Effect of Unbalance Force on the Dynamic Response and Stability of Cracked Rotors

JI Li¹, WANG Shuai², ZHU Chao¹, LI Kun¹

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Abstract

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Full Text

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JI Li1, WANG Shuai2, ZHU Chao1, LI Kun1

(1. School of Transportation and Logistics Engineering, Wuhan University of Technology, 430070 Wuhan, China; 2. School of Mechanical Engineering, Hefei University of Technology, 230009 Hefei, China)

Abstract: Studying the dynamic behavior and stability of cracked rotor systems can provide support for rotor crack fault diagnosis and design optimization, and is also an important issue in the field of rotor dynamics. Most existing studies construct crack models based on the assumption that gravity is dominant, thereby ignoring the influence of unbalance force. In many cases, the ratio of unbalance force to gravity is considerable and may affect the crack breathing effect. To reveal the influence of unbalance force on the nonlinear dynamic response and stability of cracked rotor systems, this study uses the finite element method to construct a dynamic analysis model of the rotor system. Combined with a breathing crack model that considers the influence of unbalance force, the system dynamic equations with periodically time-varying coefficients are obtained. To achieve efficient solution of the high-dimensional system of periodically time-varying differential equations, an efficient method for computing the system dynamic response and eigenvalues is derived based on the Hill method, and stability is judged from the real parts of the system eigenvalues. Furthermore, the influence characteristics of unbalance force on the dynamic response and stability of a cracked rotor passing through the 1/3, 1/2, and 1 times critical speeds are investigated. The analysis shows that unbalance force significantly affects the crack breathing behavior, thereby influencing the stiffness characteristics and resonance frequencies of the cracked rotor system and leading to marked changes in the unstable regions of the system.

Keywords: cracked rotor; unbalance force; nonlinear dynamics; stability; Hill method

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JI Li1, WANG Shuai2, ZHU Chao1, LI Kun1

(1. School of Transportation and Logistics Engineering, Wuhan University of Technology, 430070 Wuhan, China;
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Abstract: Studying the dynamic behavior and stability of cracked rotor systems can provide support for rotor crack fault diagnosis and design optimization, and is also an important issue in the field of rotor dynamics. Most of the existing literature constructs crack models based on the assumption of gravity dominance, ignoring the impact of unbalanced forces. In many cases, the ratio of unbalanced force to gravity is considerable and may have an impact on the crack breathing effect. In order to reveal the influence of unbalanced force on the nonlinear dynamic response and stability of the cracked rotor system, this study uses the finite element method to construct a dynamic analysis model of the rotor system. Combined with the breathing crack

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Corresponding author: WANG Shuai, Associate Professor. E-mail: shuaiwang@hfut.edu.cn
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a model that accounts for the influence of unbalance force is obtained, yielding a periodic time-varying system dynamics equation with variable coefficients. To efficiently solve high-dimensional periodic time-varying differential equations, an efficient method for computing the dynamic response and eigenvalues of the system is derived based on the Hill method, and stability is determined from the real parts of the system eigenvalues. Furthermore, the influence of unbalance force on the dynamic response and stability of the cracked rotor at 1/3, 1/2, and 1 times the critical speed is investigated. The analysis shows that unbalance force significantly affects the breathing behavior of the crack, thereby influencing the stiffness characteristics and resonant frequency of the cracked-rotor system and causing marked changes in the system' s unstable regions.

Key words: cracked rotor; unbalance force; nonlinear dynamics; stability; Hill method

Large rotating machines such as gas turbines and aeroengines usually operate under complex and harsh conditions involving heavy loads, high temperatures, high pressures, high rotational speeds, and alternating loads. Together with the influence of residual stresses introduced during manufacturing, these conditions make fatigue cracks prone to initiate in shafts after a certain period of operation. Crack faults are common faults in rotor systems and are characterized by being difficult to detect, easy to propagate, and highly destructive [1-3]. In view of the severe economic losses and social impacts caused by rotor fracture accidents, the dynamic modeling, analysis, and diagnosis of rotor crack faults have long been among the key research topics in rotor dynamics over recent decades. They have attracted sustained attention from many scholars in China and abroad, and abundant research achievements have been obtained [4-10].

Crack modeling is the key issue in modeling rotor crack faults. The accuracy of the crack model directly determines the reliability of the analysis results, while the key to crack modeling lies in simulating the breathing effect. In early breathing-crack models, the strain-energy release-rate method was often adopted, together with linear fracture mechanics theory, to calculate crack-induced stiffness changes [11-12]. On this basis, square-wave models [13], cosine models [14], and models unifying open-crack, square-wave, and cosine models [15] were developed. Because the above models are overly simplified and have difficulty describing the actual breathing behavior of cracks, some scholars, based on the gravity-dominance assumption, proposed various more accurate breathing-crack models, including models based on the neutral axis and the area moment of inertia [16], models based on the crack-closure line [17], and models based on multiple spring–damper elements [18]. In recent years, with advances in computer hardware and software, some scholars have used three-dimensional finite-element models of rotors, combined with contact modeling methods [19] or cohesive-zone models [20], to study the complex breathing behavior of rotor cracks. After decades of development, considerable progress has been made in rotor crack modeling methods. However, most existing studies are based on the gravity-dominance assumption; that is, they consider only gravity while neglecting the influence of unbalance force on the crack breathing effect. For rotor systems with good dynamic balancing performance, the gravity-dominance assumption can reflect the moment distribution at the cracked section with reasonable accuracy. However, for rotors that have been in service for a long time and suffer from severe wear, corrosion, or fouling, the influence of unbalance force is difficult to ignore.

