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Abstract

To investigate the distribution characteristics of cavities within saturated sites, traditional local linear optimization algorithms have been widely used for inversion problems. However, inversion problems often face inherent challenges such as nonlinearity and non-uniqueness. To this end, a global nonlinear optimization algorithm, the sparrow search algorithm based on quantum computations and multi-strategy enhancement (QMESSA), is introduced to achieve efficient and accurate parameter optimization. This algorithm avoids the strong dependence of inversion on the initial model, increases the population mutation probability by constructing an adaptive T-distribution, effectively avoids local optima, and enhances the global optimization capability and local refined search capability of the population. The forward modeling process adopts the indirect boundary element method (IBEM), which effectively reduces the computational dimension while significantly improving computational efficiency and accuracy. Circular, elliptical, and dumbbell-shaped cavities in a saturated half-space are used as examples to verify the effectiveness and stability of the combination of QMESSA and IBEM for inversion. The results show that QMESSA can rapidly and accurately invert the locations and scales of cavities, and that the inversion results for all parameters have high accuracy. In addition, the global search capability of the algorithm compensates to some extent for the limitations caused by insufficient prior information, demonstrating good application potential in the inversion of cavities in saturated sites.

Full Text

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Abstract: To investigate the distribution characteristics of cavities within saturated sites, traditional local linear optimization algorithms have been widely used in inversion problems. However, inversion problems often face inherent challenges such as nonlinearity and non-uniqueness. To this end, a global nonlinear optimization algorithm—the sparrow search algorithm based on quantum computations and multi-strategy enhancement (QMESSA)—is introduced to achieve efficient and accurate parameter optimization. This algorithm avoids the strong dependence of inversion on the initial model. By constructing an adaptive T-distribution to increase the population mutation probability, it effectively avoids local optima and strengthens both the global optimization capability and the local fine-search capability of the population. The forward analysis adopts the indirect boundary element method (IBEM), which effectively reduces the computational dimensionality while significantly improving computational efficiency and accuracy. Taking circular, elliptical, and dumbbell-shaped cavities in a saturated half-space as examples, the effectiveness and stability of combining QMESSA with IBEM for inversion are verified. The results show that QMESSA can rapidly and accurately invert the location and scale of cavities, and that the inversion results for the various parameters have high accuracy. In addition, the global search capability of the algorithm compensates, to a certain extent, for the limitation of insufficient prior information, demonstrating good application potential in cavity inversion for saturated sites.

Keywords: inversion of subsurface cavities in saturated sites; inversion algorithm; indirect boundary element method; QMESSA; elastic-wave exploration

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Abstract: To explore the distribution characteristics of cavity in saturated sites, traditional local optimization algorithms are commonly employed to solve inverse problems. However, such problems often involve inherent challenges, including nonlinearity and multiple solutions. To address these issues, the sparrow search algorithm based on quantum computations and multi-strategy enhancement (QMESSA) is introduced to achieve efficient and accurate parameter optimization. The algorithm reduces the inversion' s

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strong reliance on the initial model, enhances the population' s mutation probability through the use of an adaptive T-distribution, effectively mitigates the risk of local optima, and improves the population' s global optimization capability and local fine-tuning efficiency. The forward modeling process employs the indirect boundary element method (IBEM), which effectively reduces computational dimensions while improving both efficiency and accuracy. The effectiveness and stability of combining QMESSA with IBEM for inversion are demonstrated through examples of circular, dumbbell-shaped, and elliptical cavities in a saturated half-space. The results show that QMESSA can quickly and accurately determine the location and scale of a cavity, producing high-accuracy results for each parameter. Furthermore, the global search capability of the algorithm compensates for the limitation of insufficient prior information and highlights its potential for applications in cavity inversion within saturated sites.

Key words: inversion of underground cavities in saturated sites; inversion algorithm; indirect boundary element method; QMESSA; elastic-wave exploration

Seismic-wave inversion is one of the core technologies in seismic-data processing and is widely used in engineering seismic design, geological advance prediction, and oil-and-gas exploration and development. As exploration and development proceed in greater depth, requirements for the accuracy of inversion results have continually increased. In particular, under complex geological conditions such as saturated sites, higher demands are placed on seismic-wave inversion technology. The seismic response characteristics of saturated sites differ markedly from those of single-phase medium sites. The dynamic equations for saturated media established by BIOT

$$1-2$$

reveal the propagation characteristics of waves in two-phase media and their essential differences from those in single-phase media, providing a theoretical foundation for studying the dynamic response of saturated soils. The interaction between pore water and the soil skeleton in saturated soil makes the propagation laws of seismic waves more complex. Therefore, developing seismic inversion techniques suitable for saturated sites is of great significance for improving the accuracy and reliability of seismic exploration.

