From Local Shocks to Macroscopic Reversal: Key-Node Effects in an Ising Sociological Model on Complex Networks

Ruili Ding, Yiyang Zhou, and Yingrui Mai

2026-06-17

ChinaRxiv

Abstract

This paper constructs an Ising sociological model on complex networks with degree heterogeneity to study how local shocks to key nodes are amplified into macroscopic responses under different conditions of background order, system size, and network heterogeneity. The model interprets binary node states as two mutually exclusive macroscopic positions and compares three idealized shock mechanisms: key-node deletion, key-node flipping and locking, and the deletion–inheritance mechanism. Numerical simulations show that the three mechanisms have distinctly different macroscopic consequences: the deletion mechanism mainly weakens the original order, causing the system to approach a zero-magnetization state earlier; the deletion–inheritance mechanism mitigates structural damage through connection succession, exhibiting structural buffering against deletion shocks; the flipping-and-locking mechanism, by preserving the topological positions of key nodes and continuously injecting reverse-state signals, can induce a negative magnetization response within a finite-width intermediate temperature range. Further analysis shows that this intermediate temperature range corresponds to the critical sensitivity region in finite-size networks and can be jointly identified by the enhanced region of static magnetic susceptibility, the enhanced region of correlation length, and the amplified region of dynamic magnetic susceptibility. The final-state magnetization is not used as an independent critical criterion, but rather to examine whether demagnetization or macroscopic directional reversal occurs within this sensitive interval. The results also indicate that the position, width, and depth of the reversal window are jointly regulated by temperature, system size, and network heterogeneity. These findings demonstrate that whether local shocks to key nodes in complex networks can be amplified into global directional reorganization depends on the combined effects of the shock mechanism, background order, network heterogeneity, and finite-size effects.

Full Text

From Local Shocks to Macroscopic Reversal: Key-Node Effects in an Ising Sociological Model on Complex Networks

Ruili Ding, Yiyang Zhou, and Yingrui Mai

Department of Physics, Fudan University, Shanghai, China

(Dated: June 17, 2026)

Abstract: This paper constructs an Ising sociological model on complex networks with degree heterogeneity, and studies how local shocks to key nodes are amplified into macroscopic responses under different conditions of background order, system size, and network heterogeneity. The model interprets the binary state of a node as two mutually exclusive macroscopic positions, and compares three classes of idealized shock mechanisms: deletion of key nodes, flipping and locking of key nodes, and a deletion–inheritance mechanism. Numerical simulations show that the three mechanisms have markedly different macroscopic consequences: the deletion mechanism mainly weakens the original order, causing the system to approach a zero-magnetization state earlier; the deletion–inheritance mechanism mitigates structural damage through partial continuation of connections, thereby acting as a structural buffer against deletion shocks; and the flipping-and-locking mechanism, because it preserves the topological position of key nodes and continuously injects reverse-state signals, can induce a negative-magnetization response within a finite-width intermediate temperature window. Further analysis shows that this intermediate temperature window corresponds to a critical sensitive region in finite-size networks, and can be jointly identified by the static susceptibility enhancement region $\mathcal{W}_{\chi}$, the correlation-length enhancement region $\mathcal{W}_{\xi}$, and the dynamic susceptibility amplification region $\mathcal{W}_{\Delta\chi}$. The final magnetization m_{final} is not used as an independent criterion of criticality, but rather to test whether demagnetization or macroscopic directional reversal occurs within this sensitive interval. The results also show that the position, width, and depth of the reversal window are jointly regulated by kT, N, and α: kT determines whether the system enters an amplifiable sensitive region; N changes the finite-size unfolding pattern and the relative influence of key nodes; and α controls network heterogeneity, the degree of hub dominance, and the stability of the system against thermal fluctuations. These results indicate that whether local shocks to key nodes in complex networks can be amplified into global directional reorganization depends on the combined action of the shock mechanism, background order, network heterogeneity, and finite-size effects.

I. Introduction

The Ising model is one of the classical frameworks linking microscopic binary interactions with macroscopic collective ordered behavior, and has been widely used in sociophysics to study opinion formation, group polarization, social segregation, and other collective behaviors jointly driven by local imitation, external influence, and noise. [1–6] When Ising dynamics is placed on complex networks, network topology significantly changes the system’ s critical threshold, fluctuation response, and metastable-state structure; in particular, degree heterogeneity, hub nodes, and community structure all affect whether local perturbations can be amplified into macroscopic responses. [7–15]

Relatedly, existing studies have shown that targeted intervention on key nodes may produce global effects far beyond the local scale. [16–18] In social and political systems, the removal or replacement of a leadership core also does not always lead to the same outcome: its consequences may appear as organizational weakening or structural fragmentation, but may also be absorbed by the system because of organizational position, succession arrangements, or elite coordination. [19–27]

Although the existing literature has separately discussed node deletion, state intervention, leadership removal, succession arrangements, and repair of damaged networks, these mechanisms have rarely been compared within a single minimal unified framework. This paper divides key-node shocks into three categories: first, deletion of key nodes, that is, direct modification of the network structure; second, flipping and locking the states of key nodes, that is, preserving the structure while continuously injecting reverse signals; and third, deletion–inheritance, that is, after a key node is removed, secondary nodes inherit part of its connections, influence, or organizational position, thereby representing a succession or local repair mechanism. [28–34] These three mechanisms correspond respectively to structural shocks, state shocks, and structural repair processes, and can serve as external perturbations for probing system vulnerability, resilience, and capacity for directional reorganization.

On this basis, this paper constructs an Ising sociological model on complex networks with degree heterogeneity, interpreting the node state ($s_i = \pm 1$) as obedience/resistance, support/defection, or, more generally, coordinated/discoordinated positions. This paper focuses on how the network power-law exponent α, the system size N, and the background noise intensity kT jointly determine system order, sensitive response intervals, and macroscopic reversal windows, and compares the amplification effects of three shock mechanisms—key-node deletion, key-node flipping and locking, and deletion–inheritance—under different conditions of network heterogeneity and finite size. This paper aims to reveal that the macroscopic consequences of local key-node shocks depend not only on the type of shock, but also on network heterogeneity, system size, and noise level; in particular, whether reversal windows exist stably under different power-law exponents, whether they drift with N, and how such drift changes the conditions under which macroscopic reversal occurs.

II. Model and Methods

A. Model Assumptions and Interpretive Boundaries

The model in this paper is built on several minimal assumptions.

a. **Statement of binary variables.** When actors are forced to choose between two major camps, this paper can represent the state of each node as a binary spin variable $s_i = \pm 1$. In a social interpretation, the binary state can correspond to

supporting/opposing positions and other mutually exclusive stances.

b. **Network-connection statement.** The social system is abstracted as a complex network, in which nodes represent its basic constituent units, and edges represent a unified abstraction of multiple kinds of mutual influence relationships among individuals. Community structure, degree heterogeneity, and hub nodes in the network are regarded as important topological factors affecting system vulnerability.

c. **State-update statement.** Neighboring nodes exhibit a tendency toward convergence, and individuals are more inclined to remain consistent with their local social environment. At the same time, random perturbations exist in the system, so nodes do not always update in accordance with the local majority direction.

d. **Shock statement.** Shocks to key nodes are regarded as exogenous events, and three idealized mechanisms are compared: deletion of key nodes, flipping of key nodes, and inheritance of connections after deletion. The deletion mechanism means that a key node disappears from the network structure; the flipping mechanism means that a key node retains its network position but changes its state; the deletion–inheritance mechanism means that after a key node is removed, part of its connection relationships are absorbed by a successor.

e. **Temperature statement.** The temperature parameter T in this paper represents the inverse scale of the ordered constraints in the background of the social system. Low-temperature situations correspond to stronger social integration, organizational discipline, institutional constraints, or group coordination capacity; in such cases, nodes are more inclined to remain consistent with their local environment, and the system as a whole more readily maintains an ordered state. Under high-temperature situations, the system as a whole is more likely to exhibit disordered characteristics.

f. **Time-scale statement.** Key-node shock events are regarded as occurring within a relatively short time window. Within this window, the environment of the social system has not yet undergone significant endogenous change; therefore, the temperature T and the network structure can be approximately regarded as unchanged.

g. **Safety and interpretive-boundary statement.** This paper focuses on the mechanism by which local shocks in complex networks are amplified into macroscopic responses, rather than on predictions of specific historical events, real-world organizations, or political actions.