In recent years, the influence of unbalance on the crack breathing effect has also attracted scholarly attention. CHENG et al. [21], using a Jeffcott cracked rotor as the object of study, took the included angle between the crack-opening direction and the shaft-deformation direction as the criterion for judging crack opening and closure, and investigated the effects of unbalance force on the rotor critical speed and resonance response. RUBIO et al. [22], based on a three-dimensional finite-element model of a cracked rotor, treated rotor gravity and unbalance force as quasi-static forces and studied the influence of unbalance force on the crack breathing phenomenon. HE et al. [23] established a three-degree-of-freedom bending–torsional coupled model of a cracked rotor, developed a breathing-crack model considering unbalance force based on the concept of the crack-closure line, and investigated the influence of the unbalance direction angle on the stability of bending–torsional coupled vibration during acceleration of the cracked rotor. WANG et al. [24] proposed a dynamic modeling method for cracked-rotor systems based on the combination of a three-dimensional finite-element model and a spatial contact model. Their results showed that unbalance force can significantly affect the crack breathing phenomenon, and they found that unbalance force can lead to nonstationary crack breathing. ZHAI Pengcheng et al. [25] used a rotor test rig to study the effects of crack depth and unbalance-force direction on rotor vibration characteristics. The experimental results indicated that, when the crack is relatively deep, the amplitudes at the rotational frequency and twice the rotational frequency decrease as the mass-unbalance direction angle increases. MOBARAK et al. [26] proposed a breathing-crack model considering the influence of unbalance force based on the concept of the area moment of inertia, and carried out an in-depth study of the crack breathing mechanism. They found that unbalance force has a significant influence on the crack breathing phenomenon, and that the position of the crack along the rotor axis also has a marked effect on crack breathing. AL-SHUDEIFAT et al. [27] used the finite-element method to study the influence of the direction of unbalance force on the critical speed and maximum resonance amplitude of a cracked rotor–bearing system, and experimentally investigated the vibration response characteristics of the system. These studies provide valuable guidance for clarifying the laws governing the influence of unbalance force on the crack breathing effect. However, existing studies mainly focus on breathing-crack modeling and breathing effects under the influence of unbalance force, while relatively little attention has been paid to the vibration response and stability of unbalanced cracked-rotor systems. Since unbalance is difficult to avoid in actual rotor systems, it is necessary to conduct an in-depth study

unbalance force on the dynamic response and stability of the cracked-rotor system, thereby providing useful guidance for rotor crack-fault diagnosis and operation and maintenance.

Focusing on the influence of unbalance force on the nonlinear dynamic response and stability of a cracked-rotor system, this study adopts a finite element method combined with a breathing-crack model that accounts for unbalance force, and constructs a dynamic model of the cracked-rotor system. To efficiently solve the high-dimensional periodic time-varying differential equations of the system, a rapid calculation method for system dynamic response and eigenvalues based on the Hill method is established. On this basis, the effects of the magnitude and direction of unbalance force on the vibration response and instability regions of the cracked-rotor system at 1/3, 1/2, and 1 times the critical speed are investigated.

1 Modeling of a cracked-rotor system considering unbalance force

The breathing effect of a crack is, in essence, a complex contact behavior under the combined action of multiple loads. When dynamic loads such as unbalance force are far smaller than gravity, gravity is the dominant factor determining the opening and closing behavior of the crack, i.e., the gravity-dominated case. When the unbalance force cannot be neglected relative to gravity, the gravity-dominated assumption may fail to accurately describe the breathing behavior of the crack. Existing studies show that even when the amplitude of the unbalance force is only 10% of gravity, it can still have a significant influence on the crack breathing effect and the vibration response of the rotor [25, 27]. This study adopts a crack modeling method based on the theory of moments of inertia, comprehensively considers the effects of unbalance force and gravity, and constructs a breathing-crack model that accounts for unbalance force.

Figure 1 is a definition diagram of the effective bending angle φ of a cracked shaft at the cracked section. XOY is the fixed coordinate system, and $\bar{X}Ce\bar{Y}$ is the centroidal coordinate system, where Ce is the centroid of the cracked section. F_{un} denotes the direction of the unbalance force in the rotor; $m_s g$ and $m_d g$ denote the directions of the gravity acting on the shaft and the disk, respectively; M_{un} is the unbalance moment at the cracked section; $M_{m_s g}$ and $M_{m_d g}$ are the shaft gravity moment and the disk gravity moment at the cracked section, respectively; and M_R is the vector sum of the moments. Here, θ is the shaft rotation angle, β is the included angle between the unbalance-force direction and the crack, δ is the included angle between the shaft bending direction and the Y-axis, and φ is the effective bending angle of the shaft. Because the direction of the unbalance force continuously changes as the rotor rotates, under the coupled action of gravity and unbalance force, the deformation behavior of the rotor at the cracked section becomes more complex.

