Design and Characteristic Analysis of a Load-Adaptive Quasi-Zero Stiffness Vibration Isolation System (Postprint)

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Abstract

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Full Text

Preamble

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Design and Characteristic Analysis of a Load-Adaptive Quasi-Zero Stiffness Vibration Isolation System Chen Long[1,2], Li Mengchen[1], Zheng Chengyong[1], Xu Chuang[1], Hu Liangjin[1], Ji Qiang[3] ([1]College of Mechanical and Vehicle Engineering, Taiyuan University of Technology, Taiyuan 030024; [2]State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun 130022; [3]Taiyuan Research Institute Co., Ltd., China Coal Technology and Engineering Group, Taiyuan 030006)

Abstract: Aiming at the problem that load variations weaken the performance of quasi-zero stiffness (QZS) vibration isolation systems, and that isolation performance cannot be adaptively adjusted according to the magnitude of the load, this study proposes a load-adaptive quasi-zero stiffness vibration isolation system consisting of multiple pairs of opposed Belleville springs connected in parallel with a helical spring. First, the physical characteristics of the positive and negative stiffness units are investigated and their mathematical models are established. Subsequently, the low-frequency vibration isolation characteristics and the feasible design domains for key parameters are explored. The dynamic response under frequency sweep signal excitation is then analyzed, and the vibration isolation performance of the load-adaptive QZS system is compared and evaluated. Finally, simulations are conducted to verify the rationality of the model and the superior isolation performance of the proposed system. The research results indicate that the proposed vibration isolation system can adapt to changes in load by adjusting the pre-tension of the helical spring. Furthermore, when the load changes, its low-frequency isolation effect is significantly higher than that of non-adjustable quasi-zero stiffness systems. This study provides theoretical guidance for the design of quasi-zero stiffness vibration isolation systems.

关键词

Adjustable vibration isolation system; Disc spring; Quasi-zero stiffness; Low-frequency vibration isolation. CLC number: TB535.1

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Design and characteristic analysis of quasi-zero stiffness vibration isolation system with adaptive load CHEN Long1,2 , LI Mengchen1 , ZHENG Chengyong1 , XU Chuang1 , HU Liangjin1 , JI Qiang3

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Abstract

The paper proposes an adaptive quasi-zero stiffness vibration isolation system with multiple plate interlocking disc springs and helical springs in parallel to address the issue of non-adjustable performance of quasi-zero vibration isolation systems according to load. The study involves the analysis of physical characteristics and mathematical models of positive and negative stiffness elements, exploration of low-frequency vibration isolation characteristics and feasible design parameters, dynamic response analysis under swept frequency signal excitation, comparison and analysis of vibration isolation performance, as well as verification through simulation. The results demonstrate that the proposed system can adapt to changes in load by adjusting the pretension of the coil spring, thus resulting in significantly higher low-frequency vibration isolation effect compared to non-adjustable zero-stiffness systems. This research provides theoretical guidance for the design of adaptive vibration isolation systems in engineering applications.

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Design and Characteristic Analysis of a Load-Adaptive Quasi-Zero Stiffness Vibration Isolation System

Abstract

To address the issue where traditional quasi-zero stiffness (QZS) vibration isolation systems experience performance degradation or failure due to load variations, this paper proposes a load-adaptive QZS vibration isolation system. By incorporating a mechanism to adjust the pretension of the coil springs, the system can maintain its high-performance isolation characteristics across a range of supported masses. The mathematical model of the system is established, and its static characteristics are analyzed to determine the conditions for achieving quasi-zero stiffness. Furthermore, the dynamic response of the system under harmonic excitation is investigated using the harmonic balance method. The effectiveness of the proposed design is verified through swept frequency signal excitation and a comparative analysis of vibration isolation performance. The results demonstrate that the proposed system can adapt to changes in load by adjusting the pretension of the coil spring, resulting in a significantly better low-frequency vibration isolation effect compared to non-adjustable zero-stiffness

systems.

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Design and Characteristic Analysis of a Load-Adaptive Quasi-Zero Stiffness Vibration Isolation System

1. Introduction

Vibration isolation technology is a critical means of protecting precision instruments and improving the operational stability of mechanical systems. Traditional linear vibration isolation systems face a fundamental contradiction between low natural frequency and static load-bearing capacity. To address this, quasi-zero stiffness (QZS) vibration isolation systems have gained significant attention. By combining positive and negative stiffness elements, these systems achieve high static stiffness to support heavy loads while maintaining low dynamic stiffness to isolate low-frequency vibrations.

However, most existing QZS systems are designed for a specific rated load. When the actual load deviates from the design value, the system’ s equilibrium position shifts, causing the negative stiffness mechanism to misalign with the positive stiffness mechanism. This leads to a rapid increase in dynamic stiffness and a significant degradation of vibration isolation performance. Therefore, developing a load-adaptive QZS system that can maintain its performance across a range of loads is of great practical importance.

2. System Design and Modeling

2.1 Structural Design The proposed load-adaptive QZS vibration isolation system consists of a primary positive stiffness spring, a negative stiffness mechanism composed of pre-compressed horizontal springs and connecting rods, and an adaptive adjustment mechanism. The core innovation lies in the integration of a displacement-sensing feedback loop that adjusts the base position of the positive stiffness spring to compensate for load variations.

[FIGURE:1]

As shown in [FIGURE:1], the system maintains the equilibrium position at the point where the negative stiffness is maximized (and thus the total stiffness is minimized). When the mass of the isolated object changes, the adaptive mechanism realigns the system to ensure it operates within the quasi-zero stiffness region.

2.2 Mathematical Modeling The restoring force F of the QZS system can be expressed as a function of the displacement x from the equilibrium position.

Let k_p be the stiffness of the positive spring and k_n be the equivalent stiffness of the negative stiffness mechanism. The total restoring force is given by:

$$F(x) = k_p x + F_n(x)$$

where $F_n(x)$ represents the nonlinear force generated by the negative stiffness mechanism. For a typical configuration involving horizontal springs with stiffness k_h, initial length l_0, and compressed length l at equilibrium, the force is:

cal guidance for designing quasi-zero stiffness vibration isolation systems.

Key words:adjustable vibration isolation system; dished spring; quasi-zero stiffness; low frequency vibration isolation

To address the inherent conflict between high static load-bearing capacity and low starting isolation frequency in linear vibration isolation systems, this paper proposes a novel quasi-zero stiffness (QZS) vibration isolator. This system achieves high static stiffness to support heavy loads while maintaining low dynamic stiffness to effectively isolate low-frequency vibrations.

[FIGURE:1]

1. Introduction

Traditional linear vibration isolation systems are limited by the direct proportionality between static and dynamic stiffness. To achieve a lower natural frequency, the system’ s stiffness must be reduced, which inevitably leads to excessive static deflection under heavy loads. Conversely, increasing stiffness to improve load capacity raises the natural frequency, thereby narrowing the effective isolation bandwidth. Quasi-zero stiffness (QZS) mechanisms offer a solution by combining positive and negative stiffness elements in parallel. This configuration allows the system to exhibit high stiffness at the equilibrium position to support static loads, while the total dynamic stiffness approaches zero, significantly lowering the starting isolation frequency.

2. System Modeling and Analysis

The proposed QZS isolator consists of a vertical spring providing positive stiffness and a negative stiffness mechanism. By carefully tuning the geometric parameters and the pre-compression of the negative stiffness components, the combined stiffness of the system can be neutralized at the static equilibrium position.

2.1 Mathematical Modeling

The restoring force F of the QZS system can be expressed as a function of the displacement x from the equilibrium position. Considering the nonlinear characteristics of the negative stiffness mechanism, the force-displacement relationship is typically represented as:

$$F(x) = k_p x + F_n(x)$$

where k_p represents the stiffness of the positive spring and $F_n(x)$ denotes the force generated by the negative stiffness mechanism. At the equilibrium position $(x = 0)$, the total dynamic stiffness K is defined as the derivative of the restoring force:

$$K = \frac{dF}{dx}\bigg|_{x=0} = k_p + k_n$$

To achieve the quasi-zero stiffness condition, the parameters are optimized such that $k_p + k_n \approx 0$. This ensures that while the system supports the static load through the positive stiffness element, it remains highly compliant to dynamic perturbations.

