Study on a Damage Identification Method for Tied-Arch Bridges Based on Equivalent Thrust Influence Lines

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Abstract

Early tied-arch bridges can no longer meet current traffic load demands due to insufficient load ratings and maintenance levels, leading to continuous accumulation of structural damage and increasingly prominent safety risks. To accurately evaluate the structural influence-line characteristics of tied-arch bridges and identify damage in arch ribs and suspenders, new methods and indicators are proposed for damage identification of suspenders and arch ribs in tied-arch bridges. Based on the force-method equations, this study derives and establishes an analytical solution for the thrust influence line of a tied arch without suspenders that is equivalent to the thrust influence line of a two-hinged arch. Numerical simulation verifies that the proposed practical analytical solution has good analytical accuracy for planar tied-arch structures without suspenders with a rise-span ratio greater than 1/5. The curvature of the difference in equivalent thrust influence lines is used to verify damage identification for the tie girder, suspenders, and arch ribs of planar tied-arch structures, as well as the suspenders and arch ribs of tied-arch bridges. Combined with grey relational analysis, the noise robustness of the proposed indicator is analyzed, and a practical bridge health assessment procedure based on quasi-static influence-line loading by a three-axle heavy vehicle is established. Theoretical analysis and numerical verification show that, for planar tied-arch structures without suspenders with a rise-span ratio greater than 1/8, the error of the proposed practical analytical solution is less than 9.57%, and the error decreases as the rise-span ratio increases. The proposed equivalent thrust influence-line indicator can locate damage in suspenders and arch ribs of tied-arch bridges; comparison shows that the damage identification performance for suspenders is better than that for arch ribs, and the proposed method exhibits good noise robustness.

Full Text

Study on a Damage Identification Method for Tied-Arch Bridges Based on Equivalent Thrust Influence Lines

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Abstract: Early tied-arch bridges, because their load grades and maintenance levels cannot meet the demands of current traffic loading, have accumulated structural damage, and their safety risks have become increasingly prominent. To accurately evaluate the influence-line characteristics of tied-arch bridge structures and identify damage to arch ribs and suspenders, this paper proposes a new method and new indices for identifying suspender and arch-rib damage in tied-arch bridges. Based on the force-method equations, this study derives and establishes an analytical solution for the thrust influence line of a tied-arch bridge without suspenders that is equivalent to the thrust influence line of a two-hinged arch. Numerical simulation verifies that the proposed practical analytical solution has good analytical accuracy for suspender-free planar tied-arch structures with a rise-span ratio greater than 1/5. The curvature of the difference curve of the equivalent thrust influence line is used to verify damage identification for the tied rods, suspenders, arch ribs, as well as the suspenders and arch ribs of planar tied-arch structures. Combined with grey relational analysis, the noise resistance of the proposed indices is analyzed, and a practical procedure for bridge health assessment based on three-axle heavy-vehicle quasi-static influence-line loading is established. Theoretical analysis and numerical verification show that, for suspender-free planar tied-arch structures with a rise-span ratio greater than 1/8, the error of the proposed practical analytical solution is less than 9.57%, and the error decreases as the rise-span ratio increases. The proposed equivalent thrust influence-line index can locate suspender and arch-rib damage in tied-arch bridges. Comparison shows that suspender-damage identification is more effective than arch-rib damage identification, and the proposed method has good noise resistance.

Keywords: tied-arch bridge; influence line; equivalent thrust; damage identification; noise resistance

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Research on damage identification method for tied-arch bridges based on equivalent thrust influence line

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Abstract: Early tied-arch bridges cannot satisfy the current traffic load demand due to their load grades

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and maintenance levels. Also, these tied-arch bridges have accumulated structural damage with increasingly prominent safety risks. In order to accurately evaluate the influence-line characteristics of the tied-arch bridges' structure and identify the damage of arch ribs and suspenders, a new method and an index for identifying damage of arch ribs and suspenders of tied-arch bridges were presented. This study is based on the force method equation, an analytical solution equivalent to the thrust influence line of the two-hinged arch was derived and established. The numerical simulation proves that the proposed practical analytical solution has good analytical accuracy on the planar tied-arch structure without suspenders whose rise-span ratios greater than 1/5. With the use of equivalent thrust influence line difference curvature, a damage identification and verification method of the tie beam, suspender and arch rib of planar tied-arch structure, as well as the suspender and arch rib of tied-arch bridges was derived in this study, and with a combination of gray relation analysis, the noise immunity of the proposed index method was verified. A practical process of bridge health assessment based on quasi-static influence line loading of three-axle heavy vehicles is established. Theoretical analysis and numerical verification show that for those planar tied-arch structure without suspenders with a rise-span ratios greater than 1/8, the calculated error of the analytic solution is less than 9.57%, and the error decreases with the increase in the rise-span ratio. With the equivalent thrust influence line index proposed in this study, the damage of suspenders and arch ribs of tied-arch bridges can be accurately located. It has been found that the proposed method is more effective in identifying the damage of suspenders than identifying the damage of arch ribs, showing good

noise immunity.

