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Full Text

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Abstract: Ground flutter testing is a new method for studying structural flutter. By reducing continuously distributed aerodynamic forces to several concentrated forces and loading them with shakers, the flutter response of a structure can be simulated. To study ground flutter response simulation technology for cantilever plate structures at a given wind speed, a theoretical model of a multi-point excitation system based on a cantilever plate structure was constructed. For flutter analysis of structures in a supersonic environment, an aerodynamic model of the cantilever plate was established using piston theory. To address the coupling among the channels of the multi-point excitation system, a pre-feedback method was adopted to realize decoupled control of the cantilever-plate–shaker multi-point excitation system. Finally, based on the proposed theoretical model, an experimental platform was built. The analysis results show that the established cantilever-plate ground-flutter simulation test system can complete simulation of the structural response at different Mach numbers, and that the experimental results agree very well with the theoretical calculation results. For conditions with Mach numbers of 4.3 and 4.5, the error between the experimental results and the simulation results is less than 2.5%.

Keywords: ground flutter test; multi-point excitation system; cantilever plate; decoupling control; PID control

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Abstract: Ground flutter test is a novel approach for structural flutter investigation, in which the continuously distributed aerodynamic forces are reduced to a set of discrete concentrated forces and applied through shakers, thereby enabling the simulation of the structural flutter response. To study the ground flutter response simulation technology of a cantilever plate structure at a given flow speed, a theoretical model of a multi-point excitation system based on the cantilever plate structure was developed. For flutter analysis in a supersonic environment, the aerodynamic model of the cantilever plate was established using the piston theory. To address the coupling between different channels of the multi-point excitation system, a pre-feedback method was employed to achieve decoupled control of the cantilever plate–shaker multi-point excitation system. Finally, an experimental platform was constructed based on the

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proposed theoretical model. The analysis results indicate that the developed cantilever plate ground flutter simulation test system can reproduce the structural response at different Mach numbers, with the experimental results showing very good agreement with the theoretical predictions. For the cases with Mach numbers of 4.3 and 4.5, the discrepancies between the experimental and numerical results are less than 2.5%.

Key words: ground flutter test; multi-point excitation system; cantilever plate; decoupling control; PID control

Flutter is a typical aeroelastic stability problem and exists widely in various aircraft structures. Flutter occurs due to the mutual coupling of aerodynamic force, elastic force, and inertial force, and it usually leads to large-amplitude vibration of the structure; in severe cases, it may cause direct disintegration of the structure, thereby resulting in enormous safety hazards[1]. Even if flutter is controlled in time and catastrophic consequences are avoided, it may still cause irreversible damage to the structure and affect the service life and structural safety of related products. Therefore, it is of great significance to study and analyze possible flutter problems that may occur in aircraft structures during operation, so as to effectively avoid losses of personnel and property.

At present, the experimental methods used for studying aircraft flutter problems mainly include two types: wind-tunnel tests and flutter flight tests. Among them, wind-tunnel tests generally use scaled models. Scaled models not only involve a complicated design process, but also make it difficult to ensure complete preservation of the characteristics of the actual structure. Flutter flight tests require real aircraft to be tested under actual operating conditions. This method can fully retain the real working environment of the aircraft, but it is costly and risky. Therefore, a new flutter test method has gradually been developed, namely the ground flutter simulation test method.

At present, many scholars in China and abroad have carried out research on ground flutter test systems and achieved certain results[2-4]. Kearns[5] first proposed ground flutter testing and conducted preliminary theoretical and experimental-method studies. Dhital et al.[6] studied a stepwise reduction method for high-dimensional aerodynamic loads. Existing studies divide the aerodynamic load into several parts and use numerical integration to calculate the resultant force of all parts; meanwhile, the loading position of the concentrated aerodynamic force is also optimized. Chen Haoyu et al.[7] proposed a modeling method applicable to time-varying-parameter unsteady aerodynamic models, based on a Kriging surrogate model. Li Weiming et al.[8] optimized the positions of aerodynamic interpolation points, defined objective functions for optimizing the positions of loading points and measurement points, and performed optimization using a particle swarm optimization algorithm. Song Qiaozhi et al.[9] proposed an unsteady aerodynamic reconstruction method based on computational fluid dynamics (CFD), using the transformation relationship between physical coordinates and generalized coordinates to reconstruct the aerodynamic forces. Shao Chonghui[10] used a method of reducing distributed unsteady aerodynamic forces to conduct ground flutter test simulations for a panel structure, and compared and analyzed the test results with numerical calculation results. Xu Yuntao et al.[11] established non-stationary aerodynamic models for subsonic and supersonic conditions based on the subsonic doublet-lattice method and piston theory, and established a closed-loop state-space model of the ground flutter test system. For simulation loading tests of equivalent aerodynamic loads, the equipment most commonly used is an electrodynamic shaker, and the loading methods include two types: open-loop loading and closed-loop loading[12]. Zhang Guizhe et al.[13] took a full-scale wing scaled model as the research object and used several shakers and sensors to realize simulation of the flutter response of the wing structure. The analysis results showed that considering the dynamic characteristics of the shaker loading system is of great significance for improving the effectiveness of flutter simulation. Yun et al.[14] used feedforward decoupling control to design a controller for a two-input-two-output ground flutter test system, and applied it to subsonic structural flutter simulation.

