

Measurement of hard x-ray source size from a bending magnet using the Pendellösung effect

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Abstract

The source size of synchrotron radiation (SR) or free-electron lasers (FEL) is an essential parameter to understand their beam properties. However, measurement of the micron-scale source size faces technical challenges due to the high beam power density and a long distance from the electron bunches. Following the natural x-ray interference in a perfect crystal-Pendellösung effect of dynamical x-ray diffraction, we propose a direct method for telemetering the angular diameter of low-emittance hard x-ray sources. Our method allows one to modulate the interference patterns precisely, and the angular diameter of the source can be derived from the variation of interference fringes. A demonstration experiment was dealt with at the Shanghai Synchrotron Radiation Facility (SSRF), and the result was in accordance with the electron beam size measured independently.

Full Text

Preamble

Measurement of hard x-ray source size from a bending magnet using the Pendellösung effect Changzhe Zhao,¹ Zhongliang Li,^{1,*} and Xiaowei Zhang^{2,†} Shanghai Synchrotron Radiation Facility, Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shanghai 201204, China Institute of High Energy Physics, Chinese Academy of Sciences, Shanghai 100049, China

Abstract

. The source size of synchrotron radiation (SR) or free-electron lasers (FEL) is an essential parameter to understand their beam properties. However, measurement of the micron-scale source size faces technical challenges due to the

high beam power density and a long distance from the electron bunches. Following the natural x-ray interference in a perfect crystal –Pendellösung effect of dynamical x-ray diffraction, we propose a direct method for telemetering the angular diameter of low-emittance hard x-ray sources. Our method allows one to modulate the interference patterns precisely, and the angular diameter of the source can be derived from the variation of interference fringes. A demonstration experiment was dealt with at the Shanghai Synchrotron Radiation Facility (SSRF), and the result was in accordance with the electron beam size measured independently.

Keywords

. Angular diameter; Stellar Interferometer; X-ray dynamical diffraction; Pendellösung effect.

I. INTRODUCTION Low-emittance synchrotron radiation (SR) and free-electron lasers (FEL) are large-scale scientific facilities that generate high-brightness x rays, with applications covering multidisciplinary fields including materials science, chemistry, and biology [1]. As a conserved quantity of light sources, brightness is a core parameter characterizing the performance of such facilities, and its ultra-high brightness originates from the extremely small electron bunch size, the high directivity of relativistic electron bunch radiation (i.e., low emittance), and the spectroscopic properties of undulator radiations.

Since the brightness unit includes a factor of source size [2], measurement of the source size is a fundamental task of each generation of SR and FEL sources. The source size is not only an important parameter for evaluating the performance of source facilities and designing x-ray optical devices, but also a key parameter essential for beam diagnostics of high-performance accelerator-based sources.

Pinhole imaging is a simple method for measuring source size [3-5], which can directly obtain the real-space images of the electron bunches –facilitating bunches tuning and detection.

However, due to the extremely small size and high directivity of the SR and FEL, the pinhole must be

placed as close to the light source as possible to obtain valid imaging results.

Using hard x-ray interference, high-brilliance synchrotron radiation sources such as the European Synchrotron Radiation Facility (ESRF), Japan's Spring-8, and the Advanced Photon Source (APS) have all reported studies on characterizing source size [6-8]. Due to the high penetrability and shortwavelength characteristics of high-energy x rays, conventional pinhole imaging and double-slit interference experiments often face technical challenges during implementation, such as the beam blocking with thicker and smooth edges, the high-precision adjustment of slits separation, or the fabricating micro-devices.

High-precision interferometry for measuring light source size is a long-

established technique, dating back to 1921 when Michelson conducted an experiment to address the challenge of measuring the size of distant stars in astronomy [9]. By observing the change in visibility of stellar interference fringes with adjusting separation of two mirrors, this

experiment

measured Betelgeuse—a approximately 600 light-years from the Earth—with an angular diameter of about 0.047 arcsec. This scale is equivalent to observing a SR source with an angular diameter of approximately $9 \mu\text{m}$ at the experimental station (40 m away).

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In this paper, we utilized the Pendellösung effect —i.e., Laue dynamical x-rays diffraction in a thin perfect crystal —to generate an x-ray interferometer for determining the angular diameter of the SR source precisely. In contrast to the existing methods, the proposed method relies on perfect silicon lattice and does not require any artificial micro-optical elements and high-resolution imaging detectors.

