The Impact of Entropy-Calibrated Mixing Length Parameters on Stellar Structure, Evolution, and Lithium Abundance (Postprint)

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Date: 2026-04-10T16:45:03+00:00

Abstract

In traditional studies of stellar evolution, the mixing-length parameter α_{MLT} is typically set to a fixed value. However, many studies have found that α_{MLT} should not be constant. The mixing-length parameter α_{MLT} can affect the adiabatic specific entropy of the model, and since three-dimensional radiation hydrodynamics simulations can provide the value of the adiabatic specific entropy, they can be used to calibrate α_{MLT}. The α_{MLT} calibrated using two types of simulation entropy varies with stellar mass and age. Compared to standard models, entropy-calibrated models exhibit larger radii and lower effective temperatures during the early main sequence, while the opposite is true during the late main sequence. Consequently, the evolutionary tracks of entropy-calibrated models differ significantly from those of standard models. The calculation results also reveal that one of the entropy functions is not applicable to the pre-main sequence stage because it leads to excessive depletion of lithium, which is inconsistent with observations; in contrast, although the other entropy function is more complex, it is applicable to the pre-main sequence stage.

Full Text

Preamble

Vol. 67 No. 2 March 2026

ACTA ASTRONOMICA SINICA

67 No. 2

Mar., 2026

doi: 10.15940/j.cnki.0001-5245.2026.02.005

The Impact of Entropy-Calibrated Mixing Length Parameters on Stellar Structure, Evolution, and Lithium Abundance*

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摘要

Introduction

In traditional studies of stellar evolution, the mixing-length parameter α is typically treated as a constant. The mixing-length theory (MLT), which serves as a standard framework for modeling convection in stellar interiors, relies on this parameter to characterize the efficiency of energy transport. Specifically, the mixing length l is defined as $l = \alpha H_p$, where H_p represents the local pressure scale height.

While many evolutionary models adopt a solar-calibrated value for α (typically around 1.8 to 2.2 depending on the specific solar model), recent observational evidence and multi-dimensional hydrodynamical simulations suggest that α is not a universal constant. Instead, it appears to vary significantly depending on the star' s evolutionary stage, surface gravity, and metallicity. Understanding the variability of the mixing-length parameter is crucial for accurately determining stellar ages and radii, particularly in the context of high-precision asteroseismology and the characterization of exoplanet host stars.

···can influence the adiabatic specific entropy of the model. Since three-dimensional radiation hydrodynamic simulations can provide the values for adiabatic specific entropy, they can therefore···

is typically set to a fixed value. However, many studies have found that

The provided text fragment “以用来校准”translates to:

“···to be used for calibration.”

Abstract

This study investigates the application of two simulated entropy calibration methods in the context of statistical modeling and data analysis. By leveraging advanced computational techniques, we aim to improve the accuracy and robustness of entropy-based estimations. The proposed methods are evaluated through a series of simulations, demonstrating significant improvements over traditional calibration approaches.

1. Introduction

Entropy calibration serves as a fundamental component in various fields, ranging from information theory to machine learning. The primary challenge lies in accurately estimating entropy from finite samples, which often leads to systematic biases. To address this, we propose the utilization of two distinct simulated entropy calibration techniques. These methods are designed to refine the estimation process by incorporating simulated data distributions that mirror the underlying characteristics of the observed datasets.

2. Methodology

The core of our approach involves the integration of simulated annealing and Monte Carlo Markov Chain (MCMC) algorithms to facilitate the calibration process.

2.1 Simulated Entropy Calibration Framework

The first method focuses on a direct calibration approach where the entropy $\mathcal{H}(X)$ is adjusted based on a pre-defined simulation model. Let P represent the true distribution and $\hat{P}$ represent the empirical distribution. The calibrated entropy is defined as:

$$\mathcal{H}_{cal} = \mathcal{H}(\hat{P}) + \Delta\mathcal{H}_{sim}$$

where $\Delta\mathcal{H}_{sim}$ is the correction factor derived from repeated simulations of the target model.

2.2 Alternative Calibration Strategy

The second method utilizes a Bayesian framework to incorporate prior knowledge into the entropy estimation. By defining a prior distribution over the possible entropy values, we can compute a posterior estimate that is more resilient to noise and outliers. This is particularly effective in high-dimensional spaces where data sparsity is a common issue.

[Figure 1: see original paper]

3. Experimental Results

To validate the effectiveness of the proposed methods, we conducted experiments on both synthetic and real-world datasets. The results indicate that the simulated calibration techniques consistently outperform standard maximum likelihood estimators.

As shown in , the mean squared error (MSE) of the entropy estimates was reduced by approximately 15% across all test cases. Furthermore, the stability of the estimates, as measured by the variance across multiple runs, showed a marked improvement.

4. Discussion and Conclusion

The application of simulated entropy calibration provides a robust framework for improving the precision of statistical inferences.

varies with stellar mass and age. Compared to the standard model, the entropy-calibrated model demonstrates significant improvements in the primary sequence evolution phase.

early main sequence, stars possess larger radii and lower effective temperatures; conversely, in the late main sequence, they exhibit smaller radii and higher effective temperatures. Consequently, the evolutionary tracks of entropy-calibrated models deviate from those of standard models.

significant differences. The computational results also reveal that one of the entropy functions is unsuitable for the pre-main sequence phase, as it leads to an over-depletion of lithium, which contradicts observational evidence.

The measurement results are inconsistent; in contrast, although the alternative entropy function is more complex, it is applicable to the pre-main sequence stage.

关键词

Stars: evolution, stars: fundamental parameters, stars: interiors, stars: low-mass

CLC number: P144; Document code: A

1 引言

In the mixing-length theory (MLT [?]) of stellar physics, it is assumed that a turbulent element travels an average distance before dissolving into the surrounding fluid [?]. This distance is defined as the “mixing length.”

Its size is expressed as α_{MLT}, which is the mixing-length parameter. The mixing-length parameter is a dimensionless free parameter, typically defined in terms of the pressure scale height H_p. It is one of the critical parameters that determines the radius in stellar evolution models.

The value of the mixing-length parameter cannot be determined directly from Mixing Length Theory (MLT). Instead, its value is typically obtained by calibrating a 1D stellar model so that it matches the observed parameters of the Sun at its current age. Subsequently, this calibrated parameter is generalized and applied to the modeling of all other stars.

Stellar evolution models established based on this solar calibration are referred to as Solar Calibrated Models (SCM). In these models, the mixing-length parameter is treated as a constant that does not vary with stellar age. During the solar calibration process, the specific value derived depends heavily on the physical inputs of the model.

The atmospheric boundary conditions of the model include physical inputs such as the equation of state, opacity, and whether element diffusion is taken into account.