The accuracy of seismic-wave inversion depends on the accuracy of forward wavefield modeling. For forward modeling of wave motion in saturated and two-phase media, commonly used methods include analytical and numerical methods. Analytical methods are relatively accurate, but they are applicable only to seismic-wave scattering problems in local site models with simple shapes. Numerical methods are mainly divided into domain-discretization methods

$$3-5$$

and boundary-discretization methods. For complex underground geological structures, domain-discretization methods require the introduction of artificial boundaries and involve high computational cost; moreover, mesh-partitioning schemes and differences in element size significantly increase the computational burden and bring substantial storage requirements. By contrast, the boundary element method discretizes only the boundary of the computational domain, thereby reducing the computational dimension, improving computational efficiency, and lowering storage requirements. This method automatically satisfies the radiation condition at infinity, avoids complicated artificial boundaries, and accurately simulates the scattering field through Green' s functions, yielding relatively high computational accuracy. Therefore, the boundary element method has been widely applied in seismic modeling of saturated and two-phase media[6 − 10].

For the inversion of underground parameters in saturated sites, Wang Hongwei et al.[11] proposed a time-domain full-waveform inversion method based on a porous elastic medium and used the L-BFGS algorithm to effectively invert multiple underground medium parameters in single-parameter inversion; Zhang Xinming et al.[12] adopted a wavelet multiscale-regularized Gauss–Newton method to solve the nonlinearity and non-uniqueness problems in multiparameter inversion for fluid-saturated porous media; YANG et al.[13] used the L-BFGS algorithm to realize multiparameter full-waveform inversion of porous elastic media; and ZHANG et al.[14] applied a wavelet multiscale method to the inversion of porosity in fluid-saturated porous media and solved the problem at each scale using the regularized Gauss–Newton method. The above studies mostly focus on the inversion of medium parameters in saturated sites and pay relatively little attention to the geometric distribution characteristics of underground cavities; moreover, they mostly employ local optimization algorithms. However, for the inversion of the location and geometric characteristics of underground cavities, local optimization algorithms require an initial model, have difficulty effectively handling the nonlinearity and multiplicity of solutions in multidimensional parameter models, and are prone to becoming trapped in local optimal solutions.

Therefore, this study combines the indirect boundary element method (IBEM) with a sparrow search algorithm based on quantum computation and multi-strategy enhancement (QMESSA)

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, and proposes an effective method for inverting the location and geometric characteristics of underground cavities in saturated sites. A surface harmonic concentrated load is selected as the active excitation source. Based on IBEM, accurate forward-modeling results are obtained, and optimization based on QMESSA is introduced so as to determine the location and geometric characteristics of underground cavities.

1 Computational Model

Figure 1 is a schematic diagram of the computational model for inversion of underground cavities in a saturated half-space.

Let the horizontal direction be the x-direction and the vertical direction be the y-direction. Region V is the saturated half-space. Region V satisfies the basic assumptions of being continuous, homogeneous, two-phase saturated, and isotropic, with no initial stress and with infinitesimal displacement and stress; Γ is the ground surface. In the saturated half-space, λ and μ are Lamé constants, the Poisson' s ratio is ν, the bulk modulus of the soil skeleton is K_{gs}, and the soil-particle density

is ρ_{gs}, the fluid bulk modulus is K_{f}, the fluid density is ρ_{f}, the critical porosity is n_{cr}, and the critical bulk modulus of the soil is K_{cr}. A cavity is contained in the saturated half-space; Ω is the interface between the cavity and the half-space. The major-axis radius is r_{a} (if the cavity is circular, the radius is r_{a}; if it is dumbbell-shaped, the principal radius is r_{m}), the minor-axis radius is r_{b}, the center coordinates are l_x in the x direction and l_y in the y direction, and the angle between the major axis and the horizontal direction is θ (clockwise taken as positive). A harmonic concentrated load on the ground surface is used to excite linear elastic waves; the load magnitude is $F_0 e^{\mathrm{i}\omega t}$, where F_0 is the amplitude of the harmonic concentrated load, i is the imaginary unit, and ω is the frequency of the harmonic concentrated load.

Fig. 1 Schematic diagram of the inversion calculation model for an underground cavity in a saturated site

2 Forward Modeling Method

According to elastic wave theory, the total wave field can be divided into the free field and the scattered field. The free-field part is solved with reference to the dynamic Green' s function of a homogeneous poroelastic half-plane [16], while the scattered field is solved using IBEM.

2.1 BIOT Theory of Saturated Two-Phase Media

Let the displacement of the soil skeleton in the saturated medium be u_i $(i, j = x, y)$, and let the displacement of the fluid relative to the soil skeleton be w_i. The constitutive relation for a homogeneous saturated medium can be expressed as follows [1]:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}e - \alpha\delta_{ij}p \tag{1}$$

$$p = -\alpha Me - Mw_{i,i} \tag{2}$$

where p and σ_{ij} are the pore-water pressure and the total stress component of the soil, respectively; ε_{ij} and e are the strain component and volumetric strain of the soil skeleton, respectively; μ and λ are both Lamé constants of the soil skeleton; δ_{ij} is the Kronecker delta function; and α and M are parameters characterizing the compressibility of the soil particles and pore fluid $(0 \leq \alpha \leq 1,\ 0 \leq M < \infty)$.

$$\alpha = \frac{W}{M \cdot c} + c \tag{3}$$

$$M = \frac{I}{c^2} \tag{4}$$

$$W = \frac{c\left(1 - c - K_{\mathrm{dry}}/K_{\mathrm{gs}}\right)K_{\mathrm{gs}}}{\left[\left(1 - c - K_{\mathrm{dry}}/K_{\mathrm{gs}}\right) + c \cdot K_{\mathrm{gs}}/K_{\mathrm{f}}\right]} \tag{5}$$

$$I = \frac{c^2 \cdot K_{\mathrm{gs}}}{\left[\left(1 - c - K_{\mathrm{dry}}/K_{\mathrm{gs}}\right) + c \cdot K_{\mathrm{gs}}/K_{\mathrm{f}}\right]} \tag{6}$$

where c is the porosity; K_{gs} is the bulk modulus of the soil skeleton; K_{f} is the fluid bulk modulus; and K_{dry} is the bulk modulus of the solid-phase skeleton.

