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Full Text

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Abstract: After strip filling mining, the backfill–roof system jointly bears its own gravity and the load transmitted from the overlying strata, and its mechanical state has an important influence on the stability of the surrounding rock in the stope. To analyze the influence of strip filling parameters on the stress distribution of the roof, this study, based on the trigonometric series numerical method of elasticity, derives an analytical solution for the elastic stress field of the immediate roof under strip filling mining conditions, and carries out theoretical calculations through Matlab programming. On this basis, combined with the maximum principal stress theory, a tensile failure criterion for the roof is established, the distribution characteristics of the maximum tensile stress of the roof and the locations of potential dangerous points are analyzed, and a method for determining the optimal filling spacing is proposed. The results show that under strip filling mining conditions, tensile stress concentration is likely to form in the lower region of the roof, and the dangerous points are mainly located in this region; the maximum tensile stress at the dangerous points decreases with increasing backfill width and height-to-span ratio, indicating that increasing the filling width and the roof height-to-span ratio can effectively suppress tensile failure of the roof.

Keywords: strip filling; immediate roof; stress field; analytical solution; filling optimization

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Analysis of the stress field and optimization of filling parameters for the direct top of strip filling

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Abstract: After strip backfill mining, the backfill-roof system jointly bears the self-weight-induced stress and the load transmitted from the overlying strata. Its mechanical state has an important influence on the stability of the surrounding rock in the stope. To investigate the effect of strip backfill parameters on the stress distribution of the roof, an elastic analytical solution for the stress field of the immediate roof under strip backfill mining conditions was derived using the trigonometric series solution method of elasticity. The theoretical calculation was then implemented through MATLAB programming. On this basis, a tensile failure criterion for the roof was established by incorporating the analytical stress solution with the maximum principal stress criterion, and the distribution characteristics of the maximum tensile stress and the location of potential critical points were analyzed. A method for determining the optimal backfill spacing

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was further proposed. The results show that under strip backfill mining conditions, tensile stress concentration is likely to occur in the lower part of the roof, where the critical points are mainly located. The maximum tensile stress at the critical points decreases with increasing backfill width and roof height-to-span ratio. This indicates that increasing the backfill width and the roof height-to-span ratio can help reduce the likelihood of tensile failure in the roof.

Key words: strip filling; immediate roof; stress field; analytical solution; filling optimization

Strip backfill mining uses backfilled strips to support the overlying roof strata, avoiding large-area subsidence of the overburden, thereby achieving the goals of controlling subsidence and protecting the environment

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. Compared with traditional mining methods, strip backfill mining can effectively solve the problems of "three-under" coal mining and the ecological environment of the mining field. It is of great practical significance for reducing backfilling cost, improving coal utilization, protecting the ecology, and extending the service life of mines, and has become an important development direction for green mining

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In recent years, regarding the stability of the roof–backfill system after strip backfill mining, scholars at home and abroad have conducted in-depth studies of the stress field and displacement field through mechanical analysis and experimental methods. Du Xianjie et al.

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regarded the overall structure of the "backfilled strip–immediate roof" as a finite-length beam model on an elastic foundation, and derived the deformation equation of the immediate roof. Chen Shaojie et al.

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analyzed the overall mechanical characteristics and failure modes of the "backfilled strip–immediate roof" composite through loading tests. For roof deformation, Shi Jianfei

