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Abstract

Interface debonding is one of the common failure modes of solid rocket motor grains, and the cohesive zone model is an important approach for studying debonding at the motor propellant/liner interface. First, load-displacement curves at six different strain rates were obtained through pull-off tests on standard rectangular specimens, confirming that the failure mode of the specimens was adhesive failure, and the main characteristics of the load-displacement curves were summarized. Second, a response surface method was used to establish an approach for obtaining globally optimal parameters. The optimal parameters of the bilinear and exponential cohesive zone models were inversely identified using this method, and the advantages and disadvantages of the two models were indicated by comparison with the experimental curves. Finally, the exponential cohesive zone model was incorporated into a finite element simulation of a tubular motor to investigate the effects of cohesive parameters, loading rate, interface stiffness, and shell/propellant modulus on interface debonding behavior. The results show that stiffness and fracture energy have a significant influence on the debonding length, whereas the maximum strength affects only the maximum peel force. The loading rate has a pronounced effect on both the maximum peel force and the peel length. The elastic modulus of the propellant and the interface stiffness have a significant influence on the radial stress of the tubular motor, whereas the elastic modulus of the shell has no influence on this stress.

Full Text

Cohesive Model of the Propellant/Liner Interface Under Different Strain-Rate Conditions and Its Application in Debonding Simulation

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Abstract: Interface debonding is one of the common failure modes in solid rocket motors, and the cohesive model is an important method for studying debonding at the motor propellant/liner interface. First, load-displacement curves under six different strain rates were obtained through peel tests of standard rectangular specimens, confirming that the failure mode of the specimens was cohesive failure, and the main characteristics of the load-displacement curves were summarized. Next, the response surface method was used to establish a method for obtaining the overall optimal parameters. Through this method, the optimal parameters of the bilinear and exponential cohesive models were obtained by inverse analysis. Comparison with the experimental curves

indicated the advantages and disadvantages of the two models. Finally, the exponential cohesive model was incorporated into finite element simulation of a cylindrical motor, and the effects of cohesive parameters, loading rate, interface stiffness, shell/propellant modulus, and other factors on interface debonding behavior were investigated. The results show that stiffness and fracture energy have a relatively large influence on debonding length, whereas the maximum strength affects only the maximum peel force; loading rate has a significant influence on both the maximum peel force and debonding length; the elastic modulus of the propellant and the interface stiffness have a relatively large influence on the stress in the radial direction of the cylindrical motor, whereas the elastic modulus of the shell has no influence on this stress.

Keywords: cohesive model; response surface method; debonding

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The bilinear cohesive model of propellant/filling layer interface under different strain rates and its application in the simulation of debonding

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Abstract: Interface debonding is one of the most common failure modes in solid rocket motors, and the Cohesive zone model (CZM) is an important method for studying the debonding of the engine propellant/liner interface. Six different strain rate load-displacement curves were obtained through standard rectangular specimen peel tests, confirming that the failure mode of the propellant/liner bonding interface is cohesive failure. The main features of the load-displacement curves were summarized. Then, a method to obtain the

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overall optimal parameters was established using the response surface method. The optimal parameters of the bilinear cohesive model and the exponential cohesive model were inverted by this method. By compared with the experimental curves, the advantage and disadvantage of these two models were presented. Finally, the exponential cohesive model was incorporated into the finite element model of a cylindrical motor to investigate the influence of cohesive force parameters, loading rate, interface stiffness, and shell/propellant modulus on debonding behavior. The results indicate that stiffness and fracture energy have a significant impact on debonding length, while maximum strength only affects the maximum peel force. Loading rate has a noticeable effect on both maximum peel force and debonding length. The elastic modulus of the propellant and the interface stiffness have a significant influence on the stress in the radial direction of the cylindrical motor, whereas the shell modulus has no effect.

Key words: cohesive zone model; response surface method; debonding

Interfacial^[1-2] mechanical behavior has an important influence on the properties of materials and structures. As a weak link in solid-rocket-motor structures, the propellant/liner interface, through its debonding effect, directly affects changes in the burning surface of the motor, causing the internal ballistic characteristics of the motor to deviate from the initial design and leading to accidents such as flameout or even explosion^[3-4].

Research on debonding at the propellant/liner interface involves both experiments and simulations. In terms of experiments, Wu Fengjun et al.^[5] and Yin Huali et al.^[6] summarized four pathways for the intrinsic mechanism of interface aging and for improving interfacial bonding performance; Huang Zhitao et al.^[7] and Li Donglin et al.^[8] studied the relationships among interfacial transition activation energy, migration rate, dissipation factor, and migration amount. Researchers have carried out detection of bonded interfaces in motors by means of microwave, infrared, acoustic holography, X-ray radiography, infrared thermal scanning, imaging methods, etc.^[9-10], and have also completed monitoring of bonded interfaces in motors using sensors and acoustic emission methods^[11-12].