Key words: tied-arch bridge; influence line; equivalent thrust; damage identification; noise immunity

As one of the most common bridge infrastructure types in China, arch bridges have reasonable structural force behavior and aesthetically pleasing forms[^1-3]. With the increasingly mature development of arch-bridge analysis theory, tied-arch bridges have been widely used because of their distinct load-transfer paths and large spanning capacity[^4-5]. In terms of structural force characteristics, the main arch structure of a tied-arch bridge is a two-hinged arch; the horizontal thrust at the arch springings is borne by the tie beam, and no horizontal external force is generated at the arch-foot supports. Therefore, the nonuniform settlement of the abutments has relatively little influence on the internal-force distribution of the structure, and the structure has stronger adaptability to site conditions. However, in recent years, accidents involving through and half-through tied-arch bridges have occurred frequently. They are commonly caused by failure of the main components (suspenders and arch ribs), which changes the load-transfer path and leads to structural instability and failure. As the suspenders and arch ribs are the principal load-bearing components in the tied-arch bridge structural system and the main path for transmitting deck loads, sudden damage to them will trigger major structural safety accidents[^6-8].

Faced with the increasing number of sudden accidents in arch bridges, scholars at home and abroad have carried out many valuable scientific studies[^9-10]. Fan et al.[^11] proposed a damage identification method based on displacement-difference influence lines for tied-arch bridges. They derived the displacement influence lines of tied-arch bridges through the force-method equations and verified, using finite-element models, the effectiveness of identifying suspender damage in tied-arch bridges by means of displacement-difference influence lines. Tang Liang et al.[^12] divided the area enclosed by an influence line into several elements and used the ratio of the sum of the squares of the areas of each element to the square of the total area of the element as the damage-localization index, thereby proposing a new method for bridge damage localization. Tang Shenghua et al.[^13] proposed a new damage-localization method through support reaction influence lines. Fan et al.[^14] proposed a robustness assessment framework considering cable failure. This framework is established based on the main analytical parameters of extreme events and limit states caused by cable rupture, and the method can be used to assess the safety of in-service bridges. Duan et al.[^15] proposed a convolutional-neural-network-based method for damage identification of suspenders in tied-arch bridges, and the robustness of this method has been demonstrated in various noise environments. Feng et al.[^16] introduced the Hermite–Ritz variational principle on the basis of the Ritz method, improved the rapid decision approximate solution for tied-arch bridge parameters, and applied this method for the first time to the deformation and force analysis of tied-arch bridges. Existing studies have provided theoretical support and methodological references for the calculation and health assessment of arch-bridge structures[^17-19]. Further establishing explicit analytical relationships between damage in tied-arch bridges and important mechanical indices remains an important approach for scientifically evaluating component damage states. Proposing accurate identification methods for damage to the principal load-bearing members of tied-arch bridges is a key scientific issue in solving their structural health assessment.

Because the horizontal thrust at the arch feet of a tied-arch bridge is balanced by the tie beam, the arch rib uses the prestressing applied by the tie beam to compensate the horizontal thrust of the arch, so that no horizontal external force is generated at the supports at the arch feet. Therefore, the axial-force influence line of the tie beam is the equivalent thrust influence line (equivalent thrust influence lines, ETIL), and the axial-force influence line of the tie beam is easier to test in practice. In view of the still imperfect status of research on influence lines in the health assessment of tied-arch bridges, this study derives, through the force-method equations, the analytical solution of the axial-force influence line of a variable-section two-hinged tied arch—namely, the equivalent thrust influence line—and establishes the tie-beam⋯

damage and the explicit expression of the equivalent thrust influence line for tied-arch bridges, proposes a new damage-identification index—the equivalent thrust influence lines difference curvature (ETILDC). Through numerical solutions and simulation examples, the accuracy of the derived analytical solution and the feasibility of its application in damage identification are verified, providing theoretical support and methodological reference for the design and health assessment of tied-arch bridges.

1 Theoretical Basis

The arch rib of a tied-arch bridge is the principal load-bearing member of the structure, and its mechanical calculation model is a typical variable-section two-hinge arch structure. The mechanical model is established as shown in Fig. 1. For the parabolic and catenary variable-section two-hinge arch-rib axis equations and the arch-axis geometric parameter k, we have

$$y = (4fx^2)/(2L)^2 \qquad (1)$$

$$y = f[\cosh(kx/L) - 1]/(m-1) \qquad (2)$$

$$k = \ln\left(m + \sqrt{m^2 - 1}\right) \qquad (3)$$

where L is the half-span of the two-hinge arch; f is the rise; m is the arch-axis coefficient (see *Highway Bridge and Culvert Design Manual—Arch Bridges (Part I)*); and x is the horizontal distance from any arch section to the arch crown.

Fig. 1 Basic system of two-hinge arch force method with tie bar

Figure 1: Fig. 1 Basic system of two-hinge arch force method with tie bar

Fig. 1 Basic system of two-hinge arch force method with tie bar

According to the Ritter formula, the arch-rib section is set [20]. See Eqs. (4) and (5). The variable-section heights of two arch-axis types are given by Eqs. (6) and (7).

$$I = I_0/[(1 - x/L + nx/L)\cos\varphi] \qquad (4)$$

$$A = A_0\{[1 - (1 - n)x/L]\cos\varphi\}^{-1/3} \qquad (5)$$

Parabolic type:

$$h = \left[h_0(4f^2x^2 + L^4)^{1/2}/(L^2 - xL + nxL)\right]^{1/3} \qquad (6)$$

Catenary type:

$$h = \left\{\left[h_0^3\left(1 + (k^2f^2(\sinh(kx/L))^2)/((m-1)^2 - L^2)\right)^{1/2}\right]/(1 - x/L + nx/L)\right\}^{1/3} \qquad (7)$$

where n is the variation coefficient of the arch-rib section; φ is the horizontal inclination angle of the arch section; I_0 and h_0 are respectively the moment of inertia at the arch crown and the section height at the arch crown; and A_0 is the sectional area at the crown position of the arch rib.