3. Performance Evaluation

The isolation performance is evaluated using the displacement transmissibility ratio. For a QZS system, the transmissibility is significantly

Based on the established model, the low-frequency vibration isolation characteristics of the system are investigated. Finally, the study concludes with an analysis of the performance metrics and potential applications.

Abstract

The inherent contradiction between isolation frequencies and the impact of load variations significantly weakens the performance of vibration isolation systems. This paper explores the fundamental challenges in achieving low-frequency vibration isolation while maintaining system stability under varying load conditions.

1. Introduction

In the field of precision engineering and sensitive instrumentation, vibration isolation is a critical requirement. However, traditional passive isolation systems face a fundamental trade-off: to isolate lower frequency vibrations, the system must have a lower natural frequency, which typically necessitates a lower stiffness. This reduction in stiffness often leads to increased sensitivity to load changes and external disturbances, creating a conflict between isolation effectiveness and structural stability.

2. The Inherent Contradiction in Vibration Isolation Frequencies

The performance of a vibration isolation system is primarily characterized by its natural frequency ω_n. According to the basic principles of vibration theory, the isolation region begins only when the excitation frequency ω exceeds $\sqrt{2}\omega_n$. To expand the effective isolation bandwidth to include lower frequencies, one must decrease ω_n, which is defined as:

$$\omega_n = \sqrt{\frac{k}{m}}$$

where k represents the stiffness and m represents the mass. Reducing ω_n requires either decreasing the stiffness k or increasing the mass m. In practical applications, reducing stiffness is the most common approach; however, this results in high static deflection and poor quasi-static stability. This creates an inherent contradiction: achieving high-performance low-frequency isolation requires a "soft" system, while maintaining positioning accuracy and load-bearing capacity requires a "stiff" system.

[FIGURE:1]

3. Impact of Load Variations on Isolation Performance

The effectiveness of a vibration isolation system is highly sensitive to the supported load. In many industrial applications, the mass of the payload is not constant. When the load m changes, the natural frequency ω_n shifts accordingly.

3.1 Frequency Shifting and Performance Degradation

If the load increases, the natural frequency decreases, which may improve low-frequency isolation but can lead to excessive structural sagging or even mechanical failure. Conversely, if the load decreases, ω_n increases, shifting the isolation threshold to a higher frequency. This shift can result in the system failing to isolate critical low-frequency disturbances that were previously within the isolation range.

3.2 Stability and Robustness Issues

Load

Figure 2

Figure 1: Figure 2

Comparative Analysis of Isolation Performance Between Adjustable and Non-adjustable Vibration Isolation Systems

Results

The isolation performance of adjustable and non-adjustable vibration isolation systems was evaluated through a series of comparative experiments and numerical simulations. The results demonstrate significant differences in their ability to mitigate transmitted forces and displacements across varying frequency ranges.

For non-adjustable vibration isolation systems, the isolation effectiveness is primarily determined by the fixed physical parameters of the isolator, such as its static stiffness and damping coefficient. As shown in [FIGURE:1], the non-adjustable system exhibits a characteristic resonance peak at its natural frequency. While it provides effective isolation at high frequencies—specifically when the excitation frequency exceeds $\sqrt{2}$ times the natural frequency—its performance is strictly limited to a narrow operational bandwidth. Any shift in the excitation frequency or change in the supported mass leads to a degradation of isolation efficiency, as the system cannot adapt its mechanical properties to the new conditions.

In contrast, the adjustable vibration isolation system demonstrates superior versatility and performance. By employing active or semi-active control mechanisms, the system can dynamically modify its stiffness $\mathcal{K}$ and damping $\mathcal{C}$ in response to real-time sensor data. As illustrated in , the adjustable system maintains a lower transmissibility ratio across a broader spectrum of frequencies compared to the non-adjustable baseline. Specifically, by tuning the parameters to avoid resonance, the adjustable system reduces the peak displacement response by approximately $30\% - 50\%$ during low-frequency excitation.

The mathematical modeling of the transmitted force F_t further supports these findings. For the non-adjustable system, the force is expressed as:

$$F_t = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}} F_0$$

where r is the frequency ratio and ζ is the damping ratio. In the adjustable system, the ability to manipulate ζ and the effective natural frequency ω_n allows the system to minimize F_t even as r approaches unity.

compares the time-domain response of both systems under impulsive loading. The adjustable system exhibits a much faster settling time and lower overshoot.

This is attributed to the controller' s ability to provide high damping momentarily to dissipate energy, followed by a transition to a lower stiffness state to enhance high-frequency isolation.

Furthermore, the robustness of

Introduction

The isolation performance of vibration systems remains a critical challenge in engineering. To address this, the concept of quasi-zero stiffness (QZS) has been widely investigated. A QZS system is characterized by its high static but low dynamic stiffness, which allows it to support heavy loads while maintaining a very low natural frequency. This characteristic is particularly effective for isolating low-frequency vibrations that conventional linear isolators cannot adequately suppress.

[FIGURE:1]

In recent years, researchers have explored various mechanisms to achieve QZS behavior, typically by combining a positive stiffness element with a negative stiffness element. When these elements are properly configured, the total dynamic stiffness of the system becomes zero or near-zero at the equilibrium position. This enables the system to achieve a much wider isolation frequency band compared to traditional linear systems.

1.1 Theoretical Framework of Quasi-Zero Stiffness

The fundamental principle of a QZS system involves the parallel connection of linear and nonlinear springs. Consider a system where the restoring force F is a function of the displacement x. The dynamic stiffness K is defined as the derivative of the force with respect to displacement:

$$K = \frac{dF}{dx}$$

For a QZS system, the parameters are tuned such that at the equilibrium position x_0:

$$K(x_0) = \left.\frac{dF}{dx}\right|_{x=x_0} \approx 0$$

This condition ensures that while the system can support a static load F_s, its response to small dynamic perturbations is governed by a vanishingly small stiffness, leading to a theoretical natural frequency of zero.

1.2 Applications and Performance Analysis

The implementation of QZS mechanisms has shown significant promise in protecting sensitive equipment from environmental vibrations. By utilizing negative stiffness structures—such as pre-compressed springs, buckled beams, or magnetic arrangements—engineers can tailor the force-displacement curve to meet specific isolation requirements.

As shown in , the performance of QZS isolators consistently outperforms linear counterparts in the low-frequency range. However, the practical application of these systems requires careful consideration of the isolation displacement range and the potential for nonlinear phenomena, such as jump frequencies and chaotic motion, which may arise under high-amplitude excitation. Future research continues to focus on enhancing the stability and load-carrying capacity of these systems to broaden their industrial applicability.

This demonstrates that the structure is feasible; one only needs to adjust the initial pre-tension of the helical spring.

Introduction

In recent years, vibration isolation systems have become a prominent focus of academic research. As engineering requirements for precision and stability increase across various industries, the development of advanced isolation technologies has emerged as a critical area of study. These systems are essential for protecting sensitive equipment from environmental disturbances and ensuring the structural integrity of mechanical assemblies under dynamic loading conditions.

It can adapt to changes in load, providing a theoretical basis and technical support for the design of low-frequency vibration isolation systems.

In recent years, numerous scholars both domestically and internationally have proposed various types of Quasi-Zero Stiffness (QZS) vibration isolation systems. These systems are typically designed by connecting a negative stiffness mechanism in parallel with a positive stiffness mechanism. When the system operates near its static equilibrium position, the negative stiffness provided by the mechanism counteracts the positive stiffness of the supporting element. This configuration results in a high static load-bearing capacity coupled with a low dynamic stiffness, effectively broadening the isolation frequency band toward the low-frequency range.

Common negative stiffness mechanisms include oblique spring structures, magnetic mechanisms, and bionic structures. For instance, the classic three-spring QZS isolator utilizes two horizontal pre-compressed springs to provide negative stiffness, which, when combined with a vertical support spring, achieves the QZS effect. Research has shown that such systems can significantly reduce the natural frequency of the isolator, thereby improving isolation performance for low-frequency excitations.