This study takes a cantilever plate as an example and discusses ground simulation technology for the flutter response of a cantilever plate in a supersonic environment. Based on piston theory, an aerodynamic model of the cantilever plate is established, and an equivalent model of distributed aerodynamic load is established based on the modal equivalence method. For the multi-point excitation system, because the coupling effects between loading channels affect the experimental results, this study adopts a feedforward-feedback decoupling method to complete decoupling control of the multi-point loading system for the cantilever plate. Finally, an experimental platform for the multi-point excitation system of the cantilever plate is built; at a given Mach number, simulation tests of the flutter response of the cantilever plate are performed, and the experimental results are compared with the theoretical results.

1 Ground Flutter Test Principle

A ground flutter test is a semi-physical experimental system. Sensors are arranged on the structural surface, and displacement and velocity information at a finite number of points on the structural surface is obtained in real time through the sensors. The distributed aerodynamic load acting on the structural surface is calculated and then equivalently transformed into several concentrated loads. The controller receives in real time the results from the aerodynamic-force calculation module and the output signals fed back by the force sensors, and computes the corresponding control voltage. Finally, the calculation results are

Figure 1: Ground flutter simulation test process

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Figure 2: Schematic diagram of multipoint excitation test system

Figure 2: Figure 2: Schematic diagram of multipoint excitation test system

transmitted to the shakers to complete loading on the structure. The main process of the ground flutter test system is shown in Fig. 1.

Figure 1 Ground flutter simulation test process

Fig. 1 Ground flutter simulation test process

2 Modeling of the Cantilever-Plate-Exciter System and Aerodynamic-Load Calculation

2.1 Structural Modeling of the Cantilever Plate

The cantilever-plate + exciter system for multipoint excitation is shown in Fig. 2, where the plate thickness h is a constant that does not vary with x or y. The cantilever-plate material used in this study is aluminum, with dimensional parameters: $a = 0.45$ m, $b = 0.4$ m, $h = 0.003$ m. Since the plate thickness is much smaller than its mid-plane dimensions, it can be analyzed as a thin plate. This study assumes that the research object satisfies the Kirchhoff assumptions and adopts classical plate theory for modeling. The mid-plane displacement of the cantilever plate is used to represent the displacement of an arbitrary point, and the von Kármán large-deformation nonlinear relation is adopted to express the stress–strain relationship [15].

Note: In the figure, the actual plate thickness and the enlargement ratio are 0.003 m and $2.5:1$, respectively.

Figure 2 Schematic diagram of multipoint excitation test system

Fig. 2 Schematic diagram of multipoint excitation test system

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\dfrac{\partial w_0(x,y)}{\partial x} \\ v(x,y,z) = v_0(x,y) - z\dfrac{\partial w_0(x,y)}{\partial y} \\ w(x,y,z) = w_0(x,y) \end{cases} \tag{1}$$

$$\begin{cases} \varepsilon_x = \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 - z\dfrac{\partial^2 w_0}{\partial x^2} \\ \varepsilon_y = \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 - z\dfrac{\partial^2 w_0}{\partial y^2} \\ \gamma_{xy} = \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} - 2z\dfrac{\partial^2 w_0}{\partial x \partial y} \end{cases} \tag{2}$$

where u_0, v_0, and w_0 denote the displacements of an arbitrary point on the mid-plane of the cantilever plate in the x, y, and z directions, respectively; u, v, and w denote the displacements of an arbitrary point in the plate in the x, y, and z directions, respectively; z denotes the distance between an arbitrary point in the plate and the mid-plane; and ε_x, ε_y, and γ_{xy} denote the strains at an arbitrary point in the plate.

