Study on the Response Characteristics of Vortex-Induced Vibration of Triangularly Arranged Wavy Cone Columns

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Abstract

The vortex-induced vibration characteristics of triangularly arranged wavy conical cylinders at a subcritical Reynolds number of Re=3900 were numerically simulated using overset mesh and UDF techniques. The variations in the single-degree-of-freedom vortex-induced vibration amplitude, frequency, and lift and drag coefficients of the downstream wavy conical cylinder with reduced velocity were analyzed for different cross-flow spacing ratios L/Dm=2-5, streamwise spacing ratios T/Dm=2-6, and reduced velocities Ur=4-15. The results show that, under the complex wake of the two upstream wavy conical cylinders, the vibration excitation mode of the downstream wavy conical cylinder changes from reattachment of the upstream shear layers to an alternating vortex-shedding excitation mode under wake merging. The fluctuating lift coefficient is significantly increased, effectively extending the lock-in range. Under appropriate operating conditions, the amplitude can be increased by up to 23.2% compared with that of a single wavy conical cylinder. In addition, within the lock-in range, the lift coefficient exhibits a multi-frequency phenomenon, which gradually weakens as the reduced velocity increases. This study provides theoretical support for the layout of bladeless wind turbine arrays.

Full Text

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Abstract: Using overlapping grids and UDF technology, numerical simulations were carried out on the vortex-induced vibration characteristics of triangularly arranged wavy cone columns at the subcritical Reynolds number $Re = 3\,900$. The variation laws of the amplitude, frequency, and lift/drag coefficients of the single-degree-of-freedom vortex-induced vibration of the downstream wavy cone column with the reduced velocity were analyzed for different transverse-flow spacing ratios $L/D_\mathrm{m} = 2 \sim 5$, streamwise spacing ratios $T/D_\mathrm{m} = 2 \sim 6$, and reduced velocities $U_\mathrm{r} = 4 \sim 15$. The results show that, under the action of the complex wake of the upstream double wavy cone columns, the vibration excitation mode of the downstream wavy cone column changes from the reattachment of the upstream shear layers to the alternating vortex excitation mode under wake merging; the pulsating lift coefficient is significantly increased, effectively broadening the lock-in interval. Under appropriate operating conditions, the amplitude increase can be as high as 23.2% compared with that of a single wavy cone column. Within the lock-in interval, a multi-frequency phenomenon appears in the lift coefficient, and as the reduced velocity increases, the multi-frequency phenomenon gradually weakens. This provides theoretical support for the layout of bladeless wind-turbine arrays.

Keywords: wavy cone column; triangular arrangement; wake effect; vortex-induced vibration; numerical simulation

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Research on the response characteristics of vortex induced vibration of triangular arranged wave cone columns

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Abstract: The vortex-induced vibration of triangular arranged wavy conical columns at subcritical Reynolds number $Re = 3\,900$ is numerically simulated by using overlapping grid and UDF technique. The amplitude, frequency and drag coefficient of vortex induced vibration of the downstream wavy conical column with reduced velocity were analyzed for different transverse flow direction spacing ratio $L/D_\mathrm{m} = 2-5$, forward flow direction spacing ratio $T/D_\mathrm{m} = 2-6$ and reduced velocity $U_\mathrm{r} = 4-15$. The research shows that the vibration excitation mode of the downstream wave cone column under the complex wake of the upstream double wave cone column changes from the upstream shear layer reattachment to the alternating vortex excitation mode under the wake fusion, and the pulsation lift coefficient is significantly increased, effectively extending the frequency locking interval. The amplitude increase can be as high as 23.2% compared with that of the single wave cone column under appropriate working conditions, and the lift coefficient appears multi-frequency phenomenon within the locking interval. With the increase of the

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reduced velocity, the multi-frequency phenomenon gradually weakens. The study provides theoretical support for the layout of bladeless wind turbines.

