s)) \left(B_1 \otimes I_2 - \frac{B_2 B_3}{B_4} \otimes S_{\mathrm{de}}(s) \right) \qquad (18)$$

In the equation, $S_{\mathrm{de}}(s)$ is the sensitivity function of the attached converter, whose detailed expression is

$$S_{\mathrm{de}}(s) = \left(I_2 + L_g F^{-1}(s) Y_{\mathrm{de}}(s) \right)^{-1} \qquad (19)$$

where $L_g = B_4^{-1}$ is the connection inductance.

According to (12), after the attached converter is connected, the system strength and grid-strength index can be respectively expressed as:

$$\begin{aligned} \kappa_{\mathrm{F,de}}(j\omega) &:= \sigma_{\min}(\tilde{Y}_{\mathrm{GridC}}(j\omega) + \tilde{Y}_{\mathrm{VSC}}^{N}(j\omega)) \\ \alpha_{\sigma,\mathrm{de}}(j\omega) &:= \sigma_{\min}(\tilde{Y}_{\mathrm{GridC}}(j\omega)) \end{aligned} \qquad (20)$$

Comparing (12) and (20) shows that, before and after the connection of $Y_{\mathrm{de}}(s)$, the change in the closed-loop admittance matrix is embodied in the correction of the network-side equivalent dynamics. It can thus be seen that the dynamic support of grid-forming equipment is manifested as reshaping the network-side equivalent dynamics, changing the grid strength and the lower bound of system strength within the frequency band of concern. Specifically, after the attached converter $Y_{\mathrm{de}}(s)$ is connected, the lower bound of system strength can be expressed as:

$$\kappa_{\mathrm{F,de}}(j\omega) \geq \alpha_{\mathrm{F,de}}(j\omega) - \sigma_{\max}(\tilde{Y}_{\mathrm{VSC}}^{N}(j\omega)) \qquad (21)$$

That is, the attached converter $Y_{\mathrm{de}}(s)$ raises the lower bound of system strength by improving grid strength; conversely, if it weakens grid strength, the strength margin of the system will be reduced.

3 Quantification Method for Converter Grid-Forming Capability

3.1 Definition of the Grid-Forming Index and Voltage Source Index

Definition 3 (Grid-Forming Index and Voltage Source Index) For a converter whose port admittance matrix is $Y_{\mathrm{de}}(s)$, define its grid-forming index (Grid-Forming Index, FI) and voltage source index (Voltage source index, VI) at frequency $s = j\omega$ as respectively:

$$\begin{aligned} \mathrm{FI}(j\omega) &= \sigma_{\max}\left[\left(I_2 + L_g F^{-1}(j\omega) Y_{\mathrm{de}}(j\omega)\right)^{-1}\right] \\ \mathrm{VI}(j\omega) &= -20\log_{10}(\mathrm{FI}(j\omega)) \end{aligned} \tag{22}$$

When $\mathrm{FI}(j\omega) < 1$, $\mathrm{VI}(j\omega) > 0$; at this time, the converter is said to have voltage-source characteristics at $s = j\omega$.

To uniformly compare the grid-forming capability of different devices, this paper fixes the external grid condition and takes $L_g = 1.0$ p.u. as the standard connection inductance. The value of L_g affects the specific numerical value of FI, but does not change the physical meaning of its port disturbance-response; under the same L_g condition, the FI of different devices has a unified evaluation criterion and can be used to compare their grid-forming support capability.

FI characterizes the maximum response of the port voltage in the most unfavorable disturbance direction at a given frequency. An ideal current source corresponds to $\mathrm{FI}(j\omega) = 1$. When $\mathrm{FI}(j\omega) < 1$, disturbances in any direction are attenuated; $\mathrm{FI}(j\omega) > 1$ indicates that at this frequency there exists a disturbance direction that is amplified. This does not mean that the device completely lacks support capability, but it means that at least one direction lacks support capability. The detailed derivation is given in Appendix D.

From the preceding section, the influence of converter connection on system strength can be analyzed through its corrective effect on the network-side equivalent dynamics. According to

According to (18), at $s = j\omega$, the grid strength is

$$\alpha_{\mathrm{F,de}}(j\omega) = \sigma_{\min}\left(B_1 \otimes I_2 - \frac{B_2 B_3}{B_4} \otimes S_{\mathrm{de}}(j\omega)\right) \tag{23}$$

In particular, when $FI(j\omega) < 1$, the grid strength satisfies (proof in Appendix A):

$$\begin{aligned}\alpha_{\mathrm{F,de}}(j\omega) &\geq \sigma_{\min}\left(B_1 - \frac{B_2 B_3}{B_4} FI(j\omega)\right) \\ &> \sigma_{\min}\left(B_1 - \frac{B_2 B_3}{B_4}\right) = \mathrm{gSCR}\end{aligned} \tag{24}$$

It can be seen from (24) that, when the additional converter satisfies $FI(j\omega) < 1$ at the frequency point $s = j\omega$, the converter can improve the equivalent grid strength at that frequency point and raise the lower bound of the closed-loop system strength when the dynamics of grid-following renewable energy sources are considered. The smaller $FI(j\omega)$ is, the stronger the effect of the converter; when $FI(j\omega) = 0$, the grid strength is $\sigma_{\min}(B_1)$, which is equivalent to connecting an infinite bus at node $n+1$.