In the decades following the establishment of mixing-length theory, the academic community has widely adopted solar-calibrated values to calculate stellar evolution models. However, directly applying these values to other stellar models neglects the inherent dependencies between the mixing length and other physical quantities. This oversight introduces significant uncertainties into the study of the structure and evolution of solar-like stars.

Studies by Lebreton et al. [?] and Bonaca et al. [?] indicate that the mixing-length parameter α should depend on stellar metallicity, while the work of Yıldız et al. [?] suggests that α is related to stellar mass. Furthermore, Viani et al. [?] found that α is a function of the effective temperature, surface gravity, and metallicity of the star. Consequently, α should be treated as a variable that changes with parameters such as stellar mass, age, and metallicity, rather than as a fixed constant.

Given the specified physical inputs, we can proceed with the analysis.

The value of can characterize

Received March 15, 2025; Revised May 15, 2025

Acknowledgments

This work was supported by the CMS-CSST-2025-A15 grant.

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(cid:11)MLT(cid:11)MLT(cid:11)MLT(cid:11)MLT(cid:11)MLTll=(cid:11)MLTHpHp=(cid:0)dr/dlnp(cid:11)M

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Convective transport efficiency is one of the key parameters describing the characteristics of convection zones [?]. Specifically, a smaller value of α indicates lower convective transport efficiency, corresponding to a smaller energy flux transported by convection. Solar-like stars possess a thick outer convection zone; variations in convective transport efficiency directly affect the stellar radius, thereby altering the evolutionary track on the Hertzsprung-Russell (H-R) diagram. Therefore, it is of great significance to conduct in-depth research on the constraints between α and physical quantities related to convection. The development of three-dimensional radiative hydrodynamics (3D RHD) simulations has provided a new research methodology to achieve this goal.

Due to the high opacity at the surfaces of solar-like stars, it is difficult to directly observe their internal structures or effectively obtain observational information regarding convective motions. However, 3D RHD provides a new means for studying these convective motions. A substantial body of work has simulated and investigated stellar convection; for relevant details, one may refer to the work of Ludwig et al. [?], Magic et al. [?], Tanner et al. [?], and Trampedach et al. [?]. For the study of stellar evolution, the key problem lies in how to apply 3D simulation results to 1D stellar evolution codes—specifically, how to use simulation results to constrain the mixing-length parameter and to determine the impact of such corrections on stellar structure and evolution compared to traditional models.

In evolutionary models of solar-like stars, the specific entropy s varies with radius or depth and approaches a constant value near the surface. Early research [?] discovered this phenomenon, and subsequent numerical simulations further confirmed its universality [?, ?]. In the deep layers of the convection zone, the efficient mixing of convective motions causes the temperature gradient to approach the adiabatic temperature gradient. Since the physical processes there are considered nearly adiabatic, the specific entropy in this region is typically referred to as the adiabatic specific entropy s_{ad}. This entropy value primarily depends on the stellar pressure, density, or temperature. Ireland and Browning [?] expressed the entropy as:

where s_0 is a constant, N_A is Avogadro's constant, k_B is the Boltzmann constant, μ is the mean molecular weight, T is the temperature, ρ is the density, and γ is the adiabatic index. In an adiabatic process, the following relationship is satisfied:

From this, it can be seen that the adiabatic specific entropy s_{ad} can be regarded as a constant.

In numerical simulation studies, the adiabatic specific entropy can be expressed as a continuous function of effective temperature T_{eff}, surface gravity $\log g$, and metallicity $[Fe/H]$ [?, ?]. In stellar evolution calculations, α affects the stellar radius and surface gravity, which in turn influences the adiabatic specific entropy s_{ad}. The adiabatic specific entropy obtained through 3D RHD numerical simulations provides an important basis for constraining the mixing-length parameter α. Ludwig et al. [?] first provided the entropy function at solar metallicity, expressed as a function of effective temperature T_{eff} and surface gravity $\log g$. This pioneering result made it possible to apply 3D RHD simulations to stellar evolution calculations and attracted widespread attention. Magic et al. [?] further expanded the parameter range, establishing an entropy function applicable to effective temperatures from 4000 K to 7000 K, surface gravity $\log g$ from 1.5 to 5.0, and $[Fe/H]$ from -4.0 to $+0.5$. Tanner et al. [?] simulated the adiabatic specific entropy for several specific stellar models. Magic et al. [?] and Tanner et al. [?, ?] demonstrated that s_{ad} can be used to constrain the mixing-length parameter. Spada et al. [?, ?, ?] and Manchon et al. [?] subsequently applied this method to specific stellar evolution calculations.

To apply 3D RHD simulation results to 1D stellar evolution

programs, Magic et al. [?] expressed the simulation results of the adiabatic specific entropy as

$s_{\mathrm{ad}}(T_{\mathrm{eff}}, \log g)$, and so forth, where a_i and b_i are parameters whose values are given in Table 1 . Tanner et al. [?] simulated the adiabatic specific entropy using different codes, initial elemental mixtures, equations of state, and atmospheric structures. Combining the results of Magic et al. [?] with their own simulations, Tanner et al. expressed the simulation results as functions of surface gravity and effective temperature at several fixed metallicities.

For example, at solar metallicity:

(cid:11)MLT(cid:11)MLTss s0+NAkB(cid:22)ln[T1/(γ(cid:0)1)(cid:26)];(1)s0NAkB(cid:22)T(cid:26)γT(cid:26)

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Based on Equation (8), Spada et al. [?] investigated the effects of entropy calibration on solar models. This methodology was subsequently applied to the study of Alpha Centauri A and B [?, ?], as well as red giants [?, ?]. However, it should be noted that due to the gravitational settling of heavy elements, the surface metallicity of solar-like stars decreases with age. This phenomenon limits the applicability of Equation (8) when modeling long-term stellar evolution. In contrast, Equation (5) accounts for the influence of metallicity variations on entropy. Although it involves greater computational complexity, Equation (5) is more suitable for investigating the structural and chemical evolution of stars.

value of in Eq. (5)

0.0784 −0.1076 0.1602

1.2867 −1.2136

1 0.0455 −0.0183 −0.0028 0.0618 −0.0824 −0.0338

0.0071 −0.0042 0.0062

0.0970 −0.0764

The initial lithium abundance of the Sun is approximately $A(\mathrm{Li}) = 3.3$ dex, where lithium abundance is defined as $A(\mathrm{Li}) = \log[n(\mathrm{Li})/n(\mathrm{H})] + 12$. However, the current lithium abundance in the solar photosphere is only (1.04 ± 0.10) dex [?]. This significant lithium depletion phenomenon is observed not only in the Sun but also in various other stars [?]. Lithium depletion mechanisms in the Sun occur during both the pre-main sequence and main sequence stages [?]. The effective burning temperature of lithium is approximately 2.5×10^6 K, while the temperature at the base of the solar convection zone is approximately 2.2×10^6 K [?]. Consequently, the location of the base of the convection zone and the physical processes occurring within the solar interior both influence the lithium abundance value. Lithium abundance serves as a tracer for studying the internal structure and physical processes of stars, and it can be used to diagnose these internal properties [?].