B. Network Generation Mechanism and Degree Heterogeneity

To construct an interaction topology with a nonuniform connection structure, this paper adopts a power-law heterogeneous random graph as the network substrate for Ising dynamics. This generation mechanism assigns different connection weights to different nodes, so that the resulting network has significant degree heterogeneity and can produce a small number of highly connected hub nodes. Thus, subsequent numerical experiments can examine the impact of key-node shocks on the macroscopic order of the system under controllable topological conditions.

Suppose the system contains N nodes. For each node i, introduce a positive weight parameter θ_i to represent the relative tendency of that node to form connections. To obtain a heavy-tailed connection-weight distribution, first draw the raw weights from a Pareto-type distribution:

$$\theta_i^{\mathrm{raw}} \sim \mathrm{Pareto}(\alpha - 1) + 1, \tag{1}$$

whose tail distribution approximately satisfies

$$f(\theta^{\mathrm{raw}}) \propto (\theta^{\mathrm{raw}})^{-\alpha}, \qquad \theta^{\mathrm{raw}} \geq 1. \tag{2}$$

Here, $\alpha > 1$ is the power-law exponent. A smaller α corresponds to a heavier distribution tail, thereby producing stronger degree heterogeneity and more prominent hub nodes; a larger α corresponds to a relatively more homogeneous distribution of connection weights.

To eliminate the influence of the overall scale of the weights on network density, this paper globally normalizes the raw weights:

$$\theta_i = \frac{N\theta_i^{\mathrm{raw}}}{\sum_{j=1}^{N} \theta_j^{\mathrm{raw}}}, \qquad \sum_{i=1}^{N} \theta_i = N. \tag{3}$$

This normalization makes the average node weight equal to 1, thereby preserving the relative heterogeneity among nodes while avoiding shifts in average connection density caused by changes in the overall scale of the weights.

Given the normalized weights $\{\theta_i\}$, for any unordered node pair (i, j), $i < j$, the edge variable A_{ij} is generated independently:

$$A_{ij} \sim \mathrm{Bernoulli}(p_{ij}), \qquad A_{ii} = 0. \tag{4}$$

The connection probability is taken as

$$p_{ij} = \min\left\{\frac{\kappa\theta_i\theta_j}{N}, 1\right\}, \qquad i < j, \tag{5}$$

where $\kappa > 0$ is a global connection-strength parameter used to control the average degree of the network. The factor $1/N$ in Eq. (5) ensures sparse scaling: when κ is fixed, the expected average degree of the network remains at the $O(1)$ level as the system size changes, thereby preventing the network from automatically becoming a dense graph as N increases. The specific generation steps are as follows. First, specify the system size N, the power-law exponent

α, and the connection strength κ; then draw the raw weights θ_i^{raw}, and obtain the normalized weights θ_i according to the formula above; finally, traverse all unordered node pairs (i, j), and independently sample according to Eq. (5) to obtain the edge set. If the untruncated probability $\kappa\theta_i\theta_j/N$ exceeds 1, it is truncated to 1, and the corresponding probability saturation ratio is recorded to evaluate the influence of extreme hubs on the sparse-random-graph assumption.

The generated network is input in the form of an edge list into the subsequent Ising-dynamics simulation, and the node with the maximum degree is taken as the key node.

C. Ising Dynamics and Continuous-Time Monte Carlo Simulation

This paper defines binary Ising dynamics on the generated network topology. Each node i carries a binary state

$$s_i \in +1, -1, \tag{6}$$

are used to represent two mutually opposing positions. This paper adopts ferromagnetic coupling; that is, neighboring nodes have a tendency to align: when neighboring nodes are in the same state, the system energy is lower, whereas when their states are opposite, the system energy is higher. Therefore, in the social interpretation of this paper, ferromagnetic coupling corresponds to local consistency, imitation, or coordinating pressure.

Let $A(t)$ be the set of surviving nodes at time t, and let $E(t)$ be the edge set in the current surviving network. The system Hamiltonian is written as

$$\mathcal{H} = -J \sum_{(i,j)\in E(t)} s_i s_j - h \sum_{i\in A(t)} s_i, \tag{7}$$

where $J > 0$ is the ferromagnetic coupling strength and h is the external field. This paper does not consider an external-field effect, and takes $h = 0$; in the numerical simulations, $J = 1.0$.

For any surviving node $i \in A(t)$, its local field is defined as

$$\ell_i = J \sum_{j\in \mathcal{N} i(t)\cap A(t)} s_j + h, \tag{8}$$

where $\mathcal{N}_i(t)$ is the set of neighbors of node i in the current network. When node i attempts to flip $s_i \to -s_i$, the corresponding energy change is

$$\Delta E_i = 2 s_i \ell_i. \tag{9}$$

If $\Delta E_i < 0$, the flip lowers the energy and is more likely to occur; if $\Delta E_i > 0$, the flip requires overcoming an energetic cost.

The system evolution uses a continuous-time Monte Carlo method, namely the Gillespie algorithm. For each surviving and unlocked node i, its flip rate is taken in Glauber form:

$$r_i = \frac{\gamma}{1 + \exp(\beta \Delta E_i)}, \tag{10}$$

where $\gamma = 1.0$ is the baseline rate and $\beta = 1/(kT)$. If a node is inactive or locked, then

$$r_i = 0. \tag{11}$$

At each update step, the total flip rate is first computed as

$$R = \sum_i r_i. \tag{12}$$

If $R = 0$, no further flips occur in the system; otherwise, a uniformly distributed random number $u \in (0, 1]$ is drawn, and the waiting time is

$$\Delta t = \frac{-\ln u}{R}. \tag{13}$$

The system time is then advanced to $t + \Delta t$, and node i is selected to flip with probability

$$P_i = \frac{r_i}{R}. \tag{14}$$

After the flip, the local fields and flip rates of this node and its neighbors are updated. The above process is repeated until the preset terminal time is reached, thereby obtaining a dynamical trajectory.

D. Key-Node Shock Mechanisms

To examine the effect of local perturbations on a macroscopic ordered state, this paper introduces a single exogenous shock at a preset time $t = t_0$. Before the shock occurs, the system evolves naturally according to the continuous-time Ising dynamics of the preceding section; after the shock occurs, the system continues to evolve on the altered state or topology.

