Variation of Shannon Information Entropy of Triboelectric Nanogenerators and Analysis of Their Landauer Limit

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Abstract

The triboelectric nanogenerator (TENG) is an important device for converting mechanical energy into electrical energy, yet its interfacial charge evolution and output formation process exhibit pronounced stochasticity and non-equilibrium statistical features. While existing studies primarily focus on material properties, electrical responses, and contact electrification mechanisms, there is a lack of quantitative characterization of the state evolution, the transition from high entropy to low entropy, and the associated energy constraints during the output formation process. This work innovatively introduces Shannon information entropy theory to quantify the working process of TENGs. We establish a microstate charge-state probability model under interface discretization; from a Shannon information entropy perspective, we describe the system' s evolution from disorder to order, from high entropy to low entropy, and, with the aid of Landauer' s principle, link Shannon entropy to the output energy, enabling the computation of the minimum energy bound corresponding to the entropy-reduction of the TENG. The results indicate that externally driven mechanical processes yield a dynamically evolving distribution of interfacial charges, with Shannon entropy decreasing as the system orders, thereby yielding the corresponding expression for the TENG output energy bound. The cross-disciplinary framework of "charge-state probability distribution—Shannon information entropy—energy bound" integrates classical information theory with Landauer' s principle within the extended Maxwell equations framework developed by Academician Zhonglin Wang. This approach breaks through the limitations of traditional analyses confined to a single electrical dimension, constructing a unified theory that connects microscopic charges to macroscopic energy and establishing an integrated universal analytical method spanning classical information theory, information physics, and nano-energy science.

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Preamble

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This work innovatively introduces Shannon information entropy theory to quantify the working process of TENGs. We establish a microstate charge state probability model under interface discretization; from a Shannon information entropy perspective, we describe the system' s evolution from disorder to order, from high entropy to low entropy, and, with the aid of Landauer' s principle, link Shannon entropy to the output energy, enabling the computation of the minimum energy bound corresponding to the entropy reduction of the TENG.

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This approach breaks through the limitations of traditional analyses confined to a single electrical dimension, constructing a unified theory that connects microscopic charges to macroscopic energy and establishing an integrated universal analytical method spanning classical information theory, information physics, and nano energy science.

Keywords

triboelectric nanogenerator; Shannon information entropy; Landauer principle; energy bound

1 Introduction

In 2012, the team of Academician Zhong in Wang first proposed a triboelectric nanogenerator (TENG) that enables mechanical energy electrical energy conversion based on contact electrification and electrostatic induction effects; it can convert environmental mechanical energy into electrical signals with high voltage and low current . Due to its broad material sources,

flexible structural design, sensitivity to low frequency mechanical excitation, and ease of integration with sensing units, TENG has demonstrated broad application prospects in wearable electronics, self powered sensing, environmental energy harvesting, and micro nano energy systems . Particularly in harvesting low frequency, distributed, and intermittent mechanical energy, TENG exhibits unique advantages over traditional electromagnetic and piezoelectric generators, and has thus become an important direction in current energy harvesting and powered system research Focusing on the working mechanism and output characteristics of TENG, studies have systematically explored multiple levels, including material surface electronic affinity, interfacial triboelectrification mechanisms, charge transfer behavior, device structure optimization, and equivalent circuit modeling, and have established analytical methods for key physical quantities such as open circuit voltage, short circuit current, and output power . These works lay an important foundation for understanding the macroscopic electrostatic behavior of TENG. However, unlike traditional steady state energy converters, the operating process of TENG is essentially a equilibrium interface driven by external mechanical stimulation, transitioning from high entropy to low entropy; its output depends not only on material and structural parameters but also on micro contact configurations, local charge distributions, surface defect states, fluctuations in effective contact area, and environmental disturbances, among other factors . While existing models can describe partial output energy features under specific conditions, they still lack a unified and quantifiable theoretical description of the intrinsic relationship between state evolution, uncertainty variation, and energy constraints during the formation of TENG output.

Regarding the theoretical description of TENG, Academician Zhonglin Wang extended the displacement current theory to a generalized form that includes dielectric polarization and surface charge sources, establishing an extended Maxwell framework for describing the TENG output process . By introducing an additional surface polarization term , the interface bound charges generated by triboelectrification are treated as a source term of displacement current, thereby unifying the open circuit voltage, short circuit current, and load output characteristics of TENG at the level of continuum electrodynamics . This extension not only clarifies the physical essence of TENG as a displacement current driven device but also provides a foundation for subsequent field theoretic analyses of interface charge evolution. However, the extended Maxwell equations

primarily address the electromagnetic response in continuous media and still lack direct descriptors for the spatial inhomogeneity of interface charge distribution, local configurational fluctuations, and the resultant statistical evolution features . Consequently, beyond the continuum field model, it is necessary to introduce statistical methods capable of characterizing the probability distribution of interface states and the dynamic evolution from high entropy to low entropy configurations.

From a more fundamental physical perspective, the operation of TENG is not merely a direct conversion of mechanical energy into electrical energy; internally it is accompanied by statistical evolution such as reconstruction of the interface contact state, redistribution of charge occupancy, and the progressive determination of the output response . If these microscopic states are regarded as random variables with certain probability distributions, the operating process of TENG under external mechanical work can be understood as a dynamic evolution from higher uncertainty states to lower uncertainty states, i.e., a high entropy to low entropy transition . In this framework, the variation of the device output energy reflects the contraction of the system state distribution and the enhancement of its degree of ordering . Thus, it becomes evident that to further reveal the essence of the formation of TENG output, a description based solely on traditional electronics remains inadequate, and there is an urgent need to introduce a novel mathematical perspective capable of capturing the evolution of the system' s statistical states.