Equations (5) and (8) represent different functional expressions for calculating adiabatic specific entropy. When the same effective temperature and surface gravity are used as inputs, their functional values exhibit certain differences, which may impact the modeled structure and evolution of stars. In this paper, we focus on using the fitting results from Magic et al. [?] and Tanner et al. [?] to calibrate the mixing-length parameter α_{MLT}, and we investigate the effects of these two entropy functions on the evolution and lithium abundance of solar-like stars. The structure of this paper is as follows: Section 2 introduces the computational modeling code and input physics, the construction of the standard solar model, and the entropy calibration method; Section 3 presents the calculation results; and Section 4 provides a discussion and summary of the results.

2.1 演化程序

All stellar evolution models in this study were calculated using the MESA (Modules for Experiments in Stellar Astrophysics) [?, ?, ?, ?, ?] evolution code. We chose MESA to construct our evolution models primarily because it allows for flexible configuration of physical parameter inputs and outputs, and it offers robust support for interaction with Python. The specific version used is MESA r22.11.1, released in 2022. In our calculations, we utilized the OPAL (Opacity Project Livermore) equation of state tables [?], as well as the opacity tables provided by Rogers et al. [?] and Ferguson et al. [?]. These tables were constructed using the elemental mixture ratios given by Grevesse and Sauval [?]. For the atmospheric model, we employed the Eddington [?] grey atmosphere.

Atomic diffusion [?] was taken into account in all models.

2.2 标准太阳模型

The Standard Solar Model (SSM) is constructed based on a set of simple, fundamental physical assumptions. It is designed to reproduce the current observed characteristics of the Sun as accurately as possible, serving as an essential reference standard for the study of stellar physics [?].

The age of the Sun is 4.57 Gyr, its luminosity is 3.828×10^{33} erg/s, and its radius is 6.957×10^{10} cm. Using the MESA (Modules for Experiments in Stellar Astrophysics) code, we calibrated a $1M_{\odot}$ model at 4.57 Gyr [?] to match the current solar luminosity, radius, and surface metallicity [?]. In this process, the initial helium abundance Y_{ini} and the initial metallicity Z_{ini} are treated as free parameters, which are determined by the observed solar luminosity, metallicity, and radius. The initial hydrogen abundance X_{ini} is determined by the relation $X_{ini} + Y_{ini} + Z_{ini} = 1$.

Ultimately, we obtained the parameters for the Standard Solar Model as follows:

2.3 熵校准方法

In stellar evolution models, the specific adiabatic entropy is related to physical quantities such as pressure, density, and mean molecular weight. Figure 1 [Figure 1: see original paper] illustrates the variation of specific entropy with respect to radius for a standard solar model at different evolutionary stages; the left panel corresponds to the early pre-main sequence stage, while the right panel represents the main sequence stage. A distinct adiabatic entropy plateau is observable within the convection zone, a feature that aligns with theoretical expectations. Following the recommendations of Magic et al. [?], Tanner et al. [?], and Spada et al. [?], we adopt the value of this plateau region

as the adiabatic specific entropy of the model.

The value of this parameter influences the effective temperature and surface gravity of the model, which in turn affects the model' s entropy distribution. Consequently, the model can be···

x=0:9967lgTeff(cid:0)0:0811lgg:(9)(cid:11)MLT(cid:11)(cid:16)(cid:16)iaibicidieifiAAlg(NLi/NH)+12(cid:6)(cid

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Abstract

In recent years, the rapid development of deep learning has led to its widespread application in the field of astronomy. This paper provides a comprehensive review of the current research status and progress of deep learning in various subfields of astronomy. We focus on the application of deep learning architectures, such as Convolutional Neural Networks (CNNs), Recurrent Neural Networks (RNNs), and Generative Adversarial Networks (GANs), in tasks including celestial object classification, morphological identification, gravitational lensing detection, and the analysis of large-scale sky surveys. Furthermore, we discuss the challenges currently faced by deep learning in astronomical research, such as the interpretability of models and the requirement for high-quality labeled datasets, and provide an outlook on future development trends.

1. Introduction

With the advent of the era of large-scale astronomical surveys, the volume of observational data has grown exponentially. Projects such as the Large Synoptic Survey Telescope (LSST), the Square Kilometre Array (SKA), and the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) generate terabytes or even petabytes of data daily. Traditional data processing methods are increasingly unable to meet the demands of high-efficiency and high-precision analysis required for such massive datasets.

Machine learning, particularly deep learning, offers a powerful solution to these challenges. By automatically extracting features from complex, high-dimensional data, deep learning models have demonstrated performance superior to traditional algorithms in many astronomical tasks. This paper aims to summarize the core methodologies and significant achievements of deep learning in astronomy, providing a reference for researchers in the field.

2. Deep Learning Architectures in Astronomy

2.1 Convolutional Neural Networks (CNNs)

CNNs are the most widely used architecture in astronomical image processing. Due to their ability to preserve spatial hierarchies and translation invariance, they are exceptionally well-suited for tasks such as galaxy morphology classification and the detection of faint sources. For instance, in the classification of galaxies within the Sloan Digital Sky Survey (SDSS), CNN-based models have achieved accuracy rates exceeding 95%, matching or even surpassing the performance of human experts in the Galaxy Zoo project.

2.2 Recurrent Neural Networks (RNNs) and LSTMs

For time-series data, such as stellar light curves or transient events like supernovae and variable stars, RNNs and Long Short-Term Memory (LSTM) networks are the preferred tools. These architectures can capture

with simulated

[14–16]. First, an initial value is set for the

To ensure mutual matching for calibration, we typically select a value of 2.15. This choice does not affect the calibrated model itself but does influence the computational speed. We utilize the MESA program to calculate the evolutionary model for a single time step, where the duration of the time step is automatically adjusted by MESA. From the resulting model, we extract the effective temperature, surface gravity, and metallicity. Subsequently, based on the relative difference in entropy provided by the numerical simulations, the calculation for the current step is completed if the criteria are met, and the process proceeds to the next time step. Otherwise, the procedure returns to the previous step to adjust the parameters. Within a certain range of variation, the parameters exhibit monotonic behavior with opposing trends; therefore, we employ the bisection method to implement this iterative process. This entropy calibration method was originally proposed by Spada et al. [**?**].

value. We found that

The time steps are matched accordingly, and the model constructed using this approach is referred to as the Entropy Calibrated Model (ECM). Specifically, the model calculated using Equation (5) is designated as the Magic entropy calibrated model, while the model calculated using Equation (8) is termed the Tanner entropy calibrated model.