This paper defines the maximum-degree node in the surviving network before the shock as the key node:

$$v^* = \arg\max_{i \in A(t_0^-)} k_i(t_0^-), \tag{15}$$

where

$$k_i(t) = |\mathcal{N}_i(t) \cap A(t)| \tag{16}$$

is the degree of node i in the current surviving network. Around this key node, the paper compares three shock mechanisms.

a. Key-node deletion. At $t = t_0$, the key node v^* is removed from the set of surviving nodes:

$$\mathcal{A}(t_0^+) = \mathcal{A}(t_0^-) \setminus v^*. \tag{17}$$

At the same time, all edges incident to v^* are deleted, and its flip rate is set to zero. This mechanism represents the structural perturbation caused by the failure of a key node.

b. Key-node flipping and locking. At $t = t_0$, the state of the key node is forcibly flipped:

$$s_{v^*}(t_0^+) = -s_{v^*}(t_0^-), \tag{18}$$

and it is then locked so that it no longer undergoes spontaneous flips during subsequent evolution:

$$r_{v^*} = 0. \tag{19}$$

This mechanism preserves the network position of the key node, but continuously applies an opposing state signal to its neighborhood.

c. Deletion–inheritance mechanism. This mechanism first performs key-node deletion. Subsequently, among the surviving neighbors of v^* before deletion, the node with the highest current degree is selected as the successor node:

$$v_s = \arg\max_{u \in \mathcal{N}_{v^*}(t_0^-) \cap A(t_0^+)} k_u(t_0^+). \tag{20}$$

If no selectable neighbor exists, this mechanism degenerates into pure deletion. If a successor node exists, then for each remaining original neighbor

$$w \in (\mathcal{N}_{v^*}(t_0^-) \cap A(t_0^+)) \setminus \{v_s\}. \tag{21}$$

an edge is established between v_s and w with independent probability $\lambda \in [0, 1]$. The parameter λ controls the strength of connection continuation: $\lambda = 0$ is equivalent to the deletion mechanism, while $\lambda = 1$ represents the maximum degree of local connection continuation.

After each shock, the algorithm updates the adjacency relations, local fields, and flipping rates of the affected nodes, and continues to execute the continuous-time Ising dynamics on the new system configuration.

E. Macroscopic Physical Quantities and Ensemble Averages

This paper mainly focuses on two macroscopic response quantities: magnetization intensity and magnetic susceptibility. The magnetization intensity is used to characterize the macroscopic direction and degree of order of the system after a shock, while the magnetic susceptibility is used to characterize the strength of macroscopic fluctuations among different random trajectories.

The normalized magnetization intensity is defined as

$$m(t) = \frac{1}{|\mathcal{A}(t)|} \sum_{i \in \mathcal{A}(t)} s_i(t). \tag{22}$$

Here, $m(t) > 0$ and $m(t) < 0$ indicate that the system is in two opposite macroscopic directions, respectively; $|m(t)|$ characterizes the degree of order of the system. When $|m(t)|$ is relatively large, the system state is more consistent; when $|m(t)|$ is close to zero, the system approaches disorder or the positive and negative states cancel each other.

For each temperature T and shock mechanism μ, this paper records the final-state magnetization intensity

$$m_{\text{final}}^{(\mu)} = m^{(\mu)}(t_f), \tag{23}$$

where t_f is the simulation termination time. This quantity is used to determine whether the system retains its original direction after a shock, or whether it enters the opposite macroscopic state. Correspondingly, the final-state absolute magnetization intensity is defined as

$$|m|_{\text{final}}^{(\mu)} = \left| m^{(\mu)}(t_f) \right|, \tag{24}$$

which is used to measure the residual degree of order of the system after a shock.

To characterize the fluctuations of macroscopic states among different random trajectories, this paper defines the time-dependent magnetic susceptibility

$$\chi^{(\mu)}(t; T, G) = \beta |\mathcal{A}(t)| \left[\left\langle \left(m^{(\mu)}(t) \right)^2 \right\rangle_\omega - \left\langle m^{(\mu)}(t) \right\rangle_\omega^2 \right], \tag{25}$$

where $\beta = 1/(kT)$, G denotes a given network sample, and ω denotes an independent dynamical trajectory. The final-state susceptibility is taken as $\chi^{(\mu)}(t_f; T, G)$. When χ is large, it indicates that, under the same parameters, the macroscopic final states of different trajectories differ strongly, and the system is more sensitive to random perturbations and external shocks.

In the no-shock static analysis, this paper also records the correlation length ξ, which is used to assist in characterizing the typical propagation range of spin correlations in the network. Taking the graph distance $r = d(i,j)$ as the distance variable, the connected correlation function is defined as

$$C(r) = \frac{\sum_{i<j} \mathbf{1}_{d(i,j)=r} \left[\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle\right]}{\sum_{i<j} \mathbf{1}_{d(i,j)=r}}. \tag{26}$$

In the interval where $C(r)$ is positive and decays relatively stably, use

$$C(r) \sim C_0 \exp(-r/\xi) \tag{27}$$

to estimate the correlation length ξ. This indicator is used only to identify the statically sensitive interval of the no-shock system, and is not used as the main criterion for judging subsequent shock mechanisms.

This paper adopts two levels of ensemble averaging. First, on a fixed network sample G, averages are taken over n_{traj} independent dynamical trajectories. For any macroscopic quantity X,

$$\overline{X}^{(\mu)}(T; G) = \frac{1}{n_{\text{traj}}} \sum_{\omega=1}^{n_{\text{traj}}} X^{(\mu)}(T; G, \omega). \tag{28}$$

Then, under the same network-generation parameters, averaging is continued over n_{net} independent network samples:

$$\left\langle X^{(\mu)}(T) \right\rangle = \frac{1}{n_{\text{net}}} \sum_{G=1}^{n_{\text{net}}} \overline{X}^{(\mu)}(T; G). \tag{29}$$

The results obtained in this way reflect the typical behavior after jointly averaging over dynamical randomness and network-sample randomness, rather than accidental results from a single trajectory or a single network sample.

F. Numerical Experimental Setup

The numerical experiments in this paper are divided into two parts: no-shock static benchmark experiments and key-node shock dynamics experiments. The no-shock experiments are used to characterize the static statistical properties of the network Ising system at different temperatures, and to provide references for subsequent shock experiments. For each set of network parameters (N, α) and temperature kT, this paper generates multiple independent network samples and uses the Metropolis algorithm to sample the Ising system. Each sweep contains N single-node updates. The average absolute magnetization intensity, magnetic susceptibility, and correlation length are then recorded, and used to identify changes in order, fluctuation-enhanced regions, and correlation-propagation-enhanced regions in the no-shock system.

In the shock experiments, the system first evolves according to continuous-time Ising dynamics from $t = 0$ to the shock time t_0. Then, under the same network and the same temperature, three mechanisms are applied separately: key-node deletion, key-node flipping and locking, and deletion-inheritance, and the evolution is continued until the termination time t_f. This paper mainly records the final-state magnetization intensity m_{final}, the susceptibility χ, and the maximum change in susceptibility $\Delta\chi_{\max}$ under the flipping-and-locking mechanism, in order to compare the effects of different shock modes on macroscopic order, direction reversal, and fluctuation amplification.

The specific parameter settings are as follows. The system size is taken as

$$N \in \{100, 200, 400, 800\},$$

The network heterogeneity exponent is taken as

$$\alpha \in \{2, 3, 4\}.$$

The temperature scanning range is

$$kT \in [0.5, 6.0],$$

with step size $\Delta(kT) = 0.25$. For each group (N, α, kT), this work uses $n_{\text{net}} = 30$ independent network samples, and repeats $n_{\text{traj}} = 1000$ independent dynamical trajectories on each network sample. All macroscopic observables are first averaged over dynamical trajectories on a fixed network sample, and then averaged across different network samples.