The equations of motion of the soil related to u and w can be expressed as [2]

$$\mu u_{i,jj} + \left(\lambda + \alpha^2 M + \mu\right)u_{j,ji} + \alpha M w_{j,ji} = \rho\ddot{u}_i + \rho_{\mathrm{f}}\ddot{w}_i \tag{7}$$

$$\alpha M u_{j,ji} + M w_{j,ji} = \rho_{\mathrm{f}}\ddot{u}_i + m\ddot{w}_i + b\dot{w}_i \tag{8}$$

where p is the pore-water pressure; $\rho = (1-c)\rho_{\mathrm{gs}} + c\rho_{\mathrm{f}}$ is the total density of the soil; ρ_{gs} and ρ_{f} are the mass densities of the soil particles and the fluid, respectively; $b = (Z/k)c^2$ is the coefficient of reactive viscous coupling, Z is the fluid viscosity, and k is the permeability coefficient. If internal friction is neglected, then $b = 0$. $m = \rho_{22}/c^2$ is a mass-like parameter, and $\rho_{22} = c\rho_{\mathrm{f}} - \rho_{12}$ and $\rho_{12} = -0.5(1-c)\rho_{\mathrm{f}}$ are dynamic mass coefficients.

2.2 Free Field

First consider the free field. Assume that a harmonic concentrated load acts at the origin, with frequency ω and magnitude $F_0 e^{\mathrm{i}\omega t}$. The free-field stress, soil-skeleton displacement, fluid relative displacement, and pore-water pressure are denoted by σ_i^{f}, u_i^{f}, w_i^{f}, and p^{f}, respectively, where the superscript f denotes the free field and the subscript i denotes the direction. The specific formulas are given in Appendix A.

2.3 Scattered Field

This study uses IBEM to simulate the scattered wave field. Equations are established through the boundary conditions on the ground surface and the cavity to solve for the wave-source densities, and the scattered field is then obtained.

2.3.1 Construction of the Wave Field According to single-layer potential theory, the scattered field in the saturated half-space is formed by the superposition of concentrated forces and flow sources acting on the boundary surface S $(S = \Gamma + \Omega)$. In the half-space, the soil-skeleton displacement u_i^{s}, traction t_i^{s}, fluid relative displacement w_i^{s}, and pore-water pressure p^{s} are respectively expressed as [17-20]

$$u_i^{\mathrm{s}}(x) = \int_S \left[\varphi_j(\xi) G_{ij}(x,\xi) + \varphi_F(\xi) G_{iF}(x,\xi)\right] \mathrm{d}S_\xi \tag{9}$$

$$t_i^{\mathrm{s}}(x) = \int_S \left[\varphi_j(\xi) T_{ij}(x,\xi) + \varphi_F(\xi) T_{iF}(x,\xi)\right] \mathrm{d}S_\xi \tag{10}$$

$$w_i^{\mathrm{s}}(x) = \int_S \left[\varphi_j(\xi) W_{ij}(x,\xi) + \varphi_F(\xi) W_{iF}(x,\xi)\right] \mathrm{d}S_\xi \tag{11}$$

$$p^{\mathrm{s}}(x) = \int_S \left[\varphi_j(\xi) P_j(x,\xi) + \varphi_F(\xi) P_F(x,\xi)\right] \mathrm{d}S_\xi \tag{12}$$

where $x \in V$ is the field point; $\xi \in S$ is the wave-source point; the superscript s denotes the scattered field; φ_j and φ_F are respectively the wave-source densities under the action of a concentrated force and a flow source; and G_{ij}, T_{ij}, W_{ij}, and P_j are respectively the soil···

skeleton displacement, traction, fluid relative displacement, and pore-water-pressure Green' s functions; G_{iF}, T_{iF}, W_{iF}, and P_F are, respectively, the soil-skeleton displacement, traction, fluid relative displacement, and pore-water-pressure Green' s functions under the action of a flow source. The specific expressions of the Green' s functions are given in Ref. [21]; i, j denote directions, with $i, j = x, y$.