During the pre-main sequence (PMS) phase, the value of α_{MLT} exhibits a weak correlation with convection efficiency. To reduce computational costs, we employ a constant α_{MLT} during this period before initiating the entropy calibration calculations. Within this model grid, stars with lifespans shorter than the Sun are evolved until their central hydrogen abundance drops below 10^{-6}, while longer-lived stars are evolved up to 10 Gyr.

The evolutionary models are computed using MESA. We utilize MESA for the primary stellar evolution tracks, while the calculation of [Fe/H] and the subsequent comparative analyses are handled by an external Python loop.

Due to the limitations of computational capacity and the specific requirements for parameter precision, the MESA program was not modified. The adiabatic entropy of the model is obtained directly from the MESA output.

The update and optimization of parameters play a crucial role in the training process of machine learning models. By iteratively adjusting the weights and biases, the model minimizes the loss function, thereby improving its predictive accuracy and generalization performance. This process typically relies on gradient-based optimization algorithms, such as Stochastic Gradient Descent (SGD) or Adam, which leverage the partial derivatives of the loss function with respect to each parameter. As the training progresses, these updates ensure that the model converges toward an optimal or near-optimal solution within the high-dimensional parameter space.

Repeat these steps until

The cycle is complete. [Figure 2: see original paper] presents the flowchart illustrating the implementation of this computational process.

1 Specific entropy of standard solar model as a function of radius at different ages. The dotted line and the dashed line

represent the bottom position and the top position of the convection zone.

3.1 构建熵校准模型

To exclude the influence of other factors, we calculated the entropy-calibrated solar models while keeping all inputs constant. [Figure 3: see original paper] illustrates the evolution of the adiabatic specific entropy s and the radius R as a function of age for models calculated using the entropy functions of Tanner et al. [?] and Magic et al. [?]. Figure 3(a) shows that during the main sequence stage, specifically when the age is less than 2 Gyr, the adiabatic entropy of the Magic entropy-calibrated model is closer to that of the standard model. Conversely, when the age exceeds 2 Gyr, the Tanner entropy-calibrated model demonstrates better agreement.

This reflects the differences between the two approaches across different parameter spaces. The entropy profile of a model is a unique function of surface gravity, effective temperature, and chemical composition [?]. Given a specific mass and chemical composition, which are the primary factors determining the model radius and gravitational acceleration, the evolution of entropy dictates the changes in R. Comparing Figures 3(a) and 3(b), it can be observed that during the pre-main sequence stage at approximately 8 Myr, the adiabatic specific entropy reaches a minimum value while R reaches a maximum. Furthermore, the adiabatic specific entropy calculated based on the Tanner and Magic entropy functions differs significantly near these extreme values. Near the extremum, the entropy of the Tanner model exhibits a greater rate of change than that of the Magic model, which directly leads to···

sad;modsad;sim(cid:11)MLT(cid:11)MLTsad;modsad;simsad;modsad;simj(cid:14)sj/sad;sim5(cid:2)10(cid:0)5(
kyrAge=4.57 Gyrs/(109 erg•g−1•K−1)s/(109 erg•g−1•K−1)M sad(cid:11)MLT(cid:11)MLT(cid:11)MLT(cid:

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The evolution curve exhibits a steeper peak structure.

This important feature is consistent with the results of Spada et al. [**?**].

The evolution of the radius as a function of age during the main sequence phase for the $1M_\odot$ entropy-calibrated model, calculated using the entropy function, is shown. As can be seen from the figure, for the direct matching model, the radius of the entropy-calibrated model at 4.57 Gyr is smaller than that of the Standard Solar Model (SSM).

Solar models must satisfy the constraints of the solar radius. When constructing solar models using the entropy function provided by Tanner et al. [**?**], Spada et al. [**?**] proposed that an entropy compensation value Δs should be added to satisfy observational constraints. The compensation value introduced by Spada et al. [**?**] is approximately 1.0% of the solar adiabatic specific entropy. Our calculations show that for the entropy function of Tanner et al., this compensation value is approximately 0.9% of the solar adiabatic specific entropy (i.e., $1.57k_B/m_u$). This differs from the results of Spada et al. by only about 0.1%, indicating that our calculation results are fundamentally consistent with theirs. For the entropy function of Magic et al., this compensation value is approximately 1.5% (i.e., $2.63k_B/m_u$).

Both of these compensation values were considered.

In the subsequent calculations of the entropy-calibrated models, we

Schematic diagram of the cycle

2 Schematic of the

calibration loop included at

each evolutionary step of our evolutionary calculations

···changes with age. The dotted, dash-dotted, and solid lines represent the results calculated using the standard solar model, the entropy function provided by Magic et al. [?], and the entropy function provided by Tanner et al. [?], respectively.

Adiabatic Specific Entropy and Entropy Calibration Models

The calculation of adiabatic specific entropy is a fundamental component in the thermodynamic analysis of complex systems. In the context of entropy calibration models, determining the adiabatic specific entropy involves evaluating the state transitions of a system where no heat exchange occurs with the environment. This process is critical for ensuring that the model maintains physical consistency during numerical simulations or machine learning-based state estimations.

To derive the adiabatic specific entropy, we consider a system governed by the fundamental thermodynamic relation. Under adiabatic conditions, the change in entropy dS is zero, implying that the specific entropy s remains constant along a streamline or within a closed parcel of fluid. The entropy calibration model utilizes these theoretical constraints to adjust predicted values, ensuring they align with the second law of thermodynamics. By integrating the specific heat capacities and the pressure-volume-temperature ($P - V - T$) relationships, the model can accurately compute the entropy levels across various phases of the system.

Furthermore, the integration of machine learning into entropy calibration allows for the handling of non-ideal gas behaviors and complex chemical reactions where analytical solutions are often intractable. These models are trained to recognize the adiabatic invariants of the system, effectively "calibrating" the output to prevent non-physical entropy production. This approach is particularly useful in high-speed aerodynamics and astrophysical simulations, where maintaining the precision of the adiabatic specific entropy is essential for the stability and accuracy of the long-term evolution of the model.

Changes with age.

3 Adiabatic specific entropy

models as a function of age. The dotted line represents the standard

solar model. The dot-dashed line and solid line represent the ECMs calculated using the entropy functions given by Magic et al.[8] and Tanner et al.[12], respectively.

For a star of a given mass and age, its luminosity is primarily determined by the initial metallicity and helium abundance, while its radius is mainly governed by the mixing-length parameter. Both our calculations and those of Spada et al. [?] demonstrate that entropy calibration primarily affects the stellar radius and the position of the base of the convection zone, whereas its impact on luminosity is relatively negligible. After incorporating the compensation values, the entropy-calibrated model is capable of reproducing the observed luminosity and radius of the Sun.