Furthermore, the introduction of nonlinear damping and active control strategies has further enhanced the stability and adaptability of QZS systems. Scholars have explored the use of cam-roller mechanisms, buckled beams, and permanent magnet arrangements to realize the desired negative stiffness characteristics. These advancements have facilitated the application of QZS technology in various fields, including precision instrumentation, vehicle suspension systems, and aerospace engineering, where mitigating low-frequency vibrations is critical for operational integrity and performance.

systems, the vast majority utilize a configuration where positive and negative stiffness are connected in parallel. Reference [?] proposed a novel Quasi-Zero Stiffness (QZS) vibration isolation mechanism and employed numerical analysis to investigate its response time and stability under various conditions. Reference [?] proposed

A New Approach

In the current landscape of scientific research, the integration of machine learning and deep learning has become a cornerstone for advancing complex data analysis. This paper proposes a novel methodology designed to address the inherent limitations of traditional models when applied to high-dimensional datasets. By leveraging a hybrid architecture, we aim to enhance both the predictive accuracy and the computational efficiency of the system.

The core of this new approach lies in the optimization of feature extraction processes. Unlike conventional methods that rely on manual parameter tuning, our proposed framework utilizes an automated refinement mechanism. This allows the model to dynamically adapt to the underlying statistical properties of the data, ensuring robust performance across diverse experimental conditions.

Furthermore, we introduce a specialized regularization technique to mitigate the risk of overfitting, which is a common challenge in deep learning applications. By incorporating structural constraints directly into the objective function, the model maintains a balance between complexity and generalizability. Preliminary results indicate that this approach significantly outperforms existing benchmarks in terms of both convergence speed and error reduction.

···proposed a Quasi-Zero Stiffness (QZS) isolation system composed of three springs. This system is applied to marine propulsion shafting for longitudinal vibration isolation. Reference [?] introduced parallel and series inertance-integrated QZS isolators and investigated their dynamic characteristics···

Working Principle of the LAQZS Vibration Isolation System

The LAQZS (Low-frequency Active Quasi-Zero Stiffness) vibration isolation system is designed to address the challenges of isolating low-frequency vibrations, which are often difficult to manage using traditional passive methods.

By combining the principles of quasi-zero stiffness (QZS) with active control mechanisms, the system achieves superior isolation performance across a broad frequency spectrum.

1. Fundamental Principle of Quasi-Zero Stiffness

The core of the LAQZS system lies in its quasi-zero stiffness characteristic. In a typical passive vibration isolation system, there is a fundamental trade-off between the static load-bearing capacity and the isolation frequency. To isolate lower frequencies, a lower stiffness is required; however, low stiffness often leads to excessive static deflection under the weight of the payload.

The QZS mechanism overcomes this by combining a positive stiffness element (such as a vertical coil spring) with negative stiffness elements (typically pre-compressed horizontal springs or magnetic configurations). Near the equilibrium position, the negative stiffness cancels out a significant portion of the positive stiffness. This results in a system that possesses high static stiffness to support heavy loads but exhibits very low—or "quasi-zero"—dynamic stiffness. This allows the system to achieve a very low natural frequency, significantly extending the isolation range into the low-frequency range.

2. Active Control Integration

While passive QZS systems are effective, they are highly sensitive to payload variations and environmental disturbances, which can shift the system away from the optimal zero-stiffness equilibrium point. The LAQZS system incorporates an "Active" component to mitigate these limitations.

The active control layer typically consists of sensors (such as accelerometers or displacement transducers), a controller, and actuators (such as electromagnetic or piezoelectric actuators). The working mechanism follows these steps:

- **Sensing:** Sensors detect residual vibrations or displacements of the isolated platform in real-time.
- **Processing:** The controller processes these signals using advanced algorithms (e.g., PID, skyhook damping, or robust control) to determine the necessary corrective force.
- **Actuation:** The actuators apply a counter-force to the platform, effectively "canceling out" the vibration energy that the passive QZS stage could not fully suppress.

3. Synergistic Mechanism

The LAQZS system operates through the synergy of its passive and active components. The passive QZS structure serves as the primary filter, providing high-efficiency isolation for

is shown.

Figure 2

Figure 2: Figure 2

Furthermore, the influence of primary parameters on these characteristics was revealed, and metrics were established to evaluate isolation performance. Literature [?] proposed a quasi-zero stiffness (QZS) isolator composed of a pair of torsion springs, inclined rods, and linear bearings, analyzing its dynamic characteristics through Lagrange equations and the method of averaging. Literature [?] introduced a QZS isolation system consisting of a diaphragm spring set in series with a helical spring and analyzed its static characteristics; the results demonstrated that this structure effectively reduces system stiffness. Literature [?] proposed a QZS isolation system composed of a magnetic negative stiffness spring in parallel with a folded beam, designed for vibration isolation and noise reduction in transformers.

However, the aforementioned structural configurations are all isolation systems with non-adjustable payload capacities. Consequently, they are only suitable for specific operating conditions. When the payload mass changes, because the parameters of the isolation system cannot be adjusted, the system's performance may degrade significantly.

[FIGURE:1] Schematic diagram of the overall structure of the LAQZS vibration isolation system

Diagram of LAQZS vibration isolation system overall structure

The vibration isolation performance subsequently diminishes.

Building upon this foundation, Reference [?] proposed a pneumatically adjustable type of mechanism.

The QZS (Quasi-Zero Stiffness) vibration isolation system is composed of a vertical dual-chamber adjustable pneumatic spring combined with four symmetrically arranged, initially horizontal, single-chamber adjustable pneumatic springs. This configuration maintains the system' s quasi-zero stiffness characteristics by regulating the internal air pressure of the cylinders. Reference [?] proposed an isolator consisting of electromagnetic and pneumatic springs in parallel, which maintains QZS characteristics by adjusting both pressure and coil current. Reference [?] designed a variable-load QZS vibration isolator capable of adapting to load changes by altering the spring installation angle. While these adjustable QZS vibration isolation systems are highly effective at addressing low-frequency, small-amplitude vibrations, they still suffer from drawbacks such as structural complexity and difficulties in fine-tuning vibration isolation performance.

In summary, to mitigate the degradation of vibration isolation performance caused by load variations and to simplify the design of stiffness-adjustable mechanisms, this study proposes a load-adaptive quasi-zero stiffness (LA-QZS) vibration isolator. This system is designed to maintain high-performance isolation across a range of operating conditions by automatically adjusting its mechanical properties in response to changes in the supported mass.

stiffness, LAQZS) isolation system. First, the feasible regions for key parameters are obtained by analyzing the physical characteristics of the positive and negative stiffness elements. Subsequently, during the establishment of the model …

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[FIGURE:1]

Structural Cross-Section of the LAQZS Vibration Isolation System

The structural cross-section of the LAQZS (Linear-Asymptotic Quasi-Zero Stiffness) vibration isolation system illustrates the integration of its primary mechanical components. The system is designed to achieve high-performance vibration isolation by combining positive and negative stiffness elements to reach a quasi-zero stiffness state.

As shown in [FIGURE:1], the core architecture consists of a central payload platform supported by a vertical positive stiffness spring. To counteract the positive stiffness and achieve the desired nonlinear characteristics, the system incorporates a set of pre-compressed horizontal springs or auxiliary mechanisms that provide negative stiffness. The geometric configuration of these components is precisely calibrated so that, at the equilibrium position, the total dynamic stiffness of the system approaches zero, while maintaining sufficient static load-bearing capacity.

Key structural features identified in the cross-section include: - **Load-bearing Frame:** Provides the structural rigidity necessary to house the internal isolation components. - **Positive Stiffness Element:** Typically a linear helical spring that supports the static weight of the isolated mass. - **Negative Stiffness Mechanism:** A configuration of inclined or horizontal linkages and springs that generate a restorative force acting in opposition to the main support spring. - **Damping Elements:** Integrated to suppress resonant peaks and ensure system stability during transient excitations.

This arrangement allows the LAQZS system to maintain a low natural frequency, effectively isolating low-frequency vibrations that conventional linear isolators cannot address.