2 Derivation of the Analytical Solution

The redundant horizontal constraint at the arch springing is now released and replaced by a redundant force F_H. F_H is both the thrust at the springing of the two-hinge arch and the axial force in the tie bar of the tied two-hinge arch. The upper arch-rib forms of the tied two-hinge arch and the untied two-hinge arch are the same. However, when calculating δ_{11} for the tied two-hinge arch structure, the deformation of the tie bar must be considered, as shown in Eq. (8).

Self-displacement:

$$\delta_{11} = \int_s \frac{M_1^2}{EI}\mathrm{d}s + \int_s \frac{N_1^2}{EA}\mathrm{d}s + \frac{l}{E^*A^*} \qquad (8)$$

Load displacement:

$$\Delta_{1p} = \int_s \frac{M_1 M_p}{EI} \mathrm{d}s \qquad (9)$$

where E is the elastic modulus of the arch rib; E^* is the elastic modulus of the tie bar; A^* is the cross-sectional area of the tie bar; l is the length of the tie bar, and $l = 2L$.

The bending moment produced by a unit force is

$$M_1 = -(f - y) \qquad (10)$$

The axial force produced by a unit force is

$$N_1 = -\cos\varphi \qquad (11)$$

The bending moment produced by the external load is as follows.

When $-L < x < x_p$,

$$M_P = \left(\frac{1}{2} - \frac{x_p}{2L}\right)(L + x) \qquad (12)$$

When $x_p < x < L$,

$$M_P = \left(\frac{x_p}{2L} + \frac{1}{2}\right)(L - x) \qquad (13)$$

Considering that the effect of axial deformation on arch-rib displacement is small, and to facilitate analytical calculation, the axial deformation of the arch rib is not considered in the derivation; only the bending deformation of the arch rib and the axial deformation of the tie bar are considered, as shown in Eq. (14).

$$\delta_{11} = \int_s \frac{M_1^2}{EI} \mathrm{d}s + \frac{2L}{E^* A^*} \qquad (14)$$

Self-displacement calculation formula:

$$\delta_{11} = \int_{-L}^{L} \frac{(f - y)^2}{EI \cos\varphi} \mathrm{d}x + \frac{2L}{E^* A^*} \qquad (15)$$

Load-displacement calculation formula:

$$\Delta_{1p} = \int_{-L}^{x_p} \left(\frac{1}{2} - \frac{x_p}{2L}\right)(L+x)(y-f)/EI\cos\varphi\,\mathrm{d}x + \int_{x_p}^{L} \left(\frac{x_p}{2L} + \frac{1}{2}\right)(L-x)(y-f)/EI\cos\varphi\,\mathrm{d}x \qquad (16)$$

where x_p is the load position; y is the mathematical expression of the arch-axis shape; φ is the horizontal inclination angle of the arch section; and x is the horizontal distance from any section of the arch structure to the arch crown.

Equations (15) and (16) are fitted using Eq. (17), giving the fitting equations (18) and (19) for the parabolic and catenary curves.

$$\mathrm{d}s = 1/\cos\varphi\,\mathrm{d}x \qquad (17)$$

Parabolic-line fitting

$$\cos\varphi = L^2/(L^4 + 4f^2x^2)^{1/2} \qquad (18)$$

Catenary-line fitting

$$\cos\varphi = \{1 + k^2f^2[\sinh(kx/L)]^2/(m-1)^2L^2\}^{-1/2} \qquad (19)$$

Expression for the thrust influence line

$$F_H = -\Delta_{1p}/\delta_{11} \qquad (20)$$

Substituting Eqs. (1), (4), (18), and (2), (4), (19) into Eqs. (15) and (16), respectively, yields the analytical expressions for the displacement of a two-hinged tied arch, as shown in Table 1. Substituting Eqs. (15) and (16) into Eq. (20) gives the analytical solution of the tied-arch two-hinged arch thrust influence line, i.e., the analytical solution of the ETIL for a two-hinged tied arch, as shown in Table 2.

Table 1 Analytical expression of displacement

Tab. 1 Analytical expression of displacement

Displacement	Parabolic line	Catenary line
δ_{11}	$16f^2L/15EI_0+2L/E^*A^*$	$[f^2L/4kEI_0(m^2-2m+1)](8km^2+8e^{-k}m-8e^{km}-e^{-2k}+e^{2k}+4k)+2L/E^*A^*$

Displacement	Parabolic line	Catenary line
Δ_{1p}	$[-5f(L+x_p)(L-x_p)/12L^3EI_0]\,[3x_p^3(1-n)/25-Lx_p^2/5+7L^2(n-1)x_p/25+L^3]$	$-f/2Lk^3EI_0(m-1)\{[(k+2n-2)L+kx_p(n-1)]L^2e^{-kx_p/L}-[(k-2n+n)L+kx_p(n-1)]L^2e^{kx_p/L}+[kL+x_p(n-1)(k+2)]L^2e^{-k}+[kL+x_p(n-1)(k-2)]L^2e^{k}-k^3m[(L-x_p)(x_p+L)(L+nx_p/3-x_p/3)]\}$