Entropy provides a natural theoretical tool for characterizing this process. As a core concept describing the degree of disorder in a system and the uncertainty of its state, entropy characterizes the statistical properties of the number of microstates in thermodynamics, while in classical Shannon information theory it is used to quantify the uncertainty of probability distributions For non equilibrium systems such as TENGs that combine physical randomness with statistical evolution, introducing the concept of Shannon information entropy allows the interfacial states, charge distributions, and output responses to be unified under a probabilistic description, thereby providing a quantitative account of the process by which the system transitions from disorder to order. In particular, under external mechanical excitation, if the system state progressively concentrates on fewer accessible configurations, or if its output response becomes more deterministic, the corresponding Shannon information entropy will change Therefore, Shannon information entropy can serve not only as a new perspective for characterizing the

internal state evolution of TENGs, but also as a bridge linking microscopic stochastic processes with macroscopic output characteristics.

Regarding the relationship between state changes and energy cost, Landauer' s principle provides an important theoretical basis for the link between Shannon information entropy and energy. The principle states that erasing 1 bit of information dissipates at least of energy, thereby indicating that information changes are inherently accompanied by a measurable physical . For TENGs, if the reconfiguration of interfacial states, contraction of charge distribution, or determinization of the output response i.e., the corresponding decrease in uncertainty interpreted as a generalized information reduction process, Landauer' s principle may provide a theoretical lower bound for the related energy scales . In this work, beyond conventional electrical analysis, a classical Shannon information theoretic perspective is introduced to extend the description of microcharge state evolution during the TENG output formation process, i.e., by examining the evolution of the system state probability distribution and its Shannon entropy variation, and by exploring the relation between information reduction and energy constraints.

To summarize, this paper centers on the state statistical description of TENGs and classical Shannon information theory. First, by combining typical operating modes of TENGs, a probabilistic state model is constructed to describe the statistical evolution of contact states, charge distributions, and output responses during operation; second, the concept of Shannon information entropy is innovatively introduced to characterize the process of high entropy to entropy in the TENG system and to analyze its evolution with changing operating states; on this basis, Landauer' s principle is employed to establish the minimal energy bound associated with information reduction. The aim of this work is to, beyond classical microcharge analysis, establish a unified interdisciplinary theoretical framework that connects state probability distributions, entropy variation, and energy bounds, with the goal of providing a new perspective for understanding the output mechanism of TENGs. 2 Theoretical Foundations

2.1 Shannon entropy

Entropy is a fundamental concept describing the distribution of a system' s states and is commonly used to quantify the system' s uncertainty, degree of disorder, and statistical

strictly defined information entropy as a quantitative description of the uncertainty of a probability states are relatively dispersed, the system' s uncertainty is high and the information entropy is large; conversely, when the system' s state distribution is concentrated on a few possible outcomes, its uncertainty decreases and the information entropy correspondingly diminishes.

For a discrete random variable , if its possible values are , and the corresponding probability distribution is , then the Shannon information entropy is defined as

$H (X) = - \sum p i$

Where is the information entropy of the random variable (bit); is the total number of different outcomes that the random variable may take; is the probability of the th result occurring.

where $p i = P (X = x i)$, and it satisfies

$=1$ (2 - 2)

reflects the average uncertainty of a random variable: when the probability of a particular state approaches 1, the system is nearly determined, and the entropy tends to zero; when the state probabilities approach a uniform distribution, the system is most difficult to predict, and the entropy tends to its maximum. According to the maximum entropy principle in classical Shannon information theory, this principle states that Shannon entropy attains its maximum value

when $p i =1/ n$.

$H max = \log 2 n$ (2 - 3)

Where is the maximum value of Shannon information entropy.

These results indicate that Shannon information entropy is simultaneously influenced by both the number of states and the form of their probability distribution. From a mathematical standpoint, Shannon entropy exhibits the fundamental properties of non negativity, continuity, and additivity, and thus provides a unified characterization of the state distribution of complex probabilistic

systems. It does not depend on any specific physical mechanism; rather, it relies solely on the probabilistic form of the system' s state distribution, and therefore possesses universality. For any system that can be represented by a discrete set of states and their corresponding probabilities , its uncertainty can be quantified by Equation (2 When the system undergoes the evolution of the state distribution, the change in entropy can be written as

$\Delta H = H$ final - H initial (2 - 4)

Where is change in entropy; final is entropy value of the system' s final state;

H initial is entropy value of the system' s initial state.

< 0, it indicates that the system' s uncertainty decreases, and the state distribution shifts from a more dispersed form to a more concentrated form; when > 0, it indicates that the system' s uncertainty increases. Therefore, Shannon information entropy not only characterizes the uncertainty of the system state at a given moment, but also captures the trend of evolution from disorder to order or from order to disorder in the system.

2.2 Landauer Principle

In 1961, the Landauer principle reveals a fundamental relationship between information changes and energy costs, serving as an important basis linking Shannon information entropy and energy. The principle states that the reduction of information is not without physical cost; when information in a system is erased or its state uncertainty is reduced, at least a corresponding minimum energy scale is required. For the process of erasing 1 bit of information, the minimum energy cost is

E min = k B T ln 2 (2 - 5)

Where

E min is the minimum energy required to erase 1 bit of information(J);

denotes the Boltzmann constant(1.380649$×$10 J/K); denotes the absolute temperature(K).