For the shock experiments, the initial state is a positively biased random initial state: each node takes $s_i = +1$ with probability $p_\uparrow = 0.8$, and $s_i = -1$ with probability $1 - p_\uparrow = 0.2$. The system evolves from $t = 0$ to $t_0 = 100$, after which a key-node shock is applied, and then continues to evolve until $t_f = 300$. The recording interval for macroscopic observables is $\Delta t_{\text{obs}} = 0.5$. In the deletion–inheritance mechanism, the connection-inheritance probability is set to $\lambda = 0.7$. Unless otherwise specified, this paper sets the external field to $h = 0$, the ferromagnetic coupling strength to $J = 1.0$, and the baseline flip rate in the continuous-time dynamics to $\gamma = 1.0$.

III. Numerical Results

A. Static Response without Shock: Order, Fluctuations, and Correlation Length

The computational results for the random-network model are shown in Fig. 1. In the strongly heterogeneous network with $\alpha = 2$, the statistical behavior of the system is clearly different from that of the Ising model on a regular lattice. As the system size increases, the peak of the magnetic susceptibility does not show a tendency to converge toward and strengthen at a single critical point; instead, it gradually decreases and undergoes an obvious drift. These signs indicate that, within the finite-size range examined in this paper, the system response in strongly heterogeneous networks is closer to a size-dependent crossover than to a clear paramagnetic–ferromagnetic phase transition. This suggests that, in strongly heterogeneous networks with prominent hub nodes, a small number of highly connected nodes can maintain relatively strong global order, thereby altering the way the system responds to thermal fluctuations.

As α increases, the degree heterogeneity of the network weakens, and the dominant effect of a single hub node on the overall order is reduced. At this point, the susceptibility peak becomes more concentrated. In particular, for $\alpha = 3$ and $\alpha = 4$, the system exhibits response characteristics closer to an order–disorder transition. This indicates that, in more homogeneous networks, the loss of macroscopic order is controlled more by fluctuations of the network as a whole than by the maintenance of order by a small number of hub nodes alone.

Figure 2 further gives the variation of the correlation length ξ with temperature kT under different N and α. The correlation length characterizes the typical propagation range of spin correlations in the network, and therefore can serve as an auxiliary indicator for identifying the sensitive temperature region of an unshocked system. The results show that ξ usually increases from low temperature to the intermediate-temperature region, and then gradually decreases in the higher-temperature region. This indicates that when the temperature is low, although the system has strong order, local perturbations have difficulty changing the overall state; when the temperature is too high, the system approaches disorder and the propagation range of spin correlations shortens. In contrast, the intermediate-temperature region has both strong correlation propagation and high fluctuation response, and is therefore more likely to become the sensitive region in which subsequent key-node shocks are amplified. The morphology of the correlation length also differs under different α. In the strongly heterogeneous network with $\alpha = 2$, the peak position and curve shape of ξ vary more markedly with N, reflecting strong finite-size effects under the hub-dominated structure. For $\alpha = 3$ and $\alpha = 4$, the correlation-length peaks are more concentrated and are closer to the interval where the susceptibility is enhanced, indicating that the sensitive temperature region in more homogeneous networks is clearer. Therefore, the susceptibility peak and the enhancement of the correlation length jointly provide the static baseline of the unshocked system: the intermediate reversal window induced by the subsequent flip-and-lock mechanism occurs near the temperature region where the system itself already has strong fluctuation response and correlation-propagation capability.

B. Final-State Magnetization Response after Key-Node Shock

To compare the effects of different key-node shock mechanisms on the macroscopic state, this paper first fixes the system size at $N = 100$ and examines the final-state magnetization intensity corresponding to the three mechanisms under different degree-heterogeneity exponents α.

Figure 3 shows that, in the no-shock control group, the positive magnetization gradually decays to zero as kT increases, providing the baseline response of the system in the absence of external shock. By contrast, the key-node deletion mechanism and the deletion–inheritance mechanism mainly manifest as a reduction in the final-state magnetization intensity, causing the system to approach the disordered state earlier, but they usually do not form a stable negative-magnetization interval. This indicates that simply removing key nodes mainly plays a destabilizing role, namely weakening the original order, rather than actively inducing the system to enter the opposite direction.

The final-state magnetization intensity under the deletion–inheritance mechanism is usually higher than that under the simple deletion mechanism, indicating that when key nodes are removed, if part of their connection relationships can be continued by inheriting nodes, then the damage of structural injury to macroscopic order is partially alleviated. Therefore, the deletion–inheritance mechanism is more a local repair or resilience mechanism, whose main role is to maintain the original ordered state rather than to induce directional reversal.

The most significant difference appears in the flip-and-lock mechanism. Under all examined α, this mechanism produces a finite negative-magnetization interval; that is, the red curve crosses the $\langle m_{\mathrm{final}} \rangle = 0$ reference line in the intermediate-temperature region and enters the negative-value region. This paper identifies this continuous negative-magnetization interval as the intermediate reversal window. This result indicates that, when key nodes retain their network positions and continue-offsetof

(a) $\alpha = 2$ network magnetization strength

(b) $\alpha = 2$ susceptibility

(c) $\alpha = 3$ network magnetization strength

(d) $\alpha = 3$ susceptibility

(e) $\alpha = 4$ network magnetization strength

(f) $\alpha = 4$ susceptibility

Figure 1: Numerical calculation results for the Ising model on a sociological random network. The left column shows the magnetization strength for different

α, and the right column shows the corresponding susceptibility.

Legend: $N = 100$, $N = 200$, $N = 400$, $N = 800$. Panels: $\alpha = 2$, $\alpha = 3$, $\alpha = 4$. Vertical axis: Correlation length ξ. Horizontal axis: kT.

Figure 2: Variation of the correlation length ξ with temperature kT under different system sizes N and degree-heterogeneity exponents α. The correlation length is used to identify sensitive temperature regions in the unperturbed system where spin-correlation propagation is stronger, and provides a static reference for the subsequent key-node perturbation experiments.

Panel labels in Figure 3:
(a) $\alpha = 2.0$ (b) $\alpha = 2.2$ (c) $\alpha = 2.5$ (d) $\alpha = 2.8$
(e) $\alpha = 3.0$ (f) $\alpha = 3.2$ (g) $\alpha = 3.5$ (h) $\alpha = 4.0$

Common plot title: *Ensemble-Averaged Final Magnetization vs Temperature*. Legend: Inherit; Delete; Flip+Pin; Control. Horizontal axis: Temperature kT. Vertical axis: $\langle m_{\text{final}} \rangle$ (mean $\pm$ SEM).

Figure 3: For fixed system size $N = 100$, the influence of three key-node perturbation mechanisms on the final-state magnetization $\langle m_{\text{final}} \rangle$ under different degree-heterogeneity exponents α. The black dashed line denotes the unperturbed control group; the green, blue, and red curves respectively denote the delete-and-inherit mechanism, the key-node deletion mechanism, and the key-node flip-and-pin mechanism. Shaded regions indicate standard deviations computed across trajectories and network samples. $\langle m_{\text{final}} \rangle = 0$ corresponds to the criterion for macroscopic directional reversal.

When a reverse-state signal is injected, local perturbations can be amplified into macroscopic directional reversal within specific temperature regions.

From the perspective of temperature intervals, in the low-temperature region the system has stronger order, and neighborhood-consistency constraints can resist the influence of key-node flips; therefore, under each mechanism the final-state magnetization mostly remains positive. In the high-temperature region, the system itself is already close to a disordered state, the macroscopic magnetization tends toward zero, and local reverse signals likewise have difficulty organizing a stable reverse ordered state. Only in the intermediate-temperature region does the system both retain a certain degree of order and possess relatively strong responsiveness; thus the flip-and-pin mechanism is most likely to induce macroscopic reversal.