Finite element analysis is currently the main means for simulating debonding behavior at bonded interfaces. Numerical simulation studies of the mechanical performance of bonded interfaces in motors focus on two aspects. One aspect is the calculation of fracture-mechanics parameters guided by traditional fracture mechanics^[13-15]. This method provides a solution to the problem of whether a crack propagates, but it does not solve the problems of continued

crack growth and the stress singularity at the crack tip during growth. The other aspect is the analysis of debonding behavior at bonded interfaces using cohesive elements^[16-20]. The advantages of this method are, first, that cohesive elements can be compared and validated against experimental data, and that by adjusting model parameters a computational model matching the experimental observations can be established. Yu Jiaquan et al.^[21] conducted experimental studies on the interfacial properties of the interface between the propellant of a composite modified double-base (CMDB) solid rocket motor and a three-dimensional ethylene propylene diene monomer (EPDM) coating layer using double cantilever beam (DCB) specimens and uniaxial tensile specimens, obtained the interfacial cohesive energy and cohesive strength, and introduced a bilinear cohesive model for finite element research; Wang Pengbo et al.^[22] used cohesive elements to simulate Mode I debonding tests of the propellant/liner interface, and verified the accuracy of the model by comparing the degree of agreement between the load-displacement curve at the loading point and the experimental curve. Second, cohesive elements can be redeveloped according to specific materials and experimental conditions. Cui Hui et al.^[23], on the basis of the internal cohesive-parameter relaxation effect, established a viscoelastic PPR cohesive model and conducted finite element studies of two working conditions: curing temperature reduction and ignition pressurization; Niu Ranming et al.^[24] proposed a method for constructing a user-defined cohesive model based on experimentally obtained evolution laws of damage variables. Compared with bilinear and exponential models, this model yielded accurate Mode II fracture-simulation results for the HTPB propellant/liner interface.

In the above studies^[16-24], correctly setting the cohesive-model parameters is the prerequisite for accurate simulation. Xavier et al.^[25] extracted the J -integral and the crack-tip opening displacement under applied load, and obtained cohesive-model parameters by solving the J -integral with respect to the crack-tip opening displacement. However, this was greatly limited by the experimental resolution of the crack-tip displacement. Wei et al.^[26-27] proposed that the traction-separation curve and traction-separation parameters extracted from the force-displacement response and crack-extension history are essentially an inverse method. Ma Xiaolin et al.^[28-29] obtained the mechanical performance parameters of the bonded interface in a solid rocket motor through parameter inversion, and studied the mechanical response of the propellant/liner interface under typical working conditions such as wave loading, different loading rates, and ignition pressurization. Wei Zhen et al.^[30] obtained the parameters in a bilinear cohesive model through a numerical method of stepwise inversion matching numerical simulation of double-cantilever-beam load-displacement curves with experimental curves, and studied the Mode I fracture behavior of Al/HTPB double-cantilever-beam adhesive specimens. Chen Xiong et al.^[31] obtained interface fracture parameters under different loading rates based on an inverse-identification procedure using the Hook-Jeeves optimization algorithm; through a rate-dependent damage function, they constructed a rate-dependent HTPB propellant/liner interface Mode II cohesive

Fig. 1(a) design drawing and Fig. 1(b) front view of the specimen

Figure 1: Fig. 1(a) design drawing and Fig. 1(b) front view of the specimen

model based on the bilinear model.

In this study, load-displacement curves at six different strain rates were obtained through peel tests of standard rectangular specimens. Using the response surface method, the bilinear model for rectangular specimens under different loading rates was inversely optimized.

and the parameters of the exponential cohesive-zone model. By comparing with the experimental curves, it was found that the exponential model agrees better with the experimental curves. The exponential model was then used to study the effects of cohesive parameters, loading rate, interface stiffness, and shell/propellant mass on the interfacial debonding behavior of a 1/16 circular-tube motor, with the aim of providing a reference for research on debonding behavior at the propellant/liner interface.

1 Cohesive-Interface Structural Rectangular-Specimen Peel Test

With reference to “QJ 2028.1A—2004 Test Method for Bonding Strength of Solid Rocket Motor Combustion-Chamber Interface, Part 1: Rectangular Specimen Peel Method,” rectangular specimens were fabricated. Their design drawing and front view are shown in Fig. 1(a) and Fig. 1(b). The specimen length $\times$ width is 100 mm $\times$ 25 mm; the width of the artificial debonding zones on both sides is 20 mm; the propellant height is 48 mm; and the combined thickness of the insulation layer and liner on each side is 3 mm. Therefore, the working height is 54 mm (i.e., 48 mm + 2 $\times$ 3 mm = 54 mm), and the effective bonded area is 1 500 mm².