Using Hamilton' s principle, the differential equation of motion of the cantilever plate is established, as shown in Eq. (3):

$$\int_{t_0}^{t_1} \left[\delta(T_0 - U_0) + \delta W\right] \mathrm{d}t = 0 \tag{3}$$

where T_0, U_0, and W denote, respectively, the kinetic energy, strain energy, and virtual work done by non-conservative forces of the structure. For ease of calculation, the assumed-mode method is used to express the actual displacements in terms of modes and generalized coordinates:

$$\begin{cases} u_0(x,y,t) = \mathbf{W}^{\mathrm{T}}\mathbf{q}^x = \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} W_{ij}^x(x,y)\, q_{ij}^x(t) \\ v_0(x,y,t) = \mathbf{W}^{\mathrm{T}}\mathbf{q}^y = \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} W_{ij}^y(x,y)\, q_{ij}^y(t) \\ w_0(x,y,t) = \mathbf{W}^{\mathrm{T}}\mathbf{q}^z = \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} W_{ij}^z(x,y)\, q_{ij}^z(t) \end{cases} \tag{4}$$

where q_{ij}^x, q_{ij}^y, and q_{ij}^z denote the generalized coordinates in the x, y, and z directions, respectively; $W_{ij}(x,y)$ denotes the mode function; and m and n denote the highest modal orders considered for the cantilever plate in the x and y directions, respectively. The expressions of the mode functions are as follows [16–17]:

$$\begin{cases} W_{ij}^{x}(x,y) = \cos\left(\dfrac{i\pi x}{a}\right)\sin\left(\dfrac{2j-1}{2}\right)\dfrac{\pi y}{b} \\ W_{ij}^{y}(x,y) = \cos\left(\dfrac{i\pi x}{a}\right)\sin\left(\dfrac{2j-1}{2}\right)\dfrac{\pi y}{b} \\ W_{ij}^{z}(x,y) = W_{i}^{f}(x)\,W_{j}^{c}(y) \end{cases} \qquad (5)$$

where $W_i^f(x)$ denotes the i-th free-free beam mode function:

$$W_i^f(x) = \begin{cases} W^f, & i=1 \\ W^f(1-2x/a), & i=2 \\ \cosh r_i^f x + \cos r_i^f x + \nu_i^f\left(\sinh r_i^f x + \sin r_i^f x\right), & i \geq 3 \end{cases} \qquad (6)$$

$$\nu_i^f = -\frac{\sinh r_i^f a + \sin r_i^f a}{\cosh r_i^f a + \cos r_i^f a} \qquad (7)$$

where W^f denotes, when $i \geq 3$, the value of $W_i^f(x)$ at $x = 0$, and $W_j^c(y)$ denotes the j-th cantilever-beam mode function.

$$W_j^c(y) = \cosh r_j^c y - \cos r_j^c y + \upsilon_j^c\left(\sinh r_j^c y - \sin r_j^c y\right) \qquad (8)$$

$$\upsilon_j^c = -\frac{\sinh r_j^c b - \sin r_j^c b}{\cosh r_j^c b + \cos r_j^c b} \qquad (9)$$

where r^f and r^c are the solutions of the following equations, respectively:

$$\cos(r^f b)\cosh(r^f b) - 1 = 0 \qquad (10)$$

$$\cos(r^c b)\cosh(r^c b) + 1 = 0 \qquad (11)$$

Substituting the assumed modes into Eq. (3), the differential equation of motion of the cantilever plate can be obtained as

$$M_0\ddot{q}(t) + D_0\dot{q}(t) + K_0 q(t) + f_0 = 0 \qquad (12)$$

where M_0 is the mass matrix of the cantilever-plate structure; D_0 is the damping matrix of the cantilever-plate structure; K_0 is the structural stiffness matrix; and f_0 denotes the structural nonlinear force vector. At this point, the construction of the differential-equation model of motion for the cantilever-plate structure is completed. According to the above model, response information such as the displacement and velocity of the cantilever plate under a specified external load can be calculated.

2.2 Modeling of the Exciter and Coupled System

The exciter is an important component in ground flutter tests. The reduced-order aerodynamic load needs to be applied to the structure in real time by the exciter to simulate the flutter response of the structure. In this study, the device used to apply loads to the cantilever plate is a contact-type electrodynamic exciter. During load application, because the exciter armature itself has a certain mass, stiffness, and damping, these factors must be considered when building the model, since they affect the structure [18]. In this study, during modeling the exciter is considered as a damped single-degree-of-freedom elastic oscillator. The differential-equation model of motion for a single exciter can be expressed as [19]

$$m_v \ddot{w} + k_v w + c_v \dot{w} = f_e + f_s \qquad (13)$$

where w denotes the displacement vector in the z-direction at the loading points corresponding to the exciters; m_v denotes the armature mass matrix (including the exciter armature mass, exciter push-rod mass, and force-sensor mass); k_v denotes the exciter spring-stiffness matrix; c_v denotes the exciter damping matrix; $f_e = \kappa_c I$ denotes the electromagnetic driving-force matrix of the exciter; κ_c denotes the electromechanical coupling coefficient matrix; and f_s denotes the matrix of forces exerted by the structure on the exciter.

The circuit part of the exciter mainly includes the resistor, inductor, power-amplifier input terminals, and coil. Its differential-equation model can be expressed as

$$L\dot{I} + RI + \kappa_c \dot{w} = Gu \qquad (14)$$

where I denotes the exciter current vector; L denotes the exciter inductance matrix; R denotes the exciter circuit resistance matrix; G denotes the gain-coefficient matrix of the power amplifier; and u denotes the input-voltage matrix of the power amplifier.