Key words: wavy conical cylinder; triangular arrangement; wake effect; vortex-induced vibration; numerical simulation

Vortex-induced vibration power generation realizes unsteady aerodynamic control of a wind column and effectively enhances/suppresses vibration by changing the surface shape of the wind column and expanding the "lock-in" region; moreover, multi-wind-column groups are used to increase the system energy density. Many scholars have carried out studies on this topic [1−3]. Zou Lin et al. [4] found that the maximum response amplitude of vortex-induced vibration of a wavy conical cylinder was 12.6% larger than that of a circular cylinder, and that the lock-in interval was effectively expanded. A large number of studies have also been conducted on the vortex-induced vibration of two circular cylinders [5−7]. Xu Jiahuang et al. [8] performed numerical simulations of vortex-induced vibration of two circular cylinders in tandem with two degrees of freedom, and found that different natural-frequency ratios and shear rates had a relatively large effect on the vibration amplitude of the downstream cylinder, but a relatively small effect on the vibration amplitude of the upstream cylinder. Liu Jian [9] found that, as the spacing ratio changed, the flow structure between two conical cylinders in tandem could be divided into three states: shear-layer wrapping, transition, and wake-impact states. Zhao Peng et al. [10] used CFD numerical simulation and found that the strength of flow interference between two circular cylinders in parallel was inversely proportional to the spacing ratio. Chen et al. [11]

used the immersed boundary method to numerically study vortex-induced vibration of three circular cylinders in tandem, and found that, owing to the strong interference between the wake and the cylinders, large-amplitude vibration of the downstream cylinders would be excited. Xu Jiahuang et al.

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numerically calculated the two-degree-of-freedom vortex-induced vibration of three circular cylinders in tandem, and found that the frequency lock-in region of the middle cylinder increased with increasing Reynolds number, while its dynamic response was greatly affected by the wake of the upstream cylinder. Chen et al.

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studied the vortex-induced vibration response characteristics of circular cylinders in an equilateral triangular arrangement, and found that the significant increase in the amplitudes of the two downstream cylinders mainly originated from the vorticity supplied by the upstream cylinder. Zhang Zheng et al.

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studied the wake characteristics of three circular cylinders in tandem at low Reynolds numbers and the vortex-induced vibration response of the middle cylinder in transverse vibration, and found that, for three stationary circular cylinders, under different spacing ratios the wake mode changed from the "2S" mode to the "2P" mode. Zhang Xiaona et al.

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numerically simulated the vortex-induced vibration of three rigidly coupled circular cylinders arranged in an equilateral triangle, and found that, as the reduced velocity increased, the vibration responses under each spacing ratio could be divided into an initial branch, a lower branch, and a non-lock-in region. Chen et al.

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numerically studied the vortex-induced vibration and wake patterns of two circular cylinders in parallel in the wake of a stationary upstream cylinder. They found that, at higher reduced velocities, the smaller the spacing ratio, the larger the amplitude and the wider the vibration range. Liu Qiong

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numerically studied the three-dimensional flow around a cluster of three circular cylinders in a staggered arrangement at subcritical Reynolds numbers, and found that, within the small-spacing-ratio range, the mean drag coefficient of the middle downstream cylinder was significantly smaller than that of a single-cylinder case.

It can be seen that the cylinder surface shape and the arrangement form of multiple cylinders directly affect the vortex-induced-vibration characteristics of the cylinders. Based on preliminary research by the research group, this study uses the commercial software Fluent, together with overset mesh and UDF techniques, to carry out simulation analysis of vortex-induced vibration of wavy conical cylinders in a triangular arrangement. The vortex-induced vibration of the downstream wavy conical cylinder is simplified as a single-degree-of-freedom spring-mass-damper system. The variations in vibration amplitude, lock-in interval, lift-drag characteristics, and flow structure of the downstream wavy conical cylinder under the wake action of fixed upstream double wavy conical cylinders at different spacing ratios are analyzed in depth, and its vortex-induced-vibration response characteristics are explored, with a view to providing reference data for the rational layout of bladeless wind-turbine groups.

