Crack Identification Method for Cantilever Beams Based on Frequency Contours and Particle Swarm Optimization

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Abstract

The intersection of frequency contours obtained from frequency response surfaces and natural frequencies can be used to determine the crack location and depth in beam structures. However, the traditional frequency contour method requires solving a large number of binary transcendental equations. As the number of frequency contours increases and testing noise interferes, plotting and locating the damage intersection points become time-consuming and inaccurate. To address these problems, a dynamic damage model of a cantilever beam with a single through crack is established. The natural frequencies and mode-shape curvature functions are used to solve the frequency contours of the structure, and a two-step particle swarm optimization algorithm based on natural frequency data is proposed to autonomously search for and determine the optimal intersection position, thereby sequentially identifying the crack location and depth in the cantilever beam structure. Numerical examples and experimental test results show that the proposed method avoids calculating a large number of binary transcendental equations to plot frequency contours and can effectively improve the computational efficiency of crack identification; it can also accurately determine the crack location under environmental noise interference.

Full Text

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Li Zhen, Lü Kaibo, Hu Shijie, Han Yuan, Pang Xinyu

(College of Mechanical and Vehicle Engineering, Taiyuan University of Technology, 030024 Taiyuan)

Abstract: The intersection points of frequency contours obtained from frequency response surfaces and natural frequencies can be used to determine the crack location and depth of a beam structure. However, the traditional frequency-contour method requires solving a large number of binary transcendental equations. As the number of frequency contours increases and test noise interferes, plotting and searching for damage-intersection points is time-consuming and has low accuracy. To address the above problems, a dynamic damage model of a cantilever beam containing a single through crack is established. The frequency contours of the structure are solved using the natural frequencies and mode-shape curvature functions, and a two-step particle swarm optimization algorithm based on natural-frequency data is proposed to autonomously search for and determine the optimal intersection-point position, thereby sequentially identifying the crack location and depth of the cantilever-beam structure. Numerical examples and experimental test results show that this method avoids calculating a large number of binary transcendental equations to draw frequency contours, can effectively improve the computational efficiency of crack identification, and can accurately judge the crack location even under environmental-noise interference.

Keywords: cantilever beam; crack identification; single crack; natural frequency; frequency contour; particle swarm optimization

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A crack identification method for cantilever beam based on frequency contours and particle swarm optimization

Li Zhen, Liu Kaibo, Hu Shijie, Han Yuan, Pang Xinyu

(College of Mechanical and Vehicle Engineering, Taiyuan University of Technology, 030024 Taiyuan, China)

Abstract: The intersection points of frequency contours obtained from the frequency response surfaces and natural frequencies can be used to determine the location and depth of crack in beam structure. However, the traditional frequency contour method requires solving a large number of binary transcendental equations, and it takes time and low accuracy to plot and find the intersection region with the increase of frequency contours and the interference of test noise. To address the above problems, a dynamic model of cantilever beam with a penetrated crack is established, natural frequencies and mode curvature functions are used to solve the frequency contours of the structure, and a two-step particle swarm optimization algorithm based on the natural frequency data is proposed to autonomously search and determine the optimal intersection point, and then the location and depth of crack in cantilever beam are sequentially identified. Numerical simulations and experimental tests show that the proposed method can effectively improve the com-

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Corresponding author: Lü Kaibo, Associate Professor. E-mail: lvkaibo@tyut.edu.cn
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putational efficiency of crack identification without calculating a large number of binary transcendental equations to plot frequency contours. Under the interference of environmental noise, the crack location can be accurately identified.

Key words: cantilever beam; crack identification; single crack; natural frequency; frequency contours; particle swarm optimization

Cantilever-beam structures are widely used in engineering practice, such as helicopter rotors, satellite antennas, and traffic sign poles[^1]. During service, because such structures have low stiffness and operate in complex environments, defects appearing on the surface can easily propagate into cracks and lead to failure. Compared with techniques such as acoustic emission, X-ray flaw detection, and visual inspection, dynamics-based damage-identification methods can achieve global detection; the sensing devices are inexpensive and convenient to install, and they are less affected by the environment[^2]. Such methods determine the location and severity of damage by detecting and analyzing changes in structural dynamic parameters, such as natural frequency, damping ratio, mode shape, anti-resonance frequency, and operating deflection shape[^3], thereby predicting the remaining life of the structure[^4]. Among them, natural frequency not only offers high identification accuracy, but is also sensitive to structural damage and can accurately identify damage parameters. Therefore, in-depth study of the intrinsic relationship between the natural frequencies and crack parameters of cantilever beams is of great significance for health monitoring of such structures.