This expression indicates that information processing has a definite physical lower bound, and information is no longer merely an abstract mathematical quantity, but is directly related to

energy dissipation. f the system' s evolution reduces information by A bits, the corresponding minimum energy bound can be written as

E min = k B T Aln 2 (2 - 6)

The key significance of Landauer' s principle lies in the fact that it does not provide the exact value of actual energy output or actual dissipation, but a theoretical lower bound determined by the amount of information reduced. In other words, the energy changes in real processes are typically greater than this limit and will not fall below this bound.

2.3 System

State Probability Model For any system with multiple reachable states, its operating state at a given time can be represented by a discrete state set. In discrete time description, it is written as

S = { s 1 , s 2 ,⋯, s n } (2 - 7)

Where s i is the i - th state that the system may occupy, i = { 1,2,⋯, n } .

The probability that the system is in state at time

p i (t) = P (X (t) = s i) (2 - 8)

where is the probability that the system is in state at time is the state variable of the system at time According to the definition of probability

p i (t) $ $0, $\sum$ p i

(t) =1 (2 - 9)

Thus the statistical state of the system at time t can be fully characterized by a probability vector. where is the probability vector.

This vector provides the distribution information over all reachable states and constitutes the basis for subsequent Shannon entropy calculations and energy analyses.

As the system evolves in time, its probability distribution also changes. If a discrete description is adopted, and within a time step the system transitions from state to state with probability , then the state probabilities at time satisfy

$p_j(t + \Delta t) = \sum T_{ij}$

where the transition - probability matrix $T(t)=[T_{ij}(t)]$ satisfies

$T_{ij}(t)$ \$ \$0, $\sum T_{ij}$

$(t) = 1$ (2 - 12)

This expression indicates that the next step probability distribution is determined by the current distribution and the transition relations. If external conditions acting on the system vary with time, then may also explicitly depend on time, reflecting the modulation of state evolution by external driving.

If a continuous time description is used, the evolution of state probabilities can be written in master equation form:

$d\,p_i(t)\,dt = \sum [w_{ji}(t)\,p_j(t) - w_{ij}(t)\,p_i(t)]$

$j \neq i$

where $w_{ij}(t)$ is the transition rate from state s_i to state s_j in unit time.

This equation describes the probability flux between states: denotes the flux into state from other states, and denotes the flux out of state to other states.

Discrete time and continuous time models are, in essence, statistically equivalent; they differ only in the parameterization of the evolution.

In practical modeling, the choice of the state set depends on the problem' s physical features.

If the set of reachable states is large or the distribution is clearly continuous, one may first discretize appropriately to partition it into a finite number of state intervals, with each discrete interval representing a statistical state. Consequently, the continuous evolution of a complex system can be transformed into the evolution of finite dimensional state probabilities. As long as the state partition captures the system' s principal dynamical features, the corresponding probability model can be used to characterize its statistical behavior. 2.4 The extended Maxwell Equations by Zhonglin Wang Academician Zhonglin Wang proposed in 2020 that the output process of a triboelectric nanogenerator can be described in a unified displacement current framework. For TENG, device output depends not only on conventional dielectric response but also on interfacial surface charges formed after triboelectric charging . To characterize this feature, the surface electrostatic charges

generated by triboelectric charging can be treated as an additional surface polarization term and incorporated into the expression for the displacement vector. Thus, the displacement vector can be written as

$D = \epsilon E + P + P s$ (2 - 14)

where is the conventional dielectric response term; is polarization induced by the external electric field; is the equivalent surface polarization term corresponding to the surface electrostatic charges formed by triboelectric charging.

Consequently, the displacement current density can be expressed as

$J D = \partial D \partial t = \epsilon \partial E \partial t + \partial P \partial t + \partial P s \partial t$ (2 - 15)

where is displacement current density. Under quasi static conditions, the device output is primarily governed by the time varying distribution of interfacial surface charges and their stochastic mechanical motion. Therefore, the additional displacement current term associated with constitutes the source term most directly related to the TENG output, and is also the key to linking the interfacial surface charge distribution with the device electromagnetic response.

Let the friction interface be , with interface normal vector . The local surface charge density can then be expressed as where is local surface charge density; is position vector.

This relationship indicates that the interfacial surface charge distribution can be characterized by the normal component of the surface polarization term . Accordingly, the establishment of the TENG' s induced field, the generation of displacement current, and the output to the external circuit can all be attributed to the electromagnetic response induced by the spatial distribution and temporal variation of the surface polarization source term Within this continuous field framework, the surface polarization term provides the

electromagnetic starting point for the interfacial surface charge distribution. If one further focuses on the spatial rearrangement of interfacial surface charges under mechanical actuation and their statistical evolution, it becomes necessary to introduce a discretized description atop the continuous surface charge distribution, so that the surface charge source terms in the continuous field can be transformed into local state variables suitable for subsequent probabilistic modeling and Shannon information entropy analysis. 3 Physical Model of the Triboelectric Nanogenerator 3.1 Device Structure and Operating Principle Triboelectric nanogenerators, depending on the form of relative motion, are typically categorized into four representative operating modes: vertical contact separation, lateral sliding, single electrode, and free vibrating layer modes. Among them, the vertical contact separation structure has a clear working mechanism, well defined boundary conditions, and relatively straightforward modeling; hence, this study selects the contact separation triboelectric nanogenerator as the object of investigation.