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established an elastic foundation beam model with one end fixed and the other end cantilevered, and obtained the deflection-curve equation for subsidence deformation of the roof strata under backfilling conditions. Sun Yunjiang et al. [8] considered the influence of nonuniform loading on the deformation of the backfilled roof, established a mechanical model of continuous bending of the roof strata under nonuniform loading, and obtained the mechanical conditions for continuous bending deformation of the roof strata in backfill mining. Considering that the foundation-bed coefficient in the caving zone of backfill mining is not constant, Feng Chao et al. [9] established an elastic foundation beam model with variable foundation-bed coefficient for working-face roof deformation, and analyzed the roof deflection and bending-moment distribution laws under changes in backfill stiffness. For stratified backfilling mining of extra-thick coal seams, Deng Xuejie et al. [10] established elastic foundation beam models for the movement characteristics of the roof in "single-slice" and "multi-slice" mining of extra-thick coal seams, and gave the corresponding calculation formulas for roof deflection curves. The above results focus on the analysis of roof deflection and bending moment in backfill mining, but do not provide stress-field solutions for the roof, making it difficult to accurately predict roof failure. Therefore, Feng Fan et al. [11] introduced the elastic-mechanics semi-inverse method into the mechanical analysis of overlying backfilled roofs, and derived the stress components and deflection-curve equation of an "embedded beam" under uniform loading. Many scholars have also used the semi-inverse method to obtain elastic stress analytical solutions for beams and plates with various constraint forms and materials [12 − 16]. For example, Liu Xiaobo et al. [17] simplified the roof as a thin plate, and, based on the Navier solution for a rectangular thin plate simply supported on all four edges, derived the deflection equation and stress expressions of a rectangular roof under uniformly distributed load and concentrated load, respectively.

In strip backfill mining, the backfilled strips support the roof and the load of the overlying strata. The action of the backfill on the roof can be regarded as a local load; thus the mechanical boundary conditions are relatively complex. Therefore, this study intends to establish, through elastic mechanics, a

Fig. 1 Schematic diagram of strip filling mining

Figure 1: Fig. 1 Schematic diagram of strip filling mining

mechanical model of the backfill body-roof at a strip backfill working face, and to solve the elastic stress analytical solution of the immediate roof by using the trigonometric series method, thereby determining the failure conditions of the immediate roof. A new calculation method for the optimal filling interval in strip backfilling is proposed, with the aim of providing theoretical reference for green mining by strip backfilling in mines.

1 Calculation Model of the Strip Backfill Body-Immediate Roof System

1.1 Strip Backfill Mining Method

Strip backfilling means that, after the coal seam is extracted and before roof caving occurs, backfilling materials (such as paste backfill, crushed-stone cemented backfill, full-tailings sand cemented backfill, etc.) are used to construct backfilled strips at intervals in the goaf, as shown in Fig. 1. Let the roof width be b, the roof thickness be h, and the backfill width be a; then the filling interval is $z = b - a$. The backfill body-roof system jointly bears the self-weight stress of the load-bearing system and the load transmitted by the overlying strata

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. Controlled by many factors such as rock strength, in-situ stress, and lithology, the roof may exhibit various failure types, including bedding-fracture penetration failure, coal-seam cleavage shear failure, block failure along rock bedding planes, shear collapse failure, and bending subsidence failure

$$19 - 20$$

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Fig. 1 Schematic diagram of strip filling mining

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1.2 Calculation Model of the Backfill Body-Immediate Roof

The backfill body-immediate roof system is a complex three-dimensional problem. To facilitate theoretical analysis, a two-dimensional plane model is adopted. The strike width of the stope is taken as a unit length. Using symmetry, the immediate roof is regarded as a two-dimensional elastic body, and the analysis model shown in Fig. 2 is established. q_1 is the load acting on the roof from the overlying strata, q_2 is the supporting force of the backfill body on the roof, and a is the backfill

Fig. 2 Immediate roof two-dimensional model

Figure 2: Fig. 2 Immediate roof two-dimensional model

where h is the height of the immediate roof, and b is the width of the immediate roof. F_1, F_2, F_3, F_4, M_1, and M_2 are all support reactions. This model is symmetric about $x = b/2$; therefore $F_1 = F_2$, $F_3 = F_4$.