Researchers have established dynamic models of beams containing a single crack through theoretical analysis or finite-element methods, analyzed the relationship between crack parameters and natural frequencies, and thereby achieved crack identification. Some researchers have fitted frequency response surfaces by solving binary transcendental equations composed of natural frequency, crack location, and crack depth, and combined measured natural frequencies to plot frequency contours; by analyzing the locations of their intersections, crack parameters can be obtained. Wu Guorong et al.[^5] treated a crack as a torsional spring, established a dynamic model of a beam containing a single through crack, and used an iterative algorithm to solve its frequency equation. Liu Wenguang et al.[^6] established a stiffness model for a breathing cracked beam with amplitude and crack depth as parameters, and verified through numerical examples the reliability of using the frequency-contour method to identify crack parameters. Dai Wenke et al.[^7] established the vibration equation of a cracked circular-section beam based on the finite-element method, and proposed a frequency-error function to improve the frequency-contour method; experimental results showed that the improved frequency-contour method was more feasible. Zhang et al.[^8] established a dynamic model of a cracked cantilever beam based on Timoshenko beam theory and fracture-mechanics theory, and also analyzed the influence of different noise levels on the identification accuracy of the frequency-contour method. Because of the complexity of solving binary transcendental equations and fitting frequency response surfaces, another group of researchers plotted frequency contours using natural frequencies and mode-shape curvature functions. Dahak et al.[^9] established a finite-element model of a cracked cantilever beam, used the relative change rates of natural frequencies and mode-shape curvature functions to plot frequency contours, and determined crack parameters by analyzing contour intersections; simulations and experiments showed that the first four frequency contours can accurately identify crack parameters. Ta et al.[^10] combined frequency contours and singular-value decomposition to identify crack locations, but still needed to plot frequency contours and analyze their intersections to determine crack depth. Lupu et al.[^11] studied the natural frequencies of cracked beams under non-ideal constraints, analyzed frequency contours using the linear superposition method, and used the first seven natural-frequency data to determine the crack location and depth.

Heuristic optimization algorithms benefit from efficient, parallel global search capability, can rapidly determine optimal solutions to multidimensional problems, and have been widely applied to damage identification[^4]. Guo et al.[^12] analyzed the influence of damage on natural frequencies and modal strain energy, and accurately identified the damage location of a truss structure using information-fusion technology and the particle swarm optimization (PSO) algorithm. Chen Zhen et al.[^13] improved the search position of the PSO algorithm by introducing a zero-variation-rate coefficient; numerical simulations showed that the improved PSO could accurately identify the damage location of a steel-frame structure and had strong noise resistance. Zheng Guangyao et al.[^14] combined a numerical-flow-shape method with an Elman neural network to identify cracks; numerical analysis showed that, under noise interference, the method could effectively identify crack locations in rectangular plates. On the basis of the work of Gillich et al.[^16], Tufisi et al.[^15] used a random hill-climbing algorithm to determine the equivalent damage degree of a cantilever beam; however, this algorithm requires damage information at any three points on the beam to be calculated by simulation, increasing the complexity of damage identification.

In summary, for single-crack identification in cantilever beams, crack parameters can be determined by analyzing the intersections of frequency contours. However, solving a large number of binary transcendental equations and plotting frequency contours make crack identification time-consuming and reduce accuracy. Based on the theory of solving frequency contours from natural frequencies and mode-shape curvature functions, this study proposes a two-step PSO algorithm, which uses natural-frequency data to autonomously search for the optimal intersection position, and then identifies the crack location and depth in sequence. First, a dynamic model of a cantilever beam containing a single through crack is established using fracture-mechanics theory. Next, the relationships among natural frequency, the mode-shape curvature function, and frequency contours are analyzed, and a theoretical model is established in which the two-step PSO algorithm searches for the optimal intersection position and identifies the crack location and depth successively. Finally, numerical examples and experimental tests are used to verify the reliability of the two-step PSO algorithm for identifying crack parameters.