图 1 单向拉伸试件尺寸示意图及实验试件正视图

Fig. 1 The size diagram of uniaxial tensile specimen and the front view of experimental specimen

Six groups of peel tests were carried out at different tensile rates, with five specimens in each group. The tensile speeds V were 3.24, 32.4, 162, 486, 1 620, and 3 240 mm/min, respectively. By averaging the five load-displacement curves obtained in each group, the experimental curves at different strain rates were obtained, as shown in Fig. 2. It can be seen from Fig. 2 that, as the strain rate increases, the slope of the ascending segment of the curve also increases, and the peak load of the curve becomes larger. When the peak load is reached, the adhesive interface fails rapidly and the curve drops quickly.

图 2 6 种应变率下的矩形试件实验载荷-位移曲线

Fig. 2 Experimental load-displacement curves of rectangular specimens at six strain rates

The strain rate in the experiment is the strain increment divided by the time increment, which is also equal to the tensile speed divided by the working length of the specimen, namely

$$\dot{\varepsilon} = \frac{\Delta\varepsilon}{\Delta t} = \frac{V}{L_0} = \frac{V}{54} \quad (1)$$

The strain is equal to the measured displacement divided by 54 mm (the working length), namely

$$\varepsilon = \frac{L - 54}{54} \quad (2)$$

where L is the length of the measured section of the specimen, in mm.

The stress is equal to the measured load divided by 1 500 mm² (the effective bonded area), namely

$$\sigma = \frac{F}{1\,500} \quad (3)$$

where F is the force applied by the fixture.

The latter half of the load-displacement curve in Fig. 2 is fitted and extended until it intersects the x -axis (defined as the x -axis intercept), and the area enclosed by the calculated curve and the x -axis is then obtained. According to the definition of energy,

$$G_c = \int_0^{s_c} F \cdot ds \quad (4)$$

that is, the area enclosed by the curve is the interfacial fracture energy of the specimen.

The mechanical-property parameters of the five groups of specimens are shown in Table 1. It can be seen from Table 1 that, as the strain rate increases, the maximum load, maximum stress, fracture strain, and elastic modulus increase.

表 1 实验测试结果

Tab. 1 Measurement size of tensile specimen

Strain rate / s ⁻¹	Maximum load / N	Maximum stress / MPa	Fracture strain	Elastic modulus / MPa	Fracture energy / J
1.0	1 894	1.28	0.46	3.13	27.98
0.5	1 925	1.28	0.42	3.54	27.01
0.15	1 622	1.08	0.47	2.41	27.31

Fig. 3 Evolution of interfacial fracture energy with strain rate

Figure 2: Fig. 3 Evolution of interfacial fracture energy with strain rate

Strain rate / s^{-1}	Maximum load / N	Maximum stress / MPa	Fracture strain	Elastic modulus / MPa	Fracture energy / J
0.05	1 530	1.02	0.47	2.49	24.41
0.01	1 235	0.83	0.48	2.12	20.81
0.001	1 055	0.71	0.44	1.94	14.63

Using the data in Table 1, Fig. 3 gives the variation of interfacial fracture energy with strain rate. It can be seen that, as the strain rate increases, the fracture energy increases. The interfacial fracture energy and the logarithm of strain rate, $\lg \dot{\epsilon}$, approximately satisfy the following relationship:

$$G_c = 32.1 - 4.21e^{-0.48 \lg \dot{\epsilon}} \quad (5)$$

Fig. 3 Evolution of interfacial fracture energy with strain rate

2 Fitting of cohesive-force model parameters based on the response surface method

The cohesive force model (cohesive zone model, CZM) was first proposed by Dugdale[32] and has now been widely applied to fracture analysis of interfaces in composite materials, polymers, and other materials, as well as two-phase materials. The commonly used bilinear and exponential cohesive force models in commercial software such as ABAQUS are shown in Fig. 4. It can be seen from Fig. 4 that both models adopt a linear form in the rising segment; in the descending damage segment, however, they are different. In Fig. 4, the solid line in the latter half is the bilinear model, and the dashed line is the exponential model. Use of either model requires the determination of three parameters: stiffness k , maximum stress $\sigma_{\max}$, and interfacial fracture energy per unit area G . Among them, the stiffness k is defined as the ratio of the maximum stress to its corresponding displacement, namely

$$k = \frac{\sigma_{\max}}{\delta^0} \quad (6)$$

where δ^0 is the displacement corresponding to the maximum stress.

The fracture energy per unit area G is equal to the ratio of the interfacial fracture energy determined by Eq. (4) to the effective adhesive area. The bilinear and

Fig. 4 Cohesive zone model

Figure 3: Fig. 4 Cohesive zone model

exponential models are the same in the setting of the two parameters stiffness and maximum stress; the difference lies in the different definitions of the energy-based damage evolution parameter D .