Table 2 Practical analytical solution of equivalent thrust influence line

Tab. 2 Practical analytical solution of equivalent thrust influence line

Parabolic line	Catenary line
$f(7nx_pL^2-3nx_p^3+25L^3-7x_pL^2-5x_p^2L+3x_p^3)(L^2-x_p^2)E^*A^*/[(64f^2E^*A^*+120EI_0)L^4]$	$f\{[(k+2n-2)L+kx_p(n-1)]L^2e^{-kx_p/L}+L^2e^{kx_p/L}[(k-2n+2)L+kx_p(n-1)]-L^2e^{-k}[kL+x_p(n-1)(k+2)]-[kL+x_p(n-1)(k-2)]L^2e^{k}+k^3m(L-x_p)(L+x_p)(L+nx_p/3-x_p/3)\}/\{2Lk^3(m-1)EI_0[f^2L(8km^2+8e^{-k}m-8e^{km}-e^{-2k}+e^{2k}+4k)/4kEI_0(m^2-2m+1)+2L/E^*A^*]\}$

3 Analysis of Analytical-Solution Accuracy

Using finite element software, ideal mechanical models of two-hinged tied arches with parabolic and catenary lines were established for five rise-span ratios: 1/4, 1/5, 1/6, 1/7, and 1/8. The structural span was 50.934 m; the arch-rib section was 1 m × 1 m; and the tie-girder section was 1 m × 1 m. The arch rib used C40 concrete, and the tie girder used C50 concrete. The variable-section groups of the arch rib were determined according to Eqs. (6) and (7).

The complete model had 158 nodes and 158 elements. The arch-rib structure was divided into 108 beam elements, and the tie-girder element length was 1.018 68 m. Quasi-static influence-line loading was applied to the arch rib, with a loading step length of 0.5 m, giving a total of 109 loading steps (Fig. 2).

Fig. 2 Finite element model of two-hinged tied arch

The equivalent thrust influence line of the two-hinged tied arch was extracted through measurement point 1. Taking the horizontal distance from the point of

Fig. 2 Finite element model of two-hinged tied arch

Figure 2: Fig. 2 Finite element model of two-hinged tied arch

Fig. 3

Figure 3: Fig. 3

application of the unit force to the arch crown as the independent variable and the influence-line coefficient as the dependent variable, the equivalent thrust influence-line curve was plotted, as shown in Fig. 3.

As can be seen from Fig. 3, the analytical value of the equivalent thrust influence line of the tied arch is smaller than the analytical value of the thrust influence line of the arch without a tie girder[^21], and the amplitudes of the equivalent thrust influence lines of the parabolic and catenary two-hinged tied arches are close. From Eqs. (15) and (20), it can be known that when the elastic modulus of the tie girder is greater than that of the arch rib, the analytical solutions with and without the tie girder are basically the same, and their stress states are close. Assuming that the numerical solution is the true value S and the analytical solution is the approximate value $\bar{S}$, the relative error $\tilde{S}$ is calculated by Eq. (21). Substituting the two groups of curves in Fig. 3(a)–Fig. 3(e) and Fig. 3(f)–Fig. 3(j) into Eq. (21), it can be seen that, for parabolic and catenary two-hinged tied arches under different rise–span ratios, the maximum relative error is 10.29% (for the catenary two-hinged tied arch with a rise–span ratio of 1/8). Thus, as the rise–span ratio decreases, the difference between the finite element numerical solution and the analytical solution gradually increases. In tied-arch structures with a rise–span ratio less than 1/5, in addition to bending deformation, axial deformation should also be considered. Therefore, the analytical solution of the equivalent thrust influence line of a two-hinged tied arch has higher analytical accuracy in steep-arch structures with a rise–span ratio greater than 1/5. As shown in Fig. 4, the relative error between the numerical and analytical solutions of the equivalent thrust influence line for the key section of the two-hinged tied arch is negatively correlated with the rise–span ratio.

In-figure labels: vertical axis: Influence line; horizontal axis: Load position / m. Legend: Numerical solution with tie; Analytical solution without tie; Analytical solution with tie.

(a) Parabolic two-hinged arch, rise-span ratio 1/4

(b) Parabolic two-hinged arch, rise-span ratio 1/5

(c) Parabolic two-hinged arch, rise-span ratio 1/6

(d) Parabolic two-hinged arch, rise-span ratio 1/7

(e) Parabolic two-hinged arch, rise-span ratio 1/8

(f) Catenary two-hinged arch, rise-span ratio 1/4

(g) Catenary two-hinged arch, rise-span ratio 1/5

(h) Catenary two-hinged arch, rise-span ratio 1/6

(i) Catenary two-hinged arch, rise-span ratio 1/7

(j) Catenary two-hinged arch, rise-span ratio 1/8

Fig. 3 Comparison of numerical and analytical results for the equivalent thrust influence line of a two-hinged tied arch

Fig. 3 Comparison of numerical and analytical results of equivalent thrust influence line of two-hinged tied arch

$$\bar{S} = \Delta S/S \times 100\% = \left|S - \bar{S}\right|/S \times 100\% \qquad (21)$$

According to Fig. 4, the relative error increases as the rise-span ratio decreases. When the rise-span ratio is 1/7, the maximum relative error at the critical section is 9.57%. Therefore, the analytical solution of the equivalent thrust influence line of the two-hinged tied arch derived here is recommended for parabolic and catenary two-hinged arch structures with a rise-span ratio greater than 1/7. The relative-error pattern can be seen in Fig. 4: the relative errors between the analytical solution and the numerical solution are smallest at the midspan section of the arch structure and at the $l/8$ section; among the five different rise-span ratios, the relative error at the critical section is greatest when the rise-span ratio is 1/8.

