Full-Constraint Shadow Price Representation Model for Regional Electricity Spot Market Parameters*

2026-05-28

ChinaRxiv

Abstract

Analysis of price characteristics in regional electricity spot markets is a key link in promoting the construction of a fair and transparent electricity market. In the clearing model of an electricity spot market designed using a security-constrained economic dispatch model, various market boundaries and complex constraints are intertwined and coupled, making it difficult to grasp the pricing patterns of regional spot markets. Taking the clearing model of the Southern Regional Electricity Spot Market as an example, this paper establishes the correspondence between the security-constrained economic dispatch model and the standard linear programming model. By analyzing and deriving the relationship between standard optimization model parameters and shadow prices, it proposes a full-constraint shadow price representation model for regional spot market parameters, and proves that, when an optimal solution is obtained, market parameters can be expressed as a linear combination of the shadow prices of all constraints. On this basis, considering changes in system security boundaries, an extension method for the full-constraint shadow price representation model is proposed, and a generalized analysis method for the price characteristics of regional spot markets is extracted, enabling rapid and accurate identification of the causes of price formation. The correctness of the proposed full-constraint shadow price representation model is verified through a regional spot market simulation case, the interactions among full-constraint shadow prices are clearly characterized, and suggestions are put forward for the future construction of electricity spot markets.

Full Text

Full-Constraint Shadow Price Representation Model for Regional Electricity Spot Market Parameters*

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Abstract: Analysis of price characteristics in regional electricity spot markets is a key link in promoting the construction of a fair and transparent electricity market. In electricity spot market clearing models designed using security-constrained economic dispatch, various market boundaries and complex constraints are intertwined and coupled, making it difficult to grasp the regularities of prices in regional spot markets. Taking the Southern regional electricity spot market clearing model as an example, this paper establishes the correspondence between the security-constrained economic dispatch model and the standard linear programming model. By analyzing and deriving the relationship between the parameters of the standard optimization model and shadow prices, it proposes a full-constraint shadow price representation model for regional spot market parameters, proving that, when an optimal solution is obtained, market parameters can be represented as a linear combination of the shadow prices of all constraints. On this basis, considering changes in system security boundaries, an extension method for the full-constraint shadow price representation model is proposed, and a generalized analytical method for the price characteristics of regional spot markets is distilled, enabling rapid and precise identification of the causes of price formation. A regional spot market simulation case verifies the correctness of the proposed full-constraint shadow price representation model, clearly characterizes the interactions among the shadow prices of all constraints, and puts forward recommendations for the future development of electricity spot markets.

[**Objective**] To solve the problems of complex and difficult-to-analyze price characteristics in electricity spot markets and improve the operational efficiency of spot markets.

[**Methods**] A full-constraint shadow price representation model for regional electricity spot market parameters is proposed, representing market parameters as a linear combination of the shadow prices of all constraints; on this basis, a generalized analytical method for regional spot price characteristics is distilled.

[**Results**] Rapid and accurate identification of the causes of regional spot price formation is achieved, and information on constraint shadow prices can be used to guide power planning and the design of electricity market mechanisms.

[**Limitations**] Insufficient consideration is given to scenarios involving hybrid interprovincial AC/DC interconnections within regional power grids.

[**Conclusions**] It is recommended to use the full-constraint shadow price representation model for price analysis and to guide electricity market development.

Keywords: electricity market; regional spot market; shadow price; market boundary

Classification No.: TM372

Shadow Price Model of all Constraints for Regional Power Spot Market Parameters

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Abstract: Analysis of regional electricity spot market price characteristics is a key link in promoting the construction of a fair and transparent electricity market. In the electricity spot

* This paper is one of the research outcomes of the National Key R&D Program-funded project "Key Technologies for Coordinated Dispatch and Trading of Cross-Basin Provincial/Regional Hydro-Wind-Solar-Storage Resources" (Project No.: 2024YFB2409000).

market-clearing model designed using the security-constrained economic dispatch model, various market boundaries and complex constraints are intertwined and coupled, making it difficult to grasp the patterns of regional spot market prices. Taking the clearing model of the Southern Regional Electricity Spot Market as an example, this paper establishes the correspondence between the security-constrained economic dispatch model and the standard linear programming model. By analyzing and deriving the relationship between the parameters of the standard optimization model and shadow prices, a full-constraint shadow-price representation model for regional spot-market parameters is proposed. It is proven that, under optimal solution conditions, market parameters can be expressed as a linear combination of the shadow prices of all constraints. On this basis, considering changes in system security boundaries, an extension method for the full-constraint shadow-price representation model is proposed, and a generalized analytical method for regional spot-market price characteristics is refined, enabling rapid and accurate identification of the causes of price formation. The correctness of the proposed full-constraint shadow-price representation model is verified through a regional spot-market simulation case study, which clearly depicts the mutual influences among full-constraint shadow prices. Finally, suggestions are provided for the future construction of electricity spot markets.

[**Objective**] To address the complex price characteristics of electricity spot markets and improve operational efficiency.

[**Methods**] A fully constraint-represented shadow-price model is developed for regional spot-market parameters, expressing parameters as linear combinations of the shadow prices of all constraints and thereby deriving a generalized analytical framework for regional spot-price characteristics.

[**Results**] This enables rapid and accurate identification of the factors driving regional spot prices, while information on constraint shadow prices can be used to guide power-system planning and inform electricity-market mechanism design.

[**Limitations**] Insufficient consideration has been given to AC/DC hybrid inter-provincial interconnection scenarios within regional power grids, and specific issues arising from new scenarios, such as the joint clearing of ancillary-service markets and spot markets, have yet to be analyzed.

[**Conclusions**] It is recommended that the fully constraint-represented shadow-price model be used for price analysis and to guide electricity-market development.

Keywords: Power market; Regional spot market; Shadow price; Market border

1 Introduction

Since the release of Document No. 9 on electricity reform in 2015

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, clarifying the formation mechanism of market-based electricity prices and analyzing the factors influencing electricity prices have consistently been among the key tasks in the development of China' s electricity spot markets

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China' s electricity spot markets generally operate through "full-quantity declaration and centralized competitive optimized clearing." Based on a power-grid nodal model, and taking into account various security operation constraints, transaction schedules and clearing prices are determined with the objective of minimizing total system cost. In this process, the constraints in the spot-market clearing model are intertwined and coupled, jointly affecting the formation of spot clearing prices and making it difficult to rapidly identify the causes of spot-price formation.

Characterizing the formation mechanism of electricity spot clearing prices and analyzing their characteristics not only help safeguard the fairness and transparency of electricity spot-market development, but also facilitate the use of spot-market price signals to guide the construction of the electricity market. Existing literature mainly studies the price characteristics of electricity spot markets through theoretical derivation and simulation verification.

In terms of theoretical derivation, Ref. [3] decomposed the nodal electricity price into two components—the energy price and the congestion price—by means of the Lagrangian function, and carried out congestion management based on congestion-price information. On this basis, Ref. [4] derived the sensitivity relationship between nodal prices and nodal loads. Ref. [5] preliminarily analyzed the causes of abnormal electricity prices from the perspective of power-system security constraints, but was limited to analyzing the influence of network power-flow constraints. Ref. [6] derived a unified expression for nodal prices on the generation and consumption sides, and revealed the coupled influence relationships among various types of constraints. Ref. [7] introduced constraint-violation penalty factors into the pricing model and proposed that penalty factors can effectively limit spike prices, but did not further investigate the mechanism by which penalty factors affect spot prices. Refs. [8–11], by analyzing the stepwise bidding curves on the generation and consumption sides and the dual problem of the market-clearing model, proposed a mechanism for determining multiple solutions of nodal prices. Some studies, starting from the nonconvexity of unit commitment, have proposed that nodal prices cannot reflect unit start-up and shut-down costs, resulting in a loss of benefits [12]. On this basis, nonconvex pricing methods such as integer-programming pricing [13] and convex-hull pricing [14–17], as well as pricing methods that take electricity prices as optimization variables [18,19], have been proposed to meet market operators' requirements regarding pricing properties. Against the background of the continuous deepening of electricity spot-market construction in China [20], analysis of spot prices has received increasing attention. The mechanisms by which unit operating constraints such as ramping and output-range limits [21,22], generation–load bidding [23,24], and new-energy policy boundaries [25] influence spot prices have been explored.