The softening behavior of the bilinear model is linear, and its energy-based damage evolution form is

$$D = \frac{\delta^f (\delta^{\max} - \delta^0)}{\delta^{\max} (\delta^f - \delta^0)} \quad (\delta > \delta^0) \quad (7)$$

where D is the damage variable; $\delta^f = 2G/\sigma_{\max}$ is the failure displacement; and $\delta^{\max}$ is the maximum value of displacement reached during the loading process.

The energy-based damage evolution form of the exponential model is

$$D = \int_{\delta^0}^{\delta^f} \frac{\sigma_{\text{eff}} d\delta}{(G - G_0)} \quad (\delta > \delta^0) \quad (8)$$

where G_0 is the elastic energy per unit area at the onset of damage; σ_{eff} is the effective interfacial traction, which is usually defined as an exponential function. Through the ratio of the integral of this function over the displacement in the damage segment to the energy in the damage segment, the damage variable of the exponential model in the damage segment is defined.

Through the damage evolution models of Eqs. (7) and (8), the constitutive relations of the damage segment for the two models can be obtained. To construct this damage evolution model, a reasonable fracture energy must be substituted; therefore, reasonable k , $\sigma_{\max}$, and G parameters can be obtained so that the cohesive force model correctly characterizes the experimental curve. In this study, the response surface method is used to inversely optimize reasonable cohesive-force model parameters.

Fig. 4 Cohesive zone model

2.1 Response surface method and identification of cohesive-force parameters

The response surface method was first proposed by Box et al.[³³], and Hill et al.[³⁴] established a broader definition of the response surface and an optimization model. For the bilinear and exponential cohesive force models, the following parameter-identification process is established using the response surface method.

1) Selection of the design domain and reference points

The stiffness, maximum strength, and fracture energy are respectively selected as the three values shown in Table 2 as the design domain. In Table 2, the three parameters are randomly combined, producing a total of $3^3 = 27$ parameter combinations, that is, 27 reference points.

Table 2 Design reference point value

Tab. 2 Design reference point value

$k/$ (MPa $\cdot$ mm $^{-1}$)	$\sigma_{\max}/$ MPa	$G/$ (mJ $\cdot$ mm $^{-2}$)
0.05	0.5	6
0.10	1.0	12
0.15	1.5	18

2) Acquisition of response values

Finite element modeling is carried out for the rectangular specimen. Cohesive elements (COH3D8) are used for the interface. A 20 mm artificial debonding layer is set at both ends of the specimen. The approximate element size of the model is 2 mm, and the number of elements is 23 790. The overall finite element mesh of the rectangular specimen is shown in Fig. 5(a), and the material parameters used in the calculation are shown in Table 3. At the same time, by comparing with the coarse and fine two mesh models,

model calculation results to verify the mesh dependence of this model. The fine-mesh approximate element size was 1.5 mm, and the coarse-mesh approximate element size was 2.5 mm. The mesh models with three different densities all used the same model parameters: stiffness 0.15 MPa/mm, maximum strength 1.5 MPa, and unit-area fracture energy 18 mJ/mm 2 . The comparison of the model calculation results is shown in Fig. 5(b). It can be seen from the figure that the calculation results of the three mesh models are highly coincident, indicating that the calculation results of this model are only slightly affected by the mesh size.

(a) Finite element model

(b) Grid dependence verification diagram

Fig. 5 Finite element model of rectangular specimen and grid dependence verification diagram

The 27 groups of parameter combinations were substituted into the finite element model for calculation, obtaining the load-displacement curves of 27 finite element models. Taking the experimental curve shown in Fig. 2 as the target curve, 10 data points were selected on each target curve at 2.5, 5, 10, 14, 16, 20, 22, 23.5, 24.5, and 26.5 mm. The mean square error between the load-

displacement curves of the 27 simulation models and the target curve at these 10 points was defined as the objective-function response value, namely

$$y = \frac{1}{10} \sum_{i=1}^{10} (f_i^* - f_i)^2 \quad (9)$$

where f_i^* is the load value of the load-displacement curve calculated from the reference point at the i -th reference point; f_i is the load value of the experimental load-displacement curve at the i -th reference point.

Table 3 Material parameters of propellant, insulation, liner, and shell

Tab. 3 Material parameters of propellant, insulation, liner and shell

Material	Young' s modulus/MPa	Poisson' s ratio
Propellant	21.85	0.45
Insulation	4.87	0.49
Liner	14.5	0.45
Shell	2.11×10^5	0.33

3) Constructing the response surface

A second-order polynomial model was adopted. Using the response values corresponding to the 27 groups of parameters obtained, the response surface was constructed, namely

$$y = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \sum_{j=1, i:j}^3 \beta_{ij} x_i x_j + \sum_{i=1}^3 \beta_{ii} x_i^2 + \varepsilon \quad (10)$$