The gravity moments and unbalance-force moment $M_{m_s g}$, $M_{m_d g}$, and M_{un} at the cracked section can be calculated from the force-balance relationship of the entire rotor system, and the total moments in the X- and Y-directions at the cracked section can then be obtained:

$$\begin{cases} M_X = M_{m_s g} + M_{m_d g} + M_{\mathrm{un}} \cos(\theta + \beta) \\ M_Y = M_{\mathrm{un}} \sin(\theta + \beta) \end{cases} \tag{1}$$

where M_X is the projection of the resultant moment M_R on the X-axis, and M_Y is the projection of the resultant moment M_R on the Y-axis.

After M_X and M_Y are obtained, the angle δ can be determined, and the effective bending angle φ can then be calculated. For example, when the moments $M_{m_s g} + M_{m_d g}$ and M_{un} are as shown in Fig. 1, at this time $\theta \in [0, \pi)$, $\beta \in [0, \pi)$, and $(\theta + \beta) \in [0, \pi)$. Thus $\delta = \arctan(M_Y / M_X)$, and $\varphi = \pi + \theta - \delta$ is obtained. Once the angle β is determined, the effective bending angle φ and the shaft rotation angle θ have a piecewise functional relationship. In Ref. [16], the shaft rotation angle θ is also the effective bending angle under gravity. Replacing the angle θ in Ref. [16] by the angle φ here, and using the iterative method proposed therein to calculate the centroid coordinates X_{Ce} and Y_{Ce}, the centroidal-axis sectional moments of inertia $I_{\bar{X}}(t)$ and $I_{\bar{Y}}(t)$ corresponding to the effective bending angle φ can be obtained. The distributions of the moments $M_{m_s g}$ and M_{un} vary along the axial direction, and the values of M_X and M_Y are also related to parameters such as the magnitude and direction of the unbalance force, the total shaft length, the shaft rotation angle, the shaft self-weight, the disk self-weight, the disk position, and the crack position. Therefore, under the coupled action of unbalance force and gravity, the bending-direction angle φ of the shaft at the cracked section is related to many parameters of the rotor system. As shown in Fig. 1, the coordinate origin Ce of the coordinate system $\bar{X}Ce\bar{Y}$ changes with the opening and closing of the crack. If the moments of inertia of the cracked section about the $\bar{X}$- and $\bar{Y}$-axes are denoted by $I_{\bar{X}}(t)$ and $I_{\bar{Y}}(t)$, respectively, their approximate expressions are [16]

$$\begin{cases} I_{\bar{X}}(t) \approx \bar{I}_{\bar{X}}(t) = I - I_{11} h(t) \\ I_{\bar{Y}}(t) \approx \bar{I}_{\bar{Y}}(t) = I + I_{11} h(t) + I_{22} g(t) \end{cases} \tag{2}$$

where $h(t)$ and $g(t)$ are crack breathing functions, and I, I_{11}, and I_{22} are sectional moment-of-inertia parameters.

Fig. 1 Definition of the effective bending angle φ of the cracked shaft at the cracked section

Through iterative calculation of the centroidal coordinates, the discrete numerical solutions of $I_{\bar{X}}(t)$ and $I_{\bar{Y}}(t)$ over one rotor revolution can be obtained. By transforming Eq. (2), the approximate time-domain numerical solutions of the breathing functions $h(t)$ and $g(t)$ can be obtained as

$$h(t) = \frac{I - I_{\bar{X}}(t)}{I_{11}}, \quad g(t) = \frac{I_{\bar{Y}}(t) - I - I_{11}h(t)}{I_{22}} \tag{3}$$

The finite element method is a commonly used method in dynamic modeling of rotor systems.

method, and the modeling and analysis accuracy is relatively high. In this study, the above breathing-crack model is combined with a finite element method based on Euler–Bernoulli beam theory to construct the dynamic model of the cracked rotor system. Furthermore, the time-varying stiffness matrix of the cracked shaft element has the following form:

$$K^{\mathrm{c}} = K_0^{\mathrm{c}} + K_1^{\mathrm{c}}h(t) + K_2^{\mathrm{c}}g(t) \tag{4}$$

where K_0^{c} denotes the cross-sectional stiffness matrix when the crack is completely closed; $K_1^{\mathrm{c}}h(t)$ and $K_2^{\mathrm{c}}g(t)$ are the time-varying stiffness matrices characterizing the breathing effect of the crack.