1 Governing Equations and Equations of Motion

1.1 Governing Equations

The basic governing equations of fluid mechanics are the energy equation, momentum equation, and continuity equation. The object of this study is an incompressible fluid, whose Navier–Stokes equations are

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \qquad (2)$$

where $\bar{u}_i$ is the filtered velocity component; x_i and x_j are coordinates; $\bar{p}$ is the filtered pressure; ν is the kinematic viscosity of the fluid; and τ_{ij} is the subgrid-scale stress tensor. In this study, turbulent calculation is performed using the enhanced wall treatment method in the standard k-epsilon model.

1.2 Equations of Motion

The vortex-induced vibration process of the elastically mounted wavy conical cylinder model is simplified as a spring–mass–damper model, and its vibration equations are expressed as

$$\begin{aligned} m\ddot{x} + c\dot{x} + kx &= F_x, \\ m\ddot{y} + c\dot{y} + ky &= F_y. \end{aligned} \qquad (3)$$

where m is the mass of the wavy conical cylinder; c is the system damping coefficient; k is the spring stiffness coefficient; x and y are, respectively, the displacements of the wavy conical cylinder in the streamwise and transverse directions; and F_x and F_y are, respectively, the forces acting on the wavy conical cylinder in the streamwise and transverse directions.

In the vortex-induced-vibration simulation calculation of this study, the fourth-order Runge-Kutta methods are applied, and the vibration equation of the downstream wavy conical cylinder is treated using a fully implicit format to obtain

$$f(v_y, \gamma) = \frac{F_y(t)}{m} - 2\xi\omega_0\gamma_{(t)} - \omega_0^2\gamma_{(t)} \qquad (4)$$

where $\xi = c/2\sqrt{km}$ is the system damping ratio; $\omega_0 = \sqrt{k/m}$ is the natural frequency of the system.

Then, applying the fourth-order Runge-Kutta method to Eq. (4), one obtains

$$v_{y(t_{n+1})} = v_{y(t_n)} + \frac{\Delta t}{6} \times (K_1 + 2K_2 + 2K_3 + K_4),$$

$$\gamma_{(t_{n+1})} = \gamma_{(t_n)} + \Delta t \times v_{y(t_n)} + \frac{(\Delta t)^2}{6} \times (K_1 + K_2 + K_3) \qquad (5)$$

where Δt is the time step; $K_1 \sim K_4$ are the coefficients of each iteration step, and

$$K_1 = \frac{F_y(t)}{m} - 2\xi\omega_0 v_{y(t_n)} - \omega_0^2\gamma_{(t_n)},$$

$$K_2 = \frac{F_y(t)}{m} - 2\xi\omega_0 \left[v_{y(t_n)} + \frac{\Delta t}{2} \times K_1\right] - \omega_0^2 \left[\gamma_{(t_n)} + \frac{\Delta t}{2} \times v_{y(t_n)}\right],$$

$$K_3 = \frac{F_y(t)}{m} - 2\xi\omega_0 \left[v_{y(t_n)} + \frac{\Delta t}{2} \times K_2\right] - \omega_0^2 \left[\gamma_{(t_n)} + \frac{\Delta t}{2} \times v_{y(t_n)} + \frac{(\Delta t)^2}{4} \times K_1\right],$$

$$K_4 = \frac{F_y(t)}{m} - 2\xi\omega_0 \left[v_{y(t_n)} + \Delta t \times K_3\right] - \omega_0^2 \left[\gamma_{(t_n)} + \Delta t \times v_{y(t_n)} + \frac{(\Delta t)^2}{2} \times K_2\right] \qquad (6)$$

Finally, the cross-flow motion velocity and displacement of the downstream wavy cone column are obtained.