2.3.2 Boundary Conditions and Solution

When the saturated half-space boundary is a permeable boundary, the boundary conditions are zero stress and zero pore-water pressure on the boundary surface. They are expressed as

$$t^{\mathrm{s}} + t^{\mathrm{f}} = 0 \tag{13}$$

$$p^{\mathrm{s}} + p^{\mathrm{f}} = 0 \tag{14}$$

For ease of solution, the half-space ground surface and the inner wall of the cavity are discretized into N_1 and N_2 elements, respectively, and the total number of discrete elements is N. According to the boundary conditions, substituting Eqs. (10) and (12) into Eqs. (13) and (14), and then carrying out integral discretization, the discrete system of equations is written as

$$\sum_{m=1}^{N} \left[\varphi_j(\xi_m) t_{ij}(x_n, \xi_m) + \varphi_F(\xi_m) t_{iF}(x_n, \xi_m)\right] = -t_i^{\mathrm{f}}(x) \tag{15}$$

$$\sum_{m=1}^{N} \left[\varphi_j(\xi_m) p_{ij}(x_n, \xi_m) + \varphi_F(\xi_m) p_{iF}(x_n, \xi_m)\right] = -p^{\mathrm{f}}(x) \tag{16}$$

where $n = 1, \cdots, N$.

2.4 Calculation of the Total Wave Field

After obtaining the free field and the scattered field as described above, the total displacement, total stress, fluid relative displacement, and pore-water pressure can be expressed, respectively, as

$$u_i(x) = u_i^{\mathrm{f}}(x) + u_i^{\mathrm{s}}(x) \tag{17}$$

$$\sigma_{ij}(x) = \sigma_{ij}^{\mathrm{f}}(x) + \sigma_{ij}^{\mathrm{s}}(x) \tag{18}$$

$$w_i(x) = w_i^{\mathrm{f}}(x) + w_i^{\mathrm{s}}(x) \tag{19}$$

$$p(x) = p^{\mathrm{f}}(x) + p^{\mathrm{s}}(x) \tag{20}$$

2.5 Method Verification

To verify the correctness of the method proposed in this study, the model containing a cavity is degraded into a half-space; that is, the cavity radius is taken as $r_a = 0.001$ m, and a unit harmonic concentrated load is applied on the surface of the saturated half-space. The detailed parameter settings are as follows: porosity $c = 0.3$, dimensionless frequency $\eta = 2r_a/\lambda_w = 0.5$, where λ_w is the shear-wave wavelength; viscous damping ratio $\zeta = 0.02$; Poisson' s ratio $\nu = 1/3$; bulk modulus of the soil skeleton $K_{gs} = 36$ GPa; bulk modulus of the fluid $K_f = 2$ GPa; bulk modulus of the solid skeleton $K_{\text{dry}} = 6\,168$ MPa; fluid density $\rho_f = 1\,000$ kg/m^3; $\alpha = 0.828\,7$; and $m = 7.22 \times 10^3$ kg/m^3.

It can be seen from Fig. 2 that the normalized surface displacement amplitudes of the degraded model agree well with the half-space free-field calculation results, verifying the accuracy of the method.

Fig. 2 Comparison of the saturated-cavity degradation results with the half-space free-field results

- Legend: degradation results; half-space free-field results
- Vertical axes: displacement amplitude $|u|$, displacement amplitude $|v|$
- Horizontal axis: x/a
- (a) x-direction
- (b) y-direction

3 QMESSA Inversion

3.1 Basic Principles of QMESSA

QMESSA is developed based on the Sparrow Search Algorithm (SSA). As a new type of swarm-intelligence optimization algorithm [22], the core mechanism of SSA is to simulate the foraging and anti-predation behaviors of sparrows, with the aim of searching for individual optimal solutions over the global domain and, through fitness comparisons among individual optimal solutions, screening out the global optimal solution. Its basic principles are as follows.

1) Initialization stage: randomly generate a sparrow population, and define the relevant parameters and the maximum number of iterations Q.
2) Fitness evaluation and ranking: calculate and rank the fitness of the initial population, and determine the best solution and the worst solution.
3) Position update: update the positions of discoverers, joiners, and sparrows aware of danger according to the algorithmic rules.

4) Iterative optimization: in each iteration, obtain the current optimal solution and compare it with the optimal solution from the previous iteration; if it is better, update it, otherwise keep the original solution unchanged until the stopping condition is satisfied. Finally, output the global optimal solution.

However, SSA has problems such as a nonuniform distribution during population initialization and a lack of multi-

such as poor diversity and a tendency to fall into local optima [23]. Based on this, Wu et al. [15] optimized SSA and proposed a sparrow search algorithm enhanced by quantum computing and multiple strategies (quantum computing and multi-strategy enhanced sparrow search algorithm, QMESSA). Its core contents are as follows.

1) Initial population optimization: an improved Circle chaotic mapping technique incorporating quantum computing and quantum-gate mutation is introduced, significantly improving the randomness and diversity of the initial population.