The influence of unbalance force on the crack breathing effect is reflected in the breathing functions $h(t)$ and $g(t)$. The unbalance force affects the crack opening–closing process, thereby influencing the relationship between the cross-sectional moment of inertia and the rotation angle. Since the breathing functions are discrete data sequences calculated at multiple rotation angles and have no accurate analytical expressions, considering their periodicity, this study uses Fourier series to approximate $h(t)$ and $g(t)$, as follows:

$$h(t) = \sum_{h=0}^{m} p_h \cos(h\omega t), \quad g(t) = \sum_{h=0}^{n} q_h \cos(h\omega t) \tag{5}$$

where p_h and q_h are the Fourier coefficients, respectively. By retaining a relatively large number of harmonic orders, $h(t)$ and $g(t)$ can be accurately characterized. For subsequent analysis, Euler' s formula is further used to transform the Fourier-series form in Eq. (5) into a complex exponential form:

$$h(t) = \sum_{h=-m}^{m} P_h \mathrm{e}^{jh\omega t}, \quad g(t) = \sum_{h=-n}^{n} Q_h \mathrm{e}^{jh\omega t} \tag{6}$$

where P_h and Q_h can be calculated from the Fourier coefficients p_h and q_h. Accordingly, the dynamic equation of the entire cracked rotor system can be expressed as

$$M\ddot{u} + [G(\Omega) + C_{\mathrm{s}} + C_{\mathrm{b}}]\,\dot{u} + [K_0 + K_1 h(t) + K_2 g(t) + K_{\mathrm{b}}]\,u = f(t) \tag{7}$$

where $M, G(\Omega) \in R^{N\times N}$ are the system mass and gyroscopic matrices, respectively, and N is the number of model degrees of freedom; $C_{\mathrm{b}}, K_{\mathrm{b}} \in R^{N\times N}$ are the bearing-related damping and stiffness matrices; $C_{\mathrm{s}} = \beta_{\mathrm{c}} K_0$ is the structural damping matrix of the system, with β_{c} being the structural damping coefficient; $K_0 \in R^{N\times N}$ denotes the global stiffness matrix of the uncracked shaft; $K_1 h(t), K_2 g(t) \in R^{N\times N}$ are the time-varying stiffness matrices induced by the breathing crack; and $u, f(t) \in R^N$ denote the displacement vector and external load vector, respectively.

2 Solution of the System Nonlinear Dynamic Response and Stability Analysis

The dynamic equations of the cracked rotor system have relatively high dimensionality, and the nonlinear phenomena caused by crack breathing further make the solution of the system dynamic equations difficult. This study adopts the Hill method

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to construct an efficient method for analyzing the nonlinear dynamic response and stability of the system. The Hill method

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can transform high-dimensional ordinary differential equations into algebraic equations, significantly reducing the computational effort required for analysis and solution.

2.1 Solution of the Nonlinear Dynamic Response

According to the principle of the Hill method, the displacement vector u and load vector $f(t)$ in Eq. (7) can be expressed as a superposition of harmonic components:

$$u = \sum_{k=-\infty}^{+\infty} \varphi_k \mathrm{e}^{jk\omega t}, \quad f(t) = \sum_{k=-\infty}^{+\infty} F_k \mathrm{e}^{jk\omega t} \tag{8}$$

where φ_k and F_k are the k-th order Fourier coefficient vectors, respectively. Substituting Eq. (8) into the system dynamic equation gives

$$\sum_{k=-\infty}^{+\infty} C_k \varphi_k \mathrm{e}^{jk\omega t} + K_1 \sum_{k=-\infty}^{+\infty} \sum_{h=-m}^{m} P_h \varphi_k \mathrm{e}^{j(h+k)\omega t} + K_2 \sum_{k=-\infty}^{+\infty} \sum_{h=-n}^{n} Q_h \varphi_k \mathrm{e}^{j(h+k)\omega t} = \sum_{k=-\infty}^{+\infty} F_k \mathrm{e}^{jk\omega t} \tag{9}$$

where

$$C_k = -(k\omega)^2 M + jk\omega \left[G(\omega) + C_\mathrm{s} + C_\mathrm{b}\right] + K_0 + K_\mathrm{b}.$$

By applying harmonic balance to both sides of the equation, the balance equation for an arbitrary r-th harmonic component can be obtained, namely

$$C_r \varphi_r + K_1 \sum_{h=-m}^{m} P_h \varphi_{r-h} + K_2 \sum_{h=-n}^{n} Q_h \varphi_{r-h} = F_r \tag{10}$$

Furthermore, by assembling the harmonic-balance equations of all orders, an algebraic system of equations with the Fourier coefficient vectors of each order as variables can be obtained:

$$\left[\tilde{C} + \left(\tilde{P} \otimes K_1\right) + \left(\tilde{Q} \otimes K_2\right)\right] \tilde{\Phi} = \tilde{f} \tag{11}$$

where $\tilde{C} = \mathrm{Bdiag}_{r=-\infty,\cdots,+\infty}(C_r)$, i.e., a pseudo block-diagonal matrix with C_r as the block-diagonal elements; $\tilde{\Phi}$ is the column vector composed of the coefficient vectors φ_r, and

$$\tilde{\Phi}^\mathrm{T} = \left[\cdots\ \varphi_{-2}^\mathrm{T}\ \varphi_{-1}^\mathrm{T}\ \varphi_0^\mathrm{T}\ \varphi_1^\mathrm{T}\ \varphi_2^\mathrm{T}\ \cdots\right]^\mathrm{T};$$

$\otimes$ denotes the Kronecker product; $\tilde{P}$ and $\tilde{Q}$ are matrices composed of $P_h(h = -m, \cdots, m)$ and $Q_h(h = -n, \cdots, n)$, respectively; and $\tilde{f}$ is the column vector composed of F_k, namely

$$\tilde{f} = \left[\cdots\ F_{-1}^\mathrm{T}\ F_0^\mathrm{T}\ F_1^\mathrm{T}\ \cdots\right]^\mathrm{T}.$$

For the rotor system studied here, the external load vector consists of gravity and unbalance force. Gravity corresponds to the F_0 term, while the unbalance force introduces the F_{-1} and F_1 terms.