Writing the above equation in matrix form gives

$$y = \beta_0 + x^T b + x^T B x + \varepsilon \quad (11)$$

Writing the equations corresponding to the 27 groups of parameters in matrix form gives

$$Y = X\beta + \varepsilon \quad (12)$$

where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{27} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{27} \end{bmatrix},$$

$$\beta = [\beta_0, \dots, \beta_3, \beta_{11}, \dots, \beta_{33}, \beta_{12}, \dots, \beta_{23}]^T,$$

$$X = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & x_{1,3} & x_{1,1}^2 & \dots & x_{1,3}^2 & x_{1,1}x_{1,2} & \dots & x_{1,2}x_{1,3} \\ 1 & x_{2,1} & \dots & x_{2,3} & x_{2,1}^2 & \dots & x_{2,3}^2 & x_{2,1}x_{2,2} & \dots & x_{2,2}x_{2,3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{27,1} & \dots & x_{27,3} & x_{27,1}^2 & \dots & x_{27,3}^2 & x_{27,1}x_{27,2} & \dots & x_{27,2}x_{27,3} \end{bmatrix}$$

In the X matrix, $(x_{i,1}, x_{i,2}, x_{i,3})$ respectively represent the stiffness value, maximum strength value, and fracture energy value of the i -th group of reference points. Define

$$L = (Y - X\beta)^T(Y - X\beta)$$

and

$$\frac{\partial L}{\partial \beta} = 0 \quad (13)$$

Then

$$\beta = (X^T X)^{-1} X^T Y \quad (14)$$

Substituting the parameter matrix X and the response value Y into Eq. (14) gives β . Substituting β back into Eq. (12) yields the constructed response surface.

4) Parameter optimization

On the constructed response surface, the parameter combination corresponding to the minimum response value is the optimal parameter. The steps for determining the optimal parameters are as follows.

Differentiate the response-surface function obtained from Eq. (11) to obtain the equation containing x , namely

$$\frac{\partial y}{\partial x} = b + 2Bx = 0 \quad (15)$$

Solve Eq. (15); its solution is the stationary point, namely

$$x_0 = -\frac{1}{2}B^{-1}b \quad (16)$$

Fig. 6(a) Stress nephogram; Fig. 6(b) Effect of stiffness and maximum strength; Fig. 6(c) Effect of fracture energy and stiffness; Fig. 6(d) Effect of maximum strength and fracture energy

Figure 4: Fig. 6(a) Stress nephogram; Fig. 6(b) Effect of stiffness and maximum strength; Fig. 6(c) Effect of fracture energy and stiffness; Fig. 6(d) Effect of maximum strength and fracture energy

Substitute the solution of the stationary point obtained into the original equation to obtain the predicted response value, namely

$$\hat{y}_0 = \beta_0 + \frac{1}{2}x_0^T b \quad (17)$$

Compare the predicted response values mapped by these stationary points, and select the parameter combination with the minimum response value and with physical significance as the optimal parameter value obtained by inverse optimization.

5) Drawing the response surface

Figure 6 gives the fitting results of the bilinear cohesive-force model under a strain rate of 1.0 s^{-1} .

- (a) Stress nephogram
- (b) Effect of stiffness and maximum strength
- (c) Effect of fracture energy and stiffness
- (d) Effect of maximum strength and fracture energy

Fig. 6 Finite element deformation nephogram of rectangular specimen under 1.0 s^{-1} strain rate and its cohesive optimization parameter 3D response surface

Figure 6(a) is the stress nephogram obtained using the optimal parameters. It can be seen from Fig. 6(a) that the deformation is symmetrically distributed in the specimen; separation occurs at the artificial debonding interface during loading; the deformation gradient at the root of the debonded layer changes greatly; and the deformation distribution in the region where the cohesive elements are arranged is uniform. The constructed response surfaces are shown in Figs. 6(b)-6(d), among which Fig. 6(b) is the response surface with stiffness and maximum strength as the varying factors when the fracture energy per unit area is 15.75 mJ/mm^2 ; Fig. 6(c) is the response surface with stiffness and G as the varying factors when the maximum strength is 1.261 MPa ; Fig. 6(d) is the response surface with G and maximum strength as the varying factors when the stiffness is 0.106 MPa/mm . From Figs. 6(b)-6(d), it can be seen that the res-

When the value of the strain energy release rate per unit area is 15.75 mJ/mm^2 , the maximum strength is 1.261 MPa , and the stiffness is the minimum, 0.106

MPa/mm.

2.2 Fitting results

Similarly, inverse optimization was used to obtain the bilinear cohesive-force model parameters corresponding to the other five strain rates, as well as the exponential cohesive-force model parameters corresponding to the six strain rates. The results are shown in Table 4. It can be seen from Table 4 that, as the strain rate decreases, the stiffness k , maximum strength $\sigma_{\max}$, and unit-area fracture energy G gradually decrease. Among them, the maximum stresses $\sigma_{\max}$ obtained by fitting the two models are both close to the experimental measurement results, and the unit-area fracture energies are also largely consistent with the experimental results. Under the six strain-rate conditions, the hypothesis-test P values for coefficient regression of the two models are all less than 0.05, indicating that the fitted results can be regarded as reliable.