2) Finder position-update strategy: in the original SSA, while the individual fitness decreases, its spatial position converges toward the population-optimal solution and gradually turns into a local search. Therefore, in the local search stage, the original formula cannot make full use of individuals with better fitness to conduct extensive exploration. To improve the convergence speed and accuracy of the algorithm, the original formula is improved: random constants τ and β are introduced, and in this study the value ranges of τ and β are $[1.5, 2.5]$ and $[4, 12]$, respectively; random numbers r_1, r_2, and r_3 are added, all with value ranges of $[0, 1]$, to enlarge the search range. In addition, to prevent the SSA algorithm from converging prematurely and causing the model to fall into a local optimum, the population should be made to jump out of the current search space. Although the finder position-update formula in the original SSA uses a normal distribution for mutation, it does not consider the dynamic variation in an individual' s ability to escape from a local optimum under different numbers of iterations. Therefore, this study introduces an adaptive T-distribution mutation factor to dynamically improve the algorithm' s ability to escape from local optima. When the value of the mutation factor approaches zero, local search is strengthened; when it is far from zero, global exploration capability is enhanced. Thus, a relatively large mutation-factor value is used at the initial stage of iteration to improve global optimization capability, whereas at the later stage of iteration, mutation is concentrated in the neighborhood of the optimal solution, maintaining population diversity while ensuring convergence speed. Therefore, the new finder position-update formula is expressed as [15]

$$x_{i,j}^{q+1} = \begin{cases} x_{i,j}^{q}\left[\tau(2r_1-1)\sqrt{r_2}\exp\left(\frac{-i^{\beta}}{\kappa Q}\right)+(r_3-0.5)^3\right], & R_2 < \mathrm{ST}, \\ x_{i,j}^{q} + T\left[\frac{1}{g}\cdot\exp\left(\frac{\ln t_c}{t_c}\right)^{q}\right]L, & \text{else}. \end{cases} \tag{21}$$

where q is the number of iterations; $x_{i,j}^{q}$ and $x_{i,j}^{q+1}$ respectively represent the position of the i-th sparrow in the j-th dimension during the q-th and $(q+1)$-th iterations; κ is a random number in the range $[0,1]$; R_2 is the warning value and is a random number in the range $[0,1]$; ST is the safety value, with a value range of $[0.5,1]$, and is set to 0.8 in this study based on experimental experience and trial calculations; T is the adaptive T-distribution mutation factor; t_c is the critical point in the iterative process, with the specific formula $t_c = Q/5$; g is the scaling factor, set to 10 in this study; L denotes a unit vector of dimension D, where D is the number of parameters to be solved.

3) Population elimination and update mechanism: after each iteration, the population is screened according to a precise elimination strategy; individuals with poor fitness are eliminated and replaced with newly generated individuals, effectively balancing global and local search capabilities. The precise elimination strategy formula is as follows.

$$x_{i,j}^{q+1} = x_{\text{best}}^{q} + (ub - lb)\times R \times T\left[\frac{1}{g}\cdot\exp\left(\frac{\ln t_c}{t_c}\right)^{q}\right] \tag{22}$$

where x_{best}^{q} denotes the global optimal solution at the q-th iteration; ub and lb are the upper and lower search bounds, respectively; R is the dynamic radius, and

$$R = 1 - (q/Q) \tag{23}$$

4) Innovation in the boundary-control method: a new boundary-control formula is designed to replace the traditional method, fully utilizing population information, accelerating convergence, and improving algorithm performance. The boundary-control formula is expressed as

$$x_{i,j}^{q+1} = x_{\text{best2}}^{q} + r \times (x_{\text{best}}^{q} - x_{\text{best2}}^{q}) \tag{24}$$

where x_{best2}^{q} denotes the global second-best solution at the q-th iteration; r is a random number in the range $[0,1]$.

3.2 Inversion Process

This study uses QMESSA for inversion and designs a fitness function to facilitate solving the problem. Observation points R_i are selected on the ground surface at fixed intervals, for a total of K points, and IBEM is used to calculate the surface displacement amplitude at each point. Let the surface displacement amplitude of each observation point in the true model be U_i, and the surface displacement amplitude of each observation point in the inversion model be U_i'. Therefore, the fitness function can be expressed as

$$J = \frac{1}{K}\sum_{i=1}^{K}\left(U_i' - U_i\right)^2 \tag{25}$$

After determining the QMESSA fitness function and combining it with IBEM-based forward simulation, the specific inversion process is shown in Fig. 3.

Fig. 3 Inversion process

Visible flowchart text:

- Start
- Improved Circle chaotic-map initialization combining quantum computing and quantum-gate mutation
- Determine the parameter search range according to geological data
- Start iteration and generate the population optimal position
- QMESSA
- QMESSA updates parameters
- Construct an adaptive T-distribution and update the population position
- IBEM forward simulation
- Eliminate individuals with poor fitness and generate new individuals for replacement
- Calculate the fitness value based on the surface displacement amplitude values of the true model
- Whether the conditions are met or the number of iterations is reached
  - No
  - Yes
- Whether the accuracy requirement is satisfied
  - No
  - Yes
- Output the optimal parameters
- End