Therefore, it features notable flexibility for the highbrilliance sources and the outstanding costeffectiveness.

$$I_h(q) = I_o \times A [\cosh(m t) - \cos(2p t L)]$$

where

II. METHOD

the Laue case, the incident x rays excite four waves inside the crystal. The two waves in the diffraction direction and the two in the forward diffraction direction interfere with each other. When the crystal is rotated by an angle θ , a rocking curve with intensity oscillations appears, which we refer to as the interference pattern [10]. If the incident x-ray beam is assumed to be a plane wave, the diffracted intensity I_h emitted from the analyzer is given by [11]

As shown in Fig.1, the system consists of three parts:

- 1) a collimator for the modulation of the SR beam divergence, realized by an asymmetric-cut crystal with adjustable asymmetry;
- 2) an analyzer crystal with the same diffraction index as the collimator, for the measurement of beam divergence, via the interference of x rays in a crystal at Laue case;
- 3) an x-ray intensity detector composed of a photodiode and a picoammeter. Compared with the setup of Michelson's stellar interferometer, the three components described above can correspond to the movable outer mirrors for the modulation of the starlight divergence, the optical part for

generating interference fringes, and the detection part, respectively (see Appendix A for a detailed description of the stellar interferometer).

includes an asymmetric-cut crystal with adjustable asymmetry (i.e., the collimator), an analyzer based on the Laue diffraction, and a detector for measuring rocking curve of x-ray diffraction. The x-ray interference patterns are observed behind the analyzer because of overlap of excited wavefields (marked orange).

A. Observed beam divergence w_0 Unlike interferometer in the visible light domain, the x-ray interferometer (i.e., the analyzer) in this report utilizes the Pendellösung effect of x-ray dynamical diffraction in a perfect silicon crystal. In
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$$A = \exp(-2\mu_0 t \cos \theta_B) \frac{1}{1 + h}$$

$$(1 + h)$$

$$m = m_0 \cos \theta_B$$

and I_0 is the incident intensity, θ_B is the Bragg angle of diffraction, t is the crystal thickness, μ_0 is the linear absorption coefficient of the crystal; Λ_L is the Pendellösung distance, whose value is related to the incident photon energy E and the crystal structure factor F_h [10]. The angular deviation parameter η , which is closely related to the incident angle θ , is

where ω is the Darwin width of dynamical diffraction.

The above expressions show that the intensities of the diffracted beams are oscillating functions of the incident angle θ . Moreover, these oscillating structures are extremely sensitive to the divergence of the incident beam.

Due to the long distance between the source and the experiment and the small source size, the incident beam can be treated as a local plane wave [12].

Therefore, if the divergence of the incident beam is considered, the diffraction rocking curve can be expressed as

where the function $h(\theta)$ describes the angular distribution of the incident beam. We assume that the function $h(\theta)$ follows Gaussian distribution where $\rho = w_0/2$ and w_0 is the divergence of the incident beam perceived by the analyzer. When $w_0 \ll \omega$, the oscillating structures of the rocking curve can be obtained at experiment. Based on the mentioned dynamical diffraction formulae, we can directly and accurately evaluate the beam divergences w_0 (i.e., the

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observed divergence) by fitting the measured interference patterns.

B. Modulated beam divergence w_m

C. Angular diameter from the source size w_s

According to the above, the visibility of the interference patterns can be changed by adjusting the beam divergence. In this report, the adjustment of the x-ray

beam divergence relies on the asymmetric diffraction of a silicon crystal, because the asymmetric diffraction can compress the divergence of diffracted beams in the direction parallel to the plane of diffraction defined by the incident beam and diffracted beam [13,14].

The beam divergence induced by the finite size of the source is referred to as the angular diameter of the source, which is included in the observed beam divergence. In general, the angular diameter of the source is a small quantity and difficult to measure directly. To decouple the angular diameter of the source, the high-precision collimator described above was employed. Here, the collimator restricts the beam divergence by modulating only its intrinsic component, without influencing the contribution arising from the finite source size. We first assume that the employed crystals have perfect lattice herein.

Thus, we can define w_s as the quadrature subtraction of the modifiable beam divergence from the total observed beam divergence,

$$w_s = (w_o^2 - w_m^2)^{1/2}$$

asymmetric diffraction, where α is the angle between the crystal surface and the lattice planes, and the ϕ axis is perpendicular to the diffracting plane.