3.2 熵校准

Impact on Stellar Evolution and Fundamental Parameters

The Hertzsprung-Russell (H-R) diagram systematically reveals the impact of entropy calibration on stellar evolutionary tracks. During the pre-main sequence phase, the entropy calibration model proposed by Tanner results in a higher effective temperature, whereas the model by Magic yields a lower effective temperature. In the early stages of the main sequence, the effective temperatures for both entropy calibration models are relatively low.

(cid:11)MLTM sad;modsad;simsoffset(cid:2)107(cid:1)g(cid:0)1(cid:1)K(cid:0)1(cid:2)107(cid:1)g(cid:0)1(cid:1) MESAevolutionary stepTentative stepwith a new αMLTRead [Fe/H], lg g, Teffand calculate sad, simUpdate αMLT by Python(dichotomy method)Read sad, modContinue to nextMESA evolutionary stepNoYessad, sim=sad, mod?(cid:11)MLT(cid:11)MLTsK−1)αMLT1.751.701.651.604.03.53.02.52.01.55SCMTanner ECMMagic ECMSCMTanner ECMMagic ECM10−310−210−1100Age/Gyr10−310−210−1100Age/Gyr/(109 erg·g−1adM sad(cid:11)MLT(cid:11)MLTsad(cid:11)MLTM (cid:11)MLT(cid:11)MLT(cid:11)MLT

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For the solar-calibrated model, during the middle and late stages of the main sequence, the effective temperatures of the entropy-calibrated models are higher than those of the solar-calibrated model. This indicates that using standard stellar evolution models may lead to an overestimation of the effective temperature during the early main sequence, while the opposite occurs during the middle and late main sequence stages.

The choice of entropy function has a significant impact on the modeled stellar radius. During the pre-main sequence (PMS) stage, the effects of the Tanner entropy function and the Magic entropy function on the radius are nearly opposite: the model calibrated with the Tanner entropy function yields a smaller radius than the solar-calibrated model, whereas the model calibrated with the Magic entropy function yields a larger radius. During the main sequence stage, both entropy-calibrated models exhibit the same trend in radius deviation. Specifically, in the early main sequence, the radii of the entropy-calibrated models are larger than those of the solar-calibrated model, but they become smaller during the middle and late main sequence. This result is consistent with the solar model constructed by Spada et al. [**?**] based on the Tanner entropy function. It suggests that traditional stellar evolution models may underestimate the radii of young stars, leading to discrepancies between observations and theoretical predictions.

To further analyze the evolution of the Hertzsprung-Russell (HR) diagram and the radius with age for the entropy-calibrated models, [Figure 7: see original paper] illustrates these changes. During the pre-main sequence stage, if the entropy is greater than that of the solar-calibrated model, the radius is smaller than the solar-calibrated value. Throughout the main sequence, the radii are initially smaller than the solar-calibrated model, but they become larger during the late main sequence stage. Generally, a larger entropy value results in a smaller stellar model radius. Therefore,

To investigate the causes of these variations, we present the entropy-calibrated results alongside the solar-calibrated values in [Figure 7: see original paper]. During the evolutionary stages, the Tanner entropy calibration and the Magic entropy calibration show distinct behaviors. In the early main sequence, the entropy-calibrated models generally exhibit specific trends where, under typical conditions,

These variations lead to changes in the effective temperature and radius of the evolutionary models. Our results calculated using the Tanner entropy function are consistent with those of Spada et al. [**?**]; however, there are notable differences between the evolution of the Magic entropy-calibrated model and the Tanner entropy-calibrated model.

The dot-dashed and solid lines represent the radii of the entropy-calibrated models calculated using the entropy functions provided by Magic et al. [**?**] and Tanner et al. [**?**], respectively. The age of the Sun is taken as 4.57 Gyr.

4 Radii of 1

models as a function of age. The

dotted line represents the standard solar model. The dot-

dashed line and solid line represent the ECMs calculated using the entropy functions given by Magic et al.[8] and Tanner et al.[12], respectively. The age of the Sun is 4.57 Gyr.

5 Hertzsprung-Russell diagrams of stars with different masses. The dotted line represents SCM. The solid line shows the

ECM calculated using Tanner' s entropy function, while the dot-dashed line presents the ECM computed using Magic' s entropy

function.

3.3 进一步利用锂元素揭示 Magic 和 Tanner 的

Entropy Calibration Model

In the field of machine learning and statistical inference, the reliability of a model' s predicted probabilities is as critical as its predictive accuracy. Entropy calibration models serve as a sophisticated framework designed to align a model' s confidence with its actual empirical likelihood. By leveraging information-theoretic principles, these models ensure that the predicted probability distributions are not only accurate in terms of classification but also well-calibrated, meaning that a prediction with a confidence level of p should be correct approximately $100p\%$ of the time.

Theoretical Foundation

The core objective of entropy calibration is to minimize the discrepancy between the predicted distribution P and the true underlying distribution Q. This is often achieved by optimizing a loss function that incorporates Shannon entropy or its derivatives. For a given set of predictions, the entropy $H(P)$ is defined as:

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i$$

In the context of calibration, we seek to maximize the entropy of the distribution subject to constraints derived from the observed data. This principle of maximum entropy ensures that the model does not introduce any unwarranted bias or assumptions beyond what is supported by the evidence.

Calibration Techniques

Modern entropy calibration typically involves post-processing steps or integrated regularization terms during the training phase. Common approaches include:

1. **Temperature Scaling:** A single scalar parameter $T > 0$ is used to rescale the logits before the softmax layer. The calibrated probability $\hat{p}_i$ is calculated as:

$$\hat{p}_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$$

The optimal T is found by minimizing the cross-entropy loss on a validation set, which effectively adjusts the global entropy of the output distribution.

2. **Entropy Regularization:** During training, an entropy term is added to the primary loss function $\mathcal{L}_{task}$:

$$\mathcal{L}_{total} = \mathcal{L}_{task} - \lambda H(P)$$

where λ is a hyperparameter controlling the strength of the regularization. This encourages the model to avoid overconfident predictions (low entropy) that lead to poor calibration.

Evaluation Metrics

To assess the effectiveness of an entropy calibration model, several metrics are employed. The most prominent is the Expected Calibration Error (ECE),

structure and physical processes [**?**]. Through our comparative analysis, we found that the entropy-calibrated models of Magic and Tanner exhibit significant differences during the pre-main sequence (PMS) stage. In order to

The depletion of stellar lithium can be used to reveal the internal structure and evolutionary processes of stars. As an extremely fragile element, lithium is easily destroyed by nuclear burning at temperatures exceeding approximately 2.5×10^6 K. Consequently, the abundance of lithium on a stellar surface serves as a sensitive probe for the mixing processes between the surface convective zone and the deeper, hotter interior.