Substituting Eq. (13) into Eq. (12) and coupling it with Eq. (14), the simulated loading system model considering the exciters can be obtained:

$$\begin{cases} (M_0 + M_v)\ddot{q}(t) + (D_0 + D_v)\dot{q}(t) + (K_0 + K_v)q(t) + f_0 - \varphi_s^T \kappa_c I = 0 \\ L\dot{I} + RI + \varphi_s \kappa_c \dot{q}^z(t) = Gu \end{cases} \qquad (15)$$

where φ_s is the modal displacement matrix at the loading points, which can be expressed as

$$\varphi_s = \begin{bmatrix} W_1(x_1, y_1) & W_2(x_1, y_1) & \cdots & W_k(x_1, y_1) \\ W_1(x_2, y_2) & W_2(x_2, y_2) & \cdots & W_k(x_2, y_2) \\ \vdots & \vdots & \ddots & \vdots \\ W_1(x_k, y_k) & W_2(x_k, y_k) & \cdots & W_k(x_k, y_k) \end{bmatrix} \tag{16}$$

M_v, D_v, and K_v are the exciter mass matrix, damping matrix, and stiffness matrix, respectively, and can be expressed as

$$M_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_v^z \end{bmatrix}, \qquad M_v^z = \varphi_s^T m_v \varphi_s \tag{17}$$

$$D_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & D_v^z \end{bmatrix}, \qquad D_v^z = \varphi_s^T c_v \varphi_s \tag{18}$$

$$K_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_v^z \end{bmatrix}, \qquad K_v^z = \varphi_s^T k_v \varphi_s \tag{19}$$

From Eq. (15), it can be seen that this model considers not only the influence of the mechanical-system part of the exciter on the motion of the cantilever-plate structure, but also the influence of the coupling between the mechanical system and the electrical system on the structure. For a given input-voltage signal of the power amplifier, this model can calculate the displacement, velocity, and acceleration responses of the structure, as well as the current response of the exciter circuit.

2.3 Calculation of Aerodynamic Forces and Equivalent Aerodynamic Forces

A ground flutter simulation test requires predicting the overall displacement field and velocity field of the structural surface from the structural responses at a limited number of measurement points on the measured structural surface. The modal-equivalence method is used to calculate the structural displacement and velocity:

$$\begin{cases} q = \Phi_m^{-1} w_m \\ \dot{q} = \Phi_m^{-1} \dot{w}_m \end{cases} \tag{20}$$

where q, Φ_m^{-1}, and w_m denote the structural modal-coordinate vector, the modal matrix of the measurement points, and the physical displacement vector, respectively.

After the structural modal coordinates are calculated, the displacement and velocity at any point on the structural surface can be calculated according to modal superposition.

$$\begin{cases} w_s = \Phi_s q \\ \dot{w}_s = \Phi_s \dot{q} \end{cases} \tag{21}$$

where w_s and Φ_s denote, respectively, the displacement at an arbitrary point on the structural surface and the modal matrix.

For flutter analysis of a structure in a supersonic environment, there is a simplified aerodynamic theory, namely piston theory. This theory holds that the aerodynamic load acting at any point on the structural surface is related only to the velocity of that point and is independent of other points. Therefore, every point on the structural surface can be regarded as a piston in motion. In this case, for any point on the structural surface, the aerodynamic load it experiences can be expressed as

$$f_a = \frac{4q_d\lambda}{Ma}\left(\frac{\partial w}{\partial x} + \frac{1}{V}\frac{\partial w}{\partial t}\right) w(x,y,t) \tag{22}$$

where f_a denotes the aerodynamic load acting at an arbitrary point of the structure; q_d, λ, and Ma denote the dynamic pressure, correction coefficient, and Mach number, respectively; and V denotes the wind speed.

According to Eqs. (21) and (22), the distributed aerodynamic force on the structural surface can be calculated. Subsequently, the distributed aerodynamic load must be equivalently transformed into several concentrated forces acting at points. In the equivalence process, it is assumed that the work done on the structure per unit time by the load before and after equivalence is the same, i.e., the principle of power equivalence:

$$\begin{cases} P_a = \int_0^a \int_0^b \left[\Delta p \dot{w}(x,y)\right] dxdy \\ P_e = \sum_{i=1}^{n} f_{ei} \int_0^a \int_0^b \left[\delta(x-x_{ei})\delta(y-y_{ei})\dot{w}(x,y)\right] dxdy \end{cases} \tag{23}$$

where P_a and P_e denote the distributed aerodynamic power and the equivalent force power, respectively; Δp denotes the aerodynamic force at an arbitrary point; f_{ei} denotes the i-th equivalent concentrated force; and x_{ei} and y_{ei} denote the x- and y-coordinates, respectively, of the point of application of the i-th equivalent concentrated force.