Fig. 1 Schematic diagram of wavy cone column

Figure 1: Fig. 1 Schematic diagram of wavy cone column

Fig. 2 Triangular arrangement wavy cone-column model and grid diagram

Figure 2: Fig. 2 Triangular arrangement wavy cone-column model and grid diagram

2 Computational domain and grid

2.1 Model description

This study adopts the wavy cone column model developed in the research group's previous work. The geometric shape of the wavy cone column is shown in Fig. 1[4], and its expression is defined as

$$D_z = D_{\mathrm{m}} + 2a \times \cos\frac{2\pi z}{\lambda} + 2k \times \left(z - \frac{H}{2}\right) \qquad (7)$$

where D_z is the diameter of the wavy cone column; $D_{\mathrm{m}} = 0.01$ m is the mean diameter of the wavy cone column. In Fig. 1, D_H is the maximum diameter of the cone column (with slope k), D_L is the minimum diameter of the cone column (with slope k), and H is the height of the wavy cone column, with $H/D_{\mathrm{m}} = 7$; a and λ are the wave amplitude and wavelength of the wavy cone column, respectively. The parameter values with the best vibration effect are selected, namely $k = 0.05$, $a/D_{\mathrm{m}} = 0.1$, and $\lambda/D_{\mathrm{m}} = 1.75$.

Fig. 1 Schematic diagram of wavy cone column

2.2 Computational domain and grid division

Figure 2 is a schematic diagram of the triangular arrangement wavy cone-column model and grid. Three wavy cone columns with identical geometric structures are arranged in a triangular pattern; among them, two wavy cone columns are fixed and placed upstream (hereinafter referred to as the "upstream upper column" and "upstream lower column," respectively), and one wavy cone column is placed downstream (hereinafter referred to as the "downstream column"). The downstream column is elastically mounted and undergoes only cross-flow single-degree-of-freedom vortex-induced vibration, simplified as a single-degree-of-freedom spring–mass–damping system.

(a) (b)

Fig. 2 Triangular arrangement wavy cone-column model and grid diagram

The size of the computational domain for the triangular arrangement wavy cone columns is $(30D_{\mathrm{m}} + T) \times (20D_{\mathrm{m}} + L) \times 10D_{\mathrm{m}}$, where L is the transverse spacing between the centers of the two upstream wavy cone columns, and T is the

Fig. 3 Amplitude of vortex-induced vibration of downstream column with single degrees of freedom

Figure 3: Fig. 3 Amplitude of vortex-induced vibration of downstream column with single degrees of freedom

streamwise spacing between the centers of the upstream and downstream wavy cone columns, as shown in Fig. 2(b). The origin is located at the center of the upstream upper column,

The distance from the centers of the upper and lower side columns to the inlet is $10D_\mathrm{m}$, and the distance from the center of the downstream column to the outlet is $20D_\mathrm{m}$. The distances from the upstream upper and lower side columns to the nearest side faces of the computational domain are both $10D_\mathrm{m}$, and the distances from the top and bottom surfaces of the corrugated conical column to the top and bottom surfaces of the computational domain are both $1.5D_\mathrm{m}$. A velocity inlet is adopted, with a magnitude of 5.772 m/s; a pressure outlet is adopted and set to 0 Pa. The governing equations are discretized by the finite volume method, and the PISO algorithm is used for pressure–velocity coupling. Second-order schemes are used for both spatial and temporal discretization [18]. Symmetry boundary conditions are applied on the side, bottom, and top surfaces of the computational domain, and a no-slip wall condition is applied on the surface of the corrugated conical column. The fluid medium is air, with density $\rho = 1.225\ \mathrm{kg/m^3}$ and dynamic viscosity $\mu = 1.813\ \mathrm{Pa\cdot s}$. The computational domain consists of foreground and background grids. The mesh type used in this study is an overset mesh. Around each corrugated conical column there is a refined region of $5D_\mathrm{m} \times 5D_\mathrm{m} \times 10D_\mathrm{m}$, and the mesh size is the same as that in the single corrugated conical-column vortex-induced vibration model in Ref. [19] (the validation of the computational model for vortex-induced vibration of a single corrugated conical column is detailed in Ref. [19]).

3 Results and Analysis

3.1 Amplitude characteristics of vortex-induced vibration of the downstream column

Figure 3 shows the variation curves of the maximum amplitude $Y_\mathrm{max}/D_\mathrm{m}$ of single-degree-of-freedom vortex-induced vibration of the downstream column in triangularly arranged corrugated conical columns with the cross-flow spacing ratio $L/D_\mathrm{m} = 2 \sim 5$ and the streamwise spacing ratio $T/D_\mathrm{m} = 2 \sim 6$, as a function of the reduced velocity U_r ($U_\mathrm{r} = U_\infty/f_n D_\mathrm{m}$, f_n is the natural frequency of the corrugated conical column).