Cross-section of LAQZS vibration isolation system structure

Core Structure of the LAQZS Vibration Isolation System

The core structure of the LAQZS vibration isolation system primarily consists of the mounting base and the helical components. These elements work in tandem to provide the necessary structural integrity and damping characteristics required for high-precision vibration isolation.

The system comprises several key mechanical components, including springs, U-bolts, a screw jack mechanism, a gear transmission mechanism, a stepper motor, a flange seat, disc springs, and at least two support plates. The disc springs are installed within the upper portion of the mounting seat. Furthermore, helical springs are respectively connected to the adjustable components at both ends.

The movable piston is positioned within the inner bore of the disc spring and secured via U-bolts.

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1. Introduction

In recent years, the rapid development of machine learning and deep learning has fundamentally transformed the landscape of scientific research. These computational techniques have moved beyond traditional data processing to become essential tools for modeling complex physical systems, predicting molecular properties, and optimizing engineering designs. By leveraging large-scale datasets and sophisticated neural network architectures, researchers can now address problems that were previously computationally intractable using classical numerical methods.

[FIGURE:1]

The integration of domain-specific knowledge with data-driven models represents a significant frontier in the field. Rather than treating neural networks as "black boxes," current methodologies increasingly incorporate physical constraints and governing equations directly into the learning process. This approach, often referred to as physics-informed machine learning, ensures that the model outputs remain consistent with fundamental laws such as the conservation of energy or momentum, thereby improving generalization and reliability in scientific applications.

2. Methodology and Framework

The proposed framework utilizes a multi-layer architecture designed to capture both local features and global dependencies within the input data. At the core of our approach is the optimization of a loss function that balances empirical data fit with structural regularizations. Let the input space be denoted by $\mathcal{X}$ and the target space by $\mathcal{Y}$. We seek to learn a mapping function $f : \mathcal{X} \to \mathcal{Y}$ such that the prediction error is minimized across the training distribution.

As illustrated in

, the data preprocessing stage involves normalization and feature extraction to ensure numerical stability during the training phase. We employ a stochas-

Figure 2

Figure 3: Figure 2

tic gradient descent variant to update the model parameters θ. The objective function is defined as:

$$\mathcal{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N} \|y_i - f(x_i;\theta)\|^2 + \lambda\mathcal{R}(\theta)$$

where N represents the number of samples, $\mathcal{R}(\theta)$ is the regularization term used to prevent overfitting, and λ is a hyperparameter that controls the strength of the penalty. This formulation allows for robust parameter estimation even in the presence of noisy experimental data.

3. Experimental Results and Analysis

To evaluate the performance of the proposed method, we conducted a series of benchmarks against state-of-the-art models in the field. The results, summarized in [TABLE:2

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The stepper motor shaft is keyed to a gear transmission mechanism, where the large gear is mounted onto a lead screw. The top of the lead screw is equipped with a piston connected to the lower end of a helical spring. The quasi-zero stiffness component of the vibration isolation system consists of a positive stiffness helical spring connected in parallel with a negative stiffness disc spring.

The LAQZS isolation system is developed based on the QZS (Quasi-Zero Stiffness) isolation system. By introducing a lever mechanism, it achieves a high-static-low-dynamic stiffness characteristic, which effectively broadens the vibration isolation frequency band, particularly in the low-frequency range.

[FIGURE:1]

The schematic diagram of the LAQZS system is shown in [FIGURE:1]. The system primarily consists of a vertical spring providing positive stiffness and a set of horizontal springs connected via a lever mechanism to provide negative stiffness. When the system is at its static equilibrium position, the restoring force generated by the negative stiffness mechanism counteracts the positive stiffness of the vertical spring, resulting in a quasi-zero stiffness state. This configuration allows the system to support a significant static load while maintaining a very low dynamic stiffness, which is essential for isolating low-frequency vibrations that conventional linear isolators cannot effectively address.

The mathematical model of the restoring force F can be expressed as:

$$F = k_v z + 2k_h \left(\frac{L}{a}\right)^2 \left(1 - \frac{l_0}{\sqrt{l^2 + (z \cdot \frac{a}{L})^2}}\right) z$$

where k_v and k_h represent the stiffness of the vertical and horizontal springs, respectively; L and a are the lengths associated with the lever arms; l_0 is the free length of the horizontal spring; and z denotes the vertical displacement from the equilibrium position. By adjusting the lever ratio $\frac{L}{a}$, the negative stiffness effect can be significantly amplified, further reducing the natural frequency of the system without requiring excessively large horizontal springs.

The design parameters for the LAQZS system are summarized in . Through the optimization of these geometric and structural parameters, the system can achieve a wider isolation region. As shown in (eq:stiffness), the dynamic stiffness K_d is derived by differentiating the restoring force with respect to displacement:

$$K_d = \frac{dF}{dz} = k_v + 2k_h \left(\frac{L}{a}\right)^2 \left[1 - \frac{l_0 l^2}{(l^2 + (z \cdot \frac{a}{L})^2)^{3/2}}\right]$$

This system innovatively incorporates a mechanical adjustable stiffness structure, which allows for the tuning of vibration isolation performance by altering the initial deformation of the helical springs (i.e., the initial force exerted by the springs on the system). The overall operational process is as follows: when the system load changes, the stepper motor shaft rotates a specified number of turns, which subsequently modifies the vertical pre-tension of the helical springs through a mechanical linkage. Since the helical springs are part of a quasi-zero stiffness system···

In the formula: E represents the elastic modulus of the disc spring; μ denotes the Poisson' s ratio; f_D is the deformation of a single disc spring at any given moment; C is the ratio of the outer diameter to the inner diameter, defined as $C = D/d$; and K_1 is a constant.

The negative stiffness component of a quasi-zero stiffness (QZS) vibration isolation system primarily utilizes the mechanical properties of Belleville springs (also known as disc springs). By exploiting the snap-through or post-buckling behavior of these springs, researchers can achieve a region of negative stiffness that counteracts the positive stiffness of the primary supporting elements. When these components are integrated, the resulting system exhibits a high static load-bearing capacity alongside a very low dynamic stiffness near the equilibrium position. This characteristic significantly reduces the natural frequency of the isolation system, thereby extending the effective isolation frequency range toward the low-frequency spectrum. Such mechanisms are critical for protecting sensitive equipment from low-frequency environmental vibrations that conventional linear isolators cannot effectively mitigate.

Figure 4

Figure 4: Figure 4

The nonlinear characteristics of the disc spring itself are illustrated in Figure 4

, which presents the characteristic curve of a single disc spring. In this figure, the vertical axis represents the ratio of the disc spring load F_D to the load at the flattened state F_c (defined by setting $f_D = h_0$ in Equation (1)), while the horizontal axis represents the disc spring deflection.

The ratio of the deformation f_D to the maximum deformation h_0 determines the mechanical behavior of the system. Specifically, when $h_0/t > 2$, the load-deformation curve of a single disc spring begins to exhibit a negative stiffness region [**?**].

In systems featuring positive stiffness elements, changes in the pre-tension amount directly influence the isolation performance and structural stability. This pre-tensioning force determines the initial equilibrium position of the system and dictates the linear range of the stiffness response. When the pre-tension is adjusted, the effective stiffness of the positive stiffness component shifts, which in turn alters the natural frequency of the entire isolation system.

Furthermore, the interaction between the pre-tensioned positive stiffness elements and any integrated negative stiffness mechanisms is critical for achieving high-static-low-dynamic stiffness (HSLDS) characteristics. If the pre-tension is not precisely calibrated, the system may deviate from its optimal operating point, leading to either insufficient vibration isolation at low frequencies or potential mechanical instability. Therefore, maintaining an appropriate pre-tension level is essential for ensuring that the isolation system provides robust protection against dynamic excitations while supporting the required static load.

The initial state of the vibration isolation system, namely the zero-stiffness position, determines the system' s performance. By adjusting this initial state, the vibration isolation characteristics of the system can be effectively tuned.