Equation (11) is an infinite-dimensional algebraic system and cannot be solved directly. In practical applications, it is usually truncated; that is, only lower-order harmonic coefficients are retained, while higher-order harmonic coefficients are ignored. Extensive practice shows that satisfactory analysis accuracy can be obtained when a certain number of harmonics is retained. According to the truncated algebraic equation, the harmonic coefficient vector can be calculated as

$$\hat{\Phi} = \left[\hat{C} + \left(\hat{P} \otimes K_1\right) + \left(\hat{Q} \otimes K_2\right)\right]^{-1} \hat{f} \tag{12}$$

where $\hat{\Phi}$, $\hat{C}$, $\hat{P}$, $\hat{Q}$, and $\hat{f}$ are respectively the corresponding terms in Eq. (11).

truncated form. Substituting the calculated harmonic coefficients of each order into Eq. (9) yields the steady-state vibration response of the system.

2.2 System stability analysis

The stability problem of a cracked rotor system can be reduced to an eigenvalue analysis problem; specifically, it can be determined from the real parts of the complex eigenvalues of the system. The Hill method can efficiently analyze the eigenvalues of periodic time-varying systems. Based on the Hill method, this study constructs an efficient computational method for the eigenvalues of a cracked rotor. First, the external load vector $f(t)$ in the system dynamics equation is neglected, and the displacement vector u is expressed as the product of a periodic function $\Phi(t)$ and the exponential function $e^{\lambda t}$:

$$u = \Phi(t)e^{\lambda t}, \quad \Phi(t) = \sum_{k=-\infty}^{+\infty} \varphi_k e^{jk\omega t} \tag{13}$$

where λ denotes the system eigenvalue.

Substituting the expression of u into the system dynamics equation gives

$$\sum_{k=-\infty}^{+\infty} \left(\lambda^2 L_k + \lambda N_k + G_k\right) \varphi_k e^{jk\omega t} + K_1 \sum_{k=-\infty}^{+\infty} \sum_{h=-m}^{m} P_h \varphi_k e^{j(h+k)\omega t} + K_2 \sum_{k=-\infty}^{+\infty} \sum_{h=-n}^{n} Q_h \varphi_k e^{j(h+k)\omega t} = 0 \tag{14}$$

where

$$\begin{aligned} L_k &= M, \\ N_k &= 2jk\omega M + G(\omega) + C_b + C_s, \\ G_k &= -k^2\omega^2 M + jk\omega \left[G(\omega) + C_s + C_b\right] + K_0 + K_b \end{aligned} \tag{15}$$

By expanding Eq. (14) and performing harmonic balance, the algebraic equation for an arbitrary r-th-order harmonic coefficient vector φ_r is obtained as follows:

$$\left(\lambda^2 L_r + \lambda N_r + G_r\right) \varphi_r + K_1 \sum_{h=-m}^{m} P_h \varphi_{(r-h)} + K_2 \sum_{h=-n}^{n} Q_h \varphi_{(r-h)} = 0 \tag{16}$$

Furthermore, assembling the balance equations of all orders yields the overall eigenvalue equation

$$\left[\lambda^2\tilde{L} + \lambda\tilde{N} + \tilde{G} + \left(\tilde{P}\otimes K_1\right) + \tilde{Q}\otimes K_2\right]\bar{\Phi} = 0 \tag{17}$$

where $\tilde{L}$, $\tilde{N}$, and $\tilde{G}$ have the same forms as in Eq. (11).

The eigenvalue equation in Eq. (17) is also infinite-dimensional and must be truncated to obtain the system eigenvalues. After retaining the first several orders of coefficients, the truncated eigenvalue equation can be obtained. The above equation is a quadratic eigenvalue problem and must be transformed into state-space form:

$$(\lambda c + A)\Psi = 0 \tag{18}$$

where

$$A = \begin{bmatrix} -\hat{L} & 0 \\ 0 & \hat{G} + \left(\hat{P}\otimes K_1\right) + \left(\hat{Q}\otimes K_2\right) \end{bmatrix}, \quad c = \begin{bmatrix} 0 & \hat{L} \\ \hat{L} & \hat{N} \end{bmatrix}, \quad \Psi = \begin{bmatrix} \lambda\hat{\Phi} \\ \hat{\Phi} \end{bmatrix} \tag{19}$$

In the equation, $\hat{L}$, $\hat{G}$, $\hat{N}$, $\hat{P}$, $\hat{Q}$, and $\hat{\Phi}$ are all the truncated forms of the corresponding terms in Eq. (17). Based on the complex eigenvalues of the system, the system stability is evaluated through the real parts of the eigenvalues. That is, when the real part of at least one eigenvalue is greater than zero, the system is unstable; when the real parts of all eigenvalues are negative, the system is in a stable state.