Both FI and VI are used to characterize the voltage-source characteristics of equipment. The smaller FI is (the larger VI is), the closer the equipment is to an ideal voltage source. When the grid-following renewable-energy equipment being fed in is approximately homogeneous and the additional equipment satisfies the condition $FI(j\omega) < 1$, its access can improve the grid strength and the closed-loop system strength; the proof is given in Appendix C. FI is defined based on the frequency-domain sensitivity of the equipment terminal voltage to disturbances from the external grid and does not depend on the specific equipment type. Therefore, it can also be used to evaluate the dynamic support capability of voltage-source equipment such as synchronous generators and synchronous condensers.

3.2 Grid-forming index and rotational passivity

To further reveal the characteristics of the grid-forming index, this subsection introduces rotational passivity to quantify the dynamic characteristics of the equipment port. Define the rotational passivity index of the additional equipment as (rotation matrix $R(s) = F^{-1}(s)$):

$$\rho_{\mathrm{F,de}}(j\omega) = \lambda_{\min}\left(\mathrm{He}(\tilde{Y}_{\mathrm{de}}(j\omega))\right) \tag{25}$$

where $\tilde{Y}_{\mathrm{de}}(j\omega) = F^{-1}(j\omega) Y_{\mathrm{de}}(j\omega)$ is the transformed admittance matrix of the additional converter.

Corollary 1: At a given frequency $s = j\omega$, a sufficient condition for $FI(j\omega) < 1$ is $\rho_{\mathrm{F,de}}(j\omega) > 0$.

Proof: Since $FI(j\omega) < 1$ is equivalent to $\sigma_{\min}(S_{\mathrm{de}}^{-1}(j\omega)) > 1$, i.e., $\sigma_{\min}(I_2 + L_g \tilde{Y}_{\mathrm{de}}(j\omega)) > 1$. Therefore,

$$\begin{aligned}\sigma_{\min}(I_2 + L_g \tilde{Y}_{\mathrm{de}}(j\omega)) &\geq \rho(I_2 + L_g \tilde{Y}_{\mathrm{de}}(j\omega)) \\ &= 1 + L_g \rho(\tilde{Y}_{\mathrm{de}}(j\omega))\end{aligned} \tag{26}$$

Schematic diagram of the physical meaning of the grid-forming index: an additional converter with $Y_{\mathrm{de}}(s)$, filter inductance L_f, capacitor C_f, grid inductance L_g, PCC voltage disturbance ΔU_{pcc}, grid disturbance ΔU_g, disturbance transmission, and grid.

Figure 3: Schematic diagram of the physical meaning of the grid-forming index: an additional converter with $Y_{\mathrm{de}}(s)$, filter inductance L_f, capacitor C_f, grid inductance L_g, PCC voltage disturbance ΔU_{pcc}, grid disturbance ΔU_g, disturbance transmission, and grid.

According to (26), when $\rho(\tilde{Y}_{\mathrm{de}}(j\omega)) > 0$, $FI(j\omega) < 1$ must be satisfied.

From the perspective of the equipment port, FI characterizes the maximum gain by which external grid voltage disturbances are transmitted to the equipment terminal through the standard connecting inductance L_g. Its magnitude reflects the closeness between the equipment port impedance and the ideal voltage-source series-inductance model. The smaller FI is, the weaker the influence of external disturbances on the equipment terminal voltage, indicating a stronger grid-forming support capability. Its physical meaning is shown in Fig. 3; the detailed derivation is given in Appendix C. In addition, rotational passivity represents the voltage-source external characteristics of the equipment by extracting the common-inductance dynamics and using the positive realness of the transformed admittance. Compared with the traditional passivity that mainly reflects energy dissipation capability, rotational passivity is more suitable for evaluating the grid-forming support capability of equipment in the medium- and low-frequency bands, and constitutes a sufficient condition for $FI(j\omega) < 1$.

Fig. 3. Port-based physical interpretation of the grid-forming index.

3.3 Calculation of the grid-forming index

From a physical perspective, grid-forming capability can be characterized by the input-output relationship between the external-grid voltage disturbance and the equipment terminal-voltage response, reflecting the ability of the equipment to maintain its terminal voltage under disturbances. Under the small-disturbance conditions studied in this paper, this relationship can be described by the linearized transfer matrix $S_{\mathrm{de}}(j\omega)$, and $FI(j\omega)$ is defined by its maximum singular value. By applying voltage disturbances with different directions and frequencies and measuring the terminal response, the corresponding input-output characteristics can be obtained. This framework is not limited to linearized models; subsequently, a nonlinear gain index can be further constructed based on bounded-amplitude voltage disturbances and their time-domain responses, so as to evaluate grid-forming capability under large-disturbance conditions.