Modeling of the LAQZS Vibration Isolation System

1. Introduction

Low-frequency vibration isolation remains a critical challenge in precision engineering and sensitive instrumentation. The Low-Frequency Quasi-Zero Stiffness (LAQZS) vibration isolation system is designed to address this by achieving high static load-bearing capacity while maintaining very low dynamic stiffness. This section details the mathematical modeling and theoretical framework of the LAQZS system.

2. System Configuration and Kinematics

The LAQZS system typically consists of a primary vertical supporting element combined with negative stiffness mechanisms. The core objective is to neutralize the positive stiffness of the load-bearing spring near the equilibrium position, thereby extending the isolation bandwidth toward lower frequencies.

[FIGURE:1]

As shown in [FIGURE:1], the geometric arrangement of the negative stiffness components—often realized through pre-compressed horizontal springs or magnetic configurations—plays a decisive role in the system' s overall behavior. Let the vertical displacement from the equilibrium position be denoted by x. The restoring force $F(x)$ of the system can be expressed as a combination of the linear positive stiffness k_p and the nonlinear negative stiffness $F_n(x)$.

3. Mathematical Modeling

The total potential energy U of the LAQZS system is the sum of the potential energies of the individual components. For a system utilizing horizontal springs as negative stiffness elements, the potential energy can be formulated as:

$$U = \frac{1}{2}k_p x^2 + 2 \cdot \frac{1}{2}k_h(\sqrt{L^2 + x^2} - L_0)^2$$

where k_h represents the stiffness of the horizontal springs, L is the horizontal distance at equilibrium, and L_0 is the free length of the horizontal springs. By differentiating the potential energy with respect to x, we obtain the restoring force:

$$F(x) = \frac{dU}{dx} = k_p x + 2k_h x \left(1 - \frac{L_0}{\sqrt{L^2 + x^2}}\right)$$

To analyze the system' s performance near the equilibrium position, we perform a Taylor series expansion of the force $F(x)$ around $x = 0$. Retaining terms up to the third order, we have:

$$F(x) \approx (k_p + 2k_h(1 - \frac{L_0}{L}))x + \frac{k_h L_0}{L^3}x^3$$

Modeling of Negative Stiffness in the LAQZS Vibration Isolation System

1. Introduction

Low-frequency vibration isolation is a critical challenge in precision engineering and sensitive instrumentation. The Low-frequency Quasi-Zero Stiffness

(LAQZS) vibration isolation system has emerged as a promising solution, leveraging the principle of negative stiffness to counteract positive stiffness, thereby achieving a high static load-bearing capacity with low dynamic stiffness. This paper focuses on the mathematical modeling and analysis of the negative stiffness mechanism within such systems.

2. Structural Configuration and Working Principle

The LAQZS system typically consists of a vertical supporting element providing positive stiffness and a horizontal arrangement of pre-compressed springs or mechanisms that generate negative stiffness. When the system is at its equilibrium position, the vertical components of the forces exerted by the negative stiffness mechanism cancel out a portion of the positive stiffness.

[FIGURE:1]

As shown in [FIGURE:1], the geometric configuration determines the non-linear force-displacement relationship. The total restoring force F of the system can be expressed as the summation of the positive stiffness force F_p and the negative stiffness force F_n:

$$F(x) = F_p(x) + F_n(x)$$

3. Mathematical Modeling of Negative Stiffness

The negative stiffness is primarily derived from the geometric nonlinearity of the horizontal springs. Let k_h represent the stiffness of the horizontal springs and L_0 represent their initial free length. When the system undergoes a vertical displacement x, the force generated by the negative stiffness mechanism in the vertical direction is given by:

$$F_n(x) = 2k_h \left(1 - \frac{L_0}{\sqrt{a^2 + x^2}}\right) x$$

where a is the horizontal distance from the pivot to the central axis at the equilibrium position. To analyze the stiffness characteristics, we derive the equivalent stiffness K_n by differentiating the force with respect to displacement:

$$K_n(x) = \frac{dF_n}{dx} = 2k_h \left[1 - \frac{L_0 a^2}{(a^2 + x^2)^{3/2}}\right]$$

At the equilibrium position ($x = 0$), the negative stiffness reaches its extremum:

$$K_n(0) = 2k_h(1 - \frac{L_0}{a})$$

To simplify the calculation of the load acting on the disc spring, the following assumptions are made: the cross-sectional shape of the disc spring remains unchanged when subjected to compressive loads (i.e., the influence of radial stress is neglected); the material is a linear elastic, isotropic body; the effects of friction are ignored during the calculation process; and any residual stresses resulting from the surface treatment of the disc spring are neglected.

This study utilizes disc springs without support surfaces for the calculations.

The structural schematic of a disc spring with supporting surfaces is shown in [FIGURE:3]. In this configuration, H_0 represents the free height of a single disc spring.

Characteristic Curves of Single Disc Springs

Disc springs, also known as Belleville washers, exhibit unique load-deflection characteristics that distinguish them from conventional helical springs. The relationship between the applied load and the resulting deformation is generally non-linear, governed by the geometric proportions of the spring.

1. Theoretical Foundation: The Almen-Laszlo Equation

The fundamental mathematical model used to describe the characteristic curve of a single disc spring is the Almen-Laszlo equation. This model assumes that the cross-section of the spring remains rectangular and rotates around a neutral center during deflection. The relationship between the axial load P and the deflection f is expressed as:

$$P = \frac{4E}{1-\mu^2} \cdot \frac{t^4}{K_1 D^2} \cdot \frac{f}{t} \left[\left(\frac{h_0}{t} - \frac{f}{t} \right) \left(\frac{h_0}{t} - \frac{f}{2t} \right) + 1 \right]$$

Where: - E is the Young' s modulus of the material. - μ is Poisson' s ratio. - D is the outer diameter. - t is the material thickness. - h_0 is the internal free height (total height minus thickness). - K_1 is a constant determined by the ratio of the outer diameter to the inner diameter (D/d).

2. Influence of the h_0/t Ratio

The shape of the characteristic curve is primarily determined by the ratio of the internal free height to the thickness (h_0/t). By varying this ratio, engineers can achieve different spring behaviors:

- **Linear Behavior ($h_0/t < 0.4$):** When the ratio is very small, the spring behaves almost linearly, similar to a standard washer.
- **Regressive Behavior ($0.4 < h_0/t < \sqrt{2}$):** As the ratio increases, the spring rate decreases as the deflection increases.

- **Constant Load / Flat Curve ($h_0/t \approx \sqrt{2}$):** At this specific ratio, the curve reaches a plateau where the load remains nearly constant over a significant range

Curves of single disc spring characteristics

When the ratio $h_0/t > 22$, the negative stiffness region gradually expands; however, the negative stiffness effect begins to diminish as the displacement continues to increase. This phenomenon suggests that while a higher geometric ratio promotes the onset of negative stiffness, the stability of the structural response is highly sensitive to the specific proportions of the components. In this regime, the snap-through behavior becomes more pronounced, leading to a more complex energy dissipation profile during loading and unloading cycles.

As the slope of the stiffness curve gradually increases, the characteristics of the curve become increasingly nonlinear. Consequently, achieving the objective of quasi-zero stiffness through the parallel connection of helical springs becomes difficult to realize. Therefore, when selecting the type and relevant dimensions of the disc springs, the following two constraints should be observed: first, select disc springs characterized by a gradual change in stiffness within the negative stiffness region;

the free height of the disc spring; h_0 is the maximum deformation of a single disc spring, namely

The stiffness region should be approximately linear, thereby facilitating the implementation of a linear positive stiffness helical spring.

t is the thickness of the disc spring; F_D is the load applied to a single disc spring.

It should be as sufficient as possible [**?**].

the deformation when flattened; d is the inner diameter of the disc spring; D is the outer diameter of the disc spring; t is the

The first approach involves matching it with a linear coil spring; the second approach requires selecting the negative stiffness region of the disc spring.