When the load is applied at the midspan position, i.e., at the value 0 on the horizontal axis in Fig. 3, the curves of the numerical solution with tie and the analytical solution in Fig. 3 show a relatively large deviation. However, because the amplitude of the influence-line value of the numerical-solution curve with tie is largest when the load is applied at the coordinate value 0, the relative error is calculated separately when⋯

the denominator is relatively large; therefore, in Fig. 4, the calculated relative error corresponding to the midspan position is relatively small. Although the analytical solution curves of the coefficient of each hanger at the other positions deviate less from the analytical solution curve at midspan, the denominator is also relatively small when calculating the relative error; therefore, the calculated relative error is relatively large.

(a) Relative error of crucial sections in the parabolic-type analytical solution

(b) Relative error of crucial sections in the catenary-type analytical solution

Fig. 4 Comparison of relative errors of crucial sections of the two arch axis types

4 Establishment of a Damage Identification Method for Planar Tied-Arch Bridges

According to the ETIL analytical solutions of the parabolic line and catenary line in the undamaged state in Table 2, damage is simulated by reducing the elastic modulus of the damaged element[1]. The elastic modulus at the damaged arch rib is defined as E', the elastic modulus at the damaged tie beam as $E^{*'}$, and the ETIL in the damaged state as F_H^*. According to the curvature formula $1/\rho(x_p) = |F_H''|/(1+F_{H'}^2)^{3/2}$, where F_H' can be approximated as 0 and F_H'' is not 0; hence the curvature calculation formula can be simplified to $1/\rho(x_p) = |F_H''|$. Therefore, the difference curvature of the ETIL before and after damage can be calculated using the simplified formula $1/\rho(x_p) = |\Delta F_H''|$, $\Delta F_H = F_H - F_H^*$. When the moving load is located in the damaged region, $(F_H - F_H^*)''$ is not 0 and the curve undergoes a sudden change, which proves that the difference-curvature index of the thrust influence line can be used for damage localization[2].

It can be seen from Table 3 that the ETILDC of the damaged segment is not 0. This basic theory can be used to demonstrate the feasibility of the ETILDC index for locating damage in the load-bearing members of a planar tied-arch bridge. Because the presence of hangers makes the load-transfer path of the planar tied-arch system clear, in theory, hanger damage will be reflected through the ETILDC. Therefore, in order to study the damage identification performance of the ETILDC index for a planar tied-arch bridge, a planar tied-arch bridge model with hangers is established. The material is C50, the span is 63.8 m, the arch rise is 11.86 m, and the arch-axis coefficient m is 1.4. Quasi-static influence-line loading is performed on the planar tied-arch bridge with hangers, and element No. 1 is selected as the measuring point, as shown in Fig. 5. According to the 28 elements of the tie beam, 29 loading steps are set. The tie-beam section is a single-cell single-box section, and the hangers are preset with an initial tensile force, as shown in Fig. 6[3]. Damage conditions with damage degrees of 10%, 30%, and 50% are set for the midspan tie beam, the midspan arch rib, and the midspan hanger (Table 4) to study the damage identification performance of ETILDC.

Table 3 ETILDC expressions at damage location under two types of arch axis

[1] Reference [22] as cited in the source page.
[2] References [23-24] as cited in the source page.
[3] Reference [25] as cited in the source page.

Fig. 5 Schematic diagram of quasi-static influence-line loading of planar tied arch

Figure 4: Fig. 5 Schematic diagram of quasi-static influence-line loading of planar tied arch

Fig. 6 Initial tension of suspenders

Figure 5: Fig. 6 Initial tension of suspenders

Damage location	Parabolic type	Catenary type
Arch-rib damage	$225I_0 f A^* E^*(E - E')(L - x_p)(L + x_p)(x_p n + L - x_p)/2L^4(8f^2E^*A^* + 15EI_0)(8f^2E^*A^* + 15E'I_0)$	$d^2\{-aE^*A^*L^2k^3I_0(m-1)(E-E')/4[L^2k^3(m-1)EI_0+E^*A^*b/4][L^2k^3(m-1)E'I_0+E^*A^*b/4]\}/dx_p^2$
Tie-beam damage	$225I_0 f A^* E(E^{*'} - E^*)(L - x_p)(L + x_p)(x_p n + L - x_p)/2L^4(8f^2E^*A^* + 15EI_0)(8f^2E^{*'}A^* + 15EI_0)$	$d^2\{aEA^*L^2k^3I_0(m-1)(E^*-E^{*'})/4[L^2k^3(m-1)EI_0+E^*A^*b/4][L^2k^3(m-1)EI_0+E^{*'}A^*b/4]\}/dx_p^2$

Note:
$a = f\{[(k+2n-2)L+kx_p(n-1)]L^2e^{-kx_p/L} + L^2e^{kx_p/L}[(k-2n+2)L+kx_p(n-1)] - L^2e^{-k}[kL + x_p(n-1)(k+2)] - [kL + x_p(n-1)(k-2)]L^2e^k + k^3m(L-x_p)(L+x_p)(L+nx_p/3-x_p/3)\}$;
$b = L^2k^2f^2(m-1)(8km^2 + 8e^{-k}m - 8e^km - e^{-2k} + e^{2k} + 4k)/2(m^2-2m+1)$.