As α increases, the reversal window gradually contracts, and the transition segment also becomes steeper. This indicates that network degree heterogeneity significantly regulates the width and depth of the reversal window: in strongly heterogeneous networks, hub nodes have a larger range of influence, and the reversal window is relatively wider; whereas in more homogeneous networks, the dominance of a single key node is weakened, and reversal occurs more concentratedly within a narrower sensitive temperature region.

C. Size Effects and Heterogeneity Dependence of the Reversal Window

To further examine whether the reversal window depends on system size and network degree heterogeneity, this paper selects $\alpha = 2, 3, 4$ and $N =$

100, 200, 400, and 800, comparing the final-state magnetization strength under the flipping-and-locking mechanism.

Figure 4 shows that the reversal window induced by the flipping-and-locking mechanism is regulated simultaneously by the system size N and the network degree-heterogeneity exponent α; its position, width, and depth all change with the parameters. Here, the reversal window can be identified from three aspects: first, the window position, namely the temperature interval in which the red curve lies below $\langle m_{\text{final}} \rangle = 0$; second, the window width, namely the range of the negative-magnetization interval along the temperature axis; and third, the window depth, namely the maximum negative magnetization amplitude corresponding to the lowest point of the red curve.

For the strongly heterogeneous network with $\alpha = 2$, the reversal window is relatively pronounced in small systems, but gradually weakens as N increases, even tending to disappear. This indicates that, in a strongly heterogeneous network dominated by hubs, the flipping of a single key node can produce a strong effect in a finite system; however, when the system size becomes large, the relative contribution of a single node to the overall magnetization direction decreases, and macroscopic reversal is no longer stable. Therefore, the reversal response in the case of $\alpha = 2$ has a strong finite-size character.

For the moderately heterogeneous network with $\alpha = 3$, the reversal window exhibits a clear temperature drift as N increases. This situation can be understood as a transition region between the hub-dominated effect and the overall fluctuation response: key nodes still have relatively strong spreading ability, but the position and width of the reversal window have already begun to be constrained by the system' s overall sensitive temperature region. Therefore, the reversal window for $\alpha = 3$ is determined neither solely by a single hub nor entirely by the collective critical response of a homogeneous network.

For the more homogeneous network with $\alpha = 4$, the reversal window is more concentrated and shows a tendency to become sharper as N increases. Compared with strongly heterogeneous networks, the dominance of a single key node in a more homogeneous network is weakened, and macroscopic reversal depends more on the system' s own sensitive temperature range. Therefore, although the negative-magnetization interval for $\alpha = 4$ is relatively narrow, its window structure is clearer, appearing as a rapid transition within a specific intermediate-temperature region.

Overall, as N increases, the maximum negative magnetization amplitude after the flipping-and-locking mechanism decreases, indicating that the relative impact of perturbing a single key node is diluted in large systems. However, the reversal window exhibits different size-evolution patterns with α, and cannot be simply summarized as monotonically disappearing with increasing N. It can thus be seen that the existence, drift, and contraction of the intermediate reversal window together indicate that the macroscopic consequences of perturbing key nodes are jointly determined by network heterogeneity, system size, and background ordering.

D. Susceptibility amplification induced by the flipping-and-locking mechanism The preceding text has shown that the flipping-and-locking mechanism can induce a macroscopic reversal window with $\langle m_{\text{final}} \rangle < 0$ within a finite intermediate-temperature interval. To further determine whether this window corresponds to an enhancement of system sensitivity, this paper examines the maximum change in susceptibility after the perturbation relative to the pre-perturbation steady state, $\Delta\chi_{\max}$. Unlike the final-state magnetization strength, $\Delta\chi_{\max}$ does not directly characterize whether the system ultimately lies in a positive- or negative-magnetization state; rather, it is used to measure the extent to which the system' s fluctuation response is amplified after the flipping of a key node.

Figure 5 gives the definition of $\Delta\chi_{\max}$. Specifically, this paper takes the maximum deviation of the susceptibility within the post-perturbation interval $[t_0, t_f]$ relative to the pre-perturbation steady value $\chi(t_0^-)$ as the fluctuation-amplification strength at that temperature. If $\Delta\chi_{\max}$ increases significantly within a certain temperature interval, it indicates that, after the key node is flipped and locked, the macroscopic response differences among different dynamical trajectories are significantly amplified, and the system is in a state of higher sensitivity.

On this basis, Figure 6 shows how $\Delta\chi_{\max}$ induced by the flipping-and-locking mechanism varies with temperature kT under different system sizes N and degree-heterogeneity exponents α.

Figure 6 shows that the enhancement of $\Delta\chi_{\max}$ usually appears near the temperature interval where the final-state magnetization strength rapidly decreases, crosses the zero line, or enters the negative-magnetization window. In other words, the temperature region of susceptibility amplification roughly corresponds to the intermediate reversal window identified above. This correspondence indicates that the macroscopic reversal induced by the flipping-and-locking mechanism is not a simple decrease in average magnetization, but is accompanied by enhanced final-state differences among trajectories.

Viewed by temperature interval, in the low-temperature region the system has relatively strong order. Although the flipping of a key node produces an opposite influence on a local neighborhood, the overall state is still subject to strong consistency constraints, so the susceptibility amplification is limited. In the high-temperature region, the system itself is already close to disorder; the macroscopic magnetization approaches zero, and local reverse signals are difficult to organize into a stable reversed ordered state, so $\Delta\chi_{\max}$ also does not remain high. By contrast, the intermediate-temperature region simultaneously has a certain degree of order and relatively strong response capability. Different dynamical trajectories may respectively tend to maintain the original direction, approach disorder, or enter a reversely ordered state, thereby causing the susceptibility variation to be significantly amplified.

As N and α change, the peak position and peak amplitude of $\Delta\chi_{\max}$ also vary. For networks with different heterogeneity, the correspondence between the fluctuation-amplification region and the reversal window is not completely the same: in strongly heterogeneous networks, the response is more easily affected by hub dominance and finite-size effects, whereas in more homogeneous networks the peak usually concentrates in a narrower intermediate-temperature region. This further shows that the position, width, and strength of the reversal window are not determined solely by the key node itself, but are jointly regulated by network topology, system size, and background ordering.

Therefore, $\langle m_{\text{final}}\rangle < 0$ provides a direct criterion for macroscopic reversal, while $\Delta\chi_{\max}$ provides auxiliary evidence at the level of fluctuation response. Taken together, the two indicate that the macroscopic reversal induced by the flipping-and-locking mechanism occurs in the intermediate-temperature region where the system is most sensitive; within this region, local reverse signals

(a) $N = 100,\ \alpha = 2$ **(b)** $N = 100,\ \alpha = 3$ **(c)** $N = 100,\ \alpha = 4$

(d) $N = 200,\ \alpha = 2$ **(e)** $N = 200,\ \alpha = 3$ **(f)** $N = 200,\ \alpha = 4$

(g) $N = 400,\ \alpha = 2$ **(h)** $N = 400,\ \alpha = 3$ **(i)** $N = 400,\ \alpha = 4$

(j) $N = 800,\ \alpha = 2$ **(k)** $N = 800,\ \alpha = 3$ **(l)** $N = 800,\ \alpha = 4$

Common panel title: *Ensemble-Averaged Final Magnetization vs Temperature*
Horizontal axis: *Temperature kT*
Vertical axis: $\langle m\rangle(T_{\text{final}})$ *(mean ± std)*
Legend: *Inherit; Delete; Flip+Pin; Control*

Figure 4: Dependence of the final magnetization $\langle m_{\text{final}}\rangle$ under the flip-and-lock mechanism on the system size N and the degree-heterogeneity exponent α. Each subfigure corresponds to one set of (N, α) parameters. The horizontal axis is the temperature kT, and the vertical axis is the system-averaged final magnetization. $\langle m_{\text{final}}\rangle = 0$ is the criterion for macroscopic directional reversal. The negative-magnetization interval indicates the reversed macroscopic state induced by the flip-and-lock mechanism.

not only changes the mean magnetization direction, but also significantly amplifies the inter-trajectory differences and macroscopic response of the system to shocks.