3 Influence of unbalance force on the crack breathing effect

This study adopts the finite element rotor model shown in Fig. 2 to analyze the influence of unbalance force on the crack breathing effect and on the vibration characteristics of the system. The system parameters are shown in Table 1. Sliding bearings are used to support both ends of the rotor. The unbalance force acts at the node of the right disk, and the crack location is expressed by the corresponding element number.

Fig. 2 Finite element model diagram of the cracked rotor system

Table 1 Cracked rotor system parameters

Physical quantity	Value
Total shaft length L/m	1.4
Shaft radius R/m	0.05
Crack position λ	0.7
Rotor material density $\rho/(\mathrm{kg}\cdot\mathrm{m}^{-3})$	7 850
Elastic modulus of rotor material E/GPa	210
Outer radius of disk R_0/m	0.3
Inner radius of disk R_i/m	0.05
Disk thickness h_d/m	0.12
Bearing stiffness $k/(\mathrm{N}\cdot\mathrm{m}^{-1})$	5×10^7
Bearing damping $c/(\mathrm{N}\cdot\mathrm{s}\cdot\mathrm{m}^{-1})$	1×10^3
Left disk position l_1/m	0.35
Right disk position l_2/m	1.05
System structural damping coefficient β_c/s	1×10^{-5}

Note: The crack position is $\lambda = l_0/L$, where l_0 is the distance from the crack to the left end face of the shaft; the left disk position l_1 is the distance from the center of the left disk to the left end face of the shaft; the right disk position l_2 is the distance from the center of the right disk to the left end face of the shaft.

By carrying out a convergence analysis of the response and eigenvalue-analysis results, the first nine harmonic components are retained in the system analysis to truncate the infinite-dimensional equations, i.e., $H = 9$. In approximating the crack breathing functions $h(t)$ and $g(t)$, $m = n = 7$ is selected. These parameter values ensure good convergence of the results.

Eigenvalue analysis of the uncracked rotor gives the first-order critical speed of the uncracked rotor system as $\Omega_c = 2\,058.6$ r/min. In the subsequent analysis, the rotational speed is presented in nondimensional form as $\omega = \Omega/\Omega_c$, where Ω is the actual rotor speed. To characterize the influence of crack depth, this study takes the ratio of the actual crack depth r_c to the shaft radius R as the nondimensional parameter for crack depth, namely

$$\mu = \frac{r_c}{R} \tag{20}$$

To quantify the influence of the magnitude and direction of the unbalance force, the included angle β between the unbalance force and the crack-opening direction is used to represent the crack direction. The magnitude of the unbalance force depends on the product of the system unbalance and the square of the rotational speed. In this study, the ratio ξ of the unbalance force F^c_{unb} when the rotor passes through the critical speed to its own weight F_g is used to quantify the magnitude of the unbalance force:

$$\xi = \frac{F_{\mathrm{unb}}^{c}}{F_g} \tag{21}$$

Furthermore, at any rotational speed Ω, the magnitude of the unbalance force acting on the system is $F_{\mathrm{unb}}(\Omega) = F_g \xi \Omega^2/\Omega_c^2$. The primary cause of crack breathing is the alternating tensile and compressive stresses in the section, which produces a periodic breathing phenomenon; the tensile and compressive stress regions in the section are mainly determined by the moment distribution on the cracked section. Figure 3 shows the sectional moment distributions at different rotor positions when the system is subjected separately to gravity and to the unbalance force.

Figure 3. Axial distribution of bending moment along the shaft

Fig. 3 Axial distribution of bending moments on an axis

In Fig. 3, λ_1 and λ_3 are respectively the left and right zero points of the moment due to self-weight, while λ_2 and λ_4 are respectively the left and right zero points of the unbalance moment. The ratio between the magnitude of the unbalance moment and that of the gravity moment is one of the factors determining the effective bending angle at the rotor crack, and it further affects the crack breathing behavior. That is, when the crack is located near λ_1 or λ_3, even an extremely small unbalance force can lead to unsteady breathing of the crack; whereas when the crack is located near λ_2 or λ_4, the influence of the unbalance force on crack breathing is greatly weakened. Considering engineering practice, the subsequent analysis in this study takes $\lambda = 0.7$ as an example.

The crack breathing effect can also be reflected by the sectional closure rate. The sectional closure rate is defined as $\Lambda = A_b(\theta)/A_0$, where $A_b(\theta)$ is the area of the closed region on the cracked section at rotation angle θ, and A_0 is the total area of the cracked section. Figure 4 shows the variation of the closure rate of the cracked section with the magnitude and direction of the unbalance force, where $\beta = 0$, $\mu = 0.5$, and $\lambda = 0.7$. In the figure, $\Lambda = 100\%$ and $\Lambda = 0$ correspond respectively to the fully closed and fully open states. It can be seen from Fig. 4 that the unbalance force significantly affects the opening and closing behavior of the cracked section. This is because the breathing behavior of the crack is generated under the action of rotor gravity, whereas the unbalance force rotates with the shaft, and the direction of the tensile/compressive stress it produces in the cracked section remains unchanged; therefore, the unbalance force affects the crack breathing behavior. Moreover, as the unbalance force increases, this influence becomes increasingly significant. Even when the unbalance force is only 10% of gravity, the sectional closure-rate curve changes markedly. In particular, when the unbalance force is relatively large, the cracked section may be unable to close completely. This result also reflects the necessity of considering the unbalance force when modeling rotor cracks.