In terms of simulation validation, Ref. [26] used the K-S test to analyze the statistical properties of electricity prices in the U.S. PJM (Pennsylvania—New Jersey—Maryland) day-ahead market. Ref. [27] performed simulation validation of the factors influencing spot prices using a perturbation-testing method, but the analytical efficiency of this method is relatively low. Refs. [28] and [29] located typical causes of abnormal spot prices based on correlation theory and the isolation forest algorithm. Ref. [30] verified, based on multi-agent simulation, the "counterintuitive" problem in the design of spot-pricing mechanisms. Overall, methods based on simulation validation still face the problems of lacking underlying theoretical support and having low efficiency.

With the accelerated construction of a nationally unified electricity market system [31], theoretical research on prices in the unified electricity spot market has become a topic of widespread concern in both academia and engineering practice [32]. The regional electricity spot market is an important form of the unified electricity spot market. Methods for analyzing its price characteristics are crucial for market operators to identify the causes of electricity-price formation and guide market development, and for market participants to deepen their understanding of electricity markets and optimize the formulation of trading strategies. The southern regional electricity spot market is cleared based on security constrained unit commitment (SCUC) and security constrained economic dispatch (SCED). It adopts a complete full-network nodal model, in which interprovincial priority planned transmission constraints are intertwined and coupled with the security constraints of units, interfaces, and other equipment across the entire network [33,34]. Spot prices are affected by multidimensional sensitive factors. Existing literature has rarely combined the background of regional electricity spot markets to conduct an in-depth analysis of the coupling relationships among various constraints, or to examine how constraint shadow-price information can be used to quantitatively analyze clearing-price characteristics and guide market construction.

To address the above issues, this paper first combines the background of the southern regional electricity spot market to establish the correspondence between the standard optimization model and the regional spot SCED model, and proposes a full-constraint shadow-price representation model for key parameters in the regional spot-clearing model, such as generating-unit bids and interprovincial transmission charges, thereby finely characterizing the mechanisms by which all market-clearing constraints affect spot prices. Then, considering scenarios involving changes in system operational-security boundaries, such as the addition of interfaces and line maintenance, it analyzes the mapping relationship between the regional spot-clearing model and the full-constraint shadow-price representation model, and proposes an extended full-constraint shadow-price representation model. Further, based on the full-constraint shadow-price representation model, a generalized method for analyzing spot-price characteristics is proposed, which can rapidly and accurately locate the causes of price formation, quantitatively analyze price-influencing factors, and provide guidance for the optimized design of market mechanisms. Finally, the correctness of the proposed full-constraint shadow-price representation model is verified through a regional spot-market simulation example, and suggestions are put forward for the future construction of electricity spot markets.

2 Standard Clearing Model for Regional Electricity Spot Markets

Establish the clearing model for the Southern regional electricity spot market, clarify the correspondence between the regional spot-market clearing model and the standard linear programming model, and thereby lay the foundation for constructing and extending the full-constraint shadow-price model for the regional spot market.

The clearing model for the Southern regional electricity spot market is shown in Eqs. (1)-(12):

$$\min \sum_{t=1}^{T}\sum_{i=1}^{N} C_i(P_{i,t}) + \sum_{t=1}^{T}\sum_{k=1}^{N_k} C_k P_{k,t} + \sum_{t=1}^{T}\sum_{s=1}^{N_s} M\left(SL_{s,t}^{+} + SL_{s,t}^{-}\right) \tag{1}$$

s.t.

$$\sum_{i=1}^{N_a} P_{i,t}^{a} + \sum_{m=1}^{N_{m,a}} T_{m,t}^{a} = D_t^{a} : (\lambda_t^{a}),\ t \in \{1, \dots, T\},\ a \in A \tag{2}$$

$$\sum_{i=1}^{N_a}\left(P_i^{a,\max}-P_{i,t}^{a}\right)\geq R_t^{a,U}:(\lambda_t^{a,U}),\ t\in\{1,\dots,T\},\ a\in A \tag{3}$$

$$\sum_{i=1}^{N_a}\left(P_{i,t}^{a}-P_i^{a,\min}\right)\geq R_t^{a,D}:(\lambda_t^{a,D}),\ t\in\{1,\dots,T\},\ a\in A \tag{4}$$

$$P_i^{\min}\leq P_{i,t}\leq P_i^{\max}:(\lambda_{i,t}^{\min},\lambda_{i,t}^{\max}),\ i\in\{1,\dots,N\},\ t\in\{1,\dots,T\} \tag{5}$$

$$\begin{cases}P_{i,t}-P_{i,t-1}\leq P_i^U:(\lambda_{i,t}^U)\\ P_{i,t-1}-P_{i,t}\leq P_i^D:(\lambda_{i,t}^D)\end{cases},\ i=\{1,\dots,N\},\ t=\{1,\dots,T\} \tag{6}$$

$$P_s^{\min}\leq\sum_{i=1}^{N}G_{s-i}P_{i,t}+\sum_{m=1}^{N_m}G_{s-m}T_{m,t}-\sum_{d=1}^{N_d}G_{s-d}D_{d,t}-SL_{s,t}^{+}+SL_{s,t}^{-}\leq P_s^{\max}:\\ (\lambda_{s,t}^{\min},\lambda_{s,t}^{\max}),\ t=\{1,\dots,T\},\ s=\{1,\dots,N_s\} \tag{7}$$

$$\sum_{k=1}^{N_{k,a}}P_{k,t}=\sum_{m=1}^{N_{m,a}}T_{m,t}:(\lambda_{k,t}^{a}),\ t=\{1,\dots,T\},\ a\in A \tag{8}$$

$$\sum_{t=1}^{T}P_{k,t}\geq P_k^{\min}:(\lambda_k^{\min}),\ k=\{1,\dots,N_k\} \tag{9}$$

$$P_{k,t}\geq 0:(\lambda_{k,t}^{\min}),\ k=\{1,\dots,N_k\} \tag{10}$$

$$T_m^{\min}\leq T_{m,t}\leq T_m^{\max}:(\lambda_{m,t}^{\min},\lambda_{m,t}^{\max}),\ m=\{1,\dots,N_m\},\ t=\{1,\dots,T\} \tag{11}$$

$$\begin{cases}SL_{s,t}^{+}\geq 0:(\lambda_{s,t}^{SL,+})\\ SL_{s,t}^{-}\geq 0:(\lambda_{s,t}^{SL,-})\end{cases},\ s=\{1,\dots,N_s\},\ t=\{1,\dots,T\} \tag{12}$$

where Eq. (1) is the objective function, minimizing the sum of the system’ s total generation cost, interprovincial transmission charges, and penalty costs for violations of interface constraints. T, N, N_k, and N_s denote, respectively, the number of optimization periods, the total number of generating units in the network, the number of interprovincial transaction components, and the number of interfaces [34]. C_i and C_k denote, respectively, the bidding function of generating unit i and the interprovincial transmission price of transaction component k. $P_{i,t}$ and $P_{k,t}$ denote, respectively, the output of the generating unit and the power of the transaction component. M, $SL^{+}_{s,t}$, and $SL^{-}_{s,t}$ denote, respectively, the penalty factor for violation of the interface power-flow constraint, the positive slack variable, and the negative slack variable. Equation (2) is the provincial load-balance···

balance constraint. A, N_a, and $N_{m.a}$ are respectively the set of provinces, the number of units in province a, and the number of DC tie lines connected to province a; $T^{a}_{m.t}$ is the transmission power of the connected tie line, D^{a}_{t} is the total load power of province a, and λ^{a}_{t} is the shadow price of the load-balance constraint of province a. Equation (3) is the provincial upward reserve constraint; $P^{a,\max}_{i}$ and $R^{a,U}_{t}$ are respectively the maximum output of unit i in province a and the upward reserve requirement of province a, and $\lambda^{a,U}_{t}$ is the shadow price of the upward reserve constraint. Equation (4) is the provincial downward reserve constraint; $P^{a,\min}_{i}$, $R^{a,D}_{t}$, and $\lambda^{a,D}_{t}$ are respectively the minimum output of unit i in province a, the downward reserve requirement of province a, and the shadow price of the downward reserve constraint. Equation (5) is the upper- and lower-limit constraint on unit output; $\lambda^{\min}_{i.t}$ and $\lambda^{\max}_{i.t}$ are respectively the shadow prices of the lower- and upper-limit constraints on unit output. Equation (6) is the unit ramping constraint; P^{U}_{i} and P^{D}_{i} are respectively the upward- and downward-ramping limits of the unit, and $\lambda^{U}_{i.t}$ and $\lambda^{D}_{i.t}$ are respectively the shadow prices of the upward- and downward-ramping constraints. Equation (7) is the interface power-flow constraint; $P^{\min}_{s}$ and $P^{\max}_{s}$ are respectively the lower and upper limits of the power flow on interface s, N is the number of load nodes in the entire network, and G_{s-i}, G_{s-m}, and G_{s-d} are respectively the sensitivities of unit i, interprovincial tie line m, and load d to the interface. $\lambda^{\min}_{s.t}$ and $\lambda^{\max}_{s.t}$ are respectively the shadow prices of the lower- and upper-limit constraints on the power flow of interface s. Equation (8) is the coupling constraint between transaction components and tie-line power; $N_{k.a}$ is the number of transaction components associated with province a, and $\lambda^{a}_{k.t}$ is the shadow price of the coupling constraint. Equation (9) is the interprovincial priority-scheduled electricity delivery constraint; $P^{\min}_{k}$ and $\lambda^{\min}_{k}$ are respectively the lower limit on delivered electricity and the shadow price of the constraint. Equation (10) is the nonnegativity constraint on the power of interprovincial transaction components; $\lambda^{\min}_{k.t}$ is the shadow price of the nonnegativity constraint on the power of transaction component k in period t. Equation (11) is the upper- and lower-limit constraint on the power of interprovincial DC tie lines; $T^{\min}_{m}$ and $T^{\max}_{m}$ are respectively the lower and upper limits of the tie-line transmission power, and $\lambda^{\min}_{m.t}$ and $\lambda^{\max}_{m.t}$ are respectively the shadow prices of the lower- and upper-limit constraints on transmission power. Equation (12) is the nonnegativity constraint on the slack variables of the interface power-flow constraint; $\lambda^{SL,+}_{s.t}$ and $\lambda^{SL,-}_{s.t}$ are respectively the shadow prices of the nonnegativity constraints on the positive and negative slack variables of the power flow of interface s.