(a) Cross-flow spacing ratio $L/D_\mathrm{m} = 2$

(b) Cross-flow spacing ratio $L/D_\mathrm{m} = 3$

(c) Cross-flow spacing ratio $L/D_\mathrm{m} = 4$

(d) Cross-flow spacing ratio $L/D_\mathrm{m} = 5$

Figure 3 Amplitude of vortex-induced vibration of the downstream column with single degree of freedom

Fig. 3 Amplitude of vortex-induced vibration of downstream column with single degrees of freedom

It can be seen from Fig. 3 that, under different cross-flow spacing ratios L/D_m, the maximum amplitude of the downstream column is obtained when the streamwise spacing ratio $T/D_\mathrm{m} = 2$, and as T/D_m increases, the overall vibration amplitude and the lock-in interval decrease accordingly. For the single corrugated conical column, the maximum amplitude is obtained at the reduced velocity $U_\mathrm{r} = 7$, and the reduced-velocity range of the lock-in interval is $U_\mathrm{r} = 5 \sim 10$. When $L/D_\mathrm{m} = 2$, the lock-in interval of the downstream column shifts markedly to the right. When the streamwise spacing ratio $T/D_\mathrm{m} = 2, 3$, the maximum amplitude is obtained at the reduced velocity $U_\mathrm{r} = 12$, and the lock-in interval is $U_\mathrm{r} = 7 \sim 15$. When the streamwise spacing ratio $T/D_\mathrm{m} = 4, 5, 6$, the downstream column obtains the maximum amplitude at the reduced velocity $U_\mathrm{r} = 11$. Compared with the single corrugated conical column, the lock-in interval is still significantly broadened. When the cross-flow spacing ratio $L/D_\mathrm{m} = 3, 4, 5$ and the streamwise spacing ratio $T/D_\mathrm{m} = 2, 3$, the downstream column obtains the maximum amplitude at the reduced velocity $U_\mathrm{r} = 8$. When the streamwise spacing ratio $T/D_\mathrm{m} = 4, 5, 6$, the reduced velocity corresponding to the maximum amplitude of the downstream column is ⋯

At reduced velocity $U_r = 7$, the downstream-column vibration enhancement is best when the cross-flow spacing ratio is $L/D_m = 4$ and the in-flow spacing ratio is $T/D_m = 2$; the vibration amplitude ratio is approximately 0.894. Compared with a single wavy cone column, the lock-in interval of the downstream column is significantly widened.

The widening of the lock-in interval and the increase in vibration amplitude allow the wavy cone column to harvest energy over a larger wind-speed range. Therefore, triangularly arranged wavy cone columns are expected to harvest more wind energy at an appropriate spacing ratio.

3.2 Analysis of vortex-induced vibration frequency characteristics of the downstream column

The downstream-column frequency ratio f^* is obtained by fast Fourier transformation (fast Fourier transformation, FFT), where $f^* = f_v/f_n$, f_v is the vortex-shedding frequency of the wavy cone column, and f_n is the natural frequency of the system. The variations of the vortex-shedding frequency and lift-coefficient power spectral density function with reduced velocity are shown in Fig. 4 and Fig. 5.