1 Theory of Crack Identification Based on Dynamics

1.1 Dynamic Model of a Cantilever Beam Containing a Single Through Crack

When a through crack exists in a beam, the crack can be equivalently represented as a torsional spring, as shown in Figure 1. According to fracture-mechanics theory, the torsional-spring stiffness K_T is[^5]

$$K_T = \frac{1}{c},\ c = \frac{5.346h}{EI} J\left(\frac{a}{h}\right),$$

$$\begin{aligned} J\left(\frac{a}{h}\right) = & 1.8624\left(\frac{a}{h}\right)^2 - 3.95\left(\frac{a}{h}\right)^3 + 16.375\left(\frac{a}{h}\right)^4 \\ & - 37.226\left(\frac{a}{h}\right)^5 + 76.81\left(\frac{a}{h}\right)^6 - 126.9\left(\frac{a}{h}\right)^7 \\ & + 172\left(\frac{a}{h}\right)^8 - 143.97\left(\frac{a}{h}\right)^9 + 66.56\left(\frac{a}{h}\right)^{10} \end{aligned} \tag{1}$$

where a is the crack depth; h is the beam thickness; $J\,(a/h)$ is the dimensionless local flexibility function; E is the elastic modulus; and I is the second moment of area of the cross section.

(a) Healthy cantilever-beam model

(b) Cracked cantilever-beam model

(c) Equivalent damage model

Fig. 1 Dynamic model of cantilever beam with a penetrated crack

At this point, the differential equations for free vibration of the cracked beam are

$$\begin{cases} EI\dfrac{\partial^4 v_1(x,t)}{\partial x^4} + \rho A\dfrac{\partial^2 v_1(x,t)}{\partial t^2} = 0, & (0 \le x \le x_{\mathrm{D}}) \\ EI\dfrac{\partial^4 v_2(x,t)}{\partial x^4} + \rho A\dfrac{\partial^2 v_2(x,t)}{\partial t^2} = 0, & (x_{\mathrm{D}} < x \le l) \end{cases} \tag{2}$$

where $v_1(x,t)$ and $v_2(x,t)$ are respectively the transverse displacements of segments 1 and 2 of the cracked beam; x_{D} is the crack location; ρ is the material density; A is the cross-sectional area; and l is the beam length, as shown in Fig. 1.

The continuity conditions at the crack location x_{D} are

$$\begin{cases} \varphi_1(x_{\mathrm{D}}) = \varphi_2(x_{\mathrm{D}}) \\ \varphi_1''(x_{\mathrm{D}}) = \varphi_2''(x_{\mathrm{D}}) \\ \varphi_1'''(x_{\mathrm{D}}) = \varphi_2'''(x_{\mathrm{D}}) \\ -EI\varphi_1''(x_{\mathrm{D}}) = K_{\mathrm{T}}(\varphi_1'(x_{\mathrm{D}}) - \varphi_2'(x_{\mathrm{D}})) \end{cases}$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are respectively the mode-shape functions of segments 1 and 2 of the cracked beam.

Combining the boundary conditions of the cantilever beam, the characteristic equation of the cantilever beam with a through crack is obtained as

$$1 + \cos(\lambda l)\cdot\cosh(\lambda l) = \frac{\lambda EI}{2K_{\mathrm{T}}}\cdot F(\beta) \qquad (3)$$

where $\beta = x_{\mathrm{D}}/l$; $\lambda^4 = \dfrac{\rho A}{EI}(2\pi f_{i\mathrm{D}})^2$; $f_{i\mathrm{D}}$ is the i-th natural frequency of the cracked cantilever beam; and $F(\cdot)$ is a function with β, λ, and l as independent variables. Its specific form is given in Ref. [6].