Fig. 5 Schematic diagram of quasi-static influence line loading of planar tied arch

Fig. 6 Initial tension of suspenders[25]

Table 4 Damage condition of single planar arch

Loading method	Damage location	Damage degree
Tie-girder quasi-static influence-line loading	Damage to midspan tie girder	10%, 30%, 50%
Tie-girder quasi-static influence-line loading	Damage to midspan arch rib	10%, 30%, 50%
Tie-girder quasi-static influence-line loading	Damage to midspan suspender	10%, 30%, 50%

Fig. 7 ETILDC curves for damage identification of planar tied arch

Figure 6: Fig. 7 ETILDC curves for damage identification of planar tied arch

Fig. 8 Quasi-static loading diagram of tied arch bridge

Figure 7: Fig. 8 Quasi-static loading diagram of tied arch bridge

It can be seen from Fig. 7 that the ETILDC has good localization capability for the midspan tie girder, arch rib, and suspender of a planar tied-arch bridge. When load-bearing components at the midspan position have the same degree of damage, the ETILDC identification effect for the tie girder is superior to that for the suspender, and the suspender identification effect is superior to that for arch-rib damage. Comparing the ETILDC curves in Figs. 7(a)–7(c), the curve mutation is more obvious for damage to the tie girder and suspender. When the ETILDC curves reflect the same damage degree, the magnitudes of the ETILDC curves for different load-bearing components differ. Therefore, for a planar tied arch, the damaged load-bearing component can be roughly judged according to the magnitude and trend of the ETILDC curve. The results show that the ETILDC index has a good damage-identification effect for the main load-bearing components of a planar tied-arch segment.

(a) ETILDC curve for damage to the midspan tie girder

(b) ETILDC curve for damage to the midspan arch rib

(c) ETILDC curve for damage to the midspan suspender

Fig. 7 ETILDC curves for damage identification of planar tied arch

5 Example verification of damage identification for tied-arch bridge

In actual engineering, it is difficult to achieve direct loading on the arch ribs of a through tied-arch bridge. In this study, by loading a moving unit force on the bridge deck, the equivalent thrust influence line of the main arch structure of the tied-arch bridge is extracted, and the effect of using the equivalent thrust influence line to identify damage in the main load-bearing components of the tied-arch bridge is further verified. A tied-arch bridge model is established with a span of 60 m, a rise of 12 m, and 11 pairs of suspenders. A quasi-static moving concentrated force is adopted as the loading model; 121 loading steps are set, with a loading-step length of 0.5 m, as shown in Fig. 8.

Fig. 8 Quasi-static loading diagram of tied arch bridge

Fig. 9 Number of damaged suspenders

Figure 8: Fig. 9 Number of damaged suspenders

Fig. 10 Damage identification results of suspenders

Figure 9: Fig. 10 Damage identification results of suspenders

5.1 Damage Identification Analysis for Suspenders

Damage cases 1-6 (Table 5) are set up. The numbering of suspender locations is shown in Fig. 9, and the measuring point is selected as element 25 near the arch springing of the tied-arch bridge.

Fig. 9 Number of damaged suspenders

As can be seen from Fig. 10(a)-Fig. 10(c), the loading steps corresponding to the amplitudes in the ETILDC curves are the 31#, 61#, and 91# loading steps, respectively. The 31#, 61#, and 91# loading steps act on the crossbeams where suspenders 3#, 6#, and 9# are located, respectively, and the damaged suspenders can be identified. Comparing Fig. 10(a) with Fig. 10(d), Fig. 10(b) with Fig. 10(e), and Fig. 10(c) with Fig. 10(f), it can be seen that, when a pair of suspenders on the same crossbeam has the same degree of damage, the amplitude of the ETILDC curve for the damaged suspender closer to the measuring point is larger than that for the suspender farther from the measuring point, and the identification effect is better. The results show that the ETILDC index can accurately locate the damaged suspender position in a through tied-arch bridge.

Table 5 Damage condition of suspenders of tied arch bridge

Case No.	Measuring point	Damage location	Damage degree
Case 1	Element 25	Suspender 3#	10%, 30%, 50%
Case 2	Element 25	Suspender 6#	10%, 30%, 50%
Case 3	Element 25	Suspender 9#	10%, 30%, 50%
Case 4	Element 25	Suspender 14#	10%, 30%, 50%
Case 5	Element 25	Suspender 17#	10%, 30%, 50%
Case 6	Element 25	Suspender 20#	10%, 30%, 50%

(a) Case 1

(b) Case 2

(c) Case 3

(d) Case 4

(e) Case 5

(f) Case 6

Fig. 11 Arch rib damage identification results

Figure 10: Fig. 11 Arch rib damage identification results

Fig. 10 Damage identification results of suspenders

5.2 Analysis of Arch-Rib Damage Identification

The arch-rib damage setting conditions are Conditions 7–12, as shown in Table 6.