Let P_a and P_e be equal, and use generalized coordinates to express the displacement and velocity in physical coordinates; the equivalent concentrated force can then be calculated as

$$\begin{cases} f_e = \varphi_s^{-1} f_{\text{aero}} \\ f_{\text{aero}} = \begin{bmatrix} \int_0^a \int_0^b \Delta p W_1(x,y)\, dxdy \\ \int_0^a \int_0^b \Delta p W_2(x,y)\, dxdy \\ \vdots \\ \int_0^a \int_0^b \Delta p W_k(x,y)\, dxdy \end{bmatrix} \end{cases} \tag{24}$$

where f_{aero} denotes the generalized aerodynamic-force vector.

In summary, the aerodynamic load corresponding to the structural response can be obtained from a finite number of measurement points on the structural surface, and the equivalent concentrated forces can then be obtained.

3 Controller Design and Control-Effect Verification for the Multi-Point Excitation System

In a ground flutter test, in order for the exciters to apply an appropriate equivalent concentrated load to the structure, the corresponding voltage signal must be calculated in real time by the controller and transmitted to the power amplifier. At the same time, force sensors must be used to monitor in real time the actual forces applied to the structure by the exciters, and these forces must be fed back to the controller to correct the input voltage signals. However, for a multi-point excitation system, the force applied to the structure by each exciter affects the output forces of the other exciters, which causes certain errors between the actual output of each exciter and the desired force signal. Therefore, the controller design must consider not only the error between the actual signal and the desired signal, but also the elimination of coupling among the channels.

The exciter output measured by the sensors in the multi-point excitation system can be calculated by

$$F_f = \kappa_c I - M_v \ddot{q}^z - D_v \dot{q}^z - K_v q^z \tag{25}$$

where F_f denotes the vector of forces exerted by the exciters on the structure. Combining Eq. (15) with Eq. (25), the state-space equation can be written in the form

Fig. 3 The block diagram of multi-point excitation system with pre-feedback decoupling introduced

Figure 3: Fig. 3 The block diagram of multi-point excitation system with pre-feedback decoupling introduced

$$\begin{cases} \dot{X} = AX + Bu + N_x f_0 \\ F = CX + Du + N_s f_0 \end{cases} \tag{26}$$

where $X = \left[q^T\ \dot{q}^T\ I^T\right]^T$ denotes the system state vector; N_x and N_s denote the matrices corresponding to the nonlinear force vector in the state equation and the output equation, respectively.

The control part mainly includes a feedforward controller, a feedback controller, and a decoupling controller. The decoupling-control part adopts the feedforward feedback compensation method [20−21]. A compensation matrix $D_{\text{pre}}(s)$ is added in front of the system transfer-function matrix $G(s)$, so that the compensated system matrix $G(s)D_{\text{pre}}(s)$ becomes a diagonal matrix or a diagonally dominant matrix. The system block diagram is shown in Fig. 3.

Figure 3. Block diagram of the multi-point excitation system with pre-feedback decoupling introduced

Let

$$\begin{aligned} G_1(s) &= G(s)D_{\text{pre}}(s) \\ &= \left[G_1(s) + G_2(s)\right]\left[E - D_F(s)\right]^{-1} \end{aligned} \tag{27}$$

Solving gives

$$D_{\text{F}}(s) = -G_1(s)G_2(s) \tag{28}$$

where: $G_1(s)$ and $G_2(s)$ denote, respectively, the matrix in $G(s)$ containing only the diagonal elements and the matrix containing only the off-diagonal elements; $D_{\text{F}}(s)$ denotes the matrix on the feedback loop inside the decoupling controller; and E denotes the identity matrix.

The feedforward controller is mainly used to convert the desired force signal into the corresponding voltage signal, and its transfer-function matrix is taken as the inverse matrix of the transfer-function matrix of the coupled loading system:

$$\begin{cases} u_{\text{f}} = G_{\text{f}} f \\ G_{\text{f}} = G^{-1} \end{cases} \tag{29}$$

where: u_f denotes the input voltage-signal vector; f denotes the desired force-signal vector; G denotes the transfer-function matrix obtained by matrix transformation of the linear part of Eq. (26); and G_f denotes the transfer-function matrix of the feedforward controller.

The feedback part adopts a proportional-integral-derivative (PID) controller:

$$u_\mathrm{p} = K_\mathrm{p} e(t) + K_\mathrm{i} \int_0^t e(t)\, dt + K_\mathrm{d} \frac{\mathrm{d}e(t)}{\mathrm{d}t} \qquad (30)$$

Based on the above analysis, decoupling control of the multi-point excitation system was completed. During simulation and testing, the PID controller parameters of each channel need to be adjusted according to the feedback signals from the force sensors and the desired signals, so as to achieve a relatively low tracking error.