Fig. 4 Downstream column frequency ratio changes with reduced velocity

Fig. 4 Downstream column frequency ratio changes with reduced velocity

Figure 4: Fig. 4 Downstream column frequency ratio changes with reduced velocity

Fig. 5 Changes of downstream column frequency ratio and vortex power spectrum function with reduced velocity of triangular arranged wavy cone columns

Figure 5: Fig. 5 Changes of downstream column frequency ratio and vortex power spectrum function with reduced velocity of triangular arranged wavy cone columns

(a) Vortex-shedding frequency spectrogram for cross-flow spacing ratio $L/D_m = 2$ and in-flow spacing ratio $T/D_m = 2$

(b) Vortex-shedding frequency spectrogram for cross-flow spacing ratio $L/D_m = 4$ and in-flow spacing ratio $T/D_m = 2$

Fig. 5 Changes of downstream column frequency ratio and vortex power spectrum function with reduced velocity of triangular arranged wavy cone columns

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At a reduced velocity of $U_r = 4$, the single wavy cone column frequency ratio is almost 0. As the reduced velocity increases, within the lock-in interval the frequency ratio fluctuates around 1, but when the single wavy cone column exits the lock-in interval, the frequency ratio is 1.065. When the transverse spacing ratio is $L/D_m = 2$ and the streamwise spacing ratio is $T/D_m = 2$, as the reduced velocity increases, the downstream-column frequency ratio first increases and then decreases, ranging from 0.923 to 0.985. When the transverse spacing ratio is $L/D_m = 4$ and the streamwise spacing ratio is $T/D_m = 2$, as the reduced velocity increases, the frequency ratio basically increases linearly, ranging from 0.928 to 1.02. However, comparison with Fig. 3 shows that outside the lock-in interval the frequency ratio likewise appears near 1, but the downstream column does not vibrate. From Fig. 5 it can be observed that, at the corresponding reduced velocity, the lift power spectral density is relatively low, with an energy density of about 10^{-7}; it is even smaller and is far below the lift power spectral function value within the lock-in interval. The energy contained in the vortex shedding is insufficient to sustain the vibration of the downstream column. Meanwhile, it can be found that within the lock-in interval the lift coefficient exhibits a multi-frequency phenomenon, and as the reduced velocity increases, the multi-frequency phenomenon gradually weakens.

Fig. 6 Lifting resistance coefficient of the downstream column of triangular arranged wavy cone column

Figure 6: Fig. 6 Lifting resistance coefficient of the downstream column of triangular arranged wavy cone column

3.3 Analysis of the Lift and Drag Coefficients of the Vortex-Induced Vibration of the Downstream Column

When the downstream column vibrates, the distance between the downstream column and the upper and lower side columns changes continuously, resulting in different wake structures of the triangular-arranged wavy cone columns and thereby causing changes in the lift and drag. The variation of the lift and drag coefficients of the triangular-arranged wavy cone columns under different spacing ratios with reduced velocity is shown in Fig. 6.

(a) Mean drag coefficient

(b) Pulsating lift coefficient

Fig. 6 Lifting resistance coefficient of the downstream column of triangular arranged wavy cone column

From Fig. 6, it can be seen that Cd_{mean} and Cl_{rms} of the upper and lower side columns in the triangular arrangement of wavy cone columns do not change significantly with increasing reduced velocity. However, when the transverse spacing ratio increases, both Cd_{mean} and Cl_{rms} of the upper and lower side columns decrease. With increasing reduced velocity, the downstream-column Cd_{mean} first increases sharply and then decreases slowly. When the transverse spacing ratio is $L/D_m = 2$ and the streamwise spacing ratio is $T/D_m = 2$, the downstream-column Cd_{mean} fluctuates around a certain value at reduced velocities $U_r = 8 \sim 11$. Its Cd_{mean} is obviously smaller than that of the triangular-arranged wavy cone columns with upper and lower side columns and transverse spacing ratio $L/D_m = 4$, streamwise spacing ratio $T/D_m = 2$. This is because during vibration the downstream column is always enveloped in the shear layers of the upper and lower side columns, and the recirculation region on the leeward side of the downstream column is suppressed. The variation trend of Cl_{rms} of the triangular-arranged wavy cone columns with reduced velocity is shown in Fig. 6(b). When the downstream column has not yet begun to vibrate and has exited the lock-in interval, Cl_{rms} of the downstream columns under the two spacing ratios is almost 0. With the increase of reduced velocity, Cl_{rms} first increases sharply, and after reaching the maximum value decreases slowly.