In addition, the clearing model of the southern regional electricity spot market also includes constraints such as the system primary frequency-regulation capacity requirement and DC ramping capability limits. The complete model may be found in Reference [34].

The standard linear programming model is shown in Equations (13)-(14):

$$\min Cx \tag{13}$$

$$\text{s.t.}\ \ Ax \geq b : (\lambda) \tag{14}$$

where x is the decision-variable matrix, C is the value-coefficient matrix of the objective function, A and b are respectively the coefficient matrix of the constraints and the constant matrix of resource limits, and λ is the shadow-price matrix corresponding to the constraints.

To establish the correspondence between the standard optimization model and the regional spot-market clearing model, the standard optimization model is expressed as a multi-period optimization model:

$$\min \sum_{t=1}^{T} C_t x_t \tag{15}$$

s.t.

$$\sum_{t=1}^{T} A_t^{\text{mult}} x_t \geq b_t^{\text{mult}} : \left(\lambda^{\text{mult}}\right) \tag{16}$$

$$A_t^{\text{sing}} x_t \geq b_t^{\text{sing}} : \left(\lambda_t^{\text{sing}}\right),\ t = \{1, \cdots, T\} \tag{17}$$

where Equation (15) is the objective function, and C_t and x_t are respectively the value-coefficient row vector and the decision-variable column vector in period t. Equation (16) represents cross-period constraints; A_t^{mult}, b_t^{mult}, and λ^{mult} respectively denote the coefficient matrix, resource-limit constant matrix, and shadow-price matrix of the cross-period constraints, corresponding to such constraints in the regional spot-market clearing model as the interprovincial priority-scheduled electricity constraint and unit ramping constraints. Equation (17) represents single-period constraints; A_t^{sing}, b_t^{sing}, and λ_t^{sing} are respectively the coefficient matrix, resource-limit constant matrix, and shadow-price matrix of the single-period constraints, corresponding to other constraints in the regional spot-market clearing model except the cross-period constraints.

The value-coefficient matrix and decision-variable matrix in Equation (15) can be further decomposed as follows:

$$C_t = [C_{g,t}^{\mathrm{T}}, C_{k,t}^{\mathrm{T}}, M_t^{\mathrm{T}}] \tag{18}$$

$$x_t = [P_{g,t}^{\mathrm{T}}, P_{k,t}^{\mathrm{T}}, SL_t^{\mathrm{T}}]^{\mathrm{T}} \tag{19}$$

where $P_{g,t}$, $P_{k,t}$, and SL_t are, respectively, the column vectors of unit output variables, transaction-component power variables, and constraint-slack variables; $C_{g,t}$, $C_{k,t}$, and M_t are, respectively, the column vectors of value coefficients corresponding to unit output, transaction-component transmitted power, and constraint slack.

Thus far, the correspondence between the variables and parameters of the standard optimization model and those of the regional spot market clearing model has been established. Any conclusion derived from the standard optimization model is therefore applicable to the regional spot market clearing model.

3 Full-Constraint Shadow-Price Representation Model

First, a shadow-price representation model for the parameters of the standard optimization model is constructed, yielding a general conclusion that the parameters of the optimization model can be represented by constraint shadow prices. Further, a full-constraint shadow-price representation model based on the parameters of the regional electricity spot market is constructed.

3.1 Full-Constraint Shadow-Price Representation Model for Standard Linear Programming Parameters

Based on the standard linear programming model in Eqs. (13)-(14), its Lagrangian function is written as shown in Eq. (20):

$$L(x, \lambda) = Cx - \lambda(Ax - b) \tag{20}$$

Rewriting Eq. (20) in scalar form gives Eq. (21):

$$L(x, \lambda) = \sum_m C_m x_m - \sum_n \lambda_n \left(\sum_m A_{n,m} x_m - b_n \right) \tag{21}$$

In Eq. (21), the subscripts m and n are used to define the decision variables and constraints, respectively. According to the KKT (Karush–Kuhn–Tucker) conditions, setting the partial derivative of the Lagrangian function with respect to the independent variable x_m to zero yields the necessary condition for the optimal solution:

$$\frac{\partial L}{\partial x_m} = C_m - \sum_n \lambda_n A_{n,m} = 0 \tag{22}$$

It can be seen from Eq. (22) that the value coefficient C_m can be expressed as a linear combination of the shadow prices λ_n of all constraints, as shown in Eq. (23):

$$C_m = \sum_n \lambda_n A_{n,m} \tag{23}$$

The constraint coefficient $A_{n_0,m}$ can then be expressed as a multivariate fractional function of the shadow prices λ_n of all constraints, as shown in Eq. (24):

$$A_{n_0,m} = \frac{C_m - \sum_{n \neq n_0} \lambda_n A_{n,m}}{\lambda_{n_0}} \tag{24}$$

The standard optimization model in Eqs. (13)-(14) covers all market clearing models based on linear programming. Therefore, the following conclusion can be drawn: for any linear-programming market clearing model, when the optimal solution is obtained, its value coefficients can be represented as linear combinations of full-constraint shadow prices, and all constraint coefficients can be represented as multivariate fractional functions of full-constraint shadow prices.

3.2 Full-Constraint Shadow-Price Representation Model for Regional Spot Market Parameters

According to Eq. (18), the value coefficients of the objective function of the regional spot market clearing model include the generators' segment-wise bid···

prices, transmission prices for interprovincial transaction components, and penalty factors for constraint violations. Based on the derivation in Section 2.1, the value coefficients of the regional spot-market clearing model can likewise be expressed as a linear combination of shadow prices of all constraints.

The Lagrangian function of the regional electricity spot-market clearing model is written as shown in Eq. (25):