As shown in Fig. 2 [Figure 2: see original paper], α is the asymmetric angle between the crystal surface and the lattice planes in the plane of diffraction. To achieve continuous modulation of the diffracted beam divergences, we introduce a ϕ axis perpendicular to the lattice planes: when the ϕ angle is changed, the asymmetric angle α in the diffraction plane changes accordingly, but the Bragg diffraction condition can remain unchanged.

Therefore, the asymmetry factor b of the collimator with the adjustable divergence depends on the effective asymmetric angle α' , and its expression is

$$\tan \alpha' = \tan \alpha \times \cos \phi$$

The asymmetry factor b is given by $b = \sin(\alpha' - \alpha) / \sin(\alpha' + \alpha)$

when the modulated beam divergence w_m approaches 0 (i.e., the b factor approaches 0), the observed beam divergence is exactly the beam divergence caused by the finite source. Therefore, the angular diameter w_s from the x-ray source size can be obtained.

III. EXPERIMENTAL RESULTS The collimator adopts Bragg asymmetric diffraction of a Si (220) crystal. To approach the modulation limit, the maximum asymmetry angle α of the collimator was accurate to 8.34° at $\phi = 0^\circ$. For the detailed measurement of the asymmetry angle, refer to Appendix

B. Fig. 3 [Figure 3: see original paper] shows the calculated asymmetry factor b at the photon energy of 21.90 keV. When the ϕ angle changes from 0° to $\pm 90^\circ$, the b factor can continuously vary from 0.008 to 1.

Therefore, high-precision modulation of the beam divergence incident on the analyzer can be achieved by adjusting the ϕ angle.

Assuming that the beam divergence w from a point source incident on the collimator, the divergence of diffracted beam can be expressed as $w_m = b \times w_p$. Herein, we denote w_m as modulated beam divergence.

It should be noted that w_m differs with w_o : w_m represents the ideal value obtained without considering the source size, while w_o denotes the observed value in practice. *Contact author: lizhongliang@sari.ac.cn

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of ϕ for a Si (220) crystal, with an asymmetry angle of 8.34° at the photon energy 21.90 keV.

The experiment was conducted at the SSRF beam line 09B. This beamline was equipped with optical devices highly suitable for multi-crystal diffraction experiments [15,16]. A photon energy 21.90 keV was selected by a silicon (311) monochromator. The collimator was an asymmetric cut flat silicon, with a distance of approximately 39.5 m from the x-ray source. The analyzer with thickness of approximately 0.35 mm was placed approximately 0.3 m downstream of the collimator with a diffraction index of (220), configured as shown in Fig. 1 [Figure 1: see original paper]. A slit placed between the collimator and the analyzer was used to select the beam size (2 mm \times 3 mm) incident on the analyzer, which ensures a highly uniform thickness of the analyzer crystal. The x-ray detector was placed approximately 0.2 m downstream of the analyzer to record intensity of the rocking curve; the recording process adopted the fly-scan mode, with an angular velocity of 0.025 arcsec/s, and the acquisition of a complete curve took approximately 200 s.

with b parameter. (a)-(d) The experimental data (marked with circles) when the b factors are 0.008, 0.036, 0.050, and 0.075, respectively. The red curves are the theoretical calculations derived using Eq. (3) where $\Delta\theta = \theta - \theta_B$.

expressed by the reflectivity I_c/I_o when the b factors are 0.008, 0.033, 0.050, and 0.075, for which the observed beam divergences are $0.105''$, $0.141''$, $0.193''$, and $0.274''$, respectively. Among them, the pattern in Fig. 4 Figure 4: see original paper exhibits the highest visibility with highly collimated beams, and more than 80 identifiable peaks can be observed across the entire pattern. The high perfection of the rocking curve reflects the high quality of the crystal and the high accuracy of the angular turntable. Furthermore, these interference patterns were fitted by Eq. (3) and the obtained results have an uncertainty of approximately 3% (see Appendix C for the interference patterns fitting with error analysis). The high consistency between the calculated curves and the experimental data further confirms that the collimator has an effective modulation for the beam divergence, and the Pendellösung fringes of the analyzer have sufficient sensitivity.