A contact separation triboelectric nanogenerator typically comprises two dielectric layers with distinct triboelectric properties and their back electrodes, with the electrodes connected through an external circuit to form a closed loop. Under external mechanical excitation, the two triboelectric layers undergo periodic contact and separation, enabling conversion from mechanical to electrical energy.

When the two friction layers come into contact, due to differences in surface electron affinity, charge transfer occurs at the interface, establishing equal and opposite bound charges on the surfaces. As the layers gradually separate, the electrostatic field established by the interfacial charges induces a potential difference between the upper and lower electrodes, driving directed migration of free electrons in the external circuit and thus generating a current output. When the layers approach and reestablish contact, the potential difference between the electrodes reverses, electrons migrate in the opposite direction, and the current direction reverses accordingly.

Consequently, the device exhibits alternating voltage and current outputs under periodic contact separation motion.

From a mechanistic perspective, the electrical output of a contact separation triboelectric nanogenerator results from the coupling of triboelectrification, electrostatic induction, and charge

transfer through the external circuit. Its output behavior is determined by the establishment of interfacial charges, the variation of interlayer distance, and the evolution of the induced potential between electrodes, and is jointly influenced by factors such as surface charge density, effective contact area, dielectric thickness, and material electrical parameters. 3.2 Classical Electrical Model of Triboelectric Nanogenerators A contact separation triboelectric nanogenerator can be equivalently modeled as a parametric electrical system comprising interfacial bound charges and a time varying capacitance. Under external mechanical excitation, the interlayer spacing varies with time, and the device' s equivalent capacitance, induced potential, and transferred charge evolve accordingly. For this operating mode, the separation distance , transferred charge , and terminal voltage can be taken as the fundamental physical variables describing the evolution of the device state.

Let the effective contact area of the two friction layers be , the interfacial surface charge density be , and the effective dielectric thickness be ; the device' s equivalent capacitance at a separation distance x can be expressed as

$$C (x) = \epsilon 0 S x + d 0$$

where is the equivalent capacitance when the separation distance is x; is the vacuum permittivity.

This formula indicates that as the separation distance increases, the device capacitance decreases gradually, and the mechanical displacement directly modulates the electrical state.

Under open circuit conditions, the interface bound charges induce an electrostatic potential between the top and bottom electrodes, and the open circuit voltage can be written as

V oc (x) = σ x ϵ 0

where is the open circuit voltage. When the external circuit is connected, free charges will transfer under the drive of the induced potential. Let the transferred charge in the external circuit be ; the instantaneous voltage across the device terminals can be written as where is the instantaneous voltage across the two terminals of the device.

Substituting the expressions of (3 1) and (3 2) yields This relation indicates that the terminal voltage is determined by two contributions: one from the potential due to charge transfer on the time varying capacitance, and another from the induced potential due to the interface bound charges. The variation of the mechanical displacement only changes the capacitance but also alters the induced potential, thereby causing the device output to exhibit a state dependent dynamic evolution.

If the external load is a resistor , the output current and output voltage satisfy

I = dQ

dt , V = R dQ

is the output current; is the output voltage. Thus, the governing equation under load conditions is obtained

dt = - Q C (x) + V oc (x) (3 - 6)

dt = - Q (x + d 0) ϵ 0 S + σ x ϵ 0

Therefore, the output response of a contact separation triboelectric nanogenerator is essentially a process of electrical state evolution driven by mechanical displacement, with the interfacial charge density, separation distance, equivalent capacitance, and transferred charge jointly determining its instantaneous output characteristics. 3.3 Statistical Modeling of TENG Output Evolution The aforementioned physical model and equivalent electrical relations provide a basic description of the output process for contact separation triboelectric nanogenerators (TENGs), but in practical operation devices do not strictly evolve along a single deterministic trajectory. This arises because interfacial contact and separation behaviors are influenced by multiple factors, including microscopic contact configurations, local charge distribution, surface defect states, and environmental perturbations, ca using the device to exhibit variability and discreteness even under identical external excitations. Consequently, the output of a triboelectric nanogenerator can be described not only by deterministic electrical variables but also by pronounced statistical evolution characteristics.

From a state descriptive perspective, the separation distance , transferred charge terminal voltage , and surface charge density can serve as the primary physical quantities characterizing the device output state. Let the system state at time The device' s operation can thus be interpreted as the continuous evolution of state variables within a given state space. If the continuous state space is discretized into a finite set of reachable states , the probability that the system is in state at time can be expressed as

p i (t) = P (z (t) = s i) (3 - 9)

and satisfies

p i (t) $ $0, $\sum$ p i i

(t) =1 (3 - 10)

Within this formulation, the device output can be further represented as the evolution of the state probability distribution over time. When the probability distribution is broad, the system output exhibits greater uncertainty; when the state concentrates into a small number of reachable states, the output exhibits higher determinism. Accordingly, the output process of a triboelectric nanogenerator can be described as a dual process of electrical state evolution under external mechanical actuation and the evolution of its statistical distribution.

This statistical description rests on the aforementioned physical model: the equivalent electrical model provides the fundamental charge constraints among state variables, while the probability distribution characterizes the discreteness and fluctuations of these variables during actual operation. The construction of the state space and the evolution of its probability distribution lay the groundwork for subsequently introducing Shannon information entropy to describe changes in system uncertainty and, further, to analyze the energy limits associated with entropy reduction, thereby providing a modeling foundation.