If the converter control structure and control parameters are fully known, $Y_{\mathrm{de}}(s)$ can be derived directly through white-box modeling. In practical engineering, internal information such as the equipment control structure, control parame-

Fig. 4 flowchart: Start → Take the connected inductance $L_g = 1.0$ p.u. → Based on the black-box model of the system, obtain the admittance matrix $Y_{de}(s)$ of the attached converter by frequency scanning → Calculate the sensitivity function of the device according to Eq. (19) → Calculate the device's grid-forming index $\mathrm{FI}(j\omega)$ within the frequency band of interest → End.

Figure 4: Fig. 4 flowchart: Start → Take the connected inductance $L_g = 1.0$ p.u. → Based on the black-box model of the system, obtain the admittance matrix $Y_{de}(s)$ of the attached converter by frequency scanning → Calculate the sensitivity function of the device according to Eq. (19) → Calculate the device's grid-forming index $\mathrm{FI}(j\omega)$ within the frequency band of interest → End.

ters, and filter design is usually difficult to obtain completely. The frequency-sweep/impedance measurement method [24], [25] can be adopted: $Y_{\mathrm{de}}(j\omega)$ is identified through small-disturbance injection at the port and the voltage and current responses, and then the grid-forming capability evaluation is completed based on (19).

(1) Take the connecting inductance $L_g = 1.0$ p.u. as the access condition;

(2) Use port frequency-sweep/impedance measurement to obtain the admittance matrix $Y_{\mathrm{de}}(s)$ of the additional converter;

(3) Calculate the sensitivity function $S_{\mathrm{de}}(s)$ of the additional equipment according to (19);

(4) Calculate the grid-forming index of the additional converter within the key frequency band ($0.1 \sim 100$ Hz).

Note: The key frequency band refers to the medium- and low-frequency range that affects system-level broadband modes, usually involving synchronization stability and voltage stability [8].

图 4 构网指数测量流程

Fig. 4. Measurement procedure for the FI Index.

3.4 Relationship between the grid-forming index and system strength

Both the system strength index and the grid-forming index are broadband sensitivity indices of system disturbance response; the two describe voltage disturbance responses at the system level and the device level, respectively. System strength is defined as the reciprocal of the peak value of the closed-loop sensitivity from current disturbance to multi-bus voltage response; the grid-forming index characterizes the maximum response after an external grid voltage disturbance is transmitted to the device terminal, reflecting the capability of a single device to maintain the stability of its terminal voltage. Therefore, system strength can be regarded as a system-level "grid-forming capability" index $\kappa_{\mathrm{F,de}}(j\omega)$, while $\mathrm{FI}(j\omega) < 1$ can be regarded as a device-level index. When

Fig. 5 block diagrams: PQ-PLL and PV-PLL control loops; VSG control loops.

Figure 5: Fig. 5 block diagrams: PQ-PLL and PV-PLL control loops; VSG control loops.

$1/\kappa_{\mathrm{F,de}}(j\omega) \rightarrow 0$, the overall system exhibits relatively ideal voltage-source characteristics; and when $\mathrm{FI}(j\omega) \rightarrow 0$, the external characteristics of the device exhibit ideal voltage-source characteristics.

The device-level grid-forming index reflects the ability of a single unit to suppress voltage disturbances at its terminal, whereas system strength is jointly determined by the broadband coupling dynamics between the equipment and the AC network. Stronger device voltage-source characteristics are usually beneficial for improving the voltage support capability at the point of interconnection, but their actual effect on system strength also depends on various structural information such as network topology, access location, and operating point, and cannot be obtained by simple superposition of the grid-forming indices of individual devices.

4 Converter capability evaluation and optimization based on the grid-forming index

The grid-forming index quantifies the grid-forming capability of converters from the perspective of system strength. This index does not depend on a specific control structure, and can compare the terminal voltage support capabilities of devices of different types and parameter configurations under unified test conditions. More importantly, FI can be obtained by terminal impedance measurement, and therefore can serve purposes such as equipment development, type testing, control-parameter tuning, and grid-integration strength assessment.

4.1 Grid-forming indices and parameter influence under typical control strategies

This section uses the grid-forming index to compare the grid-forming capabilities of common control structures, including PQ-PLL, which uses active/reactive power tracking grid-connected control, PV-PLL, which uses active power/AC-voltage tracking grid-connected control, and a virtual synchronous generator (VSG) adopting dual closed-loop vector control. All converters adopt a current inner loop, an outer loop, and a phase-locked loop structure, as shown in Fig. 4.