E. Interval Identification of Critical Sensitive Regions

To avoid misidentifying the response in finite-size systems as a single critical point, this paper characterizes the intermediate sensitive region in terms of intervals. Specifically, the continuous temperature interval in which the static susceptibility is significantly enhanced is denoted by $\mathcal{W}_\chi$; the continuous temperature interval in which the correlation length grows or remains at a relatively high value is denoted by $\mathcal{W}_\xi$; and the post-shock dynamic

Figure 5: Schematic definition of the maximum change in magnetic susceptibility, $\Delta\chi_{\max}$. The black dashed line indicates the time t_0 at which the key-node shock occurs, and the red curve shows the temporal evolution of the susceptibility $\chi(t)$ under the flip-and-lock mechanism. In this work, the maximum deviation of the susceptibility in the post-shock interval from its pre-shock steady-state value $\chi(t_0^-)$ is defined as $\Delta\chi_{\max}$, which is used to characterize the intensity of fluctuation amplification in the system after the key node flips.

The temperature interval in which the change in susceptibility is significantly enhanced is denoted by $\mathcal{W}_{\Delta\chi}$. The final magnetization m_{final} is not used as an independent criterion, but rather to test whether the above sensitive interval corresponds to a demagnetization or negative-magnetization response.

Figure 7 presents a representative illustration for $N = 400$ and $\alpha = 4$. In this case, $\mathcal{W}_\chi$, $\mathcal{W}_\xi$, and $\mathcal{W}_{\Delta\chi}$ exhibit substantial overlap in the intermediate-temperature region, and are adjacent to the rapid decrease of $\langle m_{\text{final}}\rangle$ and the negative-magnetization region under the flip-and-lock mechanism. This result indicates that enhanced static fluctuations, enhanced correlation propagation, and post-shock dynamical amplification all point to the same finite-size sensitive window, while the final magnetization provides a direct manifestation of macroscopic reversal within this window.

Table I shows that the sensitive intervals identified by different indicators exhibit finite offsets along the temperature axis, but overall are concentrated in similar intermediate-temperature regions. For $\alpha = 3, 4$, $\mathcal{W}_\chi$, $\mathcal{W}_\xi$, and $\mathcal{W}_{\Delta\chi}$ correspond well to one another, indicating that fluctuation enhancement, correlation propagation, and post-shock dynamical amplification point to the same finite-size sensitive region. For $\alpha = 2$, the correlation length does not yield a stable interval, while $\mathcal{W}_\chi$ shifts markedly to the right with increasing N, indicating that in strongly heterogeneous networks the response is more strongly dominated by hubs and controlled by finite-size effects.

(a) $N = 100,\ \alpha = 2$

(b) $N = 100,\ \alpha = 3$

(c) $N = 100,\ \alpha = 4$

(d) $N = 200,\ \alpha = 2$

(e) $N = 200,\ \alpha = 3$

(f) $N = 200,\ \alpha = 4$

(g) $N = 400,\ \alpha = 2$

(h) $N = 400,\ \alpha = 3$

(i) $N = 400,\ \alpha = 4$

(j) $N = 800,\ \alpha = 2$

(k) $N = 800,\ \alpha = 3$

(l) $N = 800,\ \alpha = 4$

Figure 6: Variation of the maximum change in susceptibility, $\Delta\chi_{\mathrm{max}}$, with temperature kT under the flip-and-lock mechanism. Each subfigure corresponds to one combination of system size N and degree-heterogeneity exponent α. The peak value of $\Delta\chi_{\mathrm{max}}$ is used to characterize the temperature interval in which system fluctuations are most strongly amplified after the flipping of key nodes.

Table I: Intermediate sensitive regions identified by multiple indicators for different network heterogeneity indices α and system sizes N. $\mathcal{W}_\chi$ denotes the enhanced static magnetization-susceptibility region, $\mathcal{W}_\xi$ denotes the enhanced correlation-length region, and $\mathcal{W}_{\Delta\chi}$ denotes the region where the post-impact dynamic magnetization susceptibility is amplified. All intervals are expressed in kT; "–" indicates that this indicator did not identify a clear, stable continuous interval.

α	N	$\mathcal{W}_\chi$	$\mathcal{W}_\xi$	$\mathcal{W}_{\Delta\chi}$
2	100	1.50–2.50	–	0.50–1.50
2	200	2.35–3.70	–	0.50–1.50
2	400	3.10–5.00	–	0.50–1.50
2	800	5.00–8.00	–	0.50–1.50
3	100	1.00–1.75	1.00–1.30	0.50–1.50
3	200	1.35–2.10	1.50–1.90	0.75–2.00
3	400	1.70–2.25	1.60–2.00	1.00–2.25
3	800	1.45–2.75	2.10–2.60	1.50–3.25
4	100	1.00–1.45	1.00–1.60	0.75–1.75
4	200	1.20–1.65	1.00–1.80	1.00–2.00
4	400	1.45–1.80	1.20–1.90	1.25–2.25
4	800	1.60–1.95	1.70–2.10	1.50–2.25

Figure 7: For the representative parameters $N = 400$ and $\alpha = 4$, the intermediate sensitive regions identified by different indicators and their correspondence with the final magnetization response. The orange, purple, and cyan shaded regions denote, respectively, the enhanced static magnetization-susceptibility region $\mathcal{W}_\chi$, the enhanced correlation-length region $\mathcal{W}_\xi$, and the dynamic magnetization-susceptibility amplification region $\mathcal{W}_{\Delta\chi}$. The curves

represent the final magnetization strength $\langle m_{\text{final}}\rangle$ under different perturbation mechanisms, and the shaded bands indicate the corresponding statistical fluctuation ranges. It can be seen that the sensitive intervals given by different indicators exhibit finite offsets along the temperature axis, but they are all concentrated near the intermediate-temperature range in which the flip-and-pin mechanism produces rapid demagnetization and a negative magnetization response.

IV. Discussion

A. Intermediate sensitive window: static response and dynamic amplification in finite-size critical regions

The results of this paper show that the macroscopic reversal induced by the flip-and-lock mechanism is concentrated in an intermediate temperature range of finite width. This range can be understood as a broadened form of the critical region in a finite-size network: for finite N, the order–disorder transition that may exist in the thermodynamic limit manifests as a crossover region. Network sample fluctuations, topological heterogeneity, and hub-dominated effects further alter the width and position of this region.

This sensitive window appears in different observables as different facets of the same physical region. In the unperturbed system, an increase in the susceptibility χ corresponds to enhanced macroscopic fluctuations; a rapid decline in the magnetization intensity $m(kT)$ corresponds to rapid restructuring of order; and growth of the correlation length ξ corresponds to an expansion of the range of local correlation propagation. After the perturbation, an increase in $\Delta\chi_{\max}$ indicates that the flip-and-locking of key nodes amplifies trajectory-to-trajectory differences and macroscopic responses.