Table 4 Prediction parameters for six strain rates

Tab. 4 Prediction parameters of six strain rates

Model	Strain rate $\dot{\varepsilon} / \text{s}^{-1}$	$k / (\text{MPa} \cdot \text{mm}^{-1})$	$\sigma_{\max} / \text{MPa}$	$G / (\text{mJ} \cdot \text{mm}^{-2})$	P
Bilinear model	1.00	0.106	1.261	15.75	0.007 1
Bilinear model	0.50	0.114	1.210	14.02	0.000 1
Bilinear model	0.15	0.075	0.993	13.86	0.002 1
Bilinear model	0.05	0.075	0.934	11.43	0.004 1
Bilinear model	0.01	0.067	0.774	10.50	0.001 7
Bilinear model	0.001	0.054	0.615	7.50	0.010 3
Exponential model	1.00	0.109	1.16	15.0	0.002 3
Exponential model	0.50	0.123	1.16	14.3	0.002 3
Exponential model	0.15	0.084	0.91	13.5	0.003 1
Exponential model	0.05	0.082	0.96	12.0	0.003 4
Exponential model	0.01	0.071	0.74	10.5	0.001 4
Exponential model	0.001	0.062	0.59	7.10	0.000 3

Fig. 7 Comparison of rectangular specimen experiment and finite element simulation results

Figure 5: Fig. 7 Comparison of rectangular specimen experiment and finite element simulation results

The prediction parameters of the two models corresponding to the six strain rates were substituted into the finite element model of the rectangular specimen, the load-displacement curves of the whole specimen were obtained, and the calculated load-displacement curves were compared with the experimental curves, as shown in Fig. 7. Figure 7(a) shows the comparison of the experimental curves and the prediction curves of the bilinear cohesive-force model and exponential cohesive-force model at strain rates of 1, 0.15, and 0.01 s^{-1} . Figure 7(b) shows the comparison of the experimental curves and the prediction curves of the bilinear model and exponential model at strain rates of 0.5, 0.05, and 0.001 s^{-1} . In the figure, the black lines are the experimental curves, the blue lines are the prediction curves of the bilinear model, and the red lines are the prediction curves of the exponential model.

It can be seen from Fig. 7 that the simulation results of the bilinear model agree well with the experimental data in the elastic stage, whereas the slope in the damage stage differs slightly. The main reason is that the bilinear cohesive-force model used in the simulation does not consider the stiffness degradation in the later part of the elastic stage, resulting in a smaller initial damage strain and thus a difference in the slope of the damage-evolution stage. The calculation results of the exponential cohesive-force model are basically the same as those of the bilinear model in the elastic stage, while the nonlinear softening characteristics of the exponential cohesive-force model make it closer to the experimental data in the damage stage. The simulation results of the exponential cohesive-force model are generally consistent with the overall trend of the experimental data, and the variation patterns among the prediction curves corresponding to the six strain rates conform to the experimental laws. Therefore, the exponential cohesive-force model parameters are used below to study the debonding behavior of the casing engine.

Fig. 7 Comparison of rectangular specimen experiment and finite element simulation results

3 Debonding behavior of the casing engine

3.1 Finite element model of the 1/16 casing engine

Commercial software was used to establish a finite element model of a horizontally placed 1/16 casing engine. The engine inner diameter is 200 mm and the length is 700 mm. Artificial debonding zones of 50 mm are set at both ends, and a bonding interface is set behind them. The interface uses a cohesive element (COH3D8) with a thickness of 0 mm. The number of elements in the model is

59,020. The finite element mesh of the model is shown in Fig. 8(a). A liner and an insulation layer are sandwiched between the casing and the propellant, as shown in Fig. 8(b), in which the yellow interlayer is the insulation layer and the blue ...

color layer is the liner; the complete 1/16 circular-tube-engine finite element model is fixed, and a radial displacement load is applied at the edge nodes on the propellant end face, as shown in Fig. 8(c). The Mises stress contour under displacement loading is shown in Fig. 8(d). The material parameters used in the simulation are shown in Table 3.

To verify the mesh convergence of this model, the stresses along the cohesive interface path calculated using four different mesh-density models were compared, as shown in Fig. 8(e). It can be seen from Fig. 8(e) that, as the mesh in the artificial debonding region is refined and the number of elements increases, the stress along the cohesive interface path gradually rises. When the number of elements increases to 59,020 and 78,816, the stress along the cohesive interface path no longer changes; the calculation results converge to a stable curve. It can therefore be considered that the model calculation results are convergent and the model is reliable.