4 Information

Entropy Based TENG State Modeling and Energy Limit Analysis

4.1 Definition of State Variables

The output formation process of a contact separation mode triboelectric nanogenerator is essentially determined by the combined effects of interfacial charge establishment, variation in

interlayer spacing, and charge transfer to the external circuit. Among these, the distribution of interfacial charges resides at the initial link of the entire energy conversion chain, directly affecting the formation of the induced potential, the evolution of transferred charge, and the response characteristics of the terminal voltage. Meanwhile, in actual device operation, the output is also influenced by factors such as microscopic contact configuration variations, local defect state trapping, surface roughness fluctuations, and environmental perturbations, which cause the interfacial charge to exhibit spatial nonuniformity and pronounced statistical discreteness. Consequently, the output process of a triboelectric nanogenerator is not only an electrical response but can also be regarded as a statistical distribution of interfacial charge states evolving under external mechanical drive.

Because the real interface exhibits continuity, spatial heterogeneity, and local fluctuations in its charge distribution, direct treatment in continuous space leads to quantitative analysis of its state becoming intractable due to the excessive complexity of the contributing factors. By contrast, partitioning the interface into a finite number of local regions while preserving the principal spatial distribution characteristics discretizes the continuous charge distribution problem into a finite dimensional state distribution problem, enabling the statistical evolution of interfacial charges to be expressed in terms of state probabilities. Consequently, the originally complex problem degenerates into a linear recurrence, substantially reducing complexity.

To this end, this paper partitions the friction interface into N local regions, denoted as

$S = \{ s\ 1\ ,\ s\ 2\ ,\cdots,\ s\ N\ \}\ (\ 4\ -\ 1\ )$

where is the th local region of the interface. Through interface discretization, the charge states distributed continuously over the entire friction surface are transformed into a finite set of states over the local regions, thereby endowing the interfacial charge distribution with a discrete representation.

Let the area of region , and the local surface charge density at time . The local charge amount in that region can be represented as

$q\ i\ (\ t\ ) = \sigma\ i\ (\ t\ )\ A\ i\ (\ 4\ -\ 2\ )$

Accordingly, the charge distribution over the entire friction interface can be written as where is the charge distribution state across the local regions at time

Given that we are concerned with the distribution characteristics of interfacial charges across local regions rather than the algebraic result after cancellation of charge signs, define the interfacial total charge as

$Q\ tot\ (\ t\ ) = \sum$

where is the interfacial total charge. With this definition, the interfacial charge distribution can be described by the charges in each local region together with their total, thereby converting the continuous interfacial charge distribution problem into a distribution problem in a finite dimensional state space. Consequently, the device’ s interfacial charge evolution under external mechanical drive can be reduced to a state transition process in a discrete state space, providing the state basis for constructing the local charge state probabilities.

4.2 Local Charge

State Probability Model On the basis of interface discretization and the definition of the state space, the interfacial charge distribution can be further represented as the state probability distribution over each local region. Since the output formation process of a contact separation triboelectric nanogenerator is governed by the spatial redistribution of interfacial charges, it is necessary to introduce a probabilistic description on the discretized interface to characterize the relative occupancy of each local region within the overall charge distribution. Accordingly, the continuous spatial distribution of interfacial charges can be transformed into a probability distribution of local charge states, thereby providing a mathematical foundation for subsequent uncertainty quantification.

Substituting Equations 2) and (4 4) yields:

$$q_i(t) \quad Q_{tot}(t) = \quad \sigma_i(t) \quad A_i \sum \quad N \; j=1 \; \sigma_j(t) \quad A_j$$

$$p_i(t) =$$

This expression satisfies the constraints specified by Equation (3 Thus, the interfacial charge distribution can be represented as a time varying probability vector where is the time varying probability vector.

The foregoing formulation converts the problem of charge distribution on a continuous

interface into a problem of probability distributions over a finite number of local regions, enabling the spatial heterogeneity and local occupancy characteristics of interfacial charges to be described in a unified form.

Under external mechanical actuation, the contact state, separation distance, and local electric field distribution all vary with time, and interfacial charges are continually redistributed among local regions; hence is essentially a time evolving state probability vector. If a discrete representation is adopted, within a time step T, the local charge state probabilities satisfy the relations described by quations (2 11) and (2 Correspondingly, the evolution of the entire interface' s state probability can be written in matrix form

$$P(t + \Delta t) = T(t) P(t) \quad (4-7)$$

where is the state transition matrix. This matrix characterizes the reorganization of interfacial charges among local regions under external mechanical driving; its explicit form depends on the contact separation motion, the local electric field distribution, and the interfacial charge transport characteristics. Consequently, the interface charge evolution of a contact separation type frictional nanogenerator can be uniformly described as the evolution of the state probability distribution over time. This probabilistic model not only inherits the discretized interface state space definition but also provides a direct basis for introducing Shannon entropy to characterize the uncertainty of the interfacial charge distribution.

In this framework, changes in the stochastic mechanical driving of the interfacial charge distribution can be further transformed into an evolution problem for Shannon entropy, thereby enabling entropy reduction and energy analysis.

4.3 Shannon Entropy

Based Characterization of Interfacial Charge Distribution After the local charge state probability model is established, the uncertainty of the interfacial charge distribution can be further quantified by Shannon entropy. Since the probability vector describes the relative distribution of the total interfacial charge across local regions, it can be regarded as the statistical state representation of the system at time T.