Therefore, when the crack location x_{D} and crack depth a are known, solving Eq. (3) gives the natural frequency $f_{i\mathrm{D}}$ of the cracked cantilever beam.

1.2 Crack identification by the frequency-contour method

Define the relative change rate $\Delta f_{i\mathrm{D}}$ of the i-th natural frequency before and after the occurrence of a crack as

$$\Delta f_{i\mathrm{D}} = \frac{f_{i\mathrm{U}} - f_{i\mathrm{D}}}{f_{i\mathrm{U}}} \qquad (4)$$

where $f_{i\mathrm{U}}$ is the i-th natural frequency of the healthy cantilever beam.

The functional relationship among the cantilever-beam crack location x_{D}, crack depth a, and the first four orders of $\Delta f_{i\mathrm{D}}$ is shown in Fig. 2.

The distribution function of the i-th modal strain energy $U_i(x)$ of the healthy cantilever beam is proportional to the square of the modal curvature, and its expression is given in Eq. (5). The distribution functions of the first four modal strain energies $U_i(x)$ of the healthy cantilever beam are shown in Fig. 3.

$$U_i'(x) = \frac{1}{2} EI \left[\varphi_{i\mathrm{U}}''(x)\right]^2 \qquad (5)$$

where $\varphi_{i\mathrm{U}}''(x)$ is the i-th modal curvature function of the healthy cantilever beam.

(a) First order

(b) Second order

(c) Third mode

(d) Fourth mode

Fig. 2 Relationships among crack location, crack depth, and natural frequency of a cantilever beam

Fig. 3 Modal strain energy distribution of a healthy cantilever beam

From Fig. 2, it can be seen that the change in the i-th-order natural frequency caused by a crack at x_D is proportional to the i-th-order modal strain energy at that location. Meanwhile, the crack depth a determines the degree of change in the natural frequencies of each order. Fig. 4(a) shows the relationship between the normalized amplitude of Δf_{2D} with crack location x_D and the second-order modal strain energy; Fig. 4(b) shows Δf_{2D} for different crack depths.

Therefore, the crack parameters of a cantilever beam can be obtained by solving from the i-th-order Δf_{iD} and the normalized mode-shape curvature $\varphi_{iU}''(x)$ as follows [10,16]:

$$c_i(x) = \ln \frac{\Delta f_{iD}}{\left[\varphi_{iU}''(x)\right]^2} \qquad (6)$$

where $c_i(x)$ represents the i-th frequency contour, and $\varphi_{iU}''(x)$ is the analytical mode-shape curvature function of the healthy cantilever beam.

For the first s ($s \geq 3$) orders, the abscissa of the intersection point of $c_i(x)$ is the crack location x_D, and the ordinate is the damage factor C, with C satisfying $C = c_1(x_D) = \cdots = c_s(x_D)$.

After the crack location x_D is determined, Eq. (3) becomes a one-dimensional transcendental equation. Then, by combining Eq. (4) and Eq. (6), the function relationship between different crack depths a' and $c_1(x_D)$ is obtained by interpolation, namely $c_1(x_D) = G(a')$, as indicated by the red curve $c_1(x_D = 0.5l)$ in Fig. 2(a). Finally, the crack depth a is determined using the damage factor C.

(a) Relationship between the second-order modal strain energy of the cantilever beam and Δf_{2D}

(b) Δf_{2D} of the cantilever beam for different crack depths

Fig. 4 Relationship between the second modal energy and crack parameters of a cantilever beam

1.3 Two-step PSO algorithm for crack identification

The PSO algorithm used in this study is carried out in two steps. Step 1: search previous-stage optimal intersection position of $c_i(x)$, determine the crack position x_D and the damage factor C. Step 2: solve $c_1(x_D) = G(a')$, and use the damage factor C to search for and determine the crack depth a.