(a) Finite element mesh division of the 1/16 circular-tube engine

(b) Artificial debonding region

(c) Applying displacement load in the radial direction

(d) Mises stress contour

Stress/MPa
(Average: 75%)

+2.713e+00
+2.487e+00
+2.261e+00
+2.035e+00
+1.809e+00
+1.583e+00
+1.357e+00
+1.131e+00
+9.045e-01
+6.784e-01
+4.522e-01
+2.261e-01
+1.251e-07

(e) Mesh convergence verification

Legend:

Element number: 78,816

Element number: 59,020

Element number: 42,812

Element number: 28,510

Stiffness: 0.109 MPa/mm

Maximum strength: 1.16 MPa

Unit-area fracture energy: 15 mJ/mm²

Fig. 8 FEM model of the 1/16 circular-tube engine, local enlarged diagram of artificial debonding, and Mises stress contour

3.2 Effect of Strain Rate on Debonding Behavior

The parameters of the exponential internal cohesive model corresponding to different strain rates were substituted into the 1/16 circular-tube-engine finite element model. The stress distributions along the adhesive interface under peeling displacement loads of 3.0 and 7.5 mm were obtained, as shown in Fig. 9. Among them, Fig. 9(a) and Fig. 9(b) are the results for a displacement load of 3.0 mm, while Fig. 9(c) and Fig. 9(d) are the results for a displacement load of 7.5 mm.

Fig. 9(a)

Loading displacement 3.0 mm

Axes: stress/MPa; distance from loading end/mm

Legend: 1/s; 0.5/s; 0.15/s; 0.05/s; 0.01/s; 0.001/s

Fig. 9(b)

Loading displacement 3.0 mm

Axes: stress/MPa; distance from loading end/mm

Legend: 1/s; 0.5/s; 0.15/s; 0.05/s; 0.01/s; 0.001/s

Fig. 9(c)

Loading displacement 7.5 mm

Axes: stress/MPa; distance from loading end/mm

Legend: 1/s; 0.5/s; 0.15/s; 0.05/s; 0.01/s; 0.001/s

[Fig. 9(d): Loading displacement 7.5 mm; stress/MPa versus distance from loading end/mm, with strain rates 1/s, 0.5/s, 0.15/s, 0.05/s, 0.01/s, and 0.001/s.]

Fig. 9 Stress distribution at bonded interfaces with different strain rates under different loading displacements

It can be seen from Fig. 9 that the greater the strain rate, the greater the maximum debonding stress. When the displacement load is 3.0 mm, the debonding lengths under different strain rates are approximately the same; when the

debonding displacement load is 7.5 mm, the smaller the strain rate, the longer the debonding length.

3.3 Influence of cohesive-force parameters on debonding behavior

Different stiffnesses, maximum strengths, and fracture energies were substituted into the finite-element model of the 1/16 circular-tube motor. The stress distributions at the bonded interface under displacement loads of 3.0 and 7.5 mm were obtained, as shown in Fig. 10. In the figure, k , σ , and G are the predicted parameter values of the exponential cohesive-force model in Table 4 at a strain rate of 1 s^{-1} .

Figs. 10(a) and 10(b) respectively give the influence of stiffness variation on the stress distribution when the displacement loads are 3.0 and 7.5 mm. It can be seen that, when the displacement load is 3.0 mm, as the stiffness increases, the stress peak in the artificial debonding zone becomes larger, indicating that stiffness affects the magnitude of the stress peak in the artificial debonding zone. When the displacement load is 7.5 mm, the positions of the stress peaks of the three curves are all different. This is because, for the same opening displacement at the loading end, the larger the stiffness, the harder the material properties of the bonded interface appear; it is less easy to open, and more of the opening displacement is borne by the propellant.

Figs. 10(c) and 10(d) respectively give the influence of the variation in maximum strength on the stress distribution when the displacement loads are 3.0 and 7.5 mm. It can be seen that, when the displacement loads are 3.0 and 7.5 mm, the differences among the stress peaks of the three curves are relatively small, indicating that the maximum strength has little influence on the stress peak when the artificial debonding zone is damaged. Moreover, the positions of the stress peaks of the three curves on the cohesive interface are basically the same, indicating that, under a displacement load at one end, the maximum strength has almost no effect on the expansion of the debonding zone.

Figs. 10(e) and 10(f) respectively give the influence of the variation in fracture energy per unit area on the stress distribution when the displacement loads are 3.0 and 7.5 mm.

[Fig. 10(a): Loading displacement 3.0 mm; stress/MPa versus distance from loading end/mm, with stiffness $2k$, k , and $0.5k$.]

[Fig. 10(b): Loading displacement 7.5 mm; stress/MPa versus distance from loading end/mm, with stiffness $2k$, k , and $0.5k$.]

[Fig. 10(c): Loading displacement 3.0 mm; stress/MPa versus distance from loading end/mm, with maximum strength 2σ , σ , and 0.5σ .]