4 Numerical Example Analysis

To verify the effectiveness of combining IBEM with QMESSA for inversion

To verify its stability and accuracy, this study first adopts a circular cavity in a saturated half-space as the computational model, solves for its position and radius, and preliminarily verifies the effectiveness and accuracy of the method. Then, an elliptic cavity is used as the computational model, and its position, semimajor and semiminor axes, and inclination angle are solved, further demonstrating the stability and accuracy of the algorithm. Finally, a dumbbell-shaped cavity is used as the computational model, and its position, principal radius, shape-control parameter, and inclination angle are solved, further verifying the applicability of the algorithm to different geometric shapes. Unless otherwise specified, the computational parameters in the examples of this study are as follows: porosity $c = 0.3$, bulk modulus of the soil skeleton $K_{\mathrm{gs}} = 36$ GPa, soil-particle density $\rho_{\mathrm{gs}} = 2\,650\ \mathrm{kg/m^3}$, fluid bulk modulus $K_{\mathrm{f}} = 2$ GPa, fluid density $\rho_{\mathrm{f}} = 1\,000\ \mathrm{kg/m^3}$, critical porosity $n_{\mathrm{cr}} = 0.36$, critical bulk modulus of the soil $K_{\mathrm{cr}} = 200$ MPa, Poisson' s ratio $\nu = 1/3$, viscous damping ratio $\zeta = 0.02$, and cavity-center coordinates $l_x = -1$ m, $l_y = 2$ m. The dimensionless frequency is $\eta = r_a/\lambda_{\mathrm{w}} = 0.5$.

In this study, the inversion accuracy is evaluated according to the relative error, specifically,

$$E = \left(1 - \frac{|P' - P|}{|P|}\right) \times 100\% \tag{26}$$

where E is the accuracy, P' is the inverted parameter value, and P is the true parameter value.

4.1 Inversion of a circular underground cavity

First, two sets of inversions are carried out for a circular cavity with radius $r_a = 1$ m.

Table 1 lists the search ranges and inversion results for the cavity position and radius.

Table 1 Statistical table of parameter search range and inversion results for a circular cavity ($r_a = 1$ m)

No.	Parameter	Search range	Inversion result	Accuracy/%
1	r_a/m	$0.50 \sim 2.00$	1.02	98.00
1	l_x/m	$-2.00 \sim 2.00$	-1.01	99.00
1	l_y/m	$1.00 \sim 4.00$	2.03	98.50
2	r_a/m	$0.50 \sim 2.00$	0.99	99.00
2	l_x/m	$-2.00 \sim 2.00$	-0.99	99.00
2	l_y/m	$1.00 \sim 4.00$	1.99	99.50

Figure 4 shows the iterative process of the two inversions. The results indicate that both inversions found the global optimum in the 20th iteration, and that by the 4th iteration the population' s optimal position was already close to the true model, demonstrating that QMESSA has good global search capability. Introducing the adaptive T distribution can effectively increase the population mutation probability, enhance the ability to jump out of local optima, and reduce the number of iterations. In terms of accuracy, Table 1 shows that for the underground cavity model with radius $r_a = 1$ m, the accuracy of each parameter in the two inversion results exceeds 98%, with an average of 98.83%.

Fig. 4 Inversion iteration diagram of circular cavity ($r_a = 1$ m)

(a) Sequence 1

(b) Sequence 2

4.2 Inversion of a regular elliptic underground cavity

Table 2 lists the search ranges, inversion results, and accuracies of the inversion parameters for an elliptic cavity ($r_a = 0.8$ m, $r_b = 0.4$ m, $\theta = 0$).

Table 2 Statistical table of parameter search range and inversion results for a regular elliptic cavity ($r_a = 0.8$ m, $r_b = 0.4$ m)

No.	Parameter	Search range	Inversion result	Accuracy/%
1	r_a/m	$0.60 \sim 1.60$	0.80	100.00
1	r_b/m	$0.30 \sim 0.80$	0.40	100.00
1	l_x/m	$-2.00 \sim 2.00$	-1.01	99.00
1	l_y/m	$1.20 \sim 4.00$	2.01	99.50
2	r_a/m	$0.60 \sim 1.60$	0.80	100.00
2	r_b/m	$0.30 \sim 0.80$	0.41	97.50
2	l_x/m	$-2.00 \sim 2.00$	-1.01	99.00
2	l_y/m	$1.20 \sim 4.00$	2.01	99.50

Figure 5 gives the iterative process of the two sets of inversions in Table 2. It can be seen from the figure that, in the two inversions, the result of the first iteration has a relatively large error compared with the true model; however, after introducing the adaptive T distribution, the population mutation probability is increased.

···and its ability to escape local optima, enabling subsequent iterations to rapidly approach the true model and ultimately find the global optimum at the 25th iteration.

Fig. 5 Inversion iteration diagram of a regular elliptical cavity ($r_a = 0.8$ m, $r_b = 0.4$ m, $\theta = 0$)
(a) No. 1; (b) No. 2

4.3 Inversion of an Inclined Elliptical Underground Cavity

Table 3 gives the search ranges, inversion results, and accuracies of the inversion parameters for an inclined elliptical cavity ($r_a = 0.8$ m, $r_b = 0.4$ m, $\theta = -\pi/6$).

Figure 6 shows the iterative processes for the two inversions in Table 3. The results indicate that cases No. 1 and No. 2 find the global optimum at the 22nd and 24th iterations, respectively. In both inversions, the population-best position is already close to the true model parameters at the 5th iteration, demonstrating that the algorithm has good global search capability.

From the combined analysis of Fig. 5 and Fig. 6, it can be seen that QMESSA has strong global search capability. Although the result of the first iteration in Fig. 5 has a relatively large error compared with the true model, subsequent iterations rapidly approach the true model. The introduction of the adaptive T-distribution increases the probability of population mutation, improves the ability to escape local optima, and reduces the number of iterations. According to Tables 2 and 3, the accuracies of the inverted parameters for the underground elliptical-cavity model are all greater than 97%, with an average accuracy of 98.84%.