In Step 1, the PSO algorithm constructs the objective function $f(m, n)$ using the crack position m and the damage factor n as parameters, i.e.,

$$f(m,n) = \sum_{i=1}^{s} \min\left[D_i(m,n)\right] \qquad (7)$$

where

$$D_i(m,n) = (m-x)^2 + \left[n - c_i(x)\right]^2, \quad x \in [0, l];$$

m is the search dimension for the crack position; n is the search dimension for the damage factor C; s is the number of frequency contours; and $\min[D_i(m,n)]$ is the square of the distance between point (m, n) and the i-th frequency contour.

When the relative crack depth $a/h > 0.5$, the beam structure can be regarded as having failed. Therefore, by calculating $G(a' = 0.5h)$ for different crack positions, one obtains $c_1(x_D) < 0$. Thus, the parameter settings for Step 1 of the PSO algorithm are as shown in Table 1.

Table 1 Parameters of two-step PSO

Tab. 1 Parameters of two-step PSO

Parameter name	Step 1	Step 2
Particle dimension	2	1
Number of particles	100	10
Number of iterations	500	100
Swarm factor rand(0, 1)	2	2
Self factor rand(0, 1)	2	2
rand(0, 1)	rand(0, 1)	rand(0, 1)
Objective function	rand(0, 1)	rand(0, 1)
rand(0, 1) range	rand(0, 1)	–

Parameter name	Step 1	Step 2
rand(0, 1) range	rand(0, 1)	–
rand(0, 1) range	–	rand(0, 1)

Note: rand(0, 1) is a random number uniformly distributed over [0, 1].

After Step 1 of PSO identifies the crack position x_D and the damage factor C, Step 2 of the PSO algorithm constructs the objective function $g(q)$ using the crack depth q as the parameter, i.e.,

$$g(q) = \min\left(|C - G(q)|\right) \qquad (8)$$

In the two PSO searches, the inertia weight factors are both obtained by linear decrement:

$$\omega(t) = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}} t \qquad (9)$$

where $\omega_{\max} = 0.8$; $\omega_{\min} = 0.4$; $t_{\max}$ is the maximum number of iterations; and t is the t-th iteration.

Therefore, for the identification of a single crack in a cantilever beam, the identification process of the two-step PSO algorithm is shown in Fig. 5. In this process, considering the influence of environmental changes and noise in actual testing, an error range of 1% is set for $\Delta f_{i\mathrm{D}}$.

```
Start
↓
Test to obtain f_U
↓
Monitor f_i
↓
Decision: Δf_iD < 1%, i
  No → No crack → return to monitoring f_i
  Yes → Crack identification
          ↓
          Calculate c_i(x)
          ↓
          [Two-step PSO]
          Step 1 PSO
          ↓
          Output x_D = m, C = n
          ↓
          Solve c_1(x_D)=G(a )
          ↓
```

```
            Step 2 PSO
            ↓
            Output a = q
            ↓
            Output x_D
↓
End
```

Fig. 5 Flow diagram of crack identification for cantilever beam

2 Numerical Examples

2.1 Parameter settings for cantilever beams with cracks

In the numerical examples, a total of four cantilever-beam samples with cracks are set up. Their mechanical and dimensional parameters are shown in Table 2, and their crack-parameter information is shown in Table 3.

Table 2 Parameters of cantilever beam

Tab. 2 Parameters of cantilever beam

Parameter name	Parameter setting
Elastic modulus E/Pa	2.1×10^{11}
Density $\rho/(\mathrm{kg \cdot m^{-3}})$	7 860
Poisson' s ratio ν	0.3
Length l/mm	235
Width w/mm	16
Thickness h/mm	6

In actual testing, natural frequencies are inevitably affected by random noise [8]. Therefore, different levels of noise are set for samples No. 2–4 in Table 3, as follows:

$$f_i(1-\varepsilon) \le f_i^* \le f_i(1+\varepsilon) \qquad (10)$$

where f_i is the ideal natural frequency of the ith mode; f_i^* is the natural frequency of the ith mode after noise is added; and ε is the noise level.

Table 3 Crack parameters

Tab. 3 Parameters of crack

Group	x_D/mm	a/mm	ε
1	47	2	0
2	47	3	1/100

Group	x_D/mm	a/mm	ε
3	47	3	3/1 000
4	78	0.5	1/1 000

2.2 Crack identification and result analysis

The natural-frequency data of sample No. 1 are shown in Table 4. Because they are not disturbed by noise, the frequency contours intersect at a single point, and the two-step PSO algorithm can accurately identify the crack location x_D and crack depth a. The identification results are shown in Fig. 6.