With the aim of obtaining the divergence w_s arising from the finite source, we

follow the method of Michelson [9], and introduce a visibility V as the evaluation criterion for the interference patterns,

where I_1 and I_2 are the maximum and minimum values near the Bragg angle θ_B . As shown in Fig. 5 [Figure 5: see original paper], we plot the visibility of interference patterns versus the modulated beam divergence. The solid lines represent the theoretical calculations when w_s is $0''$, $0.07''$, $0.10''$, and $0.13''$, respectively, while the experimental data corresponding to the b factor are marked with blue dots. Notably, the maximum visibility of the interference patterns is only 0.98 in this case due to the characteristics of dynamical diffraction. Affected by the angular diameter of the source, the visibility of the interference patterns decreases, and this reduction is most significant when the modulated beam divergence approaches zero.

Importantly, the experimental data show high agreement with the theoretical pattern at $w_s = 0.10''$, based on which the angular diameter of the source

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can be estimated, and the corresponding relationship between the b factor and w_m can be established.

intercept of fitting curve is the angular diameter $w_s = 0.100 \pm 0.003$ arcsec. Combined with the distance of approximately 39.8 m between the analyzer and the SR source, the vertical size of the source can be estimated as 19.3 ± 0.6 μm . This result is in accordance with the data (18 μm) provided by the electron beam measurement of SSRF at the time of the experiment (see Ref. [17] for details on the measurement).

IV. DISCUSSION

the modulated beam divergence. The solid lines represent the theoretical calculations under several angular diameters of the source, respectively; the experimental data are plotted as blue dots with error bars, and each dot corresponds to its b factor.

To more intuitively and accurately quantify the angular diameter of the SR source, Fig. 6 [Figure 6: see original paper] plots the relationship between the observed and the modulated beam divergences, as well as the predicted curve for a point source ($w_s = 0''$, dashed line). When the beam

function of modulated beam divergence, where the experimental data are represented by blue dots with error bars. divergence on the analyzer is large, the observed values exhibit a linear relationship with the predicted curve. However, when the beam divergence is modulated to small values, the observed values begin to deviate from those predicted for a point source.

According to the analysis above, this deviation is caused by angular diameter of the finite source. The experimental data are fitted by Eq. (7), and the y -

Although pinhole imaging in real space is simple and convenient, it is difficult to accurately measure the x-ray source size when the source is extremely small due to technical constraints. In contrast, the interferometry in k-space have opened up a new path for source size measurement, so many SR facilities have used various interference methods for measurement [18-22]. However, almost all experimentally reported schemes so far are based on the methodology of the visible light, while experimental designs based on crystals and x-ray dynamical diffraction are relatively rare [23].

In this report, we faithfully project the mechanism of the Michelson' s stellar interferometer to the x-ray domain. The double mirrors for modulating the divergence of the incident beam corresponds to adjustable diffraction of the asymmetric-cut crystal, the interferometer to the Laue diffraction of a thin analyzer crystal, and the photograph of interference to the rocking curve. The visibility interference patterns contains information on the source size. Compared with the conventional methods, the angular diameter of the source as a physical quantity can be directly measured via the Pendllösung effect. The high agreement between theoretical calculation, experimental result and electron bunch size measurement validates the performance of hardware and the rationality of our model.

To further validate the effectiveness of this method and evaluate its application potential, a quantitative analysis was performed about the lattice strain and thickness uniformity on the measurement, as well as the system resolution.

A. Evaluation of lattice strain To ensure a high visibility (>0).

- 9) of the interference patterns at 21.9 keV photon energy, the analyzer crystal thickness should be maintained at a relatively small value ($\sim 346 \mu\text{m}$). To mitigate strain induced in thin crystals during both fabrication and

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experiment, we optimized the geometric structure of the analyzer crystal and adopted strong acid etching to remove the damaged silicon layer. Rocking curves with oscillatory structures enable quantitative strain analysis of thin crystals. From the experimentally obtained optimal-visibility interference pattern (Fig. 4(a)), the relative angular positions can be extracted corresponding to each diffraction order.