$$
\begin{aligned}
&L(P_g, P_k, SL, \lambda) \\
=&\sum_{t=1}^{T}\sum_{i=1}^{N} C_i(P_{i,t}) + \sum_{t=1}^{T}\sum_{k=1}^{N_k} C_k P_{k,t} + \sum_{t=1}^{T}\sum_{s=1}^{N_s} M(SL_{s,t}^{+} + SL_{s,t}^{-}) \\
&- \sum_{t=1}^{T}\sum_{a\in A} \lambda_t^a \left(\sum_{i=1}^{N_a} P_{i,t}^a + \sum_{m=1}^{N_m} T_{m,t}^a - D_t^a\right) - \sum_{t=1}^{T}\sum_{a\in A} \lambda_t^{a,U}\left[\sum_{i=1}^{N_a}(P_i^{a,\max} - P_{i,t}^a) - R_t^{a,U}\right] \\
&- \sum_{t=1}^{T}\sum_{a\in A} \lambda_t^{a,D}\left[\sum_{i=1}^{N_a}(P_{i,t}^a - P_i^{a,\min}) - R_t^{a,D}\right] - \sum_{t=1}^{T}\sum_{i=1}^{N} \lambda_{i,t}^{\min}(P_{i,t} - P_i^{\min}) \\
&- \sum_{t=1}^{T}\sum_{i=1}^{N} \lambda_{i,t}^{\max}(P_i^{\max} - P_{i,t}) - \sum_{t=1}^{T}\sum_{i=1}^{N} \lambda_{i,t}^{U}(-P_{i,t} + P_{i,t-1} + P_i^U) \\
&- \sum_{t=1}^{T}\sum_{i=1}^{N} \lambda_{i,t}^{D}(P_{i,t} - P_{i,t-1} + P_i^D) - \sum_{t=1}^{T}\sum_{s=1}^{N_s} \lambda_{s,t}^{\min}(P_{s,t} - P_s^{\min} + SL_{s,t}^{-} - SL_{s,t}^{+}) \\
&- \sum_{t=1}^{T}\sum_{s=1}^{N_s} \lambda_{s,t}^{\max}(-P_{s,t} + P_s^{\max} + SL_{s,t}^{+} - SL_{s,t}^{-}) \\
&- \sum_{t=1}^{T}\sum_{a\in A} \lambda_{k,t}\left(\sum_{k=1}^{NKa} P_{k,t}^a - \sum_{m=1}^{NMa} T_{m,t}^a\right) - \sum_{k=1}^{N_k} \lambda_k^{\min}\left(\sum_{t=1}^{T} P_{k,t} - P_k^{\min}\right) \\
&- \sum_{t=1}^{T}\sum_{m=1}^{N_m} \lambda_{m,t}^{\min}(T_{m,t} - T_m^{\min}) - \sum_{t=1}^{T}\sum_{m=1}^{N_m} \lambda_{m,t}^{\max}(-T_{m,t} + T_m^{\max}) \\
&- \sum_{t=1}^{T}\sum_{s=1}^{N_s} \left(\lambda_{s,t}^{SL,+} SL_{s,t}^{+} + \lambda_{s,t}^{SL,-} SL_{s,t}^{-}\right)
\end{aligned} \tag{25}
$$

where P_g, P_k, and SL are, respectively, matrices composed of the outputs of all generating units in all periods, the powers of all transaction components, and the slack variables of all interface-flow constraints. $P_{s,t}$ is the transmission power of interface s in period t, and can be expressed as Eq. (26):

$$
P_{s,t} = \sum_{i=1}^{N} G_{s-i} P_{i,t} + \sum_{m=1}^{N_m} G_{s-m} T_{m,t} - \sum_{d=1}^{N_d} G_{s-d} D_{d,t} \tag{26}
$$

As in the construction method for the full-constraint shadow-price representation model of the standard optimization model, according to the KKT conditions, setting the partial derivatives of the Lagrangian function with respect to the decision variables equal to zero gives the necessary conditions for the optimal solution:

Taking the partial derivative with respect to unit output $P_{i,t}$ gives Eq. (27):

$$\frac{\partial C_i(P_{i,t})}{\partial P_{i,t}} = \lambda_t^a - \lambda_t^{a,U} + \lambda_t^{a,D} + \lambda_{i,t}^{\min} - \lambda_{i,t}^{\max} - \lambda_{i,t}^U + \lambda_{i,t}^D + \lambda_{i,t+1}^U - \lambda_{i,t+1}^D - \sum_{s=1}^{N_s} G_{s-i}(\lambda_{s,t}^{\max} - \lambda_{s,t}^{\min}) \tag{27}$$

where $\partial C_i(P_{i,t})/\partial P_{i,t}$ denotes the marginal offer of unit i in period t. It can be seen from Eq. (27) that the marginal offer of unit i can be expressed as a linear combination of the shadow prices of the load-balance constraint of its province, the positive- and negative-reserve constraints of its province, the upper and lower limits on unit output, the upward- and downward-ramping constraints of the unit in adjacent periods, and the upper and lower limits on interface flow.

Taking the partial derivative with respect to the interprovincial transaction-component power $P_{k,t}$ gives Eq. (28):

$$C_k = \lambda_{k,t}^s + \lambda_{k,t}^r + \lambda_k^{\min} \tag{28}$$

where $\lambda_{k,t}^s$ and $\lambda_{k,t}^r$ are, respectively, the shadow prices of the coupling constraints between the transaction component in the sending-end province and the tie-line power, and between the transaction component in the receiving-end province and the tie-line power. Since the coupling constraint between the transaction component and the tie-line power flow does not correspond to an actual boundary for secure system operation, its physical meaning is not clear. Therefore, the shadow price of this constraint can be eliminated through other optimality conditions. Taking the partial derivative of the Lagrangian function with respect to the interprovincial DC tie-line power $T_{m,t}$ associated with transaction component k yields Eq. (29):

$$\lambda_t^s - \lambda_t^r + \sum_{s=1}^{N_s} G_{s-m}\left(\lambda_{s,t}^{\max} - \lambda_{s,t}^{\min}\right) + \lambda_{k,t}^s + \lambda_{k,t}^r - \lambda_{m,t}^{\min} + \lambda_{m,t}^{\max} = 0 \tag{29}$$

Substituting Eq. (29) into Eq. (28) eliminates the shadow prices of the coupling constraints between the transaction component and the tie-line power, giving Eq. (30):

$$C_k = \lambda_t^r - \lambda_t^s + \sum_{s=1}^{N_s} G_{s-m}\left(\lambda_{s,t}^{\min} - \lambda_{s,t}^{\max}\right) + \lambda_{m,t}^{\min} - \lambda_{m,t}^{\max} + \lambda_k^{\min} \tag{30}$$

It can be seen from Eq. (30) that the transmission price C_k of interprovincial transaction component k can be represented as a linear combination of the shadow prices of the load-balance constraints of the receiving-end and sending-end provinces, the upper and lower section-flow constraints, the upper and lower interprovincial tie-line power constraints, and the lower bound constraint on the delivered energy of the transaction component. It should be noted that if the powers $T_{m,t}$ of multiple interprovincial tie lines ($m \in M$) are associated with transaction component k, then the Lagrangian function in Eq. (25) is differentiated with respect to the power of any tie line in the set M, and Eqs. (29) and (30) both hold.

Taking the derivative with respect to the slack variable $SL^{+}_{s,t}$ of the upper section-flow constraint yields Eq. (31):

$$M = \lambda^{\max}_{s,t} - \lambda^{\min}_{s,t} + \lambda^{SL,+}_{s,t} \tag{31}$$

It can be seen from Eq. (31) that the penalty factor for relaxing the section-flow constraint can be represented as a linear combination of the shadow prices of the upper and lower section-flow constraints and the nonnegativity constraint of the slack variable. Similarly, taking the derivative with respect to the slack variable $SL^{-}_{s,t}$ of the lower section-flow constraint yields Eq. (32):

$$M = \lambda^{\min}_{s,t} - \lambda^{\max}_{s,t} + \lambda^{SL,-}_{s,t} \tag{32}$$

From the above analysis, all value coefficients in the clearing objective function of the regional electricity spot market can be represented by linear combinations of the shadow prices of all constraints.

In summary, the theoretical innovation of the proposed all-constraint shadow-price representation model can be further refined as follows: existing studies mainly differentiate the load in the market-clearing model to derive the relationship between clearing prices and price components such as energy and congestion; in contrast, the proposed all-constraint shadow-price representation model further differentiates all decision variables, deriving equality relationships between market parameters and the shadow prices of all constraints, and also reflecting the exact relationships among the shadow prices of all constraints. Since market parameters are already determined before the model is solved, after the optimization model is established, the linear superposition sum of all constraint shadow prices is also determined. This conclusion helps rapidly identify the causes of price formation, carry out audits of the clearing model, and analyze the impact of changes in market parameters on clearing results.

4 Extended All-Constraint Shadow-Price Representation Model

From Eqs. (27)-(32), it can be seen that, in the linear relationship between regional spot-market parameters and the shadow prices of all constraints, the coefficients of the shadow prices are constants. However, when the system security boundary changes, the constraints of the clearing model may increase or decrease, the objective function may change, and the coefficients of the shadow prices may also change accordingly. To improve the applicability of the proposed all-constraint shadow-price representation model, this chapter constructs a mapping relationship between the regional spot-market clearing model and the all-constraint shadow-price representation model when the system security boundary changes, such as in the maintenance mode, under section-control requirements, and when market entities enter or exit the market, so as to realize synchronous updating of the clearing model and the shadow-price model. The cases are divided into "constraint addition"

for the three scenarios of "constraint reduction" and "objective-function change," and proposes extended forms of the all-constraint shadow-price representation model.