After the natural-frequency data are contaminated by noise, the intersection of the frequency contours changes into a triangular region. Meanwhile, as the number of natural-frequency modes increases, the number of frequency contours increases, and the triangular region also increases accordingly. Reference [9] determined the intersection location by plotting the first four frequency contours and calculating the average of the vertices of the minimum triangle, thereby identifying the crack parameters. In this section, the method in Ref. [9] and the two-step PSO algorithm are used to identify the crack parameters. For the two-step PSO algorithm, the first 4 and first 6 natural-frequency data are used, respectively, for crack identification. Considering the randomness of noise, 10 groups of random noise are added to samples No. 2–4, and the identification results of the two methods are statistically analyzed. The identification results for samples No. 2 and No. 3 are shown in Fig. 7.

Table 4 First six natural frequencies and Δf_{iD} of the first sample

Tab. 4 First six natural frequencies and Δf_{iD} of the first sample

Group	f_1	f_2	f_3	f_4	f_5	f_6
Natural frequency of healthy sample/Hz	90.708	568.499	1 591.973	3 119.457	5 156.656	7 703.153
Natural frequency of sample No. 1/Hz	88.496	568.392	1 580.388	3 061.581	5 077.806	7 673.090
Δf_{iD}/%	2.439	0.019	0.728	1.856	1.529	0.390

Fig. 7(a) Crack-location identification results

Figure 1: Fig. 7(a) Crack-location identification results

Fig. 7(b) Crack-depth identification results

Figure 2: Fig. 7(b) Crack-depth identification results

(a) Crack identification by the first-step PSO

(b) Crack-depth identification by the second-step PSO

Fig. 6 Crack identification of the first sample

When the natural-frequency data are disturbed by low-level noise, in the crack identification of sample No. 3, using the first four natural frequencies, the two-step PSO algorithm has a greater advantage in identifying x_D. In the identification of some values of a, the accuracy decreases, but the results can still serve as a basis for judging the crack depth. Sample No. 4 also shows similar results; however, it is a micro-crack, and both identification methods exhibit reduced accuracy. Under high-level noise interference, the two-step PSO algorithm maintains relatively high accuracy in identifying x_D for sample No. 2, but the identification of a shows a larger error than that in Ref. [9]. Meanwhile, in most cases, increasing the number of natural-frequency modes can improve the identification accuracy of the algorithm.

In summary, the two-step PSO algorithm can use natural-frequency data to identify crack parameters, and it avoids plotting frequency contours and calculating the intersection location. Under noise interference, the algorithm can identify x_D with relatively high accuracy; when noise increases, the identification of a is more susceptible to interference and its accuracy decreases.

(a) Crack-location identification results

(b) Crack-depth identification results

Fig. 7 Crack identification results for the second and the third samples

3 Experimental Testing and Results Analysis

3.1 Experimental Design

To further verify the reliability of the proposed algorithm for crack identification, modal testing and crack-identification verification were carried out on cracked cantilever-beam models. Referring to the dimensions of the beam specimens in Ref. [6], the experimental specimens were determined to have length, width, and thickness of 250, 16, and 6 mm, respectively. A milling-machine fixture was used to clamp a length of 15 mm to simulate the fixed constraint of a cantilever beam. The beam specimens were made of 45 steel, and the crack parameters were set as shown in Table 5.

Tab. 5 Crack parameters of cantilever beams

Group	x_D/mm	a/mm
1 (healthy)	–	–
2 (crack 2)	85	2
3 (crack 3)	85	3

In the modal test, both the force hammer and the acceleration sensor were products of PCB, with model numbers 086C03 and 352C34, respectively; the excitation and response data were recorded by a CoCo80 dynamic signal analyzer. The cantilever beam was excited with the force hammer at 50 mm from the fixed end, and the acceleration sensor placed at the free end acquired the response. At the same time, the acquisition instrument recorded the excitation and response signals, and the sampling rate was set to 10.24 kHz. Each cantilever-beam specimen was excited 5 times; 3 groups of data were selected and averaged, and the frequency-response function was calculated. Taking the third group of tests as an example, the test layout, excitation and response data, and frequency-response function are shown in Fig. 8.