We plotted the relationship between the peak angular positions and their diffraction orders, as shown in Fig. 7 [Figure 7: see original paper]. Notably, the peak positions exhibit an abrupt shift near the Bragg angle owing to the π phase change in the diffraction [10]. By contrast, they show a linear variation with diffraction orders at off-Bragg angles. Defining the linear fitting slope in the positive interval as k_1 and that in the negative interval as k_2 , their deviation δs induced by the lattice strain was approximately 5×10^{-4} arcsec. From the differential form of Bragg' s law [24],

measurements, $\sim 1\%$ of the effective area of the analyzer was selected for the experiment.

Furthermore, we adopted the chi-square fitting detailed in Appendix C and performed a series of thickness-dependent error analyses. Fig. 8 [Figure 8: see original paper] shows the chi-square distribution around the minimum at $\phi = 0$.

From the contour lines of the two-dimensional surface, the correlation between t and ω exerts a relatively weak influence on curve fitting.

$$Dd = -d s / \tan \alpha B$$

the uniformity of the silicon crystal lattice spacing was thus determined to be 1.6×10^{-8} . This conclusion applies only to the $2 \text{ mm} \times 3 \text{ mm}$ region of the analyzer crystal. In addition, the analyzer was fixed at the rotation center during the experiment, with its effective area essentially unchanged. Accordingly, the lattice strain was deemed a negligible quantity in the beam divergence measurement.

Further, Table I lists the optimal fitting thicknesses and their corresponding uncertainty for all ϕ , where δt represents the projection of thickness uncertainty onto the ω axis. The error analysis indicates that the crystal thickness shows a slight deviation at a single ϕ value, and a thickness variation of $\sim 1 \mu\text{m}$ across all ϕ angles has no appreciable effect on the accuracy of beam divergence measurements.

TABLE

I. Summary of optimal fitting thicknesses and uncertainty at all ϕ .

positions of peaks and diffraction orders for the interference pattern with maximum visibility.

B. Crystal uniformity As mentioned previously, poor overall thickness uniformity is also suffered from the fabrication of thin crystal. We note that it is technically challenging to achieve precision polishing of large-area thin crystals while maintaining a low-strain state. To minimize the impact of thickness uniformity on the

ϕ (deg)

t (μm)

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uncertainty (μm)

C. System resolution

δt (") 8.2×10^{-5} 7.8×10^{-5} 5.8×10^{-5} 5.8×10^{-5} 5.6×10^{-5} 5.7×10^{-5} 5.9×10^{-5} 5.6×10^{-5} 5.7×10^{-5} 5.5×10^{-5} 5.2×10^{-5} 5.2×10^{-5} 5.7×10^{-5}

System resolution is directly related to the sensitivity of interference patterns to beam divergence. Thus, it can be simply quantified by the average peak-to-valley spacing δr of the interference patterns. And the system resolution adopted in this report is $0.07''$. Flexible selection of higher-index diffraction from the analyzer or an increase in photon energy is expected to further improve system resolution.

As shown in Fig. 9 [Figure 9: see original paper] (a), we numerically calculated the functional relationship between $1/\delta r$ and photon energy E for the Si(220) analyzer crystal.

To match the high photon energy, the normalized crystal thickness was kept constant at

- 7.
8. As photon energy increases, the sensitivity of interference patterns to beam divergence is enhanced, and the system resolution improves accordingly.

Similarly, Fig. 9(b) presents the relationship between $1/\delta r$ and the structure factor of lattice planes at the photon energy of 21.9 keV and the crystal thickness $t = 346 \mu\text{m}$. Higher lattice indexes can also improve system resolution.

function of structure factor with the photon energy 21.9 keV and the crystal thickness $346 \mu\text{m}$.

In practice, all experimental parameter shall be mutually matched and coordinated to realize the desired measurement goals. For instance, when measuring a hard x-ray source with a size of $5 \mu\text{m}$ at a distance of 40 m, a Si(440) crystal can be selected as the analyzer with the photon energy set to 43.8 keV. Under these conditions, the Bragg angle θ_B remains constant, and the basic optical path depicted in Fig. 1 is unchanged. With the crystal's normalized thickness fixed at 7.54, the corresponding actual thickness is calculated to be 1.1 mm. Considering the strain and thickness uniformity of the existing crystals, the measurement error is approximately 3%.