Fig. 2 Immediate roof two-dimensional model

2 Stress-Field Analysis of the Immediate Roof

The support action of the backfill on the roof slab is a local load; therefore, the trigonometric series method is adopted to solve the roof-slab stress. To satisfy the boundary conditions of this problem, the stress function Φ is taken as [21]

$$\Phi = \Phi_1 + \Phi_2 \qquad (1)$$

where Φ_1 and Φ_2 are both particular solutions of the biharmonic equation. To make Φ_1 satisfy the normal-stress conditions on the upper and lower boundaries and the shear-stress conditions on the left and right boundaries, and Φ_2 satisfy the normal-stress conditions on the left and right boundaries, let

$$\Phi_1 = \sum_{m=1}^{\infty} \sin\alpha_m x \left[A_m \sinh(\alpha_m y) + B_m \cosh(\alpha_m y) + C_m y \sinh(\alpha_m y) + D_m y \cosh(\alpha_m y)\right] + \sum_{m=1}^{\infty} \cos\alpha'_m x \left[A'_m \sinh\right.$$

$$\Phi_2 = Ey^3 + Gy^2 \qquad (3)$$

In the equations, A_m, B_m, C_m, D_m, α_m, A'_m, B'_m, C'_m, D'_m, α'_m, E, and G are all unknown parameters. Because the compatible equation is a linear system and satisfies the superposition principle, Φ also satisfies the compatible equation.

2.1 Solution of the Stress Function Φ_1

For the stress function Φ_1, the stress boundary conditions on the upper and lower surfaces are

$$(\sigma_y)_{y=0} = -q_1,$$

$$(\sigma_y)_{y=h} = \begin{cases} 0, & 0 \le x < (b-a)/2, \\ -q_2, & (b-a)/2 \le x < (b+a)/2, \\ 0, & (b+a)/2 \le x \le b, \end{cases}$$

$$(\tau_{xy})_{y=0} = 0, \quad (\tau_{xy})_{y=h} = 0 \qquad (4)$$

According to Saint-Venant’ s principle, the stress boundary conditions on the left and right sides are

$$(\sigma_x)_{x=0} = 0, \quad (\sigma_x)_{x=b} = 0,$$

$$\int_0^h (\tau_{xy})_{x=0}\, dy = F_1, \quad \int_0^h (\tau_{xy})_{x=b}\, dy = -F_2 \qquad (5)$$

To satisfy the normal-stress boundary conditions in Eq. (5), set in Eq. (2)

$$A'_m = B'_m = C'_m = D'_m = 0, \quad \alpha_m = \frac{m\pi}{b}\ (m = 1, 2, 3, \cdots),$$

then the stress components can be simplified as

$$\begin{cases} \sigma_x^1 = \sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \sin\dfrac{m\pi x}{b} \left[\left(A_m + \dfrac{2b}{m\pi} D_m \right) \sinh\dfrac{m\pi y}{b} + \left(B_m + \dfrac{2b}{m\pi} C_m \right) \cosh\dfrac{m\pi y}{b} + C_m y \sinh\dfrac{m\pi y}{b} + D_m y \right. \\ \sigma_y^1 = -\sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \sin\dfrac{m\pi x}{b} \left(A_m \sinh\dfrac{m\pi y}{b} + B_m \cosh\dfrac{m\pi y}{b} + C_m y \sinh\dfrac{m\pi y}{b} + D_m y \cosh\dfrac{m\pi y}{b} \right), \\ \tau_{xy}^1 = -\sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \cos\dfrac{m\pi x}{b} \left[\left(B_m + \dfrac{b}{m\pi} C_m \right) \sinh\dfrac{m\pi y}{b} + \left(A_m + \dfrac{b}{m\pi} D_m \right) \cosh\dfrac{m\pi y}{b} + D_m y \sinh\dfrac{m\pi y}{b} + C \right. \end{cases}$$

Substituting the boundary conditions Eq. (4) into Eq. (6) gives

$$\begin{cases} \sum\limits_{m=1}^{\infty} m^2 \cos\dfrac{m\pi x}{b} \left(A_m + \dfrac{b}{m\pi} D_m \right) = 0, \\ \sum\limits_{m=1}^{\infty} m^2 \cos\dfrac{m\pi x}{b} \left[\left(B_m + \dfrac{b}{m\pi} C_m \right) \sinh\dfrac{m\pi h}{b} + \left(A_m + \dfrac{b}{m\pi} D_m \right) \cosh\dfrac{m\pi h}{b} + D_m h \sinh\dfrac{m\pi h}{b} + C_m h \cosh \right. \\ \dfrac{\pi^2}{b^2} \sum\limits_{m=1}^{\infty} m^2 \sin\dfrac{m\pi x}{b} B_m = q_1, \\ \sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \sin\dfrac{m\pi x}{b} \left(A_m \sinh\dfrac{m\pi h}{b} + B_m \cosh\dfrac{m\pi h}{b} + C_m h \sinh\dfrac{m\pi h}{b} + D_m h \cosh\dfrac{m\pi h}{b} \right) = \begin{cases} 0, & 0 \le x < \\ q_2, & \dfrac{b-a}{2} \le \\ 0, & \dfrac{b+a}{2} \le \end{cases} \end{cases}$$