4.1 Constraint Increase

The model adds an arbitrary constraint, as shown in Eq. (33):

$$\sum_{t=1}^{T}\sum_{i=1}^{N} a_{i,t}P_{i,t} + \sum_{t=1}^{T}\sum_{k=1}^{N_k} b_{k,t}P_{k,t} + \sum_{t=1}^{T}\sum_{s=1}^{N_s}\left(c_{s,t}^{+}SL_{s,t}^{+} + c_{s,t}^{-}SL_{s,t}^{-}\right) \geq C^{\text{add}} : \left(\lambda^{\text{add}}\right) \tag{33}$$

where $a_{i,t}$ is the constraint coefficient associated with time period t and unit i; $b_{k,t}$ is the constraint coefficient associated with time period t and transaction component k; $c_{s,t}^{+}$ and $c_{s,t}^{-}$ are respectively the constraint coefficients associated with the upper-limit slack variable and lower-limit slack variable of interface s in time period t; and C^{add} and λ^{add} are respectively the right-hand-side constant and the shadow price of the newly added constraint.

Add the Lagrangian term corresponding to Eq. (33) to Eq. (25), and then, according to the KKT conditions, take derivatives with respect to each decision variable in turn to obtain the extended all-constraint shadow-price representation model. Specifically:

Taking the derivative with respect to unit output $P_{i,t}$, Eq. (27) is extended as:

$$\begin{aligned}\frac{\partial C_i(P_{i,t})}{\partial P_{i,t}} = {} & \lambda_t^{a} - \lambda_t^{a,U} + \lambda_t^{a,D} + \lambda_{i,t}^{\min} - \lambda_{i,t}^{\max} - \lambda_{i,t}^{U} + \lambda_{i,t}^{D} + \lambda_{i,t+1}^{U} - \lambda_{i,t+1}^{D} \\ & - \sum_{s=1}^{N_s} G_{s-i}\left(\lambda_{s,t}^{\max} - \lambda_{s,t}^{\min}\right) + a_{i,t}\lambda^{\text{add}}\end{aligned} \tag{34}$$

Taking the derivative with respect to the interprovincial transaction component power $P_{k,t}$, and substituting it into Eq. (29), Eq. (30) is extended as:

$$C_k = \lambda_t^r - \lambda_t^s + \sum_{s=1}^{N_s} G_{s-m}\left(\lambda_{s,t}^{\min} - \lambda_{s,t}^{\max}\right) + \lambda_{m,t}^{\min} - \lambda_{m,t}^{\max} + \lambda_k^{\min} + b_{k,t}\lambda^{\text{add}} \quad (35)$$

Taking the derivatives with respect to the slack variables $SL_{s,t}^{+}$ and $SL_{s,t}^{-}$ of the upper- and lower-limit constraints of the interface flow, Eqs. (31) and (32) can be extended respectively as Eqs. (36) and (37):

$$M = \lambda_{s,t}^{\max} - \lambda_{s,t}^{\min} + \lambda_{s,t}^{SL,+} + c_{s,t}^{+}\lambda^{\text{add}} \quad (36)$$

$$M = \lambda_{s,t}^{\min} - \lambda_{s,t}^{\max} + \lambda_{s,t}^{SL,-} + c_{s,t}^{-}\lambda^{\text{add}} \quad (37)$$

The mapping relationship between constraint increase in the clearing model and the all-constraint shadow-price representation model is explained through a typical scenario: in actual power-system operation, adding a security interface is a typical scenario of constraint increase. If this security interface is related to unit i but unrelated to the interprovincial transaction component, then in the newly added constraint Eq. (33), the constraint coefficients $b_{k,t}$, $c_{s,t}^{+}$, and $c_{s,t}^{-}$ are all 0; $a_{i,t}$ represents the sensitivity of the unit to the newly added interface; C^{add} represents the flow limit of the newly added interface; and λ^{add} represents the shadow price of the newly added interface flow constraint.

4.2 Constraint Reduction

The mapping relationship between constraint reduction in the clearing model and the all-constraint shadow-price representation model is explained through two typical scenarios.

Unit maintenance: If unit i is under maintenance and exits the regional spot-market optimization, then the variables related to unit i are removed from the constraints of the clearing model; in the all-constraint shadow-price representation model, the KKT condition Eq. (27) related to unit i is also removed.

Line maintenance: If a line related to interface s is under maintenance, then the flow constraint for interface s is removed from the clearing model; in the all-constraint shadow-price representation model, the terms related to interface s in Eqs. (27) and (30), namely $G_{s-i}(\lambda_{s,t}^{\max} - \lambda_{s,t}^{\min})$ and $G_{s-m}(\lambda_{s,t}^{\min} - \lambda_{s,t}^{\max})$, are removed; and Eqs. (31) and (32) related to interface s are removed.

At the same time, maintenance on any transmission line will cause changes in the network connectivity relationships and impedance characteristics, thereby leading to changes in the sensitivities of the remaining sections in Eqs. (27) and (30) to the sending- and receiving-end nodes of generating units and interprovincial tie lines.

4.3 Changes in the objective function

The mapping relationship between changes in the objective function and the full-constraint shadow-price representation model is explained through two typical scenarios: market entry or exit of generating units, and the addition of slack-penalty costs.

Market entry or exit of generating units. If a non-market generating unit i participates in the regional spot-market optimization, then in the clearing model, the generation cost corresponding to the output of unit i in each period is added to the objective function in Eq. (1). In the full-constraint shadow-price representation model, the equality relationship obtained by taking the derivative of the newly added Lagrangian function Eq. (25) with respect to the output variable $P_{i,t}$ of unit i in each period is as shown in Eq. (27). When unit i exits the market, the generation cost of unit i in each period is removed from the objective function of the clearing model; in the full-constraint shadow-price representation model, the equality relationship obtained by taking the derivative of the Lagrangian function with respect to $P_{i,t}$ is also removed accordingly.

Addition of slack-penalty cost. Suppose that a slack variable SL_i is added to any constraint. A slack-penalty cost $M_i SL_i$ is added to the objective function, and the constraint related to the slack variable is as shown in Eq. (38):

$$\sum_{t=1}^{T}\sum_{i=1}^{N} a_{i,t}P_{i,t} + \sum_{t=1}^{T}\sum_{k=1}^{N_k} b_{k,t}P_{k,t} + SL_j \geq C_j^{\text{add}} : (\lambda_j^{\text{add}}) \tag{38}$$

where C_i^{add} and λ_i^{add} are respectively the resource-limitation coefficient and shadow price of the constraint related to the slack variable. In the full-constraint shadow-price representation model, taking the derivative of the Lagrangian function Eq. (24) with respect to the slack variable SL_i yields Eq. (39):

$$M_j - \lambda_j^{\text{add}} - \lambda_j^{+} = 0 \tag{39}$$

where λ_i^{+} is the shadow price of the nonnegativity constraint on the slack variable SL_i.

By adding Eq. (39) to the full-constraint shadow-price representation model, synchronous updating of the clearing model and the shadow-price model under the scenario of newly added penalty costs is achieved.

5 Generalized Method for Analyzing Price Characteristics in Regional Spot Markets

Based on the full-constraint shadow-price representation model constructed in Chapter 3, a generalized analysis method is proposed for the formation mechanism of price characteristics in regional power spot markets, and the characteristics of typical regional spot clearing prices are analyzed.

5.1 Generalized analysis method

The proposed generalized analysis method for price characteristics in regional spot markets is based on the full-constraint shadow-price representation model of the clearing model obtained through duality theory. According to the shadow-price information in the market clearing results, it enables rapid identification of the causes of price formation, as shown in Fig. 1:

Figure diagram text

- **Regional spot-market clearing model**
  - Objective function: minimization of total system cost
  - Boundary conditions:
    - Generating-unit bids
    - Operating characteristics of units and the network
    - Interprovincial priority schedules
- Market clearing →
- **Clearing results**
  - Cleared generation of units
  - Nodal electricity prices
  - Constraint shadow prices
- Constraint update
- Dual transformation; change in security boundaries
- **Full-constraint shadow-price representation model**
  - Unit-bid parameters
  - Interprovincial transmission-price parameters
  - Constraint-violation penalty-factor parameters
  - Equality relationships
- Mapping expansion
- Equality-term rearrangement →
- Substitute ↓
- **Identification of causes of price formation**
  - Nonzero shadow prices
  - Binding constraints

Fig. 1 Generalized analytical method for the price-formation mechanism in regional spot markets

In Fig. 1, the methodology for analyzing price characteristics in regional spot markets can be summarized as follows:

Taking minimization of total system cost as the objective, and taking generating-unit bids, the operating characteristics of units and the network, and interprovincial priority schedules as the boundaries, a regional electricity spot-market clearing model is constructed;

Based on duality theory, a full-constraint shadow-price representation model is constructed through the Lagrangian function, establishing equality relationships for three types of parameters: generating-unit bids, interprovincial transmission prices, and constraint-violation penalty factors;

If the system security boundaries change, the constraints of the regional spot-market clearing model are updated according to the latest boundary information, and an extended full-constraint shadow-price representation model is obtained based on the mapping relationships constructed in Chapter 3;

Regional spot-market clearing is performed to obtain the cleared generation of units, nodal electricity prices, and the shadow prices of each constraint. These are substituted into the rearranged equalities in the full-constraint shadow-price representation model to locate nonzero shadow prices, thereby identifying the binding constraints that affect price characteristics as well as the coupling relationships among the binding constraints.