Correspondingly, the Shannon entropy of the interfacial charge distribution is defined as

H (t) = - $\sum$ p i

This expression reflects the average uncertainty of the interfacial charge distribution, and its value is jointly determined by the state probabilities across the local regions. When the interfacial charge is distributed relatively uniformly among regions, the probability distribution is more dispersed, the system state uncertainty is higher, and the corresponding Shannon entropy is larger; when the interfacial charge gradually concentrates in a subset of regions, the probability distribution becomes more concentrated, the system state uncertainty decreases, and the corresponding Shannon entropy decreases. Thus, Shannon entropy can serve as a statistic to characterize the degree of disorder and the level of ordering of the interfacial charge distribution.

For a given discrete region count , the range of Shannon entropy values satisfies

$0\leq$ H (t) $\leq$ log 2 N (4 - 9)

Among them, when the interfacial charge is completely concentrated in a single local region, there is In this case, the Shannon entropy attains its minimum value

H min =0 (4 - 11)

When the interfacial charge is equally distributed across all regions, there is

p i (t) = 1 N , i =1,2,$\cdots$, N (4 - 12)

According to the maximum entropy theorem of information theory, the Shannon entropy reaches its maximum

H max = log 2 N (4 - 13)

This demonstrates that Shannon entropy provides a mathematically continuous representation of the transition of the interfacial charge distribution from highly concentrated to highly dispersed, and its numerical variation directly corresponds to changes in the system state uncertainty.

In a contact separation mode triboelectric nanogenerator, the interfacial charge distribution is not statically invariant but continually evolves under external mechanical driving. Therefore, is essentially a time varying quantity, and its evolution reflects the dynamic pattern of the interfacial charge state under mechanical action from dispersed to concentrated, or from concentrated to dispersed, i.e., the dynamic variation between high entropy and low entropy. If, during a given stage, the Shannon information entropy decreases with time, this indicates that the

interfacial charge distribution tends toward convergence and the system state uncertainty decreases; conversely, if the Shannon information entropy increases, it indicates that the interfacial charge distribution tends to diverge and the system state uncertainty rises.

To facilitate comparisons across different discrete scales, one can further introduce a normalized Shannon information entropy, defined as

H˜ (t) = H (t) log 2 N (4 - 14)

thereby satisfying

0≤ H˜ (t) $ $1 (4 - 15)

After normalization, can be used to eliminate the influence of the number of region partitions on the absolute value of the Shannon information entropy, making the charge distribution states across different interfaces comparable.

Thus, the Shannon information entropy of the interfacial charge distribution not only provides a quantitative characterization of the local charge state probability distribution but also endows the interfacial charge evolution process with a computable measure of uncertainty. On this basis, under external mechanical driving, the reorganization process of the interfacial charge distribution can be further described as the evolution of Shannon information entropy, thereby providing direct grounds for subsequent analysis of the system' s entropy reduction behavior and its corresponding energy analysis.

4.4 Entropy

- Reduction Process under External Mechanical Driving

During the operation of a contact separation triboelectric nanogenerator, externally applied mechanical work continuously alters the interface contact state, the interlayer spacing, and the local electric field distribution, thereby driving the redistribution of interfacial charges among local regions. Since Shannon information entropy characterizes the uncertainty of the interfacial charge distribution, the evolution of the local charge state probabilities under mechanical driving can be further mapped onto the evolution of Shannon information entropy. As the interfacial charge distribution evolves from more dispersed states toward more localized states, the system' s uncertainty decreases, manifested as a reduction in Shannon information entropy.

Let the system' s initial state probability distribution at a given evolutionary stage be initial and the terminal state distribution be final ; the corresponding initial entropy and terminal entropy

are respectively

(i.e., Δ H = H final - H initial).

Δ H red = [- $\sum$ p i (initial)

After simplification, we obtain

Δ H red = $\sum$ p i (final)

log 2 p i (initial) (4 - 16)

final final

log 2 p i (initial) (4 - 21)

H initial = - $\sum$ p i (initial)

H final = - $\sum$ p i (final)

Thus, the Shannon entropy change during this evolution can be computed by quations (2 Frictional nanogeneration is a process from high entropy to low entropy; therefore this work focuses only on the HH case. That is, the interfacial charge distribution transitions from a state of higher uncertainty to a state of lower uncertainty. To facilitate subsequent exposition, the entropy reduction quantity can be further defined as

Δ H red = - Δ H = H initial - H final (4 - 18)

Clearly, when the system undergoes an ordering evolution, there is

Δ H red >0 (4 - 19)

Substituting the Shannon entropy expression Equation 1) yields that the entropy reduction of the interfacial charge distribution between the initial and final states is

log 2 p i (initial)] - [- $\sum$ p i (final)

log 2 p i (final) - $\sum$ p i (initial)

This expression provides the entropy reduction expression for the evolution of the interfacial charge distribution from the initial state to the terminal state, and its value is jointly determined by the changes in the state probabilities of the local regions.

If described in a time varying form, then within the time interval , the local entropy reduction of the interfacial charge distribution can be written as

Δ H red (t) = H (t) - H (t + Δ t) (4 - 22)

H (t + Δ t) < H (t) (4 - 23) this indicates that an entropy reduction occurred within this time interval, consistent with the

conclusion of quation (4 22). From this, it is evident that the temporal evolution of Shannon entropy not only reflects the direction of the rearrangement of the interfacial charge distribution but also quantitatively characterizes the degree to which the system evolves from disorder to order under mechanical driving.