Solving Eq. (7) gives

$$\begin{cases} A_m = -\dfrac{b}{m\pi} D_m \\ B_m = \dfrac{2b}{m^2\pi^2} \displaystyle\int_0^b q_1 \sin\dfrac{m\pi x}{b}\, dx \\ C_m = \left[\left(B_m \cosh\dfrac{m\pi h}{b} + \dfrac{2b}{m^2\pi^2} \displaystyle\int_{\frac{b-a}{2}}^{\frac{b+a}{2}} q_2 \sin\dfrac{m\pi x}{b}\, dx \right) \times h \sinh\dfrac{m\pi h}{b} \right. \\ \qquad \left. + \dfrac{2b}{m^2\pi^2} \displaystyle\int_{\frac{b-a}{2}}^{\frac{b+a}{2}} q_2 \sin\dfrac{m\pi x}{b}\, dx \times \sinh\dfrac{m\pi h}{b} \times \left(h\cosh\dfrac{m\pi h}{b} - \dfrac{b}{m\pi}\sinh\dfrac{m\pi h}{b} \right) \right] \\ \qquad \div \left[\left(h\sinh\dfrac{m\pi h}{b} \right)^2 - \left(\dfrac{b}{m\pi}\sinh\dfrac{m\pi h}{b} + h\cosh\dfrac{m\pi h}{b} \right) \times \left(h\cosh\dfrac{m\pi h}{b} - \dfrac{b}{m\pi}\sinh\dfrac{m\pi h}{b} \right) \right] \\ D_m = -C_m \times \left(\dfrac{b}{m\pi}\sinh\dfrac{m\pi h}{b} + h\cosh\dfrac{m\pi h}{b} \right) \times \dfrac{1}{h\sinh\frac{m\pi h}{b}} \\ \qquad - \dfrac{2b}{m^2\pi^2} \displaystyle\int_0^b q_1 \sin\dfrac{m\pi x}{b}\, dx \times \sinh\dfrac{m\pi h}{b} \times \dfrac{1}{h\sinh\frac{m\pi h}{b}} \end{cases} \tag{8}$$

Substituting Eq. (8) into Eq. (6), and then into Eq. (5), verifies the shear-stress boundary conditions. After simplification, it is found that the equilibrium equation is identically satisfied; that is, the stress function Φ_1 satisfies the shear-stress boundary conditions on the left and right sides.

In the trigonometric-series solution, trigonometric functions are superposed to approximate the local load q_2. The order of the trigonometric series has a relatively large influence on the accuracy of fitting the local uniformly distributed load. Taking the plate thickness $h = 1$ m, the plate width $b = 14$ m, the width of the filling body $a = 7$ m, and the load of the filling body on the roof plate $q_2 = 10^5$ Pa as an example, fitting curves for the uniformly distributed load q_2 are plotted for trigonometric-series orders $m = 10, 100, 200, 2\,000$, as shown in Fig. 3. The fitted curve becomes closer to 0 Pa in the intervals 0 m $\leq x <$ 3.5 m and 10.5 m $< x \leq$ 14 m, and closer to 10^5 Pa in the interval 3.5 m $\leq x \leq$ 10.5 m; hence the fitting accuracy becomes higher.