To ensure that the helical spring can be embedded within the inner bore of the disc spring, the dimensions of the disc spring must not be too small. However, disc springs are characterized by an excessively high sensitivity to compressive loads—that is, their stiffness is inherently large. Consequently, when the dimensions of the disc spring are increased, the resulting force generated by the disc spring also increases.

exceeds the force exerted by the helical spring, thereby causing the LAQZS (Linear-Aperiodic Quasi-Zero Stiffness) isolation system···

Structure Diagram of a Disc Spring Without Supporting Surfaces

The disc spring without supporting surfaces is a fundamental mechanical component characterized by its truncated conical shape. Its geometric configuration and mechanical behavior are defined by several key parameters that determine its load-bearing capacity and deflection characteristics.

[FIGURE:1]

As illustrated in the structure diagram, the primary dimensions of a disc spring without supporting surfaces include the outer diameter D, the inner diameter d, and the material thickness t. The overall height of the unloaded spring is denoted by H_0, while the internal free height is represented by h_0, where $h_0 = H_0 - t$. These geometric ratios, particularly the ratio of the internal height to the thickness (h_0/t), are critical in defining the spring' s characteristic curve, which can range from linear to strongly non-linear or even regressive.

In practical applications, these springs are often stacked in various configurations—such as in series, parallel, or compound arrangements—to achieve specific force-displacement requirements. The absence of supporting surfaces (flats) means that the contact between the spring and the loading plates occurs at the inner and outer edges, which must be accounted for in the stress analysis and fatigue life calculations according to standard analytical models such as those provided by Almen and László.

without support surface

(1 - μ) K 1 D

Therefore, this study aims to reduce the negative stiffness by altering the combination of disc springs.

êë t - t

The stiffness of the structure is a critical factor. By utilizing a combined series-parallel configuration, the deformation can be increased while maintaining a constant compressive load [**?**]. The schematic diagram of the disc spring assembly is shown in [FIGURE:5].

- 1 úú K1 = · C 1 lnC

Abstract

In this paper, we propose a novel deep learning-based numerical method for solving high-dimensional backward stochastic differential equations (BSDEs). By leveraging the structural properties of BSDEs, we reformulate the problem into a sequence of local minimization subproblems based on a temporal discretization. At each time step, we employ a deep neural network to approximate the unknown solution, effectively overcoming the “curse of dimensionality” inherent

in high-dimensional partial differential equations (PDEs). Numerical experiments demonstrate that our method achieves high accuracy and computational efficiency for various high-dimensional nonlinear BSDEs, including those arising from derivative pricing in finance and optimal control problems.

1. Introduction

High-dimensional backward stochastic differential equations (BSDEs) play a crucial role in various fields, including mathematical finance, stochastic control, and nonlinear partial differential equations (PDEs). Since the seminal work of Pardoux and Peng [**?**], the theory of BSDEs has been extensively developed. However, finding analytical solutions for nonlinear BSDEs is generally impossible, necessitating the development of efficient numerical algorithms.

Traditional numerical methods, such as those based on binomial trees, finite differences, or Monte Carlo simulations combined with regression, often face the "curse of dimensionality." As the dimension d increases, the computational cost grows exponentially, making these methods impractical for problems where d exceeds 10. In recent years, the rapid development of deep learning has provided new tools for solving high-dimensional problems. By using deep neural networks (DNNs) as universal function approximators, researchers have proposed several "deep" solvers, such as the Deep BSDE method [**?**].

In this paper, we introduce an improved deep learning approach that treats the BSDE as a multi-stage optimization problem. Unlike global optimization methods that train a single network across the entire time horizon, our approach utilizes a step-by-step training strategy. This local optimization framework enhances the stability of the training process and reduces the accumulation of errors over long time horizons.

2. Problem Formulation

Consider the following BSDE on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}; \{\mathcal{F}_t\}_{0\leq t\leq T})$:

$$\begin{aligned} dX_t &= \mu(t, X_t)dt + \sigma(t, X_t)dW_t \\ dY_t &= -f(t, X_t, Y_t, Z_t)dt + Z_t dW_t \end{aligned}$$

When friction is neglected, a disc spring assembly consisting of six identical parameters arranged in opposing directions can be analyzed. According to the foundational theory of disc springs, the characteristic relationship between the load F and the deformation f of a single disc spring is expressed by the Almen-Laszlo equation:

$$F = \frac{4E}{1-\mu^2}\frac{t^4}{KD^2}\frac{f}{t}\left[\left(\frac{h_0}{t}-\frac{f}{t}\right)\left(\frac{h_0}{t}-\frac{f}{2t}\right)+1\right]$$

In this expression, E represents the modulus of elasticity, μ is Poisson’ s ratio, t denotes the thickness of the spring, D is the outer diameter, h_0 is the initial cone height, and K is a calculation coefficient related to the diameter ratio.

When six identical disc springs are arranged in an opposing (series) configuration, the total load F_{total} acting on the stack remains equal to the load on an individual spring ($F_{total} = F$). However, the total deformation of the system, f_{total}, is the sum of the deformations of each individual spring. For $n = 6$ springs in series, the relationship is defined as:

$$f_{total} = 6f$$

Consequently, the equivalent stiffness of the assembly is reduced to one-sixth of the stiffness of a single disc spring. This configuration is typically employed in engineering applications where a large stroke is required under a relatively constant or specific load profile. In the absence of friction, the loading and unloading curves of this 6-spring stack are identical, exhibiting no hysteretic behavior.

Stacked in a series configuration, the relationship between the compressive load and the resulting deformation is given by $F'_D = F_D$, $f'_D = 6f_D$, and $H'_0 = 6H_0$.

In the formula, F, f, and H represent the load, deformation, and free height of the disc spring after stacking, respectively. Correspondingly, F_D, f_D, and H_0 denote the load, deformation, and free height of the disc spring prior to stacking.

Design and Characteristic Analysis of a Load-Adaptive Quasi-Zero Stiffness Vibration Isolation System

1. Introduction

Vibration isolation technology is a critical means of ensuring the stable operation of precision equipment and improving the comfort of mechanical systems. Traditional linear vibration isolation systems face a fundamental contradiction between low natural frequency and high static load-bearing capacity. To address this, researchers have proposed the concept of Quasi-Zero Stiffness (QZS). By combining a positive stiffness element with a negative stiffness element, the system can achieve high static stiffness to support heavy loads while maintaining extremely low dynamic stiffness near the equilibrium position. This results in a low natural frequency and a broad isolation frequency band.

However, traditional QZS vibration isolation systems are often designed for a specific rated load. When the external load deviates from the design value, the system’ s equilibrium position shifts, causing the positive and negative stiffness elements to no longer cancel each other out effectively. This leads to a significant increase in dynamic stiffness and a degradation of isolation performance. Therefore, developing a load-adaptive QZS vibration isolation system that can maintain its low-stiffness characteristics across a range of loads is of great practical significance.

2. System Design and Modeling

The proposed load-adaptive QZS vibration isolation system consists of a primary positive stiffness spring and a negative stiffness mechanism composed of pre-compressed horizontal springs and linkage structures. To achieve load adaptability, an adjustment mechanism is integrated to modify the configuration of the negative stiffness components or the pre-load of the positive stiffness element.

[FIGURE:1]

The schematic diagram of the system is shown in [FIGURE:1]. Let k_p be the stiffness of the vertical positive stiffness spring and k_n be the stiffness of the horizontal springs. The restoring force F of the system relative to the displacement x from the equilibrium position can be expressed as:

$$F(x) = k_p x + 2k_n \left(1 - \frac{l_0}{\sqrt{a^2 + (h-x)^2}}\right)(h-x)$$

where l_0 is the free length of the horizontal springs, a is the horizontal distance from the pivot to the center, and h is the vertical distance. By performing a Taylor series expansion around the equilibrium position $x = 0$, the non-dimensional stiffness $\hat{k}$ is derived as:

$$\hat{k} = \frac{d\hat{F}}{d\hat{x}}\Big|_{\hat{x}=0}$$

Other relevant parameter values for the helical spring are shown in .

5 Diagram of disc spring combination

The comparison of the stiffness of the disc spring stack before and after the combination is shown in Figure 6 [FIGURE:6].

Tab. 1

Helical Spring Design Parameter Table

The design and calculation of helical springs require precise specification of geometric and material parameters to ensure mechanical performance and reliability. The following table summarizes the primary design parameters used in the engineering analysis of helical compression and extension springs.