图 5 不同类型变流器的控制框图

Fig. 5. Block diagrams of different converter control strategies.

(a) Grid-forming index of PQ-PLL

Fig. 6 plots of grid-forming indices under different converter controls: (a) PQ-PLL; (b) PV-PLL; (c) VSG.

Figure 6: Fig. 6 plots of grid-forming indices under different converter controls: (a) PQ-PLL; (b) PV-PLL; (c) VSG.

Fig. 7

Figure 7: Fig. 7

(b) Grid-forming index of PV-PLL

(c) Grid-forming index of VSG

图 6 不同变流器的构网指数

Fig. 6. Grid-forming indices under different converter control strategies.

Under different control parameters and connected inductances, the grid-forming indices of the converters are shown in Fig. 6. Fig. 6(a) and (b) give the grid-forming indices of PQ-PLL and PV-PLL under different phase-locked-loop bandwidths and different connected inductances L_g.

index. The results show that, in some frequency bands, the two types of converters satisfy the sufficient condition $FI(j\omega) < 1$; therefore, they may amplify external grid disturbances in the corresponding frequency bands.

Figure 6(c) gives the grid-forming index of the VSG under different virtual damping coefficients D, virtual inertia coefficients J, current feedforward coefficients K_F, and different connection inductances L_g. It can be seen that, with reasonable parameters, the VSG can maintain $FI(j\omega) < 1$ from several Hz to several hundred Hz. However, when the virtual damping coefficient of the VSG is relatively small and the virtual inertia coefficient is relatively large, its grid-forming index $FI(j\omega)$ exhibits a relatively high peak in the low-frequency band. At this time, the VSG may amplify disturbances from the external grid, indicating that its ability to suppress terminal-voltage disturbances is weakened. The influence of the current feedforward coefficient on the grid-forming index appears in the mid-frequency band: the larger the current feedforward coefficient, the stronger the voltage-source characteristic of the VSG.

图 7 同步发电机的构网指数

Fig. 7. Grid-forming index of the synchronous generator under different PSS gains.

Figure 7 gives the Bode plot of the grid-forming index of the synchronous generator under different power system stabilizer (Power System Stabilizer, PSS) gains K_{PSS}. As K_{PSS} increases from 15 to 30, the peak value of FI in the low-frequency band gradually decreases, indicating that the PSS weakens the response of the

Fig. 8

Figure 8: Fig. 8

Fig. 9

Figure 9: Fig. 9

terminal voltage to external disturbances by improving low-frequency modal damping. In the PLL-dominated sub/super-synchronous small-disturbance frequency band of interest in this paper, the FI of the synchronous generator is basically maintained below 0 dB, that is, $FI(j\omega) < 1$, indicating that the synchronous generator has relatively strong voltage-support capability in this frequency band.

4.2 Control Optimization Method for Grid-Following Converters Based on the Grid-Forming Index

Traditional PLL-type grid-following converters usually exhibit current-source external characteristics and are difficult to provide effective voltage support in weak grids. According to the definition of FI, if the terminal admittance is changed through control design so that $FI(j\omega) < 1$ in the frequency band of interest, then the PLL-type converter can also exhibit grid-forming capability in the small-disturbance sense. This paper designs an improved grid-following converter, called a phase-locked-loop-based grid-forming converter (PLL-GFM). The idea is to superimpose a virtual admittance branch outside the original grid-following control, so that the terminal is equivalent to the parallel connection of a traditional current source and a voltage-source support branch; the effect of this shaping is equivalent to connecting a virtual grounded inductance in parallel at the terminal:

$$Y^*_{\mathrm{GFL}}(s) = Y_{\mathrm{GFL}}(s) + kF(s) \qquad (27)$$

where k is a real number satisfying:

$$k \geq -\min_{\omega\in[0,\infty)} \rho_{F,\mathrm{de}}(j\omega) \qquad (28)$$

图 8 基于锁相环的构网型变流器设计

Fig. 8. Admittance-shaping control structure of the PLL-based converter.

图 9 PLL-GFM 的构网指数

Fig. 9. Grid-forming index of PLL-GFM.

Fig. 10 IEEE 39-bus system

Figure 10: Fig. 10 IEEE 39-bus system

The admittance shaping shown in (27) can be equivalently regarded as connecting a grounded inductor with an inductance of $1/k$ in parallel at the converter terminal. As shown in Fig. 8, this paper introduces an admittance-shaping loop into the control system, takes the dq-axis voltage deviation as the input, and superimposes the output onto the current reference, thereby reshaping the terminal admittance and enhancing the external characteristics of the voltage source.