It can be seen from Fig. 3 that when $m = 10$, the fitting accuracy for the uniformly distributed load q_2 is not high; in particular, in the intervals 0 m $\leq x <$ 6 m and 8 m $\leq x \leq$ 14 m, the actual load cannot be fitted accurately. When $m = 100$, relatively large errors occur in the small regions near $x = 3.5$ m and $x = 10.5$ m. When $m = 2\,000$, the fitting accuracy is very high. Considering that an excessively high trigonometric-series order greatly increases the calculation time, $m = 200$ is therefore adopted; this both ensures computational accuracy and maintains computational efficiency.

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Fig. 3 Fitting the q_2 curve of uniformly distributed load with different orders

2.2 Solution of stress function Φ_2

Solve for Φ_2. The model is shown in Fig. 4. Applying the Saint-Venant principle, the equivalent stress boundary conditions are written as

$$\begin{cases} \int_0^h (\sigma_x)_{x=0}\, dy = F_3 \\ \int_0^h (\sigma_x)_{x=b}\, dy = F_4 \\ \int_0^h (\sigma_x)_{x=0} \left(y - \frac{h}{2}\right) dy = M_1 \\ \int_0^h (\sigma_x)_{x=b} \left(y - \frac{h}{2}\right) dy = M_2 \end{cases} \tag{9}$$

Fig. 4 Model corresponding to stress function Φ_2

From Eq. (3), the stress components are known to be

$$\begin{cases} \sigma_x^2 = 6Ey + 2G \\ \sigma_y^2 = 0 \\ \tau_{xy}^2 = 0 \end{cases} \tag{10}$$

Substituting Eq. (9) into Eq. (10) and simplifying the simultaneous equations gives

$$E = \frac{2M_1}{h^3}, \quad G = \frac{F_3}{2h} - \frac{3M_1}{h^2} \tag{11}$$

Substituting Eq. (11) into Eq. (3), the stress function is obtained as

$$\Phi_2 = \frac{2M_1}{h^3} y^3 + \left(\frac{F_3}{2h} - \frac{3M_1}{h^2}\right) y^2 \tag{12}$$

Thus, the stress components are

$$\begin{cases} \sigma_x^2 = \dfrac{12M_1}{h^3}y + \dfrac{F_3}{h} - \dfrac{6M_1}{h^2} \\ \sigma_y^2 = 0 \\ \tau_{xy}^2 = 0 \end{cases} \tag{13}$$

2.3 Solution of the Stress Function Φ

Superposing the stress solutions of Eq. (6) and Eq. (13), the stress solution of the immediate roof is obtained as

$$\begin{cases} \sigma_x = \sigma_x^1 + \sigma_x^2 = \sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \sin\dfrac{m\pi x}{b} \left[\left(A_m + \dfrac{2b}{m\pi}D_m\right)\sinh\dfrac{m\pi y}{b} + \left(B_m + \dfrac{2b}{m\pi}C_m\right)\cosh\dfrac{m\pi y}{b} + C_{my}\sinh \dfrac{m \\ \qquad + D_{my}\cosh\dfrac{m\pi y}{b}\right] + \dfrac{12M_1}{h^3}y + \dfrac{F_3}{h} - \dfrac{6M_1}{h^2} \\ \sigma_y = \sigma_y^1 + \sigma_y^2 = -\sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \sin\dfrac{m\pi x}{b} \left(A_m\sinh\dfrac{m\pi y}{b} + B_m\cosh\dfrac{m\pi y}{b} + C_{my}\sinh\dfrac{m\pi y}{b} + D_{my}\cosh\dfrac{m\pi y}{b}\right) \\ \tau_{xy} = \tau_{xy}^1 + \tau_{xy}^2 = -\sum\limits_{m=1}^{\infty} \dfrac{m^2\pi^2}{b^2} \cos\dfrac{m\pi x}{b} \left[\left(B_m + \dfrac{b}{m\pi}C_m\right)\sinh\dfrac{m\pi y}{b} + \left(A_m + \dfrac{b}{m\pi}D_m\right)\cosh\dfrac{m\pi y}{b} + D_{my}\sin \\ \qquad + C_{my}\cosh\dfrac{m\pi y}{b}\right] \end{cases}$$

3 Comparative Analysis of Different Analytical Results

To analyze the reliability of the analytical results, the parameters in Table 1 are selected for calculation. Using Eq. (14), the stress distribution diagram of the immediate roof is plotted (tension is taken as positive and compression as negative).