Parameter Name	Symbol	Unit	Description
Wire Diameter	d	mm	The diameter of the wire used to wind the spring.
Mean Coil Diameter	D	mm	The average diameter of the spring coil, calculated as $(D_{outer} + D_{inner})/2$.
Spring Index	C	-	The ratio of mean coil diameter to wire diameter $(C = D/d)$.
Active Coils	n	-	The number of coils that deflect under load.
Total Coils	n_t	-	The total number of coils, including end coils for seating.
Free Length	H_0	mm	The overall length of the spring in its unloaded state.
Pitch	p	mm	The center-to-center distance between adjacent active coils.
Spring Rate (Stiffness)	k	N/mm	The force required to deform the spring by a unit distance.

Parameter Name	Symbol	Unit	Description
Shear Modulus	G	MPa	The material property representing the rigidity of the spring wire.
Maximum Load	F_{max}	N	The maximum permissible force the spring can withstand without permanent deformation.
Shear Stress	τ	MPa	The internal stress generated in the wire under load, often corrected by the Wahl factor K.

Technical Considerations for Parameter Selection

When determining the spring index C, it is generally recommended to maintain a value between 4 and 12. A value of $C < 4$ makes the spring difficult to manufacture due to high curvature, while $C > 12$ may result in a spring that is prone to buckling or instability.

The spring rate k is fundamentally governed by the relationship:

$$k = \frac{Gd^4}{8D^3n}$$

This equation demonstrates that the stiffness is highly sensitive to the wire diameter d and the mean coil diameter D.

Coil spring design data sheet

N / mm

ratio of internal to external diameter

Combined Stiffness Analysis of the LAQZS Isolation System

Since the positive stiffness helical spring and the negative stiffness disc spring are arranged in a parallel configuration, the total restoring force of the Quasi-Zero Stiffness (QZS) vibration isolation system can be expressed as:

$$F = F_p + F_n$$

By substituting the mechanical models of the individual components into the equation above, the total restoring force F of the system is derived as follows:

$$F = k_p z + 2k_n \left(L - \sqrt{a^2 + z^2}\right) \frac{z}{\sqrt{a^2 + z^2}}$$

In this expression, k_p represents the stiffness of the positive stiffness helical spring, while k_n denotes the stiffness of the negative stiffness disc spring. The variable z represents the displacement from the equilibrium position, L is the free length of the disc spring, and a is the horizontal distance from the pivot point to the center of the spring assembly.

To determine the equivalent stiffness of the combined system, we differentiate the total restoring force F with respect to the displacement z:

$$K = \frac{dF}{dz} = k_p + 2k_n \left[1 - \frac{La^2}{(a^2 + z^2)^{3/2}}\right]$$

To achieve the characteristic quasi-zero stiffness property at the static equilibrium position ($z = 0$), the system parameters must satisfy specific conditions. Setting $z = 0$ in the stiffness equation, we obtain the initial stiffness K_0:

$$K_0 = k_p + 2k_n \left(1 - \frac{L}{a}\right)$$

For the system to exhibit zero stiffness at the equilibrium position ($K_0 = 0$), the relationship between the positive and negative stiffness components must be tuned such that:

$$\frac{k_p}{2k_n} = \frac{L}{a} - 1$$

This condition ensures that the high static load-bearing capacity provided by the helical spring is offset by the negative stiffness of the disc springs at the design point, resulting in a system with high static but low dynamic stiffness. This characteristic is essential for improving vibration isolation performance, particularly in the low-frequency range.

The total load F acting on the vibration isolation system is equal to the sum of the loads borne by the disc spring assembly and the helical springs. This relationship is expressed as:

Regarding its stiffness, adopting a nested (parallel) arrangement can significantly reduce the sensitivity of the disc spring to deformation. This configuration allows the stiffness variation to become more gradual, which facilitates better matching with the corresponding positive-stiffness helical spring. Ultimately, this synergy is synergy is conducive to achieving the low-frequency isolation performance required for the LAQZS (Linear-Asymmetric Quasi-Zero-Stiffness) vibration isolation system.

Modeling the Positive Stiffness of the LAQZS Vibration Isolation System

1. Introduction

The positive stiffness component is a fundamental element of the Linear-Asymmetric Quasi-Zero Stiffness (LAQZS) vibration isolation system. Accurate modeling of this stiffness is crucial for predicting the system' s overall dynamic behavior and isolation performance. This section details the theoretical derivation and mathematical representation of the positive stiffness characteristics within the proposed isolation framework.

2. Theoretical Framework for Positive Stiffness

In the LAQZS system, the positive stiffness is typically provided by a primary elastic element, such as a linear helical spring or a specialized flexure mechanism. Unlike symmetric systems, the LAQZS configuration accounts for potential asymmetries in the loading conditions or geometric arrangements.

The restoring force F_p generated by the positive stiffness element can be expressed as a function of the displacement x from the equilibrium position. For a linear spring with a stiffness constant k_p, the relationship is defined as:

$$F_p = k_p x$$

However, in high-precision or large-stroke applications, the inherent nonlinearity of the material or geometry may necessitate a higher-order approximation. In such cases, the positive stiffness force is modeled using a Taylor series expansion:

$$F_p(x) = \sum_{n=1}^{N} k_{pn} x^n$$

where k_{pn} represents the n-th order stiffness coefficient. For most practical LAQZS analyses, considering terms up to the third order provides sufficient accuracy for capturing the hardening or softening effects of the positive stiffness component.

3. Geometric Configuration and Force Analysis

The integration of the positive stiffness element within the LAQZS assembly requires a detailed force balance analysis. [FIGURE:1] illustrates the schematic diagram of the isolation system, highlighting the orientation of the positive stiffness spring relative to the negative stiffness mechanisms.

[FIGURE:1]

When the system undergoes a vertical displacement x, the potential energy U_p stored in the positive stiffness element is given by:

$$U_p = \frac{1}{2} k_p x^2$$

The corresponding instantaneous stiffness K_p is derived by taking the second derivative of the potential energy with respect to displacement:

$$K_p = \frac{d^2 U_p}{dx^2} = k_p$$

In the context of the LAQZS system

Based on the characteristics of quasi-zero stiffness (QZS) structures composed of positive and negative stiffness elements in parallel, the vertical load of the system is the sum of the loads borne by the disc spring and the helical spring. To further improve the vibration isolation performance of the LAQZS system, the stiffness of the positive-stiffness helical spring should be determined based on the slope of the negative-stiffness region of the disc spring [**?**]. When the compression of the disc spring reaches h_0 (i.e., when the disc spring is flattened), the stiffness of the helical spring k_2 should be equal to the negative of the disc spring’ s stiffness $-k_1$ at that point:

$$k_2 = -k_1 = (1 - \mu^2) K_1 D^2$$

(1 - μ) K 1 D

F = nα êê

6 Comparison of stiffness before and after disc spring involution

Abstract

In this paper, we investigate the existence of solutions for a class of fractional p-Laplacian equations with a generalized Choquard term and a critical Sobolev exponent. By utilizing the mountain pass theorem, the concentration-compactness principle, and refined analytical estimates, we establish the existence of at least one non-trivial solution for the problem under suitable conditions.

1. Introduction

In recent years, fractional differential equations have attracted significant attention due to their wide range of applications in physics, biology, and finance. In particular, the fractional p-Laplacian operator, denoted by $(-\Delta)_p^s$, has been extensively studied as a non-local generalization of the classical p-Laplacian. This operator arises naturally in the study of non-local diffusion processes and quasi-geostrophic flows.

The study of equations involving the Choquard term, also known as the Hartree-type nonlinearity, dates back to the work of Pekar in describing the quantum mechanics of a polaron at rest. When combined with critical growth terms, these problems present significant mathematical challenges due to the loss of compactness in the Sobolev embeddings. The concentration-compactness principle, introduced by Lions, has become a fundamental tool for addressing these issues in the context of critical exponents.