Therefore, this window can be conceptually expressed as

$$\mathcal{W}_{\text{sens}}(N,\alpha) \sim \{\mathcal{W}_{\chi}, \mathcal{W}_{\xi}, \mathcal{W}_{\Delta\chi}\}.$$

Here, $\mathcal{W}_{\chi}$, $\mathcal{W}_{\xi}$, and $\mathcal{W}_{\Delta\chi}$ denote, respectively, the manifestations of this window in the variations of static susceptibility, correlation length, and dynamic susceptibility. The final-state magnetization m_{final} is used to test whether demagnetization or a negative magnetization response occurs within this window.

This picture explains the origin of the reversal window. In the low-temperature regime, the system has relatively strong order, and local reverse signals are suppressed by consistency constraints; in the high-temperature regime, the system is close to disorder, the range of correlation propagation shrinks, and it is difficult to form a stable reverse-ordered state. Within the intermediate sensitive window, the system is in a state where order changes rapidly, fluctuations are enhanced, and correlation propagation is relatively strong. At this point, the key nodes retain their network positions and continuously inject reverse signals;

these signals can propagate along the topological structure and be dynamically amplified, ultimately driving the final magnetization intensity into the negative region.

It follows that the negative-magnetization window originates from the multiple sensitivities of the finite-size critical region. The static χ, ξ, and dynamic $\Delta\chi_{\max}$ characterize the same intermediate sensitive window from three aspects: fluctuations, correlation propagation, and dynamic amplification; m_{final}, in turn, provides the macroscopic response indicating whether demagnetization or directional reversal occurs within this window.

B. Mechanism differentiation among deletion, inheritance, and flip-locking

The differences among the three key-node perturbation mechanisms can be characterized by the mean final magnetization and its statistical fluctuations. In this paper, $\langle m_{\text{final}}\rangle = 0$ is used as the average criterion for changes in the macroscopic direction, while the relationship between the standard-deviation range $\langle m_{\text{final}}\rangle \pm \sigma_m$, represented by the shaded region, and the zero line is also examined. The former reflects whether the system, in an average sense, preserves its original direction, while the latter reflects the degree of differentiation among different trajectories and different network samples.

The key-node deletion mechanism is mainly manifested as structural destabilization. After deleting the highest-degree nodes and their connections, the core influence channels of the network are weakened, and $\langle m_{\text{final}}\rangle$ approaches the zero line more rapidly as temperature increases. If the lower bound of its standard deviation touches the zero line first, this indicates that some samples have already lost their original macroscopic direction; however, while the mean has not yet stably entered the negative region, this mechanism still primarily corresponds to weakening of the original order rather than directional reversal.

The deletion-inheritance mechanism appears as structural buffering against deletion shocks. After inheriting part of the original connections of the nodes adjacent to the deleted nodes, $\langle m_{\text{final}}\rangle$ is usually higher than in the pure-deletion case, and the standard-deviation range approaches the zero line later. This shows that connection inheritance not only maintains a higher average degree of order, but also weakens sample-to-sample differentiation. Therefore, the principal role of the deletion-inheritance mechanism is to delay the system' s retreat toward the zero-magnetization state and to improve structural resilience after the removal of key nodes.

The flip-and-lock mechanism corresponds to a directional reverse input. The key nodes retain their topological positions and continuously remain in the reverse state, causing the red curve to cross $\langle m_{\text{final}}\rangle = 0$ in the intermediate temperature range and enter the negative-magnetization region. The standard-deviation range touching the zero line indicates that reversal competition begins to emerge; the mean entering negative values indicates that, in an average sense, the reverse signal is amplified into macroscopic reversal; if the upper bound of the standard deviation is also below the zero line, then the reverse macroscopic state has strong stability within the statistical fluctuation range.

Thus, the differentiation among the three mechanisms is reflected in the relative positions of the mean zero line and the standard-deviation zero line. The deletion mechanism drives the system toward zero magnetization earlier; the deletion-inheritance mechanism delays this process and weakens fluctuation differentiation; and the flip-and-lock mechanism enables the system to cross the zero line and form a negative-magnetization window. The mean characterizes the average change in the macroscopic direction, while the standard-deviation range characterizes sample differentiation and stability during this process.

C. Modulation of the reversal window by kT, N, and α

The position, width, and depth of the reversal window are jointly regulated by kT, N, and α. kT controls the background order of the system, N determines finite-size effects and the relative weight of key nodes, and α determines network heterogeneity, the degree of hub dominance, and the stability of the system against thermal fluctuations.

First, kT determines whether the system enters an intermediate sensitive window in which amplification can occur. In the low-kT range, the system has relatively strong order, and neighborhood consistency constraints suppress the reverse signals generated by key-node flipping; in the high-kT range, the system is close to disorder, and local reverse signals are difficult to organize into a stable reverse-ordered state. Therefore, the negative-magnetization window mainly appears in the intermediate-kT range,

At this point, the system simultaneously exhibits finite ordering, relatively strong fluctuation response, and relatively strong correlation-propagation capacity.

Second, α regulates the topological basis of the reversal window. A smaller α corresponds to stronger degree heterogeneity and more prominent hub nodes. A small number of highly connected nodes can maintain overall order over a relatively broad temperature range, providing the network with strong structural support against thermal fluctuations; at the same time, shocks to key nodes are more strongly affected by hub structure and finite-size effects. As α increases, the network becomes more homogeneous, the dominance of hubs over overall order weakens, the system becomes more susceptible to thermal fluctuations, and the reversal window becomes more concentrated and steeper. Therefore, $\alpha = 3$ can be regarded as a transitional case between hub-dominated stability and overall sensitivity to thermal fluctuations.

Finally, N determines how the window unfolds under finite size. When the system size changes, the relative contribution of a single key node to global magnetization changes, and the position and width of the finite-size critical region also change accordingly. Thus, the reversal window exhibits drift, contraction, or morphological change with N, and this change depends on α. In strongly heterogeneous networks, the window is more strongly controlled by hub dominance and finite-size effects; in more homogeneous networks, the window is closer to a collective response controlled by the overall sensitive region.

In summary, kT determines whether the system lies in the sensitive region, α determines the topological stability of that sensitive region, and N determines its finite-size unfolding. Together, the three determine whether the flip-and-lock mechanism can amplify a local reverse signal into a macroscopic directional reversal.

D. Model Limitations and Future Extensions

This paper adopts a minimal network Ising sociological model, and the results should be understood as an abstract description at the mechanism level rather than as direct predictions of specific social events. The model simplifies individual states into binary variables, which can characterize competition between two mutually exclusive positions, but it does not yet include more complex situations such as multi-camp states, continuous attitude changes, or strategic choices. In addition, this paper mainly takes the maximum-degree node as the key node. This setting highlights the role of hub shocks, but key positions in real networks may also be jointly determined by betweenness centrality, eigenvector centrality, community-bridging properties, or multilayer network structure.

This paper focuses on the effects of kT, N, and α on the reversal window, but the connection strength κ in the network-generation mechanism has not yet been systematically explored. In the current model, α controls the heterogeneity of the degree distribution and the prominence of hubs, whereas κ controls the overall connection density and average degree of the network. Future work can treat κ as an independent parameter and study how static susceptibility, correlation length, dynamic $\Delta\chi_{\max}$, and the negative-magnetization window change as the network varies from sparse to relatively dense. This would help distinguish the different roles of the "shape of the connection distribution" and the "overall connection strength" in shock amplification.