5.2 Typical Cases of Price-Characteristic Analysis

Based on the price-characteristic analysis methodology proposed in Section 4.1, typical price characteristics of the regional spot market are analyzed from three perspectives: the clearing quantity-price relationship of generating units, the price-difference relationship between sending-end and receiving-end provinces, and the shadow price of interface-flow constraint violations.

(1) Clearing quantity-price relationship of generating units

The bid of generating unit i and the nodal electricity price at the node where it is located are often not equal. According to the definition of nodal electricity price, taking the partial derivative of the Lagrangian function in Eq. (25) with respect to load $D_{d,t}$ yields Eq. (40):

$$\lambda_{d,t}^{\mathrm{LMP}} = \lambda_t^a + \sum_{s=1}^{N_s} G_{s-d} \left(\lambda_{s,t}^{\min} - \lambda_{s,t}^{\max} \right) \tag{40}$$

where $\lambda_{d,t}^{\mathrm{LMP}}$ is the nodal electricity price of load d in period t, expressed as a linear combination of the energy price and congestion price of load d. If generating unit i and load d are located at the same node, then taking the partial derivative of Eq. (25) with respect to the unit output can likewise obtain the nodal electricity price [21].

Therefore, rearranging Eq. (27) gives Eq. (41):

$$\lambda_{i,t}^{\mathrm{LMP}} = \lambda_t^a + \sum_{s=1}^{N_s} G_{s-i}\left(\lambda_{s,t}^{\min} - \lambda_{s,t}^{\max}\right)$$
$$= \frac{\partial C_i(P_{i,t})}{\partial P_{i,t}} + \lambda_t^{a,U} - \lambda_t^{a,D} - \lambda_{i,t}^{\min} + \lambda_{i,t}^{\max} + \lambda_{i,t}^{U} - \lambda_{i,t}^{D} - \lambda_{i,t+1}^{U} + \lambda_{i,t+1}^{D} \tag{41}$$

where $\lambda_{i,t}^{\mathrm{LMP}}$ is the locational marginal price at the node where unit i is located in time period t.

Equation (41) intuitively reveals why a unit' s clearing price deviates from its offer. The discussion is as follows:

If, in time period t, the system upward reserve constraint, the unit upper output limit constraint, the unit ramp-up constraint, and the unit ramp-down constraint in time period $t+1$ are binding, the resulting shadow prices will cause the node price of the unit to be higher than the unit' s offer. Specifically, the unit cannot continue increasing its output, so any additional load demand at the node where it is located must be balanced by the output of other, higher-priced units, producing the phenomenon of "a high clearing price but no dispatch."

If, in time period t, the system downward reserve constraint, the unit lower output limit constraint, the unit ramp-down constraint, and the unit ramp-up constraint in time period $t+1$ are binding, the resulting shadow prices will cause the node price of the unit to be lower than the unit' s offer. Specifically, the unit cannot continue reducing its output, so any additional load demand at the node where it is located must be balanced by the output of other, lower-priced units, producing the phenomenon of "a low clearing price but being dispatched."

With the proposed price analysis method, the reasons why a unit's clearing price deviates from its offer can be quickly located, without repeatedly performing clearing back-calculations or manually checking whether constraints are binding, thereby significantly improving the efficiency of price analysis.

(2) Relationship between the sending-end and receiving-end provincial price spread

The difference between the energy prices of the sending-end and receiving-end provinces is not always equal to the interprovincial transmission price. By rearranging Equation (30), we obtain:

$$\lambda_t^r - \lambda_t^s = C_k - \sum_{s=1}^{N_s} G_{s-m}\left(\lambda_{s,t}^{\min} - \lambda_{s,t}^{\max}\right) - \lambda_{m,t}^{\min} + \lambda_{m,t}^{\max} - \lambda_k^{\min} \tag{42}$$

It can be seen from Equation (42) that the difference between the energy-price spread of the sending-end and receiving-end provinces and the interprovincial transmission price is determined by the shadow prices of constraints related to interfaces, DC operation, and interprovincial priority schedules. The discussion is as follows:

Regarding interface constraints, the congestion price of interface s affects the energy-price spread between the sending-end and receiving-end provinces. The energy price reflects the locational marginal price at the balancing node; therefore, the concrete manifestation of changes in the sending–receiving provincial price spread caused by interface congestion is as follows: the congestion price is converted, through sensitivity factors, to the sending-end or receiving-end node of the interprovincial tie line, and is then further transmitted to the receiving-end or sending-end province. Thus, the energy-price spread between the sending-end and receiving-end provinces is essentially formed by the superposition of the differences between the locational marginal prices of the balancing node and the terminal nodes of the interprovincial tie line.

Regarding DC operation constraints, if the DC transmission power reaches its upper limit and power cannot continue to be increased to the receiving-end province, the additional load in the receiving-end province can only be balanced by high-priced units within the province, which appears as a widening of the sending–receiving provincial price spread. If the DC transmission power reaches its lower limit, then even if the offers of units in the sending-end province, after adding the interprovincial transmission fee, have no price advantage over units in the receiving-end province, transmission to the receiving-end province still cannot continue to be reduced; the additional load within the receiving-end province is balanced by low-priced units within the province, which appears as a narrowing of the sending–receiving provincial price spread.

Regarding interprovincial priority-schedule constraints, the fact that the energy constraint of an interprovincial priority schedule reaches its lower limit means that even if units in the sending-end province have no price advantage, power must still be increased to the receiving-end province; specifically, this appears as a narrowing of the sending–receiving provincial price spread.

(3) Shadow prices of interface power-flow constraints

In the operation of the regional spot market, if a section power-flow constraint is violated, its shadow price will be equal to the violation penalty factor M. From the analysis of Eq. (12), when a section power-flow constraint is violated, $SL^{+}_{s.t}$ or $SL^{-}_{s.t} > 0$, and the shadow price of the nonnegativity constraint on the slack variable, $\lambda^{SL,+}_{s.t}$ or $\lambda^{SL,-}_{s.t} = 0$. According to Eqs. (31) and (32), at this time $\lambda^{\max}_{s.t}$ or $\lambda^{\min}_{s.t} = M$.

5.3 Practical Value of the Method for Analyzing Price Characteristics

In summary, the proposed generalized method for analyzing the price characteristics of regional spot markets has the following practical value:

1) It can use the shadow-price information of each constraint to rapidly and accurately characterize the mechanism by which spot prices are formed, thereby significantly improving the efficiency of price analysis. According to the mapping relationship of the primal-dual model, a synchronous correction and updating model can be established in which, when the spot-market clearing model changes, the shadow-price representation model for all constraints is updated accordingly. Thus, when market boundaries change—for example, during unit commissioning or line maintenance—the expressions for spot-market parameters can be updated synchronously.

2) It can provide theoretical support for the optimized design of market mechanisms. From the perspective of optimization, the physical meaning of shadow prices is the total system cost saved by relaxing a unit constraint limit, reflecting the available resources for secure operation of the power system, such as the scarcity of reserve capacity and section transmission capacity, as well as the degree of influence of interprovincial transmission plans on the spot market. The price signals contained therein can provide a basis for designing market compensation and settlement mechanisms, and can also guide the planning and expansion of power sources and power grids.

3) It can significantly improve the efficiency of correctness verification for power spot market clearing models. Verification of power spot market clearing models is an important step in ensuring the fairness and transparency of spot markets. Existing verification methods mainly construct similar clearing models and combine them with simulated operating data to assess the similarity between simulated clearing results and actual clearing results; their analytical efficiency is relatively low, and accuracy is difficult to guarantee. In scenarios where the clearing model remains unchanged, the all-constraint shadow-price representation model does not need to repeatedly carry out spot-market clearing calculations; instead, it only needs to perform iterative calculations based on spot-market parameters and shadow-price information to verify whether the clearing model is reasonable. Both analytical efficiency and accuracy are significantly higher than those of existing methods, which helps improve the fairness and transparency of power spot market construction.