In summary, decoupling control and flutter-response simulation of the cantilever-plate multi-point excitation system can be realized. Considering that Eq. (24) requires inverse calculation of the modal matrix at the loading points, the number of loading points must be consistent with the number of modes. Therefore, it is first necessary to determine m and n in Eq. (4). The first two modal frequencies of the cantilever plate were calculated for different values of m and n, and the results are shown in Table 1. Considering that an increase in the number of loading points will lead to an increase in the number of exciters and sensors, thereby increasing the difficulty of controlling the equivalent aerodynamic force and the test cost, in this study m was taken as 4 and n as 1. The first four modal frequencies of the cantilever-plate structure are shown in Table 2, and the corresponding number of loading points is also taken as 4.

Table 1 Modal convergence analysis
Tab. 1 Modal convergence analysis Unit: Hz

Modal order	Modal number $m \times n$	Modal number $m \times n$	Modal number $m \times n$	Modal number $m \times n$	Modal number $m \times n$	Modal number $m \times n$
Modal order	2×1	3×1	4×1	5×1	2×2	4×2
1	16.34	16.32	16.12	16.31	16.34	16.12
2	36.26	39.27	38.68	38.68	36.26	36.08

As shown in Fig. 4, under the two Mach-number conditions of 4.3 and 4.5, the flutter responses of the cantilever plate obtained after loading with the distributed aerodynamic force and with the equivalent aerodynamic force were calculated separately. It can be seen that the cantilever-plate multi-point-excitation flutter simulation system established in this study has a good simulation effect.

Figure 4(a)

Figure 4: Figure 4(a)

Figure 4(b)

Figure 5: Figure 4(b)

Table 2 The first 4 modal frequencies of the cantilever plate
Tab. 2 The first 4 modal frequencies of the cantilever plate

Modal order	Modal frequency/Hz	Modal order	Modal frequency/Hz
1	16.32	3	110.28
2	38.68	4	250.36

(a) Comparison of flutter responses of the cantilever plate (Mach number 4.3)

(b) Comparison of flutter responses of the cantilever plate (Mach number 4.5)

Fig. 4 Comparison of flutter response and simulated response of the cantilever plates

4 Construction of the Test Platform and Model Verification

According to the theoretical model established above, a four-point-excitation ground flutter simulation test system as shown in Fig. 5 was constructed. One edge of the cantilever plate is fixed by an aluminum fixture to realize the fixed boundary condition. Four electromagnetic exciters are installed on its lower side to apply equivalent aerodynamic force to the structure, and the exciters are connected to the plate by screws. To realize real-time measurement of the actual output force of each exciter, piezoelectric force sensors are installed between the screws and the plate. Four eddy-current displacement sensors are installed at certain distances on the upper side of the cantilever plate to measure, in real time, the displacement responses at the measurement points on the surface of the cantilever plate. Meanwhile, an acceleration sensor is fixed at the maximum-displacement location of the cantilever plate (i.e., the corner at the free-end edge) to monitor the structural response in real time. An NI (National Instruments) semi-physical simulation system is used to receive the signals from the eddy-current displacement sensors in real time and, at the same time, receive the feedback signals from the force sensors. The NI system uses the previously established aerodynamic-force equivalence algorithm to determine the equivalent aerodynamic force.

dynamics, and transmits the corresponding control voltage signal to the shaker through the power amplifier, thereby completing closed-loop control of the entire flutter-response simulation.

Fig. 5 Ground flutter simulation system test platform

Figure 6: Fig. 5 Ground flutter simulation system test platform

Fig. 6 Comparison of sweep-frequency responses of the shaker-cantilever plate simulation loading system

Figure 7: Fig. 6 Comparison of sweep-frequency responses of the shaker-cantilever plate simulation loading system

In the process of constructing the test platform, the positions of the excitation points need to be selected. In determining the excitation-point locations, this study mainly considered the following aspects: (1) avoiding the nodal lines of the modal functions of the cantilever plate at each order; (2) preventing the excitation points from being too close to the fixed boundary. The smaller the distance between an excitation point and the fixed boundary, the larger the equivalent aerodynamic load that must be applied under the same working condition, which is unfavorable to the stability of the feedback controller. Table 3 gives the coordinates of the four excitation points finally selected in this study.

Fig. 5 Ground flutter simulation system test platform

Table 3 Flutter response test excitation point coordinates

Excitation point No.	Excitation point coordinates / (m)
1	(0.117, 0.1)
2	(0.342, 0.1)
3	(0.113, 0.3)
4	(0.338, 0.3)

To verify the theoretical model of the shaker–cantilever-plate simulated loading system, sweep-frequency tests were carried out for each channel, and the responses of the theoretical model and the test results were compared. In the sweep-frequency tests, equal-amplitude signals were used; the sweep-frequency range was 0.1–150 Hz, and the sweep duration was 150 s.