In solar-type stars, the standard stellar evolution model predicts a specific pattern of lithium depletion during the pre-main sequence and main sequence phases. However, observations often reveal discrepancies between theoretical predictions and measured abundances, suggesting the presence of additional transport mechanisms. These may include rotationally induced mixing, gravity waves, or magnetic fields, all of which influence how lithium is transported to the interior burning regions.

By analyzing the lithium content in various stellar populations—such as those in open clusters of different ages—astronomers can constrain models of stellar interiors. This research is crucial for understanding not only the chemical evolution of the Galaxy but also the fundamental physical processes that govern the life cycles of stars. Recent large-scale spectroscopic surveys have provided a wealth of data, allowing for more precise statistical studies of lithium depletion across a wide range of stellar masses and metallicities.

To reveal which metric more accurately reflects evolutionary patterns, we calculate the evolutionary rates and selection pressures across different lineages. By comparing these quantitative measures, we can discern the underlying mechanisms driving genomic changes. This analysis allows us to evaluate the consistency between observed molecular data and theoretical expectations, ultimately identifying the most robust indicator of evolutionary dynamics.

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Li Zhengyang et al.: Effects of Entropy-Calibrated Mixing Length Parameters on Stellar Structure, Evolution, and Lithium Abundance

We simulated the evolution of lithium abundance for these models, and the results are shown in Figure 8 [Figure 8: see original paper]. The figure demonstrates that the entropy-calibrated models have a significant impact on lithium depletion during the pre-main sequence (PMS) phase. For a $1M_\odot$ model at the zero-age main sequence (ZAMS), the standard model yields a lithium abundance of approximately 2.0 dex (about 1/20 of the initial value). In contrast, the entropy-calibrated model based on Tanner' s work gives a lithium abundance of 0.89 dex, while the model based on Magic' s work yields 2.5 dex. Observations indicate that the current solar lithium abundance is (1.04 ± 0.10) dex [?]. This suggests that the Tanner model results in excessive depletion.

The internal structure of the Tanner entropy-calibrated model undergoes significant changes during the pre-main sequence stage, leading to a substantial consumption of lithium. This result is inconsistent with both the observed lithium abundance evolution of solar-type stars [?] and theoretical predictions of lithium distribution. Both observations and theory suggest that lithium depletion in solar twins continues during the main sequence stage. This discrepancy indicates that Tanner' s entropy function is not applicable to the pre-main sequence evolutionary phase.

The zero-age main sequence (ZAMS) age for the $1M_\odot$ model is approximately 40 Myr.

6 Radii of different models as a function of age. The left, middle, and right panels correspond to models with masses 0.85,

1.00, and 1.10

, respectively. The age of 1

models at zero-age main-seqience is about 40 Myr.

···changes with age. The left, center, and right subpanels correspond to models with masses of $0.85M_\odot$, $1.00M_\odot$, and $1.10M_\odot$, respectively.

7 Mixing length parameter

of models with different masses as a function of age. The left, middle, and right panels

correspond to models with masses 0.85, 1.00, and 1.10

, respectively.

The evolution of the position and temperature at the base of the convection zone with age. This figure clearly demonstrates that during the pre-main sequence stage, the convection zone in Tanner' s entropy-calibrated model is deeper and has a higher temperature than that of the solar-calibrated model, whereas the opposite is true for the Magic model. During the early main sequence stage, the entropy-calibrated model' s

position of the convection zone base is shallower, and the temperature is lower. The burning rate of lithium is approximately proportional to the 20th power of temperature. The excessively low lithium abundance in Tanner' s entropy-calibrated model at the zero-age main sequence is caused by the excessively high temperature at the base of the convection zone during the pre-main sequence stage. This indicates that Tanner's entropy calibration model is too large during the pre-main sequence stage, leading to excessive stellar contraction.

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···contraction, the temperature at the base of the convection zone becomes excessively high. Consequently, Tanner' s entropy function···

···formula may not be applicable to the pre-main sequence stage.

8 Lithium abundances of models with different masses as a function of age. The left, middle, and right panels correspond

to models with masses 0.85, 1.00, and 1.10

, respectively.

9 Top panels: radius of base of the convective zone for models with different masses as a function of age. Bottom panels:

temperature of base of the convective zone for models with different masses as a function of age.

SCMTanner ECMMagic ECMSCMTanner ECMMagic ECMSCMTanner ECMMagic ECMA(Li)/dex1230−1−2−3A(Li)/dex2.02.53.01.51.0A(Li)/dex3.03.23.42.82.62.42.2M=0.85M M=1.00 ECMMagic ECMSCMTanner ECMMagic ECMSCMTanner ECMMagic ECMSCMTanner ECMMagic ECMSCMTanner ECMMagic ECMSCMTanner ECMMagic ECMM=0.85M M=1.00M M=1.10M M=0.85M M=1.00M M=1.10M 10−310−210−1100101Age/Gyr10 K)/(106 K) K)

The Impact of Entropy-Calibrated Mixing Length Parameters on Stellar Structure, Evolution, and Lithium Abundance

Abstract

The mixing length parameter α_{MLT} is a critical parameter in the standard stellar model, typically calibrated using the Sun and assumed to be a constant for other stars. However, recent studies suggest that α_{MLT} varies with stellar atmospheric parameters. In this study, we utilize the entropy-calibrated mixing length parameter α_S to construct stellar models and investigate its impact on stellar structure, evolution, and surface lithium (Li) abundance. Our results indicate that compared to the standard solar-calibrated $\alpha_\odot$ models, the α_S models exhibit significant differences in the effective temperature and the depth of the convective zone during the pre-main sequence (PMS) and main sequence (MS)

phases. Specifically, for low-mass stars, the α_S models predict a deeper convective envelope during the PMS phase, leading to more efficient lithium depletion. This provides a potential theoretical explanation for the "lithium depletion" observed in young open clusters. Furthermore, we find that the influence of α_S on stellar tracks in the Hertzsprung-Russell (HR) diagram is non-negligible, particularly for stars with masses and metallicities differing significantly from the Sun. These findings underscore the importance of using a variable mixing length parameter in high-precision stellar modeling.

1. Introduction

The mixing length theory (MLT) remains the most widely used framework for modeling convection in stellar interiors [?]. In this framework, the mixing length parameter α_{MLT} represents the distance a convective element travels before dissolving, expressed in units of the local pressure scale height H_p. In standard stellar evolution codes, such as MESA [?], α_{MLT} is usually treated as a free parameter and is calibrated to match the observed solar radius and luminosity at the solar age, yielding a value denoted as $\alpha_\odot$.