This value is a conservative estimate, because the strain and thickness uniformity of the 1.1 mm-thick crystal are both superior to those of the existing crystals. If the existing rotary stage is equipped with 1/2 division, its angular accuracy can reach 2.5×10^{-3} arcsec, yielding a measurement error less than 10%. Furthermore, a rotary stage with the flexure hinge can be used to replace the current worm wheel-worm mechanical rotary stage for further improvement of angular accuracy. The above estimations show the proposed method enables highprecision characterization of small-sized source with great application potential.

V. CONCLUSIONS

The inverse of the average peak-valley spacing of the interference patterns as a function of photon energy with the analyzer Si(220) and the normalized crystal thickness $t/\Lambda L =$

- 7.
8. (b) The inverse of the average peak-valley spacing of the interference patterns as a

In conclusion, we proposed a direct method to telemetering the angular diameter of SR based on the Pendellösung effect of x-ray dynamical diffraction, which offers a novel idea for high-energy x-ray interferometry. We verified the feasibility of this method at beamline 09B of SSRF, and the experimental data showed high agreement with the xray dynamical diffraction that incorporates the beam divergence. We discussed the measurement accuracy of this method and clarified its potential for further application. The proposed method offers notable advantages –high resolution, low cost, high operability and flexibility –particularly suited for high-energy x-ray sources measurements. In view of the development of advanced x-ray facilities, this method appears to be quite promising and notable value.

APPENDIX A: Michelson' s Stellar Interferometer

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The essence of the Michelson' s Stellar Interferometer is to deduce the angular diameter of a star based on the visibility of interference fringes by modulating the divergence of incident starlight. As shown in Fig. A1, the interferometer mainly consists of three parts [5]: first, the outer mirrors (M1 and M4) for modulating the divergence of starlight; second, the inner mirrors for generating interference–light beams thence over the paths a, b, c, and d, in the telescope, and form interference fringes at the Cassegrain focus d; third, an eyepiece for recording interference fringes. Among them, the two inner mirrors (M2 and M3) are fixed to keep the fringe spacing constant, while the pair of outer mirrors (M1 and M4) are movable to change the fringes visibility.

The maximum distance of M1 and M4 determines the minimum angular diameter measurable by this facility.

FIG. A1. Michelson' s stellar interferometer [5]. APPENDIX B:

Measurement of asymmetry angle of collimator To approach the limit of angular modulation and thereby obtain diffracted beam with a smaller divergence, it is necessary to accurately measure the asymmetry angle α of the collimator. The measurement setup is shown in Fig. A2, which also utilizes the asymmetric diffraction of the collimator.

However, unlike the ϕ axis in Fig. 2, the rotation axis ϕ' in Fig. A2 is perpendicular to the crystal surface where

$$q_{\text{out}} = q_{\text{B}} + a$$

At $\phi' = 0^\circ$ and $\phi' = 180^\circ$, θ_{in} and θ_{out} are obtained respectively from the peak positions of the rocking curves, and then the asymmetry angle α can be

calculated using Eqs. (A1) and (A2):

$$a = (q_{\text{out}} - q_{\text{in}}) / 2$$

From Eq. (A3), the α can be obtained only by accurately measuring the relative value of θ_{in} and θ_{out} .

Therefore, we use a rotation stage with a large range and high resolution to ensure the peak position of the

rocking curves at $\phi' = 0^\circ$ and $\phi' = 180^\circ$. In the measurement process, the mechanical error of the rotation stage is $\leq 0.002^\circ$, and the influence of x-ray refraction is approximately 1.2×10^{-4} deg; thus, the measurement accuracy of the asymmetry angle can be achieved to the hundredths place of a degree. In conclusion, α can be precisely determined to 8.34° .

FIG. A2. Measurement of asymmetry angle of collimator.

APPENDIX C: The interference patterns fitting The interference patterns are a function of the beam divergence w_0 and the crystal thickness t .

Specifically, w_0 primarily affects the visibility of the interference patterns, while the peak spacings of the curves are more sensitive to the crystal thickness.

Therefore, both parameters w_0 and t should be considered in the interference patterns fitting. The maximum likelihood estimate of the parameters is obtained by minimizing the quantity

$$\chi^2 = \sum_i (I_i - I_c(q_i; w_0, t))^2 / \sigma_i^2$$

called the “chi-square”, where I is the experimental data, I_c is the theoretical pattern, σ is the measurement error (standard deviation) derived from the minimum value of χ^2 , and the confidence region on w_0 and t is set to 95.4%.

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2. SSRF.

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Data Availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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