Figure 4. Closure-rate curve of the cracked section

Fig. 4 Closure rate curve of crack section

Figure 5 shows the variation of the moment of inertia I_x in the X-axis direction of the cracked section with the unbalance-force direction angle β and the rotation angle θ, where $\mu = 0.5$, $\lambda = 0.7$, and $\xi = 0.6$. From Fig. 3, $M_{\mathrm{un}} > 0$. According to the tensile-compressive relationship at the crack interface, the unbalance force tends to promote the crack remaining open. In Fig. 4, as ξ increases, the angular interval over which the crack interface is completely closed gradually decreases and even disappears, which also indicates that the unbalance force here tends to keep the crack open. In Fig. 5, when $\beta = 180°$, the influence of the unbalance force on crack breathing is exactly opposite to that when $\beta = 0°$, i.e., it promotes the crack remaining closed; therefore, during a full revolution of the rotor, the crack remains almost completely closed, and the moment of inertia I_x is relatively large. In summary, the unbalance force affects the breathing behavior of cracks on the shaft, and thereby affects the stiffness of the cracked element,

Moreover, as the unbalance force increases, its influence becomes increasingly significant; the resulting stiffness variation will ultimately affect the vibration response and stability of the system.

Fig. 5 Surface projection of the crack-section moment of inertia $I_{\bar{x}}$

4 Influence of the Unbalance Force on the System' s Nonlinear Dynamic Response

On the basis of the method described above, this section analyzes the influence of the magnitude and direction of the unbalance force on the nonlinear dynamic response of the system.

4.1 Influence of the Magnitude of the Unbalance Force

Figures 6 and 7 show the vibration-response characteristics of the cracked rotor system at 1/3, 1/2, and 1 times the critical speed, where $\mu = 0.5$, $\lambda = 0.7$, and $\beta = 0°$. The figures include the balanced condition and three unbalanced conditions. The response results are the vibration amplitudes at the center point of the right disk in Fig. 2, namely node 13. As can be seen from Figs. 6 and 7, with the occurrence and continuous increase of the unbalance force, the forward-whirl peak of the rotor system gradually shifts toward lower rotational speed. From Fig. 4, it can be seen that, at this time, the unbalance force suppresses crack breathing and promotes the crack to remain open throughout the entire cycle; the stiffness of the cracked element decreases, and therefore the resonance speed decreases. The change in the reverse-whirl peak is not very obvious. As ξ increases, the resonance peak shifts slightly toward lower speed, reflecting the characteristic influence of the unbalance force on the stiffness of the cracked section. In Fig. 7, the variation of the subharmonic resonance peaks at 1/2 and 1/3 times the critical speed with ξ is relatively weak, because the unbalance force is comparatively small at subcritical speeds and has little influence on the crack-breathing effect.

To further demonstrate the influence of the magnitude of the unbalance force on the vibration response, Fig. 8 presents the relationship between the vibration-response amplitude of the cracked rotor and the excitation frequency and unbalance-force magnitude near the critical speed, where $\mu = 0.5$, $\lambda = 0.7$, and $\beta = 0°$. From Fig. 8, it can be seen that the unbalance force causes the resonance frequency to change to a certain extent, and that at 0.9 times the critical speed the resonance frequencies gradually approach and merge. These results clearly demonstrate the influence of the unbalance force on the vibration response of the cracked rotor system.

Fig. 6 Vibration response of the cracked rotor system at the critical speed

Fig. 7 Vibration response of the cracked rotor system at 1/2 and 1/3 times the critical speed

Fig. 8 Relationship between the amplitude of the cracked rotor and the excitation frequency and unbalance-force magnitude

4.2 Influence of the Direction of the Unbalance Force

Because the direction of the unbalance force has a significant influence on the stiffness of the cracked section

significant influence, and therefore it also affects the vibration response of the cracked-rotor system.

Fig. 9 shows the vibration-response characteristics of the cracked-rotor system for different directions of the unbalance force, where $\mu = 0.5$, $\lambda = 0.7$, and $\xi = 0.6$. It can be seen from Fig. 9 that the vibration-response amplitude and resonance frequency of the cracked-rotor system at the critical speed vary with the direction of the unbalance force. When $\beta = 0°$, the direction of the unbalance force is the same as the crack-opening direction; the unbalance force causes the crack to tend to remain open, leading to a relatively small overall stiffness of the cracked rotor and, correspondingly, a lower resonance frequency. When $\beta = 180°$, the direction of the unbalance force is always opposite to the crack-opening direction, causing the crack to tend to close; the stiffness at the cracked section is therefore relatively large, resulting in a higher resonance frequency. For intermediate values of the angle β, the resonance frequency also lies between these two cases.