Figure 6 compares the simulation results and test results of the displacement response signals of the cantilever plate after sweep-frequency signals were input to the four shakers, respectively.

In Fig. 6, the displacement response is the displacement response at the location of the acceleration sensor. From the comparison results, it can be seen that, for each loading channel, the peak magnitudes and peak frequencies of the theoretical model and the test results are in good agreement. This result verifies the accuracy of the theoretical model established in this study.

(a) Channel 1 sweep-frequency response

(b) Channel 2 sweep-frequency response

(c) Channel 3 sweep-frequency response

(d) Channel 4 sweep-frequency response

Fig. 7 Calculation results of the flutter boundary of the cantilever plate

Figure 8: Fig. 7 Calculation results of the flutter boundary of the cantilever plate

Fig. 6 Comparison of sweep-frequency responses of the shaker-cantilever plate simulation loading system

Based on the constructed test platform, the flutter boundary of the cantilever plate was predicted. The calculation of the flutter boundary uses motion-stability theory. First, according to Eq. (12) and Eq. (22), the aerodynamic-elastic dynamic model of the cantilever plate is obtained.

$$M_0\ddot{q}(t) + (D_0 + D_\mathrm{a})\dot{q}(t) + (K_0 + K_\mathrm{a})q(t) + f_0 = 0 \qquad (31)$$

where K_a and D_a denote the aerodynamic stiffness matrix and aerodynamic damping matrix, respectively. Omitting the nonlinear part in Eq. (31), and writing the linear part in the form of a state-space equation,

$$\dot{X}_\mathrm{s} = A_\mathrm{s}X_\mathrm{s} \qquad (32)$$

where

$$\begin{cases} X_\mathrm{s} = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \\ A_\mathrm{s} = \begin{bmatrix} 0 & E \\ -M_0^{-1}(K_0 + K_\mathrm{a}) & -M_0^{-1}(D_0 + D_\mathrm{a}) \end{bmatrix} \end{cases} \qquad (33)$$

The eigenvalues of the matrix A_s are calculated, and the maximum value among the real parts of all eigenvalues is taken. Because the matrix contains aerodynamic stiffness and aerodynamic damping, when the wind speed changes, the eigenvalues also change; thus a curve of the maximum real part of the eigenvalues versus wind speed can be obtained. When the maximum real part of the eigenvalues corresponding to a point on the curve is less than 0, the system is stable and flutter will not occur; whereas when the maximum real part of the eigenvalues corresponding to a point on the curve is greater than 0, the system is unstable at that moment, i.e., flutter occurs. Therefore, the zero point of this curve is the flutter boundary of the cantilever plate. Figure 7 shows the flutter boundary of the cantilever plate calculated using piston theory, in which the Mach number corresponding to the zero point of the real part of the eigenvalue is 4.25.

Fig. 7 Calculation results of the flutter boundary of the cantilever plate

The flutter boundary of the cantilever-plate specimen was predicted on the test platform. First, based on the experimental results of the flutter response of the

Fig. 8 Experimental results of the flutter boundary of the cantilever plate

Figure 9: Fig. 8 Experimental results of the flutter boundary of the cantilever plate

cantilever-plate specimen, the parameters in Eq. (30) were adjusted, and the control gains were determined as: $K_{\mathrm{p}} = 0.1$, $K_{\mathrm{i}} = 0$, $K_{\mathrm{d}} = 0.001$. Figures 8(a) and 8(b) show the measured flutter displacement responses of the cantilever plate at Mach numbers 3.98 and 4.09, respectively. At Mach 3.98, a pulse excitation was applied to the cantilever plate, and the observed response of the structure rapidly decayed. Therefore, it can be judged that the cantilever plate did not flutter at this time. At Mach 4.09, it can be clearly observed that flutter occurred in the structure. Combining the above two sets of test results, it can be concluded that the critical flutter speed of the cantilever plate obtained by the experimental measurement lies between Mach 3.98 and Mach 4.09. Comparing this range with the calculation result from piston theory, the relative error of the critical flutter speed of the cantilever plate obtained by experimental measurement with respect to the theoretical result is less than 6.35%.

(a) Cantilever-plate displacement response (Mach number 3.98)

(b) Cantilever-plate displacement response (Mach number 4.09)

Fig. 8 Experimental results of the flutter boundary of the cantilever plate

Under the specified Mach numbers, the established system was used to complete the simulation of the flutter response of the cantilever plate, and the test results were compared with the simulation results under the same operating conditions. Figure 9 shows the experimental and simulation results of the cantilever-plate flutter response under Mach-number conditions of 4.3 and 4.5. To quantify the error, the steady-state response was subjected to Fourier transform, and the flutter frequency and the dominant-frequency peak value were compared. Table 4 gives the comparison results of the amplitude and frequency of the flutter response. It can be seen that, for the flutter responses of the cantilever plate at different Mach numbers, the relative error between the test results of the flutter simulation experimental system established in this study and the model calculation results using piston theory is less than 2.5%. There are many reasons for this error. This study considers that the error sources mainly include the following points.