4) It can effectively analyze the impact of changes in market parameters on the clearing model, thereby facilitating market power monitoring and the optimal allocation of market parameters. On the one hand, when unit offers change, the values of each shadow price in Eq. (27) will also change accordingly. It is therefore possible to intuitively determine which constraint shadow prices are affected by changes in unit offers, and whether they further affect unit clearing prices, serving as a means for market power monitoring and evaluation. On the other hand, the market operator's configuration of parameters such as transmission prices and constraint-violation penalty factors also directly affects market clearing results. A

reduction in interprovincial transmission prices will narrow the price differences among receiving provinces; the level of interprovincial transmission prices can be determined according to each province's price-bearing capacity. The relative magnitudes of the various constraint-violation penalty factors will affect the priority of constraint relaxation, and the market operator should determine the values of various constraint-violation penalty factors according to the objectives of secure and stable operation of the power system.

6 Case Study

To demonstrate the superiority of the proposed all-constraint shadow-price representation model in rapidly identifying the causes of regional spot price formation, a simulation case of a regional power spot market comprising three provinces and six time periods is constructed for verification. The topology of the three-province system is shown in Appendix Fig. A1. Province A has relatively low load and large installed generation capacity, and surplus power can be delivered to Province B through the tie line (A5 → B6). Province B is the load center of the entire network and the hub for cross-regional power exchange; it has relatively high load, with both low-cost and high-cost units. Province C has relatively low load, but the generation cost of its in-province units is relatively high, so it receives external power through the tie line (B10 → C11). The total load demand of each province in each time period is shown in Appendix Fig. A2; the operating parameters and bidding information of generating units are shown in Appendix Table A1; and the settings for interprovincial tie lines and key sections are shown in Appendix Tables A2 and A3, respectively. Two interprovincial transaction components are set up, namely (Province A → Province B) and (Province B → Province C), with interprovincial transmission prices of 4.5 yuan/MWh and 7.5 yuan/MWh, respectively.

The clearing results of the regional spot market simulation case are shown in Fig. 2:

In-figure labels: Unit output (MW); transaction-component power (MW); period. Legend: Unit A1, Unit A2, Unit B6, Unit B8, Unit C1, Unit C8; Province A→Province B; Province B→Province C.

Figure 2. Regional spot-market clearing results

In Figure 2, in terms of cleared unit output, Unit A1 and Unit B8, which have relatively low generation costs, have relatively high cleared output, reaching their upper output limits in periods 2 and 5; Unit C8, which has the highest generation cost, has relatively low output. As can be seen from Appendix Figure 2, in periods 1, 3, 4, and 6, the system-wide load demand is relatively small, and Unit C8 operates at its lower output limit of 10 MW. In terms of the cleared power of transaction components, in periods 2 and 5 the cleared power of the transaction component Province A→Province B increases, while that of the component Province B→Province C decreases. This is because, in periods

2 and 5, the load demands of the three provinces increase by 10 MW, 100 MW, and 5 MW, respectively, compared with the preceding period; Province B has the fastest load growth and the tightest supply situation. Moreover, the interprovincial transmission tariff from Province A to Province B is lower than that from Province B to Province C. Therefore, the regional spot market minimizes total system cost by arranging increased delivery from Province A to Province B and reduced delivery from Province B to Province C.

Taking Unit B6 as an example, the shadow prices of the constraints related to the unit-output variable are listed to verify expression (27), as shown in Table 1.

Table 1. Shadow prices of constraints related to the output variable of Unit B6

Period	Shadow-price source: load-balance constraint (yuan/MWh)	Shadow-price source: positive-reserve constraint (yuan/MWh)	Shadow-price source: negative-reserve constraint (yuan/MWh)	Shadow-price source: output upper/lower-limit constraint (yuan/MWh)	Shadow-price source: ramping constraint (yuan/MWh)	Shadow-price source: interface power-flow constraint (yuan/MWh)	Sum (yuan/MWh)
1	-1006	0	0	0	2108	-1100	2
2	5317.5	0	0	0	-4215.5	-1100	2
3	-1005.5	0	0	0	2107.5	-1100	2
4	-1006	0	0	0	2108	-1100	2
5	5317.5	0	0	0	-4215.5	-1100	2
6	-1005.5	0	0	0	2107.5	-1100	2

From Figure 2 and Appendix Table A1, it can be seen that: 1) the output of Unit B6 does not reach its upper or lower limit in any period, so the shadow price of the output constraint in each period in Table 1 is 0 yuan/MWh; 2) because the up- and down-ramping rate limits of Unit B6 are both 20 MW/h, the unit' s ramping constraints are binding in all periods, generating shadow prices; 3) the sensitivities of Unit B6 to Interfaces 1 and 2 are 0.8 and 0.3, respectively, and Interfaces 1 and 2 are both in a power-flow violation state in all periods. The shadow price is determined by the penalty factor as -1000 yuan/MWh, jointly producing an interface power-flow congestion price of
$-1000 \times 0.8 - 1000 \times 0.3 = -1100$ yuan/MWh.

The results in Table 1 reflect the coupling relationships among the constraints in the spot-market clearing model. The shadow price of the interface power-flow constraint is determined by the penalty factor for interface power-flow violations, while Unit B6 reaches its ramping boundary in all periods and is unable to continue

continued adjustment to alleviate interface congestion, so that the shadow price of the interface power-flow constraint is also reflected in the ramping-constraint shadow price of unit B6. From the results for period 1 in Table 1, the nodal electricity price of unit B6 is composed of the shadow prices of the load-balance constraint and the interface power-flow constraint, and is −2106 yuan/MWh, significantly deviating from its bid of 2 yuan/MWh. The direct reason for the deviation between the bid and the cleared price of unit B6 is that the unit' s ramping constraint reaches its bound, producing a shadow price of 2108 yuan/MWh; however, the essential reason is that the high penalty cost for the interface power-flow limit violation is reflected in the congestion price, causing the nodal electricity price of unit B6 to deviate from its bid. It can be seen that the full-constraint shadow-price representation model proposed in this paper not only helps to quickly and accurately locate the causes of abnormal prices, but also clearly depicts the coupling relationships among various constraints, which is of great significance for improving the efficiency and accuracy of price analysis.

Although the shadow prices of the load-balance constraint in Province B and the ramping-constraint shadow prices of unit B6 differ across periods, the sum of the full-constraint shadow prices is always equal to the generation bid of unit B, 2 yuan/MWh, confirming the correctness of the correlation characteristics between the unit bidding parameters and the shadow prices of the constraints related to unit output.

The shadow prices of the constraints related to the cleared power variables of the transaction components, namely Province A → Province B and Province B → Province C, are listed to verify expression (30), as shown in Table 2 and Table 3:

Table 2 Shadow prices of constraints related to the cleared power variable of the transaction component Province A → Province B

Period	Shadow-price source: sending-province load-balance constraint /(yuan/MWh)	Shadow-price source: receiving-province load-balance constraint /(yuan/MWh)	Shadow-price source: interface power-flow constraint /(yuan/MWh)	Shadow-price source: tie-line power upper/lower limit constraint /(yuan/MWh)	Shadow-price source: power delivery lower-limit constraint /(yuan/MWh)	Total /(yuan/MWh)
1	2010.5	-1006	-1000	0	0	4.5

Period	Shadow-price source: sending-province load-balance constraint /(yuan/MWh)	Shadow-price source: receiving-province load-balance constraint /(yuan/MWh)	Shadow-price source: interface power-flow constraint /(yuan/MWh)	Shadow-price source: tie-line power upper/lower limit constraint /(yuan/MWh)	Shadow-price source: power delivery lower-limit constraint /(yuan/MWh)	Total /(yuan/MWh)
2	-4313	5317.5	-1000	0	0	4.5
3	2010	-1005.5	-1000	0	0	4.5
4	2010.5	-1006	-1000	0	0	4.5
5	-4313	5317.5	-1000	0	0	4.5
6	2010	-1005.5	-1000	0	0	4.5

Table 3 Shadow prices of constraints related to the cleared power variable of the transaction component Province B → Province C

Period	Shadow-price source: sending-province load-balance constraint /(yuan/MWh)	Shadow-price source: receiving-province load-balance constraint /(yuan/MWh)	Shadow-price source: interface power-flow constraint /(yuan/MWh)	Shadow-price source: tie-line power upper/lower limit constraint /(yuan/MWh)	Shadow-price source: power delivery lower-limit constraint /(yuan/MWh)	Total /(yuan/MWh)
1	1006	1.5	-1000	0	0	7.5
2	-5317.5	2.5	-1000	6322.5	0	7.5
3	1005.5	2	-1000	0	0	7.5
4	1006	1.5	-1000	0	0	7.5
5	-5317.5	2.5	-1000	6322.5	0	7.5
6	1005.5	2	-1000	0	0	7.5

In Table 2, from Fig. 2 and Appendix Table A2, it can be seen that: 1) the transmitted power of the interprovincial tie-line (A5 → B6) does not reach its upper or lower limit in any period, so the shadow price of the tie-line power upper/lower limit constraint is 0 yuan/MWh; 2) the power flow of interface 1 exceeds its limit in every period, and the congestion price is determined by the violation penalty factor. The sensitivity of the tie-line (A5 $\rightarrow$ B6) power to interface 1 is 1, thus producing a shadow price of -1000 yuan/MWh.