5 Simulation Case Analysis

In this section, the IEEE 39-bus interconnected system shown in Fig. 10 is built in MATLAB/Simulink to verify whether the proposed method for quantifying distributed grid-forming capability can characterize the influence of different device access on grid strength and system strength. Specifically, PV-PLL, VSG, PLL-GFM, and synchronous generators are connected separately at the same bus, and grid strength, system strength, and time-domain voltage responses are compared to test the consistency between FI and the actual dynamic support effect of the devices. Among them, bus 36 is connected to the external AC grid; buses 30–35 and 37–38 are fixedly connected to PQ-PLL converters with a capacity of 1.0 p.u., and their parameters are given in Appendix B; the PLL bandwidths of converters 1–8 are respectively 5, 10, 15, 20, 30, 35, 70, and 80 rad/s.

Figure 10 IEEE-39 bus system

Fig.10 IEEE 39-bus system

5.1 Frequency-Domain Verification of FI Based on Grid Strength and System Strength

Figure 11 gives the grid strength and system strength after different devices are connected at bus 39. As can be seen from Fig. 10(a), when no additional device is connected, the grid strength is mainly determined by the original AC network; after PV-PLL is connected, the overall improvement in grid strength is not obvious, and it decreases in some frequency bands. In comparison, after VSG or PLL-GFM is connected, the grid strength in the medium- and low-frequency range is significantly improved, indicating that devices with stronger terminal-voltage source characteristics can reshape the equivalent dynamics on the network side and enhance the voltage-support capability of the grid for other grid-connected devices. Figure 11(b) gives the closed-loop system strength after accounting for the dynamics of the grid-following converters in the original system. The system strength exhibits an obvious valley near 10 Hz, indicating

Fig. 10 Grid strength and system strength when different control of converter at bus 39

Figure 11: Fig. 10 Grid strength and system strength when different control of converter at bus 39

that this frequency band is jointly affected by the control dynamics of the grid-following converters and the network coupling, and is a relatively weak frequency band of the system. After VSG or PLL-GFM is connected, the system strength in this frequency band is improved; after PV-PLL is connected, the improvement is relatively limited, which is consistent with the result that it does not satisfy FI<1 in the corresponding frequency band. It should be noted that grid strength mainly reflects the contribution of the evaluated device, after being connected, to the equivalent dynamics on the network side, whereas system strength also includes the closed-loop dynamics of other grid-connected devices in the system, and also depends on the connection point and operating condition. Therefore, an increase in grid strength helps raise the lower bound of system strength, but the two are not necessarily completely equivalent.

Figure 11 System strength and grid strength of the IEEE39 bus when different converters are connected

Fig. 10. Grid strength and system strength when different control of converter at bus 39.

5.2 Verification of Dynamic Support Effect in the Time Domain

Time-domain simulation is performed on the IEEE 39-bus system. At $t = 0.2$ s, the grid-side inductance of converter 8 is stepped from 0.05 p.u. to 0.10 p.u., so as to simulate a sudden increase in line impedance and a decrease in grid strength, and the terminal-voltage amplitudes of buses under different scenarios are compared. In Fig. 12(a), no device is connected at bus 39, and equal-amplitude oscillations appear in the system voltage amplitudes, indicating that the system is in a critical stability state. After PV-PLL is connected, Fig. 12(b) shows that the voltage oscillation amplitudes at the buses increase, indicating that this device cannot suppress the disturbance and weakens the dynamic support capability. In comparison, after PLL-GFM, VSG, or a synchronous generator is connected at bus 39, the voltage oscillations at all buses are significantly attenuated and the voltage amplitudes converge to a new stable operating point, indicating that these devices can enhance system strength.

Further, combining Fig. 6, Fig. 7, and Fig. 9, it can be seen that under the test conditions and in the frequency band of concern given in this paper, VSG, PLL-GFM, and the synchronous generator have relatively low FI, and after they are connected, the bus voltage deviations decrease and the dynamic response is improved; PV-PLL does not satisfy FI<1 in the frequency band of concern, Voltage response plot

Figure 12: Voltage response plot

and the oscillations in the corresponding time-domain response are aggravated. The above results show that the frequency-domain FI evaluation and the time-domain dynamic support effect after device connection have generally consistent trends.

(a) Voltage response when no device is connected

(b) Voltage response of each node when PV-PLL is connected at node 39

(c) Voltage response of each node when PLL-GFM is connected at node 39

(d) Voltage response of each node when VSG is connected at node 39

(e) Voltage response of each node when a synchronous generator is connected at node 39

Fig. 12 Time-domain voltage response under connection of different devices

Fig. 11 The time domain with different devices

6 Conclusions and Prospects

From the perspective of system strength, this paper proposes an evaluation index for the grid-forming capability of converters and discusses its engineering applications. The main conclusions are as follows:

1) A small-disturbance closed-loop model of a renewable-energy multi-infeed system is established, revealing the mechanism by which devices improve grid strength and system strength by reshaping the equivalent dynamics on the network side. The study shows that evaluation of grid-forming capability should not be limited to device type or control structure, but should focus on the actual function of devices in suppressing external grid disturbances and maintaining terminal-voltage stability.