Fig. 10 shows the variation in the response peak and resonance frequency as the rotor passes through the critical speed for different angles β, where $\mu = 0.5$, $\lambda = 0.7$, and $\xi = 0.6$. Fig. 10 more clearly illustrates the influence of the angle β on the resonance peak and resonance frequency of the cracked-rotor system.

Fig. 9 Amplitude variation of a cracked rotor passing through the critical speed for different β.

Fig. 10 Evolution of the resonance amplitude of the rotor passing through the critical speed with β.

5 Influence of Unbalance Force on System Stability

The preceding analysis shows that the unbalance force affects the stiffness characteristics and vibration response of the cracked rotor. This section analyzes the influence of the unbalance force on the stability characteristics of the cracked rotor. Fig. 11 presents the variation in the real parts of the system eigenvalues, where $\lambda = 0.7$ and $\beta = 0°$. The speed ranges in which positive real parts appear in the figure are the unstable ranges. It can be seen from the figure that, as ξ increases, the real parts of the system eigenvalues undergo relatively pronounced changes near the critical speed. The unstable speed range gradually shifts toward lower speeds, and the unstable speed range gradually decreases.

Fig. 11 Variation in the real parts of the system eigenvalues for different ξ.

To more comprehensively demonstrate the influence of the unbalance force on the stability characteristics of the cracked-rotor system, Fig. 12 ($\lambda = 0.7$, $\beta = 0°$) shows the distribution characteristics of the instability regions of the cracked-rotor system as ξ increases from 0 to 0.6; the markers in the figure represent instability regions. It can be seen from Fig. 12 that the rotor instability regions are mainly distributed near 1 and 1/2 times the critical speed, and as ξ increases ...

As it increases, the instability regions near the critical speed shift overall toward lower rotational speeds, and their shapes gradually shorten. This phenomenon can be explained by the results shown in Figs. 4 and 5: when the β angle is 0, increasing the unbalanced force gradually reduces the range of rotation angles over which the crack can be completely closed, and may even prevent complete closure. The stiffness of the cracked section also decreases overall; consequently, the instability regions shift as a whole toward lower rotational speeds. In addition, as the unbalanced force increases, the fluctuation range of the moment of inertia of the cracked section during one rotor revolution decreases, thereby suppressing to some extent the instability phenomenon associated with small cracks.

Fig. 12 Instability-region variation diagram of the cracked rotor system for different ξ
Subfigures: (a) $\xi = 0$; (b) $\xi = 0.2$; (c) $\xi = 0.4$; (d) $\xi = 0.6$. Axes: μ versus ω.

Fig. 13 shows the variation of the instability regions of the cracked rotor under different β angles ($\lambda = 0.7$, $\xi = 0.5$). It can be seen from Fig. 13 that, as the β angle increases, the instability regions near the critical speed shift overall toward higher rotational speeds and their shapes gradually become shorter. This is because, during the increase of the β angle, the unbalanced force gradually changes from promoting crack opening to promoting crack closure; the stiffness of the cracked element on the shaft is therefore enhanced, and the overall stability of the system increases, as shown in Fig. 5. Near the 1/2 critical speed, the instability phenomenon becomes more pronounced as the β angle increases. These phenomena fully demonstrate the influence of unbalanced force on the stability of cracked-rotor systems; therefore, the influence of unbalanced force should be considered when studying crack faults in rotor systems.

Fig. 13 Instability-region variation of the cracked rotor system at different β angles
Subfigures: (a) $\beta = 0°$; (b) $\beta = 45°$; (c) $\beta = 90°$; (d) $\beta = 135°$; (e) $\beta = 180°$. Axes: μ versus ω.

6 Conclusions

By analyzing the influence of unbalanced force on the nonlinear dynamic response and stability of a cracked rotor system, this study established a finite-element analysis model for a cracked rotor system that accounts for the effect of unbalanced force. Based on the Hill method, an efficient analysis method for the system dynamic response and eigenvalues was derived. The effects of the magnitude and direction of the unbalanced force on the dynamic response and stability characteristics of the cracked rotor when passing through the 1/3, 1/2, and 1× critical speeds were investigated in depth. The main conclusions are as follows.

1) Unbalanced force significantly affects the breathing behavior of the crack, and the larger the unbalanced force, the more significant the effect. The direction angle of the unbalanced force also influences the crack-breathing effect. Under certain conditions with relatively large unbalanced force, the crack may be unable to close completely or may remain open.

2) Unbalanced force affects the amplitudes of the resonance peaks of the cracked rotor system.

and frequencies. When the unbalance force causes the cracked region to open, the natural frequencies of the system tend to decrease; when the unbalance force promotes crack closure, the natural frequencies increase.

3) Because the unbalance force affects the stiffness variation characteristics at the crack section, it also significantly influences the distribution characteristics of the unstable regions of the cracked-rotor system, including the size and location of the unstable regions. Moreover, there are also clear differences among the unstable regions under different direction angles of the unbalance force.

According to the analysis in this study, when carrying out dynamic modeling and fault-mechanism analysis of cracked-rotor systems, the potential influence of the unbalance force should be considered.

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