3.2 Crack Identification and Results Analysis

Because the bandwidth of force-hammer excitation is limited, the first 4 natural frequencies of the cantilever beam were selected for crack identification. The natural-frequency data obtained from the experiments are shown in Table 6. At the same time, due to factors such as non-ideal actual constraints and environmental noise, deviations inevitably exist between the theoretically calculated natural-frequency values and the measured results. Therefore, to improve the accuracy of crack identification, impact tests were likewise performed on undamaged cantilever-beam specimens to obtain their natural frequencies f_{iU}.

Following the crack-identification procedure in Fig. 5, the two-step PSO algorithm was used to identify cracks in cantilever beams Nos. 2 and 3. The frequency contours and the two-step PSO identification results are shown in Figs. 9 and 10.

Analysis from Table 7 shows that, owing to the presence of test noise, although the first-step PSO accurately identified x_D of cantilever beam No. 2, the second-step PSO had an error of 11.50% in identifying a; meanwhile, the two-step PSO accurately identified both x_D and a for cantilever beam No. 3.

(a) Test layout

(b) Excitation and response

(c) Frequency-response function

Fig. 8(a) Test layout

Figure 3: Fig. 8(a) Test layout

Fig. 8(b) Excitation and response

Figure 4: Fig. 8(b) Excitation and response

Fig. 8 The third modal test

Table 6 The first four natural frequencies and Δf_{iD} of cantilever beams

Tab. 6 The first fourth frequencies and Δf_{iD} of cantilever beams

Group	f_1/Hz	Δf_{1D}/%	f_2/Hz	Δf_{2D}/%	f_3/Hz	Δf_{3D}/%	f_4/Hz	Δf_{4D}/%
1 (healthy)	77.50	–	500.00	–	1 376.25	–	2 828.75	–
2 (crack 2)	76.25	1.61	492.50	1.50	1 346.25	2.18	2 827.50	0.04
3 (crack 3)	75.00	3.23	483.75	3.25	1 315.00	4.45	2 820.00	0.31

(a) First-step PSO identification of crack

(b) Second-step PSO identification of crack depth

Fig. 9 Results of crack identification for the second cantilever beam

(a) First-step PSO identification of crack

(b) Second-step PSO identification of crack depth

Fig. 10 Results of crack identification for the third cantilever beam

Table 7 Analysis of test results

Tab. 7 Analysis of test results

Group	x_D/mm	PSO identification/mm	Error/%	a/mm	PSO identification/mm	Error/%
2 (crack 2)	85	86.22	1.44	2	2.23	11.50
3 (crack 3)	85	88.32	3.91	3	2.97	1.00

4 Conclusions

1) A dynamic model of a cantilever beam containing a single through crack is established. Based on the theory of solving frequency contour lines us-

Fig. 8(c) Frequency-response function

Figure 5: Fig. 8(c) Frequency-response function

Fig. 9 Results of crack identification for the second cantilever beam

Figure 6: Fig. 9 Results of crack identification for the second cantilever beam

Fig. 10 Results of crack identification for the third cantilever beam

Figure 7: Fig. 10 Results of crack identification for the third cantilever beam

ing the natural frequencies and the curvature function of the mode shape, a two-step PSO algorithm is proposed to identify the crack location and depth in sequence. This avoids the need to compute a large number of binary transcendental equations, draw frequency contour lines, and calculate intersection locations, thereby improving the efficiency of crack identification.

2) Numerical examples and tests show that the two-step PSO algorithm can identify crack parameters using natural-frequency data. Increasing the order of the natural frequencies can effectively improve the accuracy of crack identification. Under noise interference, the algorithm can identify the crack location with relatively high accuracy, while the identification accuracy for crack depth decreases to some extent.

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Note: Figure translations are in progress. See original paper for figures.