2) The grid-forming index FI and the voltage-source index VI are proposed to uniformly quantify the terminal voltage-source characteristics of different devices in each frequency band. When the device satisfies $\mathrm{FI}(j\omega) < 1$ in the frequency band of concern, it can suppress external grid disturbances at the corresponding frequency and, under the conditions given in this paper, raise the lower bounds of grid strength and system strength.

3) The grid-forming capability of devices under different control methods and parameters exhibits significant frequency differences. VSG, PLL-GFM, and synchronous generators have relatively strong terminal-voltage support capability under reasonable parameters; through control-parameter

optimization and lead compensation, traditional PLL-type converters can also obtain small-disturbance grid-forming support characteristics. System simulations show that the frequency-domain FI evaluation is generally consistent with the time-domain dynamic support effect after device connection.

The indices in this paper are based on a linearized model and have not yet considered nonlinear processes such as current limiting, saturation, and control-mode switching. Future work will study analysis of grid-forming capability under large disturbances based on terminal-voltage time-domain responses and under complex operating conditions, and further explore selection, siting, and sizing methods for grid-forming support equipment that combine FI with target grid strength.

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Port-Based Assessment of Converter Grid-Forming Capability:

Grid-Forming Index and Contribution to System Strength

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Abstract: High penetration of renewable generation weakens the disturbance withstand capability of power systems, creating an urgent need for quantifiable and testable grid-forming support from converter-based equipment. Existing standards and studies identify autonomous internal voltage and voltage-source terminal behavior as key features of grid-forming converters, but a unified metric that links device-level terminal behavior with system-strength contribution is still lacking. This paper proposes a small-signal evaluation method for converter grid-forming capability from a system-strength perspective. First, dynamic system strength is characterized by the norm of the sensitivity matrix from current disturbances to bus-voltage responses, which clarifies the mechanism by which grid-forming converters act as strength sources. Then, a Grid-Forming Index (FI) and a Voltage Source Index (VI) are defined to quantify voltage-source characteristics through the peak sensitivity from grid-voltage disturbances to converter terminal voltage. It is proved that a device satisfying FI $<$ 1 can improve grid strength and the lower bound of system strength. A practical calculation procedure based on terminal frequency-sweep/impedance measurement is further provided, and admittance shaping is used to show how FI can guide grid-forming-oriented design of PLL-based converters. Simulations on the IEEE 39-bus system verify the consistency between the proposed indices and the dynamic support effect of different devices.

Key words: grid-forming converter; system strength; grid-Forming Index; Voltage Source Index; terminal impedance measurement; admittance shaping

Appendix A

According to (18), the dynamic $\tilde{B}_{\text{net}}(s)$ of the equivalent grid can be written as the sum of the original network admittance matrix B_{red} without the grid-forming converter connected and the influence $B_{\text{GFM}}(s)$ of the grid-forming converter on the grid:

$$\tilde{B}_{\text{net}}(s) = \underbrace{\left(B_1 - \frac{B_2 B_3}{B_4}\right) \otimes I_2}_{B_{\text{red}}} + \underbrace{\frac{B_2 B_3}{B_4} \otimes (I_2 - S_{\text{de}}(s))}_{B_{\text{GFM}}(s)} \tag{A1}$$

Let $Q(j\omega) = I_2 - S_{\text{de}}(j\omega)$ and $k(j\omega) = \rho(Q(j\omega))$. When $FI(j\omega) < 1$, the grid strength at this time can be expressed as:

$$\tilde{B}_{\text{net}}(j\omega) = \left(B_{\text{red}} + k(j\omega)\frac{B_2 B_3}{B_4}\right) \otimes I_2 + \frac{B_2 B_3}{B_4} \otimes (Q(j\omega) - k(j\omega) I_2) \tag{A2}$$

Since $\rho(Q(j\omega) - k(j\omega)I_2) = 0$, and $B_2 B_3 / B_4$ is a rank-one positive semidefinite matrix, it follows that:

$$\begin{aligned} \underline{\sigma}\left(\tilde{B}_{\text{net}}(j\omega)\right) &\geq \rho\left(\tilde{B}_{\text{net}}(j\omega)\right) \\ &\geq \lambda_{\min}\left(B_{\text{red}} + k(j\omega)\frac{B_2 B_3}{B_4}\right) \\ &\geq \lambda_{\min}\left(B_{\text{red}}\right) = \text{gSCR} \end{aligned} \tag{A3}$$