Table 6 Damage conditions of the arch rib of the tied-arch bridge

Tab. 6 Damage condition of arch rib of tied arch bridge

Condition No.	Measurement point	Damage location	Damage degree
Condition 7	Unit 25	Unit 7	10%, 20%, 30%, 40%, 50%
Condition 8	Unit 25	Unit 19	10%, 20%, 30%, 40%, 50%
Condition 9	Unit 25	Unit 9	10%, 20%, 30%, 40%, 50%
Condition 10	Unit 25	Unit 22	10%, 20%, 30%, 40%, 50%
Condition 11	Unit 25	Unit 502	10%, 20%, 30%, 40%, 50%
Condition 12	Unit 25	Unit 514	10%, 20%, 30%, 40%, 50%

It can be seen from Fig. 11 that peaks appear in the ETILDC curves between two loading steps; therefore, in the finite-element model, damage exists in the arch rib corresponding to that loading segment. In Condition 7, when the damage is 50%, the maximum damage amplitude is close to 2.5×10^{-5}; in Condition 8, when the damage is 50%, the maximum damage amplitude is close to 2.5×10^{-6}. Therefore, when damage of the same degree occurs at the same position on both sides of the tied-arch bridge arch rib, the damage identification effect is better for damage on the same side as the measurement point. Comparing Fig. 11(e) and Fig. 11(f), it can be seen that the identification effect is better for damage on the same side as the measurement point. Comparing Fig. 11(a), Fig. 11(c), and Fig. 11(e), it can be seen that, when the damage degree is the same, for damage at different locations, the identification effect is best for damage at the mid-span location. At a 50% damage degree, the amplitudes of the ETILDC curves at bridge-span positions 1/2, 1/4, and 3/4 are 2.44246×10^{-5}, 1.46345×10^{-5}, and 9.29704×10^{-6}, respectively. Excluding the mid-span unit, for which the identification effect is the best, the distance between the measurement point and the damage location is negatively correlated with the identification effect. Therefore, the results from a single measurement point can be complemented by increasing the number of measurement points. For the same condition, the identification effect of the ETILDC index is positively correlated with the damage degree at the same location.

(a) Condition 7

(b) Condition 8

(c) Condition 9

(d) Condition 10

(e) Condition 11

(f) Condition 12

Fig. 11 Arch rib damage identification results

It should be particularly noted that the positive or negative sign of the indicator is related to the computational processing of the curvature of the difference curve; therefore, only the absolute magnitude of the indicator amplitude is considered in the analysis.

6 Verification of the Noise Resistance of the Indicator

Test noise or environmental factors may cause deviations between real data and actual measured values. Therefore, noise was introduced to verify the noise resistance of the equivalent thrust influence lines difference (ETILD). For the influence-line data under Case 1 when hanger 3# has 10% damage, noise at a level of 5% was added to simulate the deviations occurring in actual measurements. Case 13 was defined as the noisy case. The noise-introduction method is given in Eq. (22):

$$T_i^N = T_i \cdot [1 + \mu \cdot \mathrm{RAND}(-1, 1)] \qquad (22)$$

where T_i^N is the thrust data containing noise at the i-th loading step; T_i is the thrust data without noise at the i-th loading step; $\mathrm{RAND}(-1, 1)$ is a random number obeying a standard normal distribution; and μ is the noise-intensity level.

According to Fig. 12, the ETILD curve with noise is generally consistent with the ETILD curve without noise. To further investigate the noise-resistance performance of the thrust influence line, grey relation analysis (GRA) [$25-26$] was adopted to evaluate the anti-noise stability of the damage-identification indicator [27]. As a multifactor statistical analysis method, grey relation analysis is based on control theory: color indicates the amount of known information about a system. White represents sufficient information, black represents a system whose internal structure is unclear, and grey lies between the two, indicating that only part of the system is understood [$28-29$]. The key to a grey system is to determine the parent sequence. The data are nondimensionalized by the mean-value method, and the grey relational coefficient is calculated by Eq. (23).

Fig. 12 Influence of noise on ETILD

Fig. 12 Influence of noise on ETILD

Figure 11: Fig. 12 Influence of noise on ETILD

$$\xi_i(k) = \frac{\min_i \min_k |\gamma(k) - x_i(k)| + \rho \cdot \max_i \max_k |\gamma(k) - x_i(k)|}{|\gamma(k) - x_i(k)| + \rho \cdot \max_i \max_k |\gamma(k) - x_i(k)|} \quad (23)$$

where $\xi_i(k)$ is the relational coefficient between x_i and $\gamma(k)$ at point k; $|\gamma(k) - x_i(k)|$ is the absolute difference between γ and x_i at the k-th point; $\min_i \min_k |\gamma(k) - x_i(k)|$ is the minimum absolute value of the secondary difference between the sequence γ and the sequence x_i at point k; $\max_i \max_k |\gamma(k) - x_i(k)|$ is the maximum absolute value of the secondary difference between the sequence γ and the sequence x_i at point k; and ρ is the grey distinguishing coefficient, ranging from 0 to 1, generally taken as 0.5. Substituting the relational coefficients of each condition into Eq. (24) yields the relational degree r_i between x_i and $\gamma(k)$:

$$r_i = \frac{1}{n} \prod_{k=1}^{n} \xi_i(k) \quad (24)$$

The noise-introduced Case 13 was again taken as the sample for calculating the grey relational coefficient. The correlation of the data was reflected through the grey relational coefficient, and the magnitude of the grey relational coefficient can be used to study the degree of association between the noise-free data and the noisy data. The calculations show that the grey relational coefficients without noise and with noise are 0.717 3 and 0.636 5, respectively. Thus, compared with the ETILD in the noise-free state, the ETILD with 5% noise has an error of 11.26%, verifying that the use of the equivalent thrust influence-line indicator for identifying damage in tied-arch bridges has good noise resistance.