3.4 Phase Transition of the Vortex-Induced Vibration of the Downstream Column

The time histories of the lift coefficient and displacement of the downstream column in the triangular-arranged wavy cone columns are shown in Fig. 7. Within

Fig. 7 Lift time history curves of downstream displacement of triangular arranged wavy conical columns with different spacing ratios

Figure 7: Fig. 7 Lift time history curves of downstream displacement of triangular arranged wavy conical columns with different spacing ratios

the lock-in interval, the downstream-column vibration response curve is always a regular sine curve, whereas the lift coefficient curve gradually changes into a curve with the characteristic of "beating" (the "beating" characteristic curve means that the lift coefficient exhibits a second dominant frequency with a relatively small energy value). When the transverse spacing ratio is $L/D_m = 2$ and the streamwise spacing ratio is $T/D_m = 2$, the displacement and lift time-history curves are "in phase" at the reduced velocity $U_r = 7$. When the reduced velocity is $U_r = 8$, the lift coefficient curve begins to show the characteristic of "beating," and the downstream-column vibration amplitude increases significantly. When the reduced velocity is $U_r = 9 \sim 14$, the lift coefficient curve exhibits an obvious "beating" characteristic, and a phase difference appears between the lift coefficient curve and the displacement time-history curve. When the downstream-column lift reaches its maximum value, the vibration displacement has not yet reached its peak, so phase transition occurs. When the reduced velocity is $U_r = 15$, the lift coefficient curve and displacement time-history curve are in an "anti-phase" state; at this time, the energy dissipation is relatively large, causing the downstream-column vortex-induced vibration to begin to exit the lock-in interval and the vibration amplitude to be relatively small. For the downstream column of the triangular-arranged wavy cone columns with transverse spacing ratio $L/D_m = 4$ and streamwise spacing ratio $T/D_m = 2$, when the reduced velocity is $U_r = 5 \sim 7$, the displacement and lift time-history curves are in an "in-phase" state. At this time, the system is at the initial position of the lock-in interval, and as the reduced velocity increases, the vibration amplitude continuously increases; when the reduced velocity ⋯

When the reduced velocity $U_r = 8$, the lift-coefficient curve begins to show the characteristic "beat" phenomenon, and at this time the vibration amplitude of the downstream column reaches its maximum; when the reduced velocity $U_r = 9 \sim 12$, a phase difference appears between the lift-coefficient curve and the displacement time-history curve; when the reduced velocity $U_r = 13$, the lift coefficient still exhibits the "beat" characteristic, but overall, the lift-coefficient curve and the displacement time-history curve are in "opposite phase."

(a) Displacement–lift time-history curves for transverse spacing ratio $L/D_m = 2$ and streamwise spacing ratio $T/D_m = 2$

(b) Displacement–lift time-history curves for transverse spacing ratio $L/D_m = 4$ and streamwise spacing ratio $T/D_m = 2$

Fig. 7 Lift time history curves of downstream displacement of triangular arranged wavy conical columns with different spacing ratios

3.5 Flow structure of vortex-induced vibration of the downstream column

Figure 8 gives the vorticity contours of sections S1 and S2 of the downstream wavy conical column within one motion period under different reduced velocities. When the transverse spacing ratio $L/D_m = 2$ and the streamwise spacing ratio $T/D_m = 2$, the shear layers in front of the upper and lower cylinders on the upstream side alternately shed vortices that strike the surface of the downstream column, and merge with the vortices shed from the downstream column. As shown in Fig. 8(a), when the reduced velocity $U_r = 7$, the downstream column is in an in-phase state. The vortex-shedding mode of section S2 is the 2C mode, whereas that of section S1 is the 2S mode; this is mainly caused by the influence of the wavy cone shape and wake interference. When the reduced velocity $U_r = 12$, the downstream column is still in a state in which the shear layers in front of the upper and lower cylinders adhere again; section S1 changes to the 2C mode, and section S2 changes to the 2T mode. At this time, the downstream column obtains the maximum vibration amplitude. When the downstream column is at reduced velocity $U_r = 15$, it is in an anti-phase state. In section S2, vortex shedding of the downstream column is enclosed within the elongated vortex-shedding envelopes above the upper-side column and below the lower-side column; whereas in section S1, the vortex shedding of the downstream column and the upper-side column [[unclear: text continues on next page]].

the long strip-like vortices shed from the lower-side column merge, at which time the vibration amplitude of the downstream column is very small.