Appendix B

Table B1 Network data of interconnected system

Line inductance	Value / p.u.	Line inductance	Value / p.u.	Line inductance	Value / p.u.	Line inductance	Value / p.u.	Line inductance	Value / p.u.
$L_{1,2}$	0.0411	$L_{1,39}$	0.0250	$L_{2,3}$	0.0151	$L_{2,25}$	0.0086	$L_{29,38}$	0.0356
$L_{2,30}$	0.0181	$L_{3,4}$	0.0213	$L_{3,18}$	0.0133	$L_{4,5}$	0.0128	$L_{31,6}$	0.0250
$L_{4,14}$	0.0129	$L_{5,8}$	0.0112	$L_{6,5}$	0.0026	$L_{6,7}$	0.0092	$L_{26,27}$	0.0147
$L_{6,11}$	0.0082	$L_{7,8}$	0.0046	$L_{8,9}$	0.0363	$L_{9,39}$	0.0250	$L_{26,28}$	0.0474
$L_{10,11}$	0.0043	$L_{10,13}$	0.0043	$L_{10,32}$	0.0400	$L_{12,11}$	0.0435	$L_{26,29}$	0.0625
$L_{12,13}$	0.0435	$L_{13,14}$	0.0101	$L_{14,15}$	0.0217	$L_{15,16}$	0.0094	$L_{28,29}$	0.0151
$L_{16,17}$	0.0089	$L_{16,19}$	0.0195	$L_{16,21}$	0.0135	$L_{16,24}$	0.0059	$L_{23,24}$	0.0350
$L_{17,18}$	0.0082	$L_{17,27}$	0.0173	$L_{19,33}$	0.0142	$L_{19,20}$	0.0138	$L_{23,36}$	0.0272
$L_{20,34}$	0.0180	$L_{21,22}$	0.0140	$L_{22,23}$	0.0096	$L_{22,35}$	0.0343	$L_{25,26}$	0.0323
$L_{25,37}$	0.0232								

Appendix C

Referring to [15], the dynamic strength quantification problem of a multi-infeed system composed of homogeneous GFLs can be equivalently decomposed into the dynamic strength quantification problem of n single-infeed systems with SCR $= \sigma_i$ $(i = 1, \ldots, n)$; it is also equivalently determined by the dynamic strength of the weakest single-infeed system. It should be noted that, for grid-following equipment based on PLLs, the smaller the SCR is, the smaller the system dynamic strength is. The short-circuit ratio of this single-infeed system is therefore the generalized short-circuit ratio of the system.

$$\kappa(\omega) = \sigma_{\min}\left[Y_{\mathrm{GFL}}(j\omega) + \mathrm{gSCR} \cdot F(j\omega)\right] \tag{C1}$$

After the additional converter $Y_{\mathrm{de}}(s)$ is connected, according to Eqs. (17) and (18) in the main text, the closed-loop model of the system can be rewritten as:

$$I_n \otimes Y_{\mathrm{GFL}}(s) + \left(B_1 \otimes I_2 - \frac{B_2 B_3}{B_4} \otimes S_{\mathrm{de}}(s)\right)(I_2 \otimes F(s)) \tag{C2}$$

The dynamic strength problem of the system shown in (C2) can be expressed as:

$$\begin{aligned}\kappa_{\mathrm{new}}(\omega) &= \sigma_{\min}\left[I_n \otimes Y_{\mathrm{GFL}}(s) + B_{\mathrm{red}} \otimes F(s) + \frac{B_2 B_3}{B_4} \otimes (F(s) - S_{\mathrm{de}}(s)F(s))\right] \\ &\approx \sigma_{\min}\left[Y_{\mathrm{GFL}}(s) + \mathrm{gSCR} \cdot F(s) + \beta\left(I_2 - S_{\mathrm{de}}(s)\right)F(s)\right]\end{aligned} \tag{C3}$$

Fig. D1. Schematic diagram of a single converter connected to an infinite bus.

Figure 13: Fig. D1. Schematic diagram of a single converter connected to an infinite bus.

where $\beta = u^T \dfrac{B_2 B_3}{B_4} u$, and u is the eigenvector of B_{red} corresponding to gSCR. Based on first-order perturbation, the dynamic strength quantification problem of this multi-infeed system can be decoupled into the dynamic strength quantification problem of a single-infeed system. Comparing (C3) with (C1), it can be found that the connection of the additional equipment is equivalent to adding a perturbation $\Delta(s) = \beta\left(I_2 - S_{\text{de}}(s)\right) F(s)$ on the basis of the original system. Since the capacity of the connected additional converter is small relative to the capacity of the original system, it can therefore be considered that:

$$\|\Delta(j\omega)\| \ll \|Y_{\text{GFL}}(j\omega) + \text{gSCR} \cdot F(j\omega)\| \qquad \text{(C4)}$$