Table 1 Basic parameters

Tab. 1 Basic parameters

h/m	b/m	a/m	q_1/Pa	q_2/Pa
1	14	7	5×10^5	10^5

Substituting $y = 0.5$ m into Eq. (14), the function of $(\sigma_y)_{y=0.5}$ with respect to x can be obtained; substituting $x = 7$ m, 10.5 m, and 14 m into Eq. (14), respectively, gives the functions of $(\sigma_x)_{x=7}$, $(\sigma_x)_{x=10.5}$, and $(\sigma_x)_{x=14}$ with respect to y. The value of $(\sigma_y)_{y=0.5}$ is symmetrically distributed about $x = 7$ m. The value of σ_y gradually increases from $x = 0$ m to $x = 7$ m, and the maximum value occurs at $x = 7$ m. As y increases, the value of $(\sigma_x)_{x=7}$ also increases; σ_x is tensile stress over the entire section, and the maximum value occurs at $y = 1$ m. For $(\sigma_x)_{x=10.5}$, as y increases, the value of σ_x first increases to the maximum compressive stress and then decreases; after decreasing to 0, it becomes tensile stress and increases to the maximum tensile stress. The variation trend of $(\sigma_x)_{x=14}$ is the same as that of $(\sigma_x)_{x=10.5}$, and its maximum tensile stress occurs at $y = 1$ m, as shown in Fig. 5. For convenience of comparison with Ref. [22], Fig. 5 plots the stress distribution schematics at the positions $(\sigma_y)_{y=0.5}$, $(\sigma_x)_{x=7}$, $(\sigma_x)_{x=10.5}$, and $(\sigma_x)_{x=14}$.

Fig. 5 Schematic diagram of stress distribution on the immediate roof

Zheng Litian et al. [22] simplified the immediate roof as a rock beam with hinged supports at both ends considering only its self-weight, and analytically obtained its stress distribution as shown in Fig. 6.

Fig. 6 Schematic diagram of stress distribution in rock beams with hinged supports at both ends in elasticity

Comparing Fig. 5 and Fig. 6, it can be found that the distribution trends of σ_x and σ_y are similar. Since Ref. [22] did not consider the load of the overlying strata on the immediate roof, whereas this model considers the load of the overlying strata, the boundary mechanical conditions on the immediate roof ···

conditions, which leads to differences in the distribution of σ_y between the two models, indicating that the elastic-mechanics analytical results in this study possess a certain reliability.

4 Evolution law analysis of direct-roof stress fields under different filling parameters

4.1 Calculation scheme

Assume that the burial depth of the working face is 400 m, the length is 200 m, and the mining height is 4 m. Based on the literature [23-26], the physical and mechanical parameters of each rock layer are comprehensively determined, as shown in Table 2. From the distance between the ground surface and the upper part of the roof, and the calculation of the overlying-rock load according to the lithology of the overlying strata, the load q_1 of the overlying strata is 10.61 MPa, and the load q_2 of the filling body on the roof is 2.12 MPa.

Using Eq. (14) and Matlab programming, the stress at any point in the roof can be calculated. To improve computational efficiency, as shown in Fig. 7, 10 points are taken at equal intervals in the span direction and 4 points are taken at equal intervals in the thickness direction. The corresponding points on the two opposite sides of the beam are connected to form 40 points, and the stresses at the 40 points in the roof are extracted for analysis.

Table 2 Physical and mechanical parameters of each rock layer

Rock layer	Rock type	Tensile strength/MPa	Density/(kg· m^{-3})	Poisson' s ratio	Thickness/m
Overlying strata	Sandy mud-stone	3.13	2 703	–	100
Overlying strata	Fine sand-stone	6.70	2 710	–	100
Overlying strata	Medium sand-stone	5.60	2 810	–	100
Overlying strata	Siltstone	3.02	2 710	–	100
Direct roof	Fine sand-stone	6.70	2 710	0.2	1∼5
Filling body	–	1.00	1 400	–	2
Floor	Sandy mud-stone	3.13	2 703	–	–

Fig. 7 Distribution of stress extraction points

In this calculation, 7 schemes are designed. Schemes 1, 2, 3, and 4 are used to study the influence of filling width a on the stress distribution of the roof; Schemes 5, 2, 6, and 7 are used to study the influence of the height-span ratio on the stress distribution of the roof, as shown in Table 3.