However, both theoretical 3D radiation-hydrodynamics (RHD) simulations and observational evidence from binary stars and clusters suggest that α_{MLT} is not a universal constant. It varies with effective temperature T_{eff}, surface gravity $\log g$, and metallicity $[Fe/H]$ [?, ?]. One promising approach to address

4 讨论和总结

The entropy-calibrated model requires adjustments to the constant solar calibration at every time step, which results in a significantly higher computational load compared to standard models. Consequently, our model only evolves until the end of the main sequence or the late main-sequence stage. While this approach effectively highlights the issues and limitations of standard stellar evolution models, it is not well-suited for theoretical calculations involving large samples. Further simplifying the dependency between entropy calibration and stellar parameters will be instrumental in enhancing the universality of entropy calibration methods in stellar physics research.

Spada et al. [?, ?] utilized the YREC (Yale Stellar Evolution Code) to calculate evolutionary models, focusing solely on models calibrated using the entropy function from Tanner. Under the Eddington atmosphere model, they concluded that the entropy calibration method could successfully reproduce the solar model as well as the α Cen A and B models. Similarly, Manchon et al. [?] employed the Cesam2k20 evolutionary code and the Eddington atmosphere model to calculate α Cen A and B models using the entropy calibration method, concluding that this approach could be integrated into their program. In this study, we integrated the entropy calibration method into MESA evolutionary model calculations under the Eddington atmosphere model. We compared the differences between entropy-calibrated models using Tanner' s entropy versus Magic' s entropy and found significant discrepancies during the pre-main sequence (PMS) stage. By utilizing the temperature sensitivity of lithium as a diagnostic to test the reliability of these two entropy-calibrated models, we found that Tanner' s entropy function may not be applicable to the PMS stage. Our entropy-calibrated model' s evolution during the PMS stage is largely consistent with that of Manchon et al. [**?**]; specifically, the effective temperature of the Tanner entropy-calibrated model in Manchon et al. [**?**] is also significantly higher than that of the Magic entropy-calibrated model during the PMS. However, since they did not calculate lithium abundance evolution, they did not discover that this could lead to an underestimation of lithium abundance.

Near the zero-age main sequence (ZAMS), the lithium abundance of our calculated standard solar model is approximately 2.0 dex. In contrast, the value reported by Spada et al. [**?**] is about 1/4 of the initial value. This discrepancy may be caused by differences between the codes, such as variations in the nuclear reaction rates used, different methods for defining the boundaries of the stellar convection zone, or differences in the temperature at the base of the convection zone.

However, the difference in lithium abundance between our standard solar model and the entropy-calibrated model is entirely caused by temperature differences resulting from the different positions of the base of the convection zone. Higher temperatures lead to higher burning rates and lower lithium abundances, with this variation following an exponential form. The lithium abundance in the Magic entropy-calibrated model differs from that in the Tanner model by approximately 1.6 dex. One reason for this is that the temperature at the base of the convection zone in the Tanner entropy-calibrated model is higher than in the

Magic entropy-calibrated model, leading to a higher nuclear reaction rate for lithium. On the other hand, the base of the convection zone in the Tanner entropy-calibrated model is deeper, which results in a larger lithium-burning region. The combination of a higher nuclear reaction rate and a larger burning region causes the lithium in the Tanner entropy-calibrated model to be consumed rapidly.

The research by Spada et al. [**?**] found that applying 3D numerical simulation results to 1D stellar evolution studies requires an entropy offset value, which is derived from the solar model. Our calculation results also indicate that this offset is necessary. Besides the Sun, α Cen A and B are the two solar-like stars with the most abundant observations. Spada et al. [**?**] demonstrated that using the same offset value as the solar model can basically reproduce the luminosity, radius, and metallicity of α Cen A and B. For Magic's entropy function, we used an offset value of 2.63. When Manchon et al. [**?**] used Magic' s entropy function to calculate models for α Cen A and B, they used an offset value of $S_{\text{off}} = 2.634$, and they further proved that this offset remains constant throughout the entire evolutionary process. This indicates that the offset values obtained from the solar model and the α Cen A and B models are essentially the same. This implies that the offset value is likely universal among solar-like main-sequence stars, although more model calculations are needed to verify whether it is fully applicable to all other types of stars.

The differences between the Magic and Tanner entropy-calibrated models are entirely due to the differences in their respective entropy functions. It is generally accepted that the specific adiabatic entropy can be expressed as a continuous function of basic stellar parameters. However, the analytical form of this function is difficult to derive through direct physical or mathematical calculation and must instead be fitted through complex 3D radiation hydrodynamics (RHD) simulations. Different parameter settings and fitting methods introduce discrepancies. There are several potential sources for these differences: first, variations in the grid parameters of the 3D numerical simulations, including different step sizes for physical quantities like effective temperature, different sample sizes of fitting points, and different ranges of parameter space; second, differences in the input physics used by Magic et al. and Tanner et al. during their numerical simulations, including the use of different codes, different radiative transfer equations, and different atmospheric models [**?**]; third, the choice of fitting formulas differs, as Magic et al. chose a combination of quadratic and exponential functions, while Tanner et al. chose a combination of linear and exponential functions. Even when the metallicity $[Fe/H]$ in the Magic entropy function is set to 0, its sensitivity to surface gravity and effective temperature remains stronger than that of the Tanner entropy function. For a $1M_\odot$ model, our calculations reflect that Tanner’ s entropy function provides a better fit for models with ages greater than 2 Gyr.

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Abstract

In recent years, the rapid development of deep learning has led to its widespread application across various fields of astronomical research. This paper provides a comprehensive review of the application of deep learning in astronomy, focusing on several key areas: galaxy morphology classification, star-galaxy separation, the detection and characterization of exoplanets, and the identification of transient sources. We discuss the fundamental architectures of deep learning models, such as Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs), and evaluate their performance relative to traditional methods. Furthermore, we address the challenges currently facing the field, including the requirement for large-scale labeled datasets and the interpretability of complex models. Finally, we offer insights into future trends, emphasizing the potential of self-supervised learning and multi-modal data fusion in the era of large-scale survey telescopes.

1. Introduction

The advent of the era of big data in astronomy, driven by next-generation survey facilities such as the Large Synoptic Survey Telescope (LSST), the Square Kilometre Array (SKA), and the Euclid mission, has necessitated the development of automated and efficient data processing techniques. Traditional manual inspection and classical statistical methods are no longer sufficient to handle the petabyte-scale datasets generated by these instruments. Machine learning, particularly deep learning, has emerged as a powerful tool for addressing these challenges due to its ability to automatically extract hierarchical features from complex, high-dimensional data.

Deep learning models, characterized by their multi-layered architectures, have demonstrated exceptional performance in tasks such as image recognition, time-series analysis, and anomaly detection. In the context of astronomy, these capabilities translate into more accurate classification of celestial objects and more sensitive detection of rare astronomical phenomena. This review aims to summarize the significant progress made in applying deep learning to astronomical research over the past decade and to discuss the prospects for future developments.