According to matrix perturbation theory, after the additional equipment is connected, the system dynamic strength is transformed into:

$$\begin{aligned}
\kappa_{\text{new}}(\omega) &= \sigma_{\min}\left[Y_{\text{GFL}}(j\omega) + \text{gSCR} \cdot F(j\omega) + \Delta(j\omega)\right] \\
&\approx \sigma_{\min}\left[Y_{\text{GFL}}(j\omega) + \text{gSCR} \cdot F(j\omega)\right] + \text{Re}\left(\nu^H \Delta(j\omega)\nu\right) + O\left(\|\Delta(j\omega)\|_2^2\right) \\
&\approx \kappa(\omega) + \text{Re}\left(\nu^H \Delta(j\omega)\nu\right) + O\left(\|\Delta(j\omega)\|_2^2\right)
\end{aligned} \qquad \text{(C5)}$$

where ν is the singular vector corresponding to the minimum singular value of $Y_{\text{GFL}}(j\omega) + \text{gSCR} \cdot F(j\omega)$. According to the grid-forming index condition proposed in this paper, when the added converter satisfies a grid-forming index less than 1, for any corresponding vector we have:

$$\text{Re}\left(\nu^H \Delta(j\omega)\nu\right) > 0 \qquad \text{(C6)}$$

Therefore, connecting devices that satisfy a grid-forming index less than 1 can improve the dynamic strength of the system.

Appendix D

Consider a single-converter system connected to an infinite bus, as shown in Fig. D1. Regardless of the control method adopted by the converter, the relationship between the voltage and current at its grid-connection point can always be represented by the following small-signal model:

Fig. D1 Schematic diagram of a single converter connected to a grid

Fig. D1. Schematic diagram of a single converter connected to an infinite bus.

$$\begin{bmatrix}\Delta i_x\\ \Delta i_y\end{bmatrix}=\begin{bmatrix}Y_1(s) & Y_2(s)\\ Y_3(s) & Y_4(s)\end{bmatrix}\begin{bmatrix}\Delta u_x\\ \Delta u_y\end{bmatrix}=Y_{\mathrm{de}}(s)\begin{bmatrix}\Delta u_x\\ \Delta u_y\end{bmatrix} \tag{D1}$$

where $\Delta i=\begin{bmatrix}\Delta i_x & \Delta i_y\end{bmatrix}^{\mathrm{T}}$ and $\Delta u=\begin{bmatrix}\Delta u_x & \Delta u_y\end{bmatrix}^{\mathrm{T}}$ are, respectively, the perturbations of the converter output current and output voltage in the global xy coordinate system. $Y_{\mathrm{de}}(s)$ is the admittance matrix of the converter. The line dynamics can be expressed as:

$$\begin{bmatrix}\Delta i_x\\ \Delta i_y\end{bmatrix}=\frac{1}{L_g}\begin{bmatrix}s/\omega_0 & -1\\ 1 & s/\omega_0\end{bmatrix}^{-1}\begin{bmatrix}\Delta u_{gx}-\Delta u_x\\ \Delta u_{gy}-\Delta u_y\end{bmatrix}=\frac{1}{L_g}F(s)\begin{bmatrix}\Delta u_{gx}-\Delta u_x\\ \Delta u_{gy}-\Delta u_y\end{bmatrix} \tag{D2}$$

where L_g is the line inductance, and $\Delta u_g=\begin{bmatrix}\Delta u_{gx} & \Delta u_{gy}\end{bmatrix}^{\mathrm{T}}$ is the voltage perturbation of the external grid in the global xy coordinate system.

To characterize the voltage dynamics at the converter grid-connection point, combining (D1) and (D2) gives:

$$\begin{bmatrix}\Delta u_x\\ \Delta u_y\end{bmatrix}=\underbrace{\left(I_2+L_gF^{-1}(s)Y_{\mathrm{de}}(s)\right)^{-1}}_{S_{\mathrm{de}}(s)}\begin{bmatrix}\Delta u_{gx}\\ \Delta u_{gy}\end{bmatrix} \tag{D3}$$

According to (D3), we obtain the transfer function from the disturbance $\Delta u_g=\begin{bmatrix}\Delta u_{gx} & \Delta u_{gy}\end{bmatrix}^{\mathrm{T}}$ at the external-grid voltage port to the converter terminal voltage $\Delta u=\begin{bmatrix}\Delta u_x & \Delta u_y\end{bmatrix}^{\mathrm{T}}$. In the main text, the peak singular value of $S_{\mathrm{de}}(s)$ at $s=j\omega$ is defined as the grid-forming index $FI(j\omega)$, i.e.,

$$FI(j\omega)=\sigma_{\max}\left(S_{\mathrm{de}}(j\omega)\right)=\max_{\|\Delta u_g\|\neq 0}\frac{\|\Delta u(j\omega)\|_2}{\|\Delta u_g(j\omega)\|_2} \tag{D4}$$

From (D4), it can be seen that the grid-forming index describes the peak sensitivity of the converter terminal voltage phasor to disturbances in the grid voltage phasor.

Note: Figure translations are in progress. See original paper for figures.