Tab. 3 Statistical table of parameter search ranges and inversion results for an inclined elliptical cavity
($r_a = 0.8$ m, $r_b = 0.4$ m, $\theta = -\pi/6$)

No.	Parameter	Search range	Inversion result	Accuracy / %
1	r_a/m	$0.60 \sim 1.60$	0.81	98.75
1	r_b/m	$0.30 \sim 0.80$	0.39	97.50
1	l_x/m	$-2.00 \sim 2.00$	-1.01	99.00
1	l_y/m	$1.20 \sim 4.00$	2.01	99.50
1	θ/rad	$-\pi/12 \sim -\pi/3$	-0.53	98.08
2	r_a/m	$0.60 \sim 1.60$	0.81	98.75
2	r_b/m	$0.30 \sim 0.80$	0.41	97.50
2	l_x/m	$-2.00 \sim 2.00$	-1.02	98.00
2	l_y/m	$1.20 \sim 4.00$	2.01	99.50
2	θ/rad	$-\pi/12 \sim -\pi/3$	-0.51	98.08

Fig. 6 Inversion iteration diagram of an inclined elliptical cavity
($r_a = 0.8$ m, $r_b = 0.4$ m, $\theta = -\pi/6$)
(a) No. 1; (b) No. 2

4.4 Inversion of an Inclined Dumbbell-Shaped Underground Cavity

Table 4 gives the inclined dumbbell-shaped underground cavity (major radius

For $r_m = 1\,\mathrm{m}$, shape-control parameter $r_c = 0.5$, and $\theta = -\pi/6$, the search ranges of the inversion parameters, inversion results, and accuracies are given. Here, the major radius r_m controls the overall size of the shape and determines the width and height of the dumbbell-shaped cavity; the shape-control parameter r_c controls the degree of contraction of the dumbbell-shaped cavity.

Table 4 (Tab. 4) Statistical table of the parameter search ranges and inversion results for a dumbbell-shaped cavity with inclination
$(r_m = 1\,\mathrm{m}, r_c = 0.5, \theta = -\pi/6)$

No.	Parameter	Search range	Inversion result	Accuracy / %
1	r_m/m	$0.50 \sim 1.50$	1.02	98.00
1	r_c	$0.25 \sim 0.75$	0.50	100.00
1	ℓ_x/m	$-1.50 \sim 1.50$	-0.99	99.00
1	ℓ_y/m	$1.50 \sim 3.00$	1.99	99.50
1	θ/rad	$-\pi/12 \sim -\pi/4$	-0.52	99.89
2	r_m/m	$0.50 \sim 1.50$	1.00	100.00
2	r_c	$0.25 \sim 0.75$	0.50	100.00
2	ℓ_x/m	$-1.50 \sim 1.50$	-0.99	99.00
2	ℓ_y/m	$1.50 \sim 3.00$	2.04	98.00
2	θ/rad	$-\pi/12 \sim -\pi/4$	-0.55	94.96

Figure 7 shows the iterative processes of the two inversions in Table 4.

(a) No. 1

(b) No. 2

Fig. 7 Inversion iteration diagram of a dumbbell-shaped cavity with inclination $(r_m = 1\,\mathrm{m}, r_c = 0.5, \theta = -\pi/6)$

It can be seen from Fig. 7 that, in the two inversions, the result of the first iteration has a relatively large error compared with the true model. However, after introducing the adaptive T-distribution, the probability of population mutation and the ability to escape local optima are improved, enabling subsequent iterations to rapidly approach the true model. Nos. 1 and 2 find the global optimal solutions at the 9th and 25th iterations, respectively. In terms of accuracy, Table 4 shows that the accuracies of the inversion parameters for the dumbbell-shaped underground cavity model are all above 90%, with an average accuracy of 98.83%.

4.5 Algorithm Comparison

To further verify the inversion performance of QMESSA, this study compares its fitness convergence process with those of the differential evolution algorithm (DE), particle swarm optimization algorithm (PSO), and genetic algorithm (GA). For each algorithm, the population size is 180, and the maximum number of iterations per run is 20. Each algorithm is run independently twice to eliminate random error. In this section, an inclined elliptical underground cavity is taken as an example, with model parameters the same as those in Section 4.3.

Figure 8 shows the fitness curves of the four algorithms in two inversion calculations.

Fig. 8 Comparison of algorithmic fitness curves

It can be seen that the initial fitness of QMESSA is significantly lower than those of DE, PSO, and GA, and approaches zero after the 15th generation, demonstrating good global and local search capabilities. This benefits from the adaptive T-distribution factor in the position-updating strategy, which increases the probability of population mutation and effectively avoids local optima. However, the DE and PSO algorithms both exhibit varying degrees of stagnation within 20 iterations, with maximum stagnation counts of 13 and 8, respectively; they fail to rapidly update the inversion model so that it fits the true model. The GA algorithm shows no obvious convergence trend in fitness. Compared with DE, PSO, and GA, QMESSA has stronger global optimization and local fine-search capabilities when computing underground cavities with coupled multiple parameters, and is less likely to fall into local optima.