When the cross-flow spacing ratio is $L/D_m = 4$ and the streamwise spacing ratio is $T/D_m = 2$, vortices freely shed from the surfaces of the upstream upper- and lower-side columns merge with the vortices shed from the downstream column, forming a new alternating vortex street. As shown in Fig. 8(b), when the reduced velocity is $U_r = 6$, the downstream column is in an in-phase state; the vortex-shedding mode on section S1 is the 2C mode. However, as the vortices shed from the downstream column develop farther downstream, the smaller vortices on section S1 disappear because their energy is dissipated, and the wake mode gradually changes into the 2S mode; the wake mode on S2 is the 2P mode. When the reduced velocity is $U_r = 8$, the downstream column enters the phase-transition stage. On section S1, the downstream column sheds, within one period, a pair of counter-rotating vortices. After merging with the vortices shed from the upstream upper- and lower-side columns, the entire flow field develops into the 2S mode, while the wake mode on section S2 is the 2T mode. When the downstream column is in an anti-phase state at the reduced velocity $U_r = 13$, the S2 wake mode is the 2P mode, and the vortex structure on section S1 is the 2C mode. As the reduced velocity increases, the shear layer of the wave-cone column gradually becomes longer, making the rolled-up vortices more difficult to shed. Moreover, the vortex shedding from the upper- and lower-side columns is approximately anti-phase, so that the lift forces cancel each other,

Fig. 8 Schematic diagram of flow structure in the movement period of the downstream wave cone column at different reduced speeds

Figure 8: Fig. 8 Schematic diagram of flow structure in the movement period of the downstream wave cone column at different reduced speeds

the vibration amplitude becomes smaller, and the system begins to leave the lock-in interval. The wakes developing from the upstream fixed upper- and lower-side columns directly interfere with the vortex-shedding form of the downstream column; with a suitable spacing ratio, the vibration amplitude and lock-in region of the downstream column are effectively increased.

(a) Cross-flow spacing ratio $L/D_m = 2$, streamwise spacing ratio $T/D_m = 2$

(b) Cross-flow spacing ratio $L/D_m = 4$, streamwise spacing ratio $T/D_m = 2$

Figure 8 Schematic diagram of the flow structure during the motion period of the downstream wave-cone column at different reduced velocities

Fig. 8 Schematic diagram of flow structure in the movement period of the downstream wave cone column at different reduced speeds

4 Conclusions

1) For the downstream column with cross-flow spacing ratio $L/D_m = 2$ and streamwise spacing ratio $T/D_m = 2$, the lock-in interval is the reduced velocity range $U_r = 7 \sim 15$; for the downstream column with cross-flow spacing ratio $L/D_m = 4$ and streamwise spacing ratio $T/D_m = 2$, the lock-in interval is the reduced velocity range $U_r = 5 \sim 13$. Compared with a single wave-cone column, the lock-in intervals are widened in both cases, and the vibration amplitude is also greatly increased.

2) By comparing the downstream-column frequency ratio, spectra, and lift-displacement time-history curves, it can be seen that outside the lock-in interval the frequency ratio is still close to 1, but the energy of the lift coefficient is insufficient to sustain the vibration of the downstream column. Within the lock-in interval, the lift-displacement time-history curve of the downstream column successively undergoes an in-phase state, a phase-transition state, and an anti-phase state; in the phase-transition state, the downstream column obtains the maximum vibration amplitude.

3) For triangularly arranged wave-cone columns with cross-flow spacing ratio $L/D_m = 2$, streamwise spacing ratio $T/D_m = 2$, and cross-flow spacing ratio $L/D_m = 4$, streamwise spacing ratio $T/D_m = 2$, the main reason for the marked differences in the lock-in interval of the downstream column is the difference in vibration excitation modes.

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