The reallocation of interfacial charges induced by contact separation motion essentially corresponds to the temporal evolution of the state probability distribution, while the entropy reduction reflects the decrease in the system' s uncertainty during this evolution. Accordingly, can serve as a quantitative indicator characterizing the transition of the interfacial charge distribution from dispersion to concentration, and be used to describe the degree of ordering of the system state under external mechanical driving. Based on this entropy reduction, the variation in the uncertainty of the interfacial charge distribution can be further related to energy scales, thereby providing a theoretical basis for subsequent analyses of energy limits. 4.5 Establishment of the energy lower bound based on the Landauer principle The Landauer principle establishes a fundamental link between information erasure and the minimum energy scale. As expressed by the relation in Equation (2 6), a reduction in Shannon entropy corresponds to a non negligible minimum energy scale, thereby providing a theoretical lower bound for energy analysis during the evolution of the system state.

For the interface charge distribution, Shannon entropy is measured in bits to quantify the uncertainty of the system state, and thus the entropy decrease of the interface charge distribution during evolution can be directly equated with the amount of information erased. Consequently, if the aforementioned entropy decrease units may be bit, nat, or Hartley; corresponding to the Landauer principle; the following will use bits) is taken as the information erasure of the system within this stage, substituting into Equation (2 6) yields the corresponding Landauer energy lower bound, which can be written as

E L = k B T Δ H red ln 2 (4 - 24)

Substitution into Equation (4 19) yields

E L = k B T (H initial − H final) ln 2 (4 - 25)

Subsequently substituting the Shannon entropy expression for the interface charge distribution Equation 22) yields

E L = k B T [- ∑ p i (initial)

log 2 p i (initial) + ∑ p i (final)

final This expression provides the minimum energy lower bound for the evolution of the interface charge distribution from the initial state to the terminal state; its value is jointly determined by the changes in the local region state probability distributions. Further substituting Equation (4 yields

E L = k B T [- $\sum$ σ i (initial) A i Q initial

log 2 (σ i (initial) A i Q initial

- $\sum$ σ i (initial) A i Q initial

log 2 (σ i (initial) A i Q initial

When the interface charge distribution evolves from a more dispersed state to a more localized state, the system' s Shannon entropy decreases, , and the corresponding Landauer lower bound is also positive; the larger the entropy reduction, the larger the associated minimum energy scale.

If described in a time varying form, within the time interval , the instantaneous energy lower bound corresponding to the local entropy reduction can be written as

E L (t) = k B T ln 2 [H (t) - H (t + Δ t)] (4 - 28)

H (t + Δ t) < H (t) (4 - 29)

there is

E L (t) >0 (4 - 30)

indicating that within this time interval the interface charge distribution has undergone an ordering evolution, which is in agreement with the conclusion of Equation (4 28), and corresponds to a positive minimum energy scale. Accordingly, the entropy reduction process of the interface charge distribution can be further transformed into an energy lower bound with explicit physical dimensions.

Thus, the discretization of the interface, the local charge state probability model, the Shannon entropy characterization, and the entropy reduction analysis can be unified into the construction of the Landauer energy lower bound. Local charge state probabilities determine the statistical shape

of the interface charge distribution, Shannon entropy characterizes the uncertainty of this distribution, and the entropy reduction induced by external mechanical driving further corresponds via the Landauer principle to a computable minimum energy scale. Consequently, the evolution of interfacial charges in a frictional nanogenerator under mechanical actuation can be described within the unified framework of "state probability distribution Shannon entropy energy lower bound." 5 Derivation of Shannon information entropy and energy lower bound in the framework of the extended Maxwell equations 5.1 From continuous surface polarization to discrete local charge states and entropy evolution driven by surface polarization sources under mechanical driving Within the extended Maxwell equations framework proposed by Academician Zhong Wang, the surface polarization term is described in a continuous field form, capturing the contribution of interfacial surface charge to the device output. If one further considers the spatial reorganization of the interfacial surface charge distribution under mechanical driving and its statistical evolution, it is necessary to convert the continuous surface charge distribution into computable discrete state variables. To this end, the friction interface is partitioned into local regions, and its state space is modeled as characteri zed by Equation Through this discretization, the surface charge states originally distributed continuously over the entire friction interface are converted into a finite set of states on local regions, thereby endowing the interfacial charge distribution with a finite dimensional representation.

Let the subinterface corresponding to the th local region be mbining Equation yields the local charge quantity in that region at time

$(\quad) = \int \quad \Gamma$

Substituting i Equation 16) yields

$(\quad) = \int \quad \Gamma$

Thus, the continuous surface polarization source in the extended Maxwell equations is transformed into a charge state variable defined on discrete local regions. This treatment preserves the principal features of the interfacial surface charge distribution while enabling the continuous field problem to be expressed in a discrete state space.

In the discrete interface representation, the interfacial surface charge distribution has been written as a state distribution over local regions; hence the evolution of the interfacial charge state is not only manifested in the change of the local charge amount but can further be described by the state probability and its Shannon information entropy H(t). Since the local charge amount is determined by the integral of the surface polarization source over each discrete region, the evolution of the uncertainty of the interfacial charge distribution is ultimately driven by the time variation of quations (5 1) and (5 2), it can be deduced that represents the th local region, is the interface normal vector. Taking the time derivative of the above equation, we have

$\mathrm{d} \quad = \int \partial \, \partial \quad \Gamma$

This equation indicates that the rate of change of the charge state in each local region is determined by the temporal variation of the surface polarization source term . Since the local charge state probability is obtained by normalizing the local charge quantity , the evolution of the state probability distribution is also controlled by the changes in Record

$= 1$

where denotes the absolute value of the local charge amount.