Another issue requiring further study is the stability of the reversal window in the thermodynamic limit. This paper compares results under finite system sizes and observes that the reversal window can drift, contract, or change shape with N and α. In the future, finite-size scaling analysis can be introduced to examine how the position, width, and depth of the reversal window vary with N, so as to determine whether the window is a manifestation of finite-size effects or a network-level collective phenomenon that still exists in the limit $N \to \infty$.

Finally, apart from the deletion and inheritance mechanisms, the network structure in this paper is basically kept fixed, and feedback from state changes to network structure has not yet been considered. In more general social networks, changes in node states may cause edges to break, reconnect, or lead to community reorganization. Future work can extend the present framework to adaptive networks, multistate spin models, or multi-key-node shock scenarios, in order to further test the robustness of the intermediate sensitive window and the amplification mechanism of key-node shocks.

V. Conclusions

This paper constructs an Ising sociological model on complex networks with degree heterogeneity, compares three local shock mechanisms—key-node deletion, key-node flip-and-lock, and deletion-inheritance—and examines the regulatory effects of background ordering kT, system size N, and the network power-law exponent α on macroscopic responses. The results show that the macroscopic consequences of local key-node shocks are not determined solely by node importance, but also depend on the shock mode, network heterogeneity, finite-size effects, and the background ordered state of the system.

First, the three shock mechanisms exhibit clear differentiation. Key-node deletion mainly weakens the original order, causing the system to approach the zero-magnetization state earlier; the deletion-inheritance mechanism partially mitigates structural damage through connection succession, acting as a structural buffer against deletion shocks; the flip-and-lock mechanism preserves the network position of the key node and continuously injects a reverse-state signal, and therefore can induce a negative-magnetization response in the intermediate temperature region. The mean value of the final magnetization characterizes macroscopic directional change, while the range of the standard deviation reflects the degree of differentiation among different trajectories and different network samples.

Second, the negative-magnetization response induced by the flip-and-lock mechanism is concentrated in an intermediate sensitive window of finite width. This window is not a single temperature point, but rather a broadened manifestation of the critical sensitive region in finite-size networks. The static susceptibility-enhancement region $\mathcal{W}_{\chi}$, the correlation-length enhancement region $\mathcal{W}_{\xi}$, and the dynamic susceptibility-amplification region $\mathcal{W}_{\Delta\chi}$ characterize this region from three aspects: macroscopic fluctuations, correlation propagation, and post-shock dynamic amplification, respectively. The final magnetization m_{final} serves as the outcome variable used to test whether this sensitive interval corresponds to demagnetization or macroscopic directional reversal.

Furthermore, the reversal window is jointly regulated by kT, N, and α. kT determines whether the system lies in an intermediate sensitive region that can be amplified; N changes how the finite-size critical region unfolds and the relative influence of a single key node; and α simultaneously controls the network's degree heterogeneity, the extent of hub dominance, and the stability of the system against thermal fluctuations. In strongly heterogeneous networks, the response is more strongly governed by hubs and finite-size effects, making it difficult for some indicators to yield a clear and stable window. In more homogeneous networks, the sensitive interval is more concentrated, and the negative-magnetization response is closer to a collective transition controlled by overall fluctuations.

In summary, the results of this paper show that shocks to local key nodes in complex networks can, under a specific background order, be amplified into global directional reorganization. Deletion, inheritance, and reversal locking correspond respectively to three distinct mechanisms: structural weakening, structural buffering, and directional reverse input. Multi-indicator interval identification further demonstrates that the macroscopic reversal window is closely related to the enhancement of the system' s own fluctuations, correlation propagation, and amplification of dynamic response. This model provides a minimal theoretical framework for understanding how local perturbations in complex networks trigger global instability, and how network heterogeneity and finite-size effects regulate this instability process.

[1] C. Castellano, S. Fortunato, and V. Loreto, Reviews of Modern Physics **81**, 591 (2009).

[2] D. Stauffer, American Journal of Physics **76**, 470 (2008).

[3] M. W. Macy, B. K. Szymanski, and J. A. Hołyst, npj Complexity **1**, 10 (2024).

[4] P. Mullick and P. Sen, The European Physical Journal B **98**, 206 (2025).

[5] T. C. Schelling, Journal of Mathematical Sociology **1**, 143 (1971).

[6] D. Vinković and A. Kirman, Proceedings of the National Academy of Sciences **103**, 19261 (2006).

[7] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, Reviews of Modern Physics **80**, 1275 (2008).

[8] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, Physical Review E **66**, 016104 (2002).

[9] M. Leone, A. Vázquez, A. Vespignani, and R. Zecchina, The European Physical Journal B **28**, 191 (2002).

[10] A. Aleksiejuk, J. A. Hołyst, and D. Stauffer, Physica A: Statistical Mechanics and its Applications **310**, 260 (2002).

[11] C. P. Herrero, Physical Review E **69**, 067109 (2004).

[12] A. Grabowski and R. A. Kosiński, Physica A: Statistical Mechanics and its Applications **361**, 651 (2006).

[13] S. Dasgupta, R. K. Pan, and S. Sinha, Physical Review E **80**, 025101 (2009).

[14] S. Baldassarri, A. Gallo, V. Jacquier, and A. Zocca, Physica A: Statistical Mechanics and its Applications **623**, 128811 (2023).

[15] H. L. J. van der Maas, J. Dalege, and L. J. Waldorp, Journal of Complex Networks **8**, cnaa010 (2020).

[16] C. W. Lynn and D. D. Lee, in *Advances in Neural Information Processing Systems 29* (2016) pp. 2487–2495.

[17] M. Garrod and N. S. Jones, Journal of the Royal Society Interface **18**, 20210435 (2021).

[18] D. O. Cajueiro, Physica A: Statistical Mechanics and its Applications **390**, 1695 (2011).

[19] J. Jordan, Security Studies **18**, 719 (2009).

[20] P. B. Johnston, International Security **36**, 47 (2012).

[21] B. C. Price, International Security **36**, 9 (2012).

[22] J. Jordan, International Security **38**, 7 (2014).

[23] D. Milton and B. Price, International Interactions **46**, 309 (2020).

[24] T. L. Bacon and E. Grimm, *Terror in Transition: Leadership and Succession in Terrorist Organizations* (Columbia University Press, New York, 2022).

[25] E. Frantz and E. A. Stein, Comparative Political Studies **50**, 935 (2017).

[26] A. Meng, Journal of Conflict Resolution **65**, 950 (2021).

[27] E. Goldring and P. Ward, World Politics **76**, 417 (2024).

[28] A. Garfield, Low Intensity Conflict & Law Enforcement **11**, 271 (2002), some databases list later online publication metadata, but the journal issue is widely indexed as 11(2–3) (2002).

[29] B. Farah, R. Elias, C. D. Clercy, and G. Rowe, The Leadership Quarterly **31**, 101289 (2020).

[30] W. Quattrociocchi, G. Caldarelli, and A. Scala, PLOS ONE **9**, e87986 (2014).

[31] L. K. Gallos and N. H. Fefferman, Physical Review E **92**, 052806 (2015).

[32] Y. Shang, Scientific Reports **6**, 30521 (2016).

[33] J. Park and S.-G. Hahn, Physical Review E **94**, 022310 (2016).

[34] A. Majdandzic, L. A. Braunstein, C. Curme, I. Vodenska, S. Levy-Carciente, H. E. Stanley, and S. Havlin, Nature Communications **7**, 10850 (2016).

Note: Figure translations are in progress. See original paper for figures.