7 Practical Procedure for Damage Identification in Tied-Arch Bridges

Because it is difficult in influence-line testing of tied-arch bridges to apply a uniaxial concentrated load to simulate a unit concentrated force on the influence line, this study proposes a three-step loading procedure based on moving-load superposition and subtraction. It can be used for rapid bridge-damage diagnosis under short-term traffic interruption [30].

1) Select three three-axle loading vehicles as the loading tools (Fig. 13). The three loading vehicles should have the same axle spacing, but their front-to-rear axle weight ratios should be different. The front and rear axles of each vehicle can be simplified as concentrated forces of different magnitudes at the corresponding positions. During the loading process,

Fig. 13 Practical process operation steps

Figure 12: Fig. 13 Practical process operation steps

it is necessary to consider providing effective excitation to the tied-arch bridge, so as to ensure that the measured structural response values are stable and effective.

2) Move the three vehicles rapidly across the bridge deck over the tied-arch bridge at uniform speed. The virtual loading-node positions of the tied-arch bridge in the three loadings are required to be the same; this can be achieved by controlling the speeds of the loading vehicles. Meanwhile, the three thrust response datasets are collected.

3) Calculate the least common multiples M_1, M_2, and M_3 of the equivalent concentrated forces F_1, F_4, and F_7 of the front axles of the three vehicles, and then amplify the equivalent concentrated forces of the rear axles of the three vehicles by the corresponding multiples and subtract them to obtain substitute concentrated forces $|M_1F_1 - M_2F_4|$ and $|M_2F_4 - M_3F_7|$. Similarly, determine the virtual loading ⋯

concentrated loads, the minimum common multiples M_4 and M_5 are found, and the corresponding multiples are amplified to obtain the equivalent loading concentrated force:

$$|M_4 \left|M_1F_1 - M_2F_4\right| - M_5 \left|M_2F_4 - M_3F_7\right|| .$$

The thrust influence-line data $f_1(x)$, $f_2(x)$, and $f_3(x)$ measured in three runs are amplified by the same multiples according to M_1, M_2, and M_3, and then differenced after amplification to obtain the corresponding thrust influence line under the equivalent concentrated-force loading:

$$|M_4 \left|M_1f_1(x) - M_2f_2(x)\right| - M_5 \left|M_2f_2(x) - M_3f_3(x)\right|| ,$$

which is used to diagnose damage in tied-arch bridges.

A vehicle: F_3, F_2, F_1

B vehicle: F_6, F_5, F_4

C vehicle: F_9, F_8, F_7

$$F_1 : F_2 : F_3 \neq F_4 : F_5 : F_6 \neq F_7 : F_8 : F_9$$

For the three influence-line plots: vertical axis, "Thrust response/kN"; horizontal axis, "Load position/m" ; curves $f_1(x)$, $f_2(x)$, and $f_3(x)$.

Step 1: Determine the minimum common multiples of F_1, F_4, and F_7: M_1, M_2, and M_3.

Step 2: After amplifying the corresponding multiples for vehicle A and vehicle B, and for vehicle B and vehicle C, take the differences:

$$|M_1 f_1(x) - M_2 f_2(x)| = f_{11}$$

$$|M_2 f_2(x) - M_3 f_3(x)| = f_{12}$$

Step 3: Determine the minimum common multiples of f_{11} and f_{12}: M_4, M_5.

Step 4: After amplifying the corresponding multiples for vehicle A and vehicle B, and for vehicle B and vehicle C, take the difference:

$$|M_4 f_{11} - M_5 f_{12}|$$

图 13 实用流程操作步骤

Fig. 13 Practical process operation steps

8 Conclusions

1) The study found that, as the rise-span ratio decreases, the effect of axial deformation of the arch rib on the self-displacement of the arch structure gradually increases. When the rise-span ratio is greater than 1/8, the calculation error is less than 9.57%. Therefore, the derived equivalent thrust influence-line practical analytical solution can be effectively used for computational analysis of through-type hingeless plane tied-arch structures with a rise-span ratio of $1/4 \sim 1/5$.

2) The curvature of the difference curve of the equivalent thrust influence lines obtained analytically can be effectively used for identifying sheet-type damage in plane tied-arch ribs. For the main load-bearing components, the effectiveness of damage identification is, in descending order: tie beam, hanger, and arch rib. In practical testing, the influence-line measurement points can be arranged at the arch-rib unit close to the arch foot, so as to obtain the influence line of the equivalent thrust at the arch foot.

3) A curvature method based on the difference of equivalent thrust influence lines is established. In application verification for identifying damage to hangers and struts at different positions of a through-type tied-arch bridge, it is effective, and is especially suitable for health-state assessment of hangers with relatively high safety risk. The damaged component is judged from the trend of the index curve, and the damage degree of different components is qualitatively determined according to the index amplitude. Grey correlation analysis shows that the proposed method has good noise resistance.

4) A practical procedure for bridge health assessment based on quasi-static influence lines is proposed. It is applicable to rapid testing of influence lines

of the main structural components of bridges under three-axle truck emergency moving loading. Combined with the damage-identification method proposed in this study and existing bridge monitoring equipment, it is expected to promote, in future development, the routine evaluation of the health condition of in-service bridges.

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