The local charge state probability is

$(\quad) = \tilde{} \quad (\quad) \quad (\quad) \; (5 - 5)$

Hence its time derivative is

d d = 1

where

= 1

If, over a given evolution interval, the sign of remains unchanged, then

d ˜ d = sgn ( ) d d (5 − 8)

Thus the time evolution of the local state probabilities is ultimately governed by

. From quation 8), the rate of change of the Shannon entropy of the interfacial charge distribution can be written as

= 1

. By further substituting , one can obtain

d d $= - \sum$ [1

= 1

Substituting quations 3) and (5 8) gives

d d $= - \sum$ [sgn ( ) $\int \partial \partial$ Γ

= 1

interfacial charge distribution. expressed as localized state, we have

= 1

= 1

This utilizes the normalization condition , which corresponds to This equation shows that the evolution rate of the Shannon entropy of the interfacial charge distribution can be written in a form driven by the surface polarization source . Accordingly, the continuous surface charge source in the extended Maxwell equations not only determines the electromagnetic output of the device but also governs the evolution of the uncertainty of the 5.2 Establishment of the energy lower bound of Landauer s principle within the theoretical framework based on extended Maxwell equations The Shannon information entropy evolution of the interfacial surface charge distribution reflects the evolution of the system s uncertainty. When the interfacial charge distribution evolves from a more dispersed state to a more localized state, the uncertainty of the system state decreases, corresponding to an entropy reduction. Let the initial time and final time of an evolution stage be , respectively; the entropy reduction of the interfacial charge distribution can be

Δ red _ M = (0) − ( ) (5 − 12)

When the interfacial charge distribution evolves from a more dispersed state to a more

Δ red _ M > 0 (5 − 13)

This corresponds to an increase in the degree of order of the system. Since Shannon information entropy is measured in bits as the unit of information uncertainty, this entropy reduction can be directly equated to the information decrease of the system during that evolution stage.

Further expanded as

$= \quad [-\sum$

$= 1$

Thereupon

$=$

$= 1$

$=$

$= 1$

scale corresponding to information reduction.

$= 1$

$= 1$

$= 1$

According to quation 31), substituting the entropy reduction expression yields

$= \quad [\ (\ 0\ ) - \ (\ \ )\ ] \ln 2$ (5 − 14)

If the entropy evolution rate form is adopted, then

Δ red _ M $= -\int$ d d

Further substituting the obtained from Eq. (5 10) yields Then rewriting the local charge rate in terms of the surface polarization term yields Thus, a continuous theoretical connection among the surface polarization source term P_s, the local charge state, the state probability distribution, Shannon information entropy, the entropy reduction, and the Landauer energy lower bound is established. The extended Maxwell equations of Academician Zhonglin Wang provide the continuous field physical basis for TENG output, whereas the discretized state probability model of the interfacial state offers a statistical formulation for Shannon entropy analysis; together, they unify the description of the triboelectric nanogenerator s interfacial charge rearrangement as a state probability evolution driven by the surface polarization source term, from which the Landauer principle yields the minimal energy

6 Conclusions

Addressing that existing TENG research emphasizes material properties and macroscopic output while paying insufficient attention to the interplay among interfacial state evolution, information reduction, and energy, this work innovatively introduces Shannon information entropy and the Landauer principle, constructing a theoretical framework of "state probability distribution Shannon information entropy minimum energy bound" to analyze, from a novel interdisciplinary perspective, the extended Maxwell equations proposed by Academician Zhonglin Wang. By discretizing the frictional interface and establishing a local charge state probability model, the continuous evolution of interfacial charge is transformed into changes in probability distributions over a discrete state space, thereby providing a clear mathematical basis for characterizing the evolution from high to low entropy in the system from the perspective of classical information theory.

Our results show that under external mechanical work driving, interfacial charges evolve from dispersed to more clustered distributions, reducing system uncertainty; Shannon information entropy decreases progressively, and the entropy reduction serves as an effective metric of the degree of charge distribution ordering. According to the Landauer principle, this entropy reduction has a definite minimum energy bound, enabling energy quantification of the output formation process on the basis of statistical description. Centered on this, we fuse Shannon information entropy, the Landauer principle, and Academician Zhonglin Wang' s extended Maxwell equations; within the displacement current framework, the interfacial surface charges are treated as surface polarization terms. Through discretization, we establish the correspondences among surface polarization sources, state probability evolution, entropy change, and energy lower bound, thereby rendering the classical information theoretic framework and the Landauer principle compatible within Academician Zhonglin Wang' s extended Maxwell equations framework.

Taken together, this work discards the conventional microscopic thinking in physics and, in a groundbreaking move, adopts a macroscopic perspective based on Shannon information entropy to establish a new theoretical framework for the triboelectric nanogenerator that describes the statistical evolution of interfacial charges, changes in uncertainty, and the ultimate energy constraint; it overcomes the limitations of traditional single electrical dimension analyses, opening an unprecedented interpretive pathw ay for the TENG field and assembling an integrated,

cross disciplinary methodology spanning classical information theory, information physics, and energy science.

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Note: Figure translations are in progress. See original paper for figures.