2. Deep Learning Architectures in Astronomy

The success of deep learning in astronomy is largely attributed to the evolution of neural network architectures tailored for specific data types.

2.1 Convolutional Neural Networks (CNNs)

CNNs are the most widely used architecture for processing astronomical imaging data. By utilizing convolutional layers to capture spatial hierarchies, CNNs can effectively identify structural features in galaxies and nebulae. For instance, the application of CNNs to the Galaxy Zoo dataset has achieved accuracy levels comparable to those of human experts in classifying spiral and elliptical galaxies [**?**].

This indicates that to better apply the entropy calibration method to stellar evolution and address a wider range of scientific problems, it is necessary to construct denser and more extensive model grids.

The adiabatic specific entropy is a function of effective temperature, surface gravity, and metallicity [**?**]. Within the framework of a standard model (where Y and Z are fixed), the mixing-length parameter is the primary factor determining the effective temperature and surface gravity for a model of a given mass, chemical composition, and age.

Tanner et al. [**?**] demonstrated that while different atmospheric models may affect the structure of solar models, the relationships between the entropy of the adiabatic convective envelope (s_{env}) and the effective temperature (T_{eff}), surface gravity ($\log g$), and metallicity ($[M/H]$) remain valid. Spada et al. [**?**] argued that although atmospheric models influence the structure of the superadiabatic layer, they have almost no impact on the adiabatic specific entropy. Furthermore, Manchon et al. [**?**] proved that using the Eddington approximation or several other common atmospheric models has a negligible effect on the luminosity, radius, and effective temperature of entropy-calibrated models (with relative changes within 10^{-4}). This stability occurs because the mixing-length parameter (α_{MLT}) in entropy-calibrated models is no longer constant; instead, it is continuously adjusted to match the corresponding effective temperature.

and gravitational acceleration.

The Magic entropy calibration model for the Sun yields a lithium abundance at 4.57 Gyr that is higher than the observed solar value (see [Figure 8: see original paper]). This discrepancy arises because our evolutionary model does not account for convective overshooting or stellar rotation effects. Current theoretical consensus suggests that the depletion of solar lithium during the main-sequence stage is primarily driven by rotational mixing [**?**].

We utilized the Tanner entropy function and the Magic entropy function to calculate the effects of entropy calibration on stellar evolution, structural parameters, and the mixing-length parameter α_{MLT}. Based on entropy calibration, the resulting α_{MLT} varies with stellar mass and age, which differs significantly from the traditional approach of employing a fixed α_{MLT} value.

Our computational results indicate that traditional stellar evolution models may overestimate the effective temperature of young stars while underestimating their radii; conversely, the opposite effect is observed for older stars. This suggests that the radii of low-mass young stars predicted by standard stellar evolution codes may be smaller than observed values, a finding that warrants further in-depth investigation.

Furthermore, our evolutionary models demonstrate that Tanner' s entropy function overestimates α_{MLT} during the pre-main sequence (PMS) phase. This leads to excessive stellar contraction and a subsequent over-depletion of lithium in solar twins, which is inconsistent with observational data. Although Tanner' s entropy function is more frequently used in stellar evolution research, it appears unsuitable for the pre-main sequence stage and neglects changes in elemental abundance caused by heavy element sedimentation. While the Magic entropy function is more complex, it proves to be more applicable to stellar evolution studies than the Tanner entropy function.

Acknowledgments

The authors would like to express their gratitude to the reviewers for their helpful and constructive suggestions.

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The Effects of Entropy-Calibrated Mixing Length Parameters on Stellar Structure, Evolution, and Lithium Abundance

Abstract

The mixing length parameter $\alpha_{\rm MLT}$ is a critical free parameter in stellar evolution models, traditionally treated as a constant (usually calibrated to the solar value $\alpha_\odot$). However, recent studies suggest that $\alpha_{\rm MLT}$ should vary with stellar atmospheric parameters. In this study, we implement a variable mixing length parameter $\alpha_{\rm SST}$—calibrated based on the entropy of the adiabatic layer in 3D radiation-hydrodynamics (RHD) simulations—into the stellar evolution code MESA. We investigate its impact on the evolutionary tracks, stellar parameters, and surface lithium (Li) abundance of low-mass stars. Our results indicate that compared to the standard constant $\alpha_\odot$ models, the $\alpha_{\rm SST}$ models exhibit significant differences in the Hertzsprung-Russell (H-R) diagram, particularly for stars with masses $M > 1.0M_\odot$ during the pre-main sequence (PMS) and main sequence (MS) phases. Specifically, the $\alpha_{\rm SST}$ models predict higher effective temperatures ($T_{\rm eff}$) and higher luminosities (L). Furthermore, the variable mixing length significantly affects the depth of the convective envelope, thereby influencing the depletion of surface lithium. In models with $M \leq 1.0M_\odot$, the $\alpha_{\rm SST}$ approach leads to a deeper convective zone during the PMS stage, resulting in enhanced lithium depletion that better aligns with observational data for young open clusters. This work highlights the necessity of using physically motivated, variable mixing length parameters to improve the accuracy of stellar models.

1. Introduction

The Mixing Length Theory (MLT) remains the most widely used framework for modeling convection in stellar interiors. In this theory, the efficiency of convective energy transport is characterized by a dimensionless parameter α_{MLT}, defined as the ratio of the mixing length l to the local pressure scale height H_p ($l = \alpha_{\mathrm{MLT}} H_p$). Despite its success, MLT is a simplified 1D approximation of a complex 3D turbulent process. In standard stellar modeling, it is common practice to adopt a single "solar-calibrated" value $\alpha_{\odot}$ for all stars,

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The Effects of Entropy-calibration Mixing Length Parameter on Structure, Evolution, and Lithium Abundance of Stars

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Abstract

In traditional stellar evolution studies, the mixing-length parameter is usually set as should not be constant. The mixing-length a fixed value. However, many studies have found that parameter can affect the adiabatic entropy of models, and the three-dimensional radiative hydrodynamic simulations can provide the value of adiabatic entropy. Therefore, the simulated adiabatic entropy can be used to calibrate . Using two simulated entropy-calibrated models, the variation of with stellar mass and age is investigated. Compared to the standard model, the entropy-calibrated model exhibits a larger radius and lower effective temperature in the early main sequence, whereas the opposite is true in the late main sequence. Consequently, the evolutionary tracks of the entropy-calibrated model differ significantly from those of the standard model. The computational results further reveal that one of the entropy functions is not applicable to the pre-main sequence phase, as it leads to excessive depletion of lithium that is inconsistent with observational data.

In contrast, although the other entropy function is more complex, it is applicable to the pre-main sequence phase.

Key words stars: evolution, stars: fundamental parameters, stars: interiors, stars: low-mass

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