Table 3 Calculation scheme

Parameter	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5	Scheme 6	Scheme 7
Filling-body width/m	9	10	11	12	10	10	10
Roof thick-ness/m	2	2	2	2	1	4	5
Roof width/m	20	20	20	20	20	20	20

Parameter	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5	Scheme 6	Scheme 7
Height-span ratio	1/10	1/10	1/10	1/10	1/20	1/5	1/4

4.2 Influence of filling width on the stress distribution of the roof

Keeping the roof thickness unchanged, the influence of filling width on the stress distribution of the roof is analyzed. The cloud maps of the maximum tensile stress σ_{r1} distribution in the roof under different filling-width conditions are plotted, as shown in Fig. 8.

The maximum tensile stress of the roof is concentrated in the lower part of the roof near the end regions on both sides, and the location of stress concentration does not change with increasing filling-body width. This indicates that the position at which the filled roof is most prone to tensile failure is the middle region between the two filling bodies. Figure 9 gives the curve of the maximum tensile stress at special points as it varies with filling width. From Fig. 9, the variation of the maximum tensile stress of the roof with filling width can be seen more clearly (the locations of the selected special points in Fig. 9 are shown in Fig. 7).

As can be seen from Fig. 9, as the filling width increases, the maximum tensile stresses at point (x_0, y_3) and point (x_4, y_3) both show a clear decreasing trend. When the filling-body width a increases from 9 m to 12 m, the maximum tensile stress at (x_0, y_3) decreases by about 26%, and that at (x_4, y_3) decreases by about 29%, indicating that increasing the filling width can effectively weaken the tensile-stress concentration in the roof plate. Overall, the greater the width of the filling body, the more sufficient its supporting effect on the roof plate, and the corresponding possibility of tensile failure of the roof plate decreases.

(a) Scheme 1 ($a = 9$ m)

(b) Scheme 2 ($a = 10$ m)

(c) Scheme 3 ($a = 11$ m)

(d) Scheme 4 ($a = 12$ m)

Fig. 8 Stress cloud map corresponding to different filling widths

Fig. 9 The variation of maximum tensile stress at special points on the top plate with filling width

4.3 Influence of the height-span ratio on the stress distribution of the roof plate

Under the condition that the filling width remains unchanged, the influence of the roof-plate height-span ratio on the stress distribution of the roof plate is further analyzed. The maximum tensile-stress σ_{r1} distribution cloud maps under different roof-plate height-span-ratio conditions are shown in Fig. 10. From Fig. 10, it can be seen that the region of maximum tensile stress in the roof plate is mainly distributed at the lower part of the roof plate; moreover, as the roof-plate height-span ratio changes, the position of the stress-concentration region does not undergo obvious migration, indicating that the dangerous region has a certain spatial stability.

Fig. 11 gives the relationship curves between the maximum tensile stress at special points and the roof-plate height-span ratio. It can be seen from Fig. 11 that the maximum tensile stresses at the special points (x_0, y_3) and (x_4, y_3) both show a decreasing trend with increasing roof-plate height-span ratio, and the variation patterns of the two are basically consistent. This indicates that, within the parameter range analyzed, increasing the roof-plate height-span ratio helps reduce the tensile-stress level of the roof plate, thereby reducing the possibility of tensile failure of the roof plate. From Fig. 10 and Fig. 11 together, it can be seen that the roof-plate height-span ratio mainly affects the numerical magnitude of the maximum tensile stress, while its influence on the location of the dangerous point is relatively limited. Regardless of how the roof-plate height-span ratio changes, the dangerous points are mainly distributed in the lower region of the roof plate near the two sides. Therefore, in the safety analysis of the roof plate in strip filling mining, attention should be focused on this region.