5 Conclusions

To improve the accuracy and efficiency of inversion for saturated-field underground cavities, this study uses the indirect boundary element method (IBEM) to perform forward wave-field simulation and introduces the global nonlinear optimization algorithm QMESSA to optimize model parameters. Through inversion calculations of cavity location and radius, the effectiveness and accuracy of the method are verified. Further inversion is carried out for the center coordinates, major and minor semi-axes, and cavity inclination angle of an elliptical cavity model; the results show that the method has good stability and reliability. Finally, calculations are performed for the location, major semi-axis, shape-control parameter, and inclination angle of a dumbbell-shaped cavity, further verifying the applicability of the algorithm under different geometric shapes. The following conclusions are obtained.

1) Combining IBEM with QMESSA for the inversion of saturated half-space underground cavities shows good stability and high accuracy even under complex conditions with strong coupling among high-dimensional parameters.

2) QMESSA avoids the need to set an initial model, has strong global search capability and high flexibility in the search range, can find the optimal solution within a relatively large parameter range, and remedies the search limitations caused by insufficient prior information.

3) Compared with DE, PSO, and GA algorithms, QMESSA has better global optimization capability in the inversion of saturated underground cavities, reduces stagnation periods and the number of iterations, while maintaining population diversity in the later stage, and is less likely to fall into local optima; it is suitable for cases with multiple parameters and strong coupling.

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Appendix A

The free-field results for a saturated half-space under a harmonic concentrated surface load are as follows.

$$u_x^{\mathrm{f}} = i\xi \left(Ae^{\gamma_1 y} + Be^{-\gamma_1 y} + Ce^{\gamma_2 y} + De^{-\gamma_2 y}\right) + \gamma_3 \left(Ee^{\gamma_3 y} - Fe^{-\gamma_3 y}\right) \tag{A1}$$

$$u_y^{\mathrm{f}} = \gamma_1 \left(Ae^{\gamma_1 y} - Be^{-\gamma_1 y}\right) + \gamma_2 \left(Ce^{\gamma_2 y} - De^{-\gamma_2 y}\right) - i\xi \left(Ee^{\gamma_3 y} + Fe^{-\gamma_3 y}\right) \tag{A2}$$

$$w_x^{\mathrm{f}} = i\xi\left[\chi_1\left(Ae^{\gamma_1 y} + Be^{-\gamma_1 y}\right) + \chi_2\left(Ce^{\gamma_2 y} + De^{-\gamma_2 y}\right)\right] + \chi_3\gamma_3\left(Ee^{\gamma_3 y} - Fe^{-\gamma_3 y}\right) \tag{A3}$$

$$w_y^{\mathrm{f}} = \chi_1\gamma_1\left(Ae^{\gamma_1 y} - Be^{-\gamma_1 y}\right) + \chi_2\gamma_2\left(Ce^{\gamma_2 y} - De^{-\gamma_2 y}\right) - i\xi\chi_3\left(Ee^{\gamma_3 y} + Fe^{-\gamma_3 y}\right) \tag{A4}$$

$$\sigma_{xx}^{\mathrm{f}} = b_1\left(Ae^{\gamma_1 y} + Be^{-\gamma_1 y}\right) + b_2\left(Ce^{\gamma_2 y} + De^{-\gamma_2 y}\right) + 2i\xi\gamma_3\left(Ee^{\gamma_3 y} - Fe^{-\gamma_3 y}\right) \tag{A5}$$

$$\sigma_{yx}^{\mathrm{f}} = 2i\xi\left[\gamma_1\left(Ae^{\gamma_1 y} - Be^{-\gamma_1 y}\right) + \gamma_2\left(Ce^{\gamma_2 y} - De^{-\gamma_2 y}\right)\right] + b_3\left(Ee^{\gamma_3 y} + Fe^{-\gamma_3 y}\right) \tag{A6}$$

$$\sigma_{yy}^{\mathrm{f}} = c_1\left(Ae^{\gamma_1 y} + Be^{-\gamma_1 y}\right) + c_2\left(Ce^{\gamma_2 y} + De^{-\gamma_2 y}\right) - 2i\xi\gamma_3\left(Ee^{\gamma_3 y} - Fe^{-\gamma_3 y}\right) \tag{A7}$$

$$p^{\mathrm{f}} = a_1\left(Ae^{\gamma_1 y} + Be^{-\gamma_1 y}\right) + a_2\left(Ce^{\gamma_2 y} + De^{-\gamma_2 y}\right) \tag{A8}$$

In the equations, u_x^{f} and u_y^{f} are respectively the x- and y-direction components of the free field u_i^{f}; w_x^{f} and w_y^{f} are respectively the x- and y-direction components of the free field w_i^{f}; ξ is the Fourier transform parameter. The expressions for the coefficients A, B, C, D, E, F, a_1, a_2, b_1, b_2, b_3, c_1, c_2, γ_1, γ_2, γ_3, χ_1, χ_2, and χ_3 in Eqs. (A1)-(A8) are given in Ref. [16].

Note: Figure translations are in progress. See original paper for figures.