1) Theoretical model modeling error: due to factors such as manufacturing errors, the dynamic characteristics of the cantilever-plate specimen used in this study are not completely consistent with the theoretical model. Meanwhile, in this study the exciter is simplified as a single-degree-of-freedom model containing a linear spring, but in reality the exciter may …

may contain nonlinear factors.

(a) Comparison of flutter responses of cantilever plate (Mach number 4.3)

(b) Comparison of flutter responses of cantilever plate (Mach number 4.5)

Fig. 9 Flutter responses of cantilever plates at different Mach numbers

Table 4 Comparison of flutter response experimental results and simulation results

Index	Experimental value	Theoretical value	Error/%
Amplitude (Mach number 4.3)	0.214 mm	0.21 mm	1.9
Flutter frequency (Mach number 4.3)	139.442 Hz	139.721 Hz	0.2
Amplitude (Mach number 4.5)	0.288 mm	0.281 mm	2.5
Flutter frequency (Mach number 4.5)	139.45 Hz	139.72 Hz	0.19

2) Sensor measurement error: The displacement sensors and force sensors used during the experiment contain unavoidable noise signals in the measurement process, and these signals affect the accuracy of the experimental results.

3) Control-system time-delay error: Ideally, the aerodynamic force should change in real time following the variation of the structural response. However, for the method adopted in this study, the control system used to generate the equivalent aerodynamic force has a time delay, causing the application of the aerodynamic force to lag behind the changes in the structural displacement and velocity.

4) Installation-clearance error: The theoretical model established in this study assumes, by default, that the boundary condition of the cantilever plate is clamped on one side and free on the other three sides. However, during the experiment, the fixture used for the fixed edge of the cantilever-plate specimen contains unavoidable factors such as clearance, which in turn causes this boundary to be not an ideal clamped condition.

The distribution of exciter positions has a significant influence on the dynamic characteristics of the test structure. To ensure that the experimental results have a certain degree of generalizability, this study randomly selected three groups of different loading-point position coordinates on the surface of the cantilever plate, and analyzed the flutter responses of the cantilever plate after changes in the loading-point positions. Table 5 gives the position coordinates of the loading points in each group. According to the different loading-point position distributions, the flutter responses of the cantilever plate under equivalent aerodynamic loading were calculated, and the results are shown in Fig. 10. It can be seen from Fig. 10 that the structural flutter responses obtained under changes in the loading-point position distribution are basically consistent. This result indicates that although changes in exciter position affect the dynamic characteristics of the structure, the simulated flutter responses obtained using the method in this study are not affected.

Table 5 Coordinates of loading point positions

Excitation-point number	Excitation-point coordinates/(m)
1	(0.099, 0.076), (0.04, 0.159), (0.042, 0.298), (0.325, 0.329)
2	(0.079, 0.166), (0.357, 0.11), (0.206, 0.257), (0.395, 0.351)
3	(0.035, 0.194), (0.44, 0.097), (0.18, 0.228), (0.32, 0.383)

(a) Mach number 4.3

(b) Mach number 4.5

Fig. 10 Flutter responses of cantilever plates under different loading point distribution positions

Although the flutter-simulation test system established in this study can accurately reproduce the calculation results of linear piston theory, linear piston theory is applicable to uniform flow fields with airflow speeds from Mach 2 to Mach 5 [22]. Therefore, for structures under operating conditions that do not meet the above conditions, the applicability of the method used in this study needs to be further analyzed.

5 Conclusion

This study investigated and implemented a multi-point excitation system and flutter-response simulation based on a cantilever plate structure, verified the effectiveness of this technology, and provided a feasible test method for ground flutter test systems. The specific conclusions are as follows.

1) A dynamic model of a four-point excitation system for a cantilever plate considering structural nonlinearity was established. In the model, the influence of the mechanical and electrical parts of the exciters on the structure was considered.

2) A control method for the four-point excitation system of the cantilever plate based on PID feedback control and feedforward-feedback compensation decoupling control was established. The responses of the ground flutter simulation system and the aeroelastic system under a given Mach number were calculated separately. The results show that the two are basically consistent, thereby theoretically verifying the accuracy of the model established in this study.

3) A cantilever-plate ground flutter response simulation system was built, and the accuracy of the theoretical model was verified through sweep-frequency tests. Comparing the theoretical calculations with the experimental results shows that the prediction error of the constructed test

platform for the critical flutter speed of the cantilever plate is lower than 6.35%. The flutter response of the cantilever plate under a given Mach number was simulated and compared with the simulation results. The results show that the constructed ground flutter simulation system can well realize the simulation of the flutter response of the cantilever plate.

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