(a) Scheme 5 ($h/b = 1/20$)

(b) Scheme 2 ($h/b = 1/10$)

(c) Scheme 6 ($h/b = 1/5$)

(d) Scheme 7 ($h/b = 1/4$)

Fig. 10 Stress cloud maps corresponding to different height-to-span ratios

Fig. 11 Changes in principal stress at key points of the top plate with height-to-span ratio

4.4 Optimal filling spacing

The principal stress at any point of the top plate is

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

The failure criterion for any point of the top plate is

$$\sigma_1 > \sigma_t$$

where σ_t is the tensile strength of the top-plate rock layer.

In Sections 4.2 and 4.3, it was found that the dangerous points of the top plate are located near the two sides at the lower part of the top plate. When the maximum tensile stress σ_{r1} at this position of the top plate is greater than σ_t, the top plate is considered to have failed.

When the thickness of the top plate, the tensile strength of the top-plate rock, the load of the overlying strata, and the supporting force of the backfill on the top plate are all known, the optimal filling spacing $z_{\max}$ can be determined; that is, under the premise that the top plate is not destroyed, the filling spacing should be increased as much as possible[^27].

According to the stress solution for the filling top plate derived in Section 2.3, the maximum tensile stress σ_{r1} at the dangerous point (x_0, y_3) or (x_9, y_3) of the top plate can be obtained. Let

$$\sigma_{\mathrm{r1}} = \sigma_t$$

then

$$\sigma_t = \frac{1}{2}\left[\sigma_x(0,h) + \sigma_y(0,h)\right] + \frac{1}{2}\sqrt{\left[\sigma_x(0,h) - \sigma_y(0,h)\right]^2 + 4\tau_{xy}(0,h)^2} \qquad (15)$$

Equation (15) contains only the unknowns a and b. Taking a and b as new variables, from the geometric relationship in Fig. 1 it can be found that

$$b - a = z \qquad (16)$$

Combining Eqs. (15) and (16), we obtain.

$$\begin{cases} \sigma_t = \frac{1}{2}\left[\sigma_x(0,h) + \sigma_y(0,h)\right] + \\ \qquad \frac{1}{2}\sqrt{\left[\sigma_x(0,h) - \sigma_y(0,h)\right]^2 + 4\tau_{xy}(0,h)^2} \\ b = a + z \end{cases}$$

According to linear programming theory, when the two curves intersect and the value of z reaches its maximum, the limiting condition is obtained. The corresponding value of z is the optimal filling spacing $z_{\max}$, as shown in Fig. 12.

Fig. 12 Schematic diagram of linear programming for determining the optimal filling spacing

Figure 5: Fig. 12 Schematic diagram of linear programming for determining the optimal filling spacing

图 12 线性规划确定最佳充填间距示意图

Fig. 12 Schematic diagram of linear programming for determining the optimal filling spacing

5 Conclusions

Based on the triangular-series solution of elasticity, this study derived an elastic analytical solution for the roof stress field under strip backfill mining conditions, and implemented the theoretical calculation through Matlab programming. On this basis, the potential dangerous zones of roof tensile failure were analyzed, and a method for calculating the optimal filling spacing was proposed. The main conclusions are as follows.

1) Under strip backfill mining conditions, the regions on both sides of the lower part of the roof are the areas where the maximum tensile stress is concentrated and are potential dangerous locations. Within the analyzed parameter range, changes in the backfill width and the roof height-span ratio have relatively little influence on the location of the dangerous point, and the position of the dangerous point is relatively stable.

2) The maximum tensile stress at the dangerous point is affected by the backfill width and the roof height-span ratio. Within a certain range, increasing the backfill width helps reduce the tensile stress level of the roof, thereby decreasing the possibility of tensile failure of the roof.

3) The method proposed in this study for calculating the optimal strip backfill spacing introduces the criterion of roof tensile failure into the process of determining the filling spacing, and can provide a theoretical reference for the parameter design of strip backfill mining.

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