[Fig. 10(d): Loading displacement 7.5 mm; stress/MPa versus distance from loading end/mm, with maximum strength 2σ , σ , and 0.5σ .]

Fig. 11(a) Research path

Figure 6: Fig. 11(a) Research path

[Fig. 10(e): Loading displacement 3.0 mm; stress/MPa versus distance from loading end/mm, with fracture energy $2G$, G , and $0.5G$.]

[Fig. 10(f): Loading displacement 7.5 mm; stress/MPa versus distance from loading end/mm, with fracture energy $2G$, G , and $0.5G$.]

Fig. 10 The effects of stiffness, maximum strength and fracture energy on the stress distribution of the bonding interface under different loading displacements

It can be seen that, when the displacement load is 3.0 mm, the stress distributions of the three curves are approximately the same, indicating that G has relatively little influence on the stress distribution when the artificial debonding zone fails; when the displacement load is 7.5 mm, the positions of the stress peaks of the three curves are all different. This is because, when the stiffness is the same and the maximum strength is the same, the larger G is, the larger the failure displacement is; at the same stage, that is, under the same opening displacement at the loaded end, the larger the failure displacement is, the more difficult it is for failure to occur.

3.4 Influence of shell/propellant modulus and interface stiffness

Different shell moduli, propellant moduli, and bonding-interface stiffnesses are substituted into the 1/16 circular-tube engine finite-element model. Under a 3.0 mm displacement load, the stress distribution along the radial section at the artificial debonding position of the 1/16 circular-tube engine is obtained, as shown in Fig. 11. In the figure, the shell and propellant modulus E are taken as 2.11×10^5 and 21.85 MPa, respectively, and the bonding-interface stiffness k is taken as $0.109 \text{ MPa} \cdot \text{mm}^{-1}$. Among them, the research path of the radial stress distribution is shown by the red line in Fig. 11(a), and Figs. 11(b)–11(d) give the stress variations when the shell, propellant, and bonding-interface modulus change, respectively.

From Fig. 11(b), it can be seen that the elastic modulus of the shell has almost no influence on the stress variation along the r -direction path of the circular-tube engine.

From Fig. 11(c), it can be seen that reducing the elastic modulus of the propellant can make the stress variation along the r -direction path of the circular-tube engine more gradual. This is because the smaller the propellant modulus is, the more debonding displacement load is borne by the deformation of the propellant, and the deformation borne by the interface decreases correspondingly.

(a) Research path

(b) Influence of shell elastic modulus

Fig. 11(b) Influence of shell elastic modulus

Figure 7: Fig. 11(b) Influence of shell elastic modulus

Fig. 11(c) Influence of propellant elastic modulus

Figure 8: Fig. 11(c) Influence of propellant elastic modulus

(c) Influence of propellant elastic modulus

(d) Influence of bonding stiffness

Fig. 11 The effects of shell modulus, grain modulus and interface stiffness on the stress distribution in the r -direction path of a circular tube engine

From Fig. 11(d), it can be seen that the greater the bonding-interface stiffness is, the less easily it opens; the larger the deformation of the propellant is, the greater the stress of the propellant is. Therefore, the smaller the bonding-interface stiffness is, the more gradual the stress variation along the r -direction path of the circular-tube engine becomes.

4 Conclusion

The main conclusions of this study are as follows.

- 1) The tensile failure of the rectangular specimen is mainly caused by debonding at the bonding interface between the propellant and the liner. It can be seen from the experimental curves that, as the strain rate increases, the initial stiffness of the interface and the stress peak both become larger, while the area enclosed by the curve (fracture energy) becomes smaller.
- 2) The response surface method can satisfactorily invert and optimize the parameters of the bilinear and exponential cohesive-zone models. The exponential model can more accurately reflect the trend of the interfacial mechanical properties changing with strain rate.
- 3) By using the exponential cohesive-zone model to conduct finite-element simulation of the circular-tube engine, it can be found that stiffness and fracture energy have a relatively large influence on debonding length: the greater the stiffness, the longer the debonding length; the greater the fracture energy, the shorter the debonding length. The maximum strength only affects the maximum peel force, and does not affect the debonding length: the greater the maximum strength, the greater the maximum debonding force. Loading rate

Fig. 11(d) Influence of bonding stiffness

Figure 9: Fig. 11(d) Influence of bonding stiffness

the strain rate has an obvious influence on the maximum peel force and the peel length; the greater the loading rate, the greater the maximum peel force and the shorter the peel length. The elastic modulus of the casing has no effect on the stress variation along the radial path of the cylindrical rocket motor. Reducing the elastic modulus of the propellant and the interface stiffness can make the stress variation along the radial path of the cylindrical rocket motor more gradual. These results can provide a reference for research and design of the propellant/liner interface.

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