In this paper, we consider the following class of fractional p-Laplacian equations:

$$(-\Delta)_p^s u + V(x)|u|^{p-2}u = \left(\int_{\mathbb{R}^N} \frac{|u(y)|^{p_{\mu,s}{}^*}}{|x-y|^\mu} dy\right) |u|^{p_{\mu,s}{}^{*-2}} u + \lambda f(x,u)$$

where $s \in (0,1)$, $p > 1$, $N > sp$, and $p_{\mu,s}^* = \frac{p(2N-\mu)}{2(N-sp)}$ is the critical exponent in the sense of the Hardy-Littlewood-Sobolev inequality. Our objective is to determine the conditions under which non-trivial solutions exist, particularly when the nonlinearity involves both critical growth and a potential function $V(x)$.

[FIGURE:1]

2. Preliminaries and Functional Framework

Before stating our main results, we introduce the necessary functional spaces and notation. Let $W^{s,p}(\mathbb{R}^N)$ be the fractional Sobolev space···

F L = k2 f

( ) +(β+1) (ft) ùúúû +k f

By differentiating Eq. (6) with respect to f, the stiffness of the LAQZS vibration isolation system can be obtained as:

Figure 2

Figure 5: Figure 2

$$K = \frac{df}{dz}$$

[FIGURE:1]

The expression for the degree k is given by:

3.2 Mechanical Model Analysis

The mechanical behavior of the system can be characterized by the following governing equation:

$$-3\beta + \beta^2 + 1 + k^2 = n\alpha$$

In this expression, f represents the deformation of the system, which consists of a combination of the main system components and the disc springs. For a helical spring assembly, the relationship between the applied load and the resulting displacement is critical for determining the overall stiffness and stability of the mechanism.

The parameters α and β are non-dimensional coefficients that describe the geometric and material properties of the spring configuration. Specifically, β relates to the ratio of the compression depth to the initial height of the disc spring, while n denotes the number of active spring elements in the stack. The constant k accounts for the specific boundary conditions and the interaction between the helical and disc spring components.

[FIGURE:1]

By analyzing the equilibrium state defined by (eq:1), we can derive the critical load thresholds required to prevent mechanical failure. This model allows for the optimization of the spring constant k to ensure that the system maintains a linear response over the intended operational range of deformation f. As shown in

, the experimental results align closely with the theoretical predictions derived from this analytical framework.

The deformation of the spring components changes synchronously; therefore, the deformation for all three is denoted by f. Here, n represents the number of disc springs, k_2 is the stiffness of the helical spring, and F_L is the force exerted by the helical spring. The parameters α, β, and K_1 are constants.

Figure 7

Figure 6: Figure 7

Figure 7

Figure 7: Figure 7

Based on Eq. (7), the stiffness curve of the vibration isolation system is plotted as shown in

.

As shown in

, the stiffness curve of the vibration isolation system intersects the zero-stiffness line.

and overlap for a certain interval. Consequently, by connecting the disc spring and the helical spring in parallel, the LAQZS isolation system exhibits quasi-zero stiffness characteristics.

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1. Introduction

In recent years, the rapid development of machine learning and deep learning has significantly transformed various scientific and engineering disciplines. These computational paradigms offer powerful tools for processing high-dimensional data, identifying complex patterns, and making accurate predictions in systems where traditional analytical models may fall short. As we explore the applications of these technologies, it becomes increasingly important to maintain a rigorous academic standard in both theoretical derivation and empirical validation.

2. Theoretical Framework

The foundation of our analysis rests on the integration of statistical mechanics and modern computational algorithms. By leveraging the representational power of neural networks, we can approximate functions that were previously considered computationally intractable. Consider a system defined by the state variable $x \in \mathbb{R}^n$. The objective is to minimize the loss function $\mathcal{L}(\theta)$, where θ represents the parameters of the model.

$$\mathcal{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N} \|f(x_i;\theta) - y_i\|^2 + \lambda R(\theta)$$

In this expression, $f(x_i;\theta)$ denotes the model output, y_i represents the ground truth labels, and $R(\theta)$ serves as a regularization term to prevent overfitting. The hyperparameter λ controls the trade-off between data fidelity and model complexity.

[FIGURE:1]

3. Methodology and Data Collection

The experimental setup was designed to ensure the reproducibility of the results. Data were collected from multiple sensors and pre-processed to remove noise and outliers. We employed a cross-validation strategy to evaluate the robustness of the proposed architecture. As shown in , the performance metrics indicate a substantial improvement over baseline methods.

The optimization process utilized the Adam optimizer with an initial learning rate of $\eta = 10^{-3}$. We observed that the convergence rate was highly sensitive to the initialization of the weight matrix W_{ij}. To address this, we implemented a Xavier initialization scheme, which maintains the variance of activations across layers.

4. Results and Discussion

The empirical results demonstrate that the proposed model effectively captures the underlying dynamics of the system. Specifically, the error residuals $\epsilon = |y - \hat{y}|$ follow a Gaussian distribution with a mean $\mu \approx 0$.

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In the formula, u represents the transmission ratio of the gear drive mechanism.

By substituting the pre-stretch amount Δx into Eq. (eq:9), the rotation of the motor shaft can be obtained.

The influence of the number of motor rotations N_d on the force-displacement curves of the vibration isolation system is illustrated in [FIGURE:8].

Stiffness Curve of the LAQZS Vibration Isolation System

1. Introduction to the LAQZS System

The Linear-Anti-spring Quasi-Zero Stiffness (LAQZS) vibration isolation system is designed to achieve high static load-bearing capacity while maintaining low dynamic stiffness. By integrating positive stiffness elements with negative stiffness mechanisms, the system can achieve a "quasi-zero stiffness" state near the equilibrium position. This characteristic significantly extends the isolation frequency range toward the low-frequency spectrum, effectively suppressing vibrations that conventional linear isolators cannot address.

2. Mathematical Modeling of Stiffness

The total stiffness of the LAQZS system, denoted as K_{total}, is the summation of the positive stiffness from the supporting springs (K_p) and the negative stiffness generated by the anti-spring mechanism (K_n). The relationship can be expressed as:

$$K_{total}(x) = K_p + K_n(x)$$

where x represents the displacement from the static equilibrium position. In a typical LAQZS configuration, the negative stiffness is non-linear and depends on the geometric configuration of the anti-spring components (such as inclined springs or buckled beams).

[FIGURE:1]

3. Analysis of the Stiffness Curve

The stiffness curve of the LAQZS system typically exhibits a "U-shape" or a "V-shape" centered at the equilibrium position ($x = 0$).

- **Equilibrium Position:** At the design equilibrium point, the negative stiffness mechanism provides maximum compensation, resulting in the minimum total stiffness. Ideally, $K_{total}(0) \approx 0$, which defines the quasi-zero stiffness region.
- **Displacement Sensitivity:** As the system deviates from the equilibrium position ($x \neq 0$), the magnitude of the negative stiffness decreases. Consequently, the total stiffness K_{total} increases. This hardening effect ensures the stability of the system under larger disturbances.
- **Linear vs. Nonlinear Regions:** Within a small range around the equilibrium point, the stiffness remains relatively constant and low. However, as shown in , the effective isolation bandwidth is highly sensitive to the precision of this stiffness matching.

4. Influence of Geometric Parameters

The shape and "flatness" of the stiffness curve are governed by several key parameters, including the initial compression of the negative stiffness springs and the linkage lengths. By adjusting the pre-tension···

Curve of LAQZS vibration isolation system stiffness

Structural Design of the LAQZS

Influence curves of the motor shaft turns N_d on the $F-f$ characteristics of the vibration isolation system.

The vibration isolation system can adjust its performance by changing the pretension Δx of the helical spring.

Figure 11

Figure 8: Figure 11

Figure 16

Figure 9: Figure 16

However, due to the constraints imposed by the inner bore of the disc spring, the helical spring can only undergo displacement in the vertical direction.

As illustrated in Figure 8, the LAQZS vibration isolation system can be tuned by adjusting the motor parameters.

This enables load adaptability, which in turn allows for the fine-tuning of its vibration isolation performance.

Curve of the effect of coil spring pretension N d on vibration isolator F-f

deformation; this necessitates the use of a mechanical

Figures

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Figure 17

Figure 10: Figure 17

Figure 18

Figure 11: Figure 18