It can be seen from Table 2 that the energy-price difference between the sending and receiving provinces includes the shadow-price component of the interface power-flow constraint. The sensitivity of the DC tie-line (A5 $\rightarrow$ B6) power to the interface is, in essence, the impact of adding one unit of load at the DC sending-end node A5 and adding one unit of generation power at the receiving-end node B6 on the interface power flow. Therefore, the product of the sensitivity of the tie-line (A5 $\rightarrow$ B6) to interface 1 and the shadow price of the interface-1 power-flow constraint reflects the congestion prices at nodes A5 and B6, that is, the difference between the nodal prices at nodes A5 and B6 and the energy prices of Provinces A and B. Even if interface 1 is located within a province, its shadow price can directly affect the energy-price difference between the sending province A and the receiving province B.

Although the energy prices of the sending and receiving provinces differ across time periods, the sum of the shadow prices associated with the transaction-component constraint (Province A $\rightarrow$ Province B) is always equal to the inter-provincial transmission price of 4.5 yuan/MWh.

From Table 3, Fig. 2, Appendix Fig. A2, and Appendix Table A2, it can be seen that in time periods 2 and 5, the loads in Provinces A and C increase slightly, while the load in Province B increases substantially, making the supply situation tight. As a result, the regional spot market reduces the cleared output of the transaction component (Province B $\rightarrow$ Province C). The power on the tie-line (B10 $\rightarrow$ C11) reaches the lower limit of 10 MW, generating a shadow price of 6322.5 yuan/MWh. The power flow of Interface 2 exceeds its limit in all time periods, while the sensitivity of the tie-line (B10 $\rightarrow$ C11) to Interface 2 is 1, generating a shadow price of -1000 yuan/MWh. The energy prices of the sending and receiving provinces differ across time periods, but the sum of all constraint shadow prices is always equal to the interprovincial transmission price of 7.5 yuan/MWh.

From Tables 1–3, it can be seen that the power flows of Interfaces 1 and 2 exceed their limits. At this point, the nonnegativity constraint on the interface-flow slack variable is inactive, and the corresponding shadow price is 0 yuan/MWh. Therefore, the congestion price is determined by the slack penalty factor as -1000 yuan/MWh, verifying the correctness of Eqs. (31)–(32).

In summary, regional spot market parameters—such as unit offers, transmission prices for transaction components, and interface-flow slack penalty factors—can all be expressed as linear combinations of the shadow prices of all constraints. Based on the above analysis, the following recommendations are proposed for electricity spot market development:

The formation mechanism of spot clearing prices should be comprehensively analyzed according to the binding status of each constraint and the information contained in shadow prices. In actual operation of electricity spot markets, various complex security constraints are intertwined and coupled, and clearing prices are often not formed by a single factor. As shown in Tables 1-3, provincial energy prices are jointly affected by unit offers, unit ramping capability, interface congestion, and other factors. It is therefore necessary to combine the system total-cost minimization objective with spot market parameter values to characterize the formation principles of clearing prices and quantitatively analyze the degree to which different factors affect clearing-price characteristics.

Shadow-price information should be used to guide market development. For example, in Table 1, the ramping constraints of Unit B6 are binding in all time periods, producing extremely high shadow prices. This indicates that improving the regulation capability of Unit B6 and allowing more units with regulation capability to participate in the market can effectively reduce the total system cost. If the interprovincial priority schedule transmission quantity constraint becomes binding and generates a shadow price, it indicates that, after the sending-province units' offers are superimposed with the interprovincial transmission price, they no longer have a competitive advantage compared with receiving-province units; that is, the interprovincial priority schedule quantity has exceeded the most economical interprovincial transmission quantity. Releasing part of the interprovincial priority schedule can alleviate this problem. It can thus be seen that the full-constraint shadow-price representation model provides price signals that directly reflect the impact of various market boundaries on spot clearing results, and can guide subsequent market optimization design.

7 Conclusion

To address the difficulty of analyzing regional spot clearing-price characteristics caused by the complex coupling of constraints in regional spot markets, this paper proposes a full-constraint shadow-price representation model for electricity spot market parameters. A mapping relationship between the standard optimization model and the regional spot market clearing model is established. Through the KKT conditions of the optimization problem, the relationship between value coefficients and constraint shadow prices is characterized and extended to the regional spot market, forming a full-constraint shadow-price representation model. A synchronous extension method for the full-constraint shadow-price representation model under changes in various market boundaries is proposed, and a unified methodology for analyzing price characteristics in regional spot markets is presented. Based on a regional spot market simulation case, the correctness of the proposed full-constraint shadow-price representation model is verified. Suggestions for future electricity spot market development are then proposed from the perspectives of identifying the causes of electricity-price formation, optimizing electricity market design, and verifying the rationality of spot clearing models. The main conclusions are as follows:

Key parameters of the regional spot market—such as unit offers, transmission prices for transaction components, and constraint slack penalty factors—can all be represented by linear combinations of the shadow prices of all constraints. The shadow prices of constraints may differ at different times, but their linear combination is always equal to the corresponding spot market parameter.

By establishing the mapping relationship between the spot-market clearing model and the full-constraint shadow-price representation model, the full-constraint shadow-price representation model can be synchronously extended when market boundaries change.

It is recommended that, in the process of developing the electricity market, the full-constraint shadow-price representation model be used to rapidly identify the causes of electricity price formation, efficiently verify the rationality of the spot-market clearing model, and provide guidance for the optimized design of the electricity market through shadow-price information.

This paper constructs a full-constraint shadow-price representation model for a regional spot market with asynchronous interprovincial interconnection. However, scenarios involving interprovincial hybrid AC/DC interconnection are not sufficiently considered. Future work will further study the shadow-price representation model for regional spot markets in hybrid AC/DC interconnected power grids.

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Appendix A Basic Data of the Case-Study System

The topology of the three-province system used in this paper is shown in Figure A1. Intra-provincial lines and inter-provincial tie-lines are represented by solid and dashed lines, respectively; the "→" symbol on an inter-provincial tie-line indicates the power-flow direction.

Figure A1 Topology of the three-province system

The load demands of Provinces A, B, and C over six time periods are shown in Figure A2:

Figure A2 Load demands of the three provinces in each time period

The operating parameters and bidding information of the generating units in the three provinces are shown in Table A1:

Table A1 Generating-Unit Operating Parameters and Bid Information

Location	$P_i^{\min}$/(MW)	$P_i^{\max}$/(MW)	P_i^{U}/(MW/h)	P_i^{D}/(MW/h)	Bid/(yuan/MWh)
A1	10	250	25	25	1.8
A2	10	150	15	15	2.2
B6	10	500	20	20	2.0
B8	10	500	15	15	1.5
C1	10	250	20	20	2.0
C8	10	180	30	30	2.5

There are a total of two interprovincial tie-lines in the three-province system; their parameters are shown in Table A2:

Table A2 Interprovincial Tie-Line Parameters

No.	Starting Node	Ending Node	$T_m^{\min}$/(MW)	$T_m^{\max}$/(MW)
1	A5	B6	10	300
2	B10	C11	10	300

Two main AC transfer interfaces are set. The power flow of interface 1 is affected by the outputs of generating units at nodes B6 and B8 and by the power of tie-line A5→B6; the power flow of interface 2 is affected by the outputs of generating units at nodes B6 and B8 and by the power of tie-line B10→C11. The sensitivity information is shown in Table A3:

Table A3 Interface Sensitivity Information

Interface Name	Device: Unit B6	Device: Unit B8	Device: Tie-Line (A5→B6)	Device: Tie-Line (B10→C11)
Interface 1	0.8	0.2	1.0	0.0
Interface 2	0.3	0.7	0.0	1.0

Author Contributions Statement:

Liang Yanjie: proposed the research idea, designed the research scheme, and revised and improved the manuscript content;

Wu Sheng: improved the research scheme and wrote the main body of the manuscript;

Xu Hekai and Tan Bowen: designed the case studies and organized the case-study data;

Peng Chaoyi and Lin Qingbiao: reviewed and revised the manuscript content.

Note: Figure translations are in progress. See original paper for figures.