A Simulation Approach for Pál-Bell Equations: The 3-D Monte Carlo-GSMP method

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Abstract

An increasing number of reactors are adopting startup procedures without an external neutron source. High-fidelity simulation of core physics during start-up is therefore essential for safety. The Pál-Bell equations describe neutron population fluctuations in weak neutron fields and provide the theoretical basis for stochastic start-up analysis. In this study, Monte Carlo methods for solving the Pál-Bell equations are developed in the NECP-MCX code. In addition to the conventional time-meshed dynamic Monte Carlo (DMC) method, a new three-dimensional Monte Carlo generalized semi-Markov process (3-D MC-GSMP) method is proposed and implemented. The new method removes the time-step dependence inherent in the time-meshed DMC method and overcomes the limited spatial description of the 0-D GSMP formulation. Calculations of burst waiting time (BWT) are performed for prompt-supercritical and delayed-supercritical benchmark reactors, including GODIVA-II, CALIBAN, and GODIVA-I. The calculated BWTs agree well with the experimental measurements and the reference-code results. With dedicated acceleration strategies, the 3-D MC-GSMP method achieves computational efficiency comparable to that of the timemeshed DMC method in prompt-supercritical problems.

Full Text

Preamble

lation Approach l-Bell Equations Monte Carlo-GSMP

method

Jiandi Qingming Jingwen Shen, Liangzhi Zheng

Abstract

increasing number reactors adopting start-up procedures without external neutron source.

High-fidelity simulation physics during start-up therefore essential safety. l-Bell equations describe neutron population fluctuations neutron fields provide theoretical basis stochastic start-up analysis. study, Monte Carlo

methods

solving l-Bell equations developed NECP-MCX code. addition conventional time-meshed dynamic Monte Carlo (DMC) method, three-dimensional Monte Carlo generalized semi-Markov process MC-GSMP)

method

proposed implemented.

method

removes time-step dependence inherent time-meshed

method

overcomes limited spatial description formulation.

Calculations burst waiting (BWT) performed prompt-supercritical delayed-supercritical benchmark reactors, including GODIVA-II, CALIBAN, GODIVA-I. calculated agree experimental measurements reference-code results. dedicated acceleration strategies, MC-GSMP

method

achieves computational efficiency comparable time-meshed

method

prompt-supercritical problems.

Index Terms l-Bell equations, Monte Carlo, generalized semi-Markov process, burst waiting time, NECP-MCX.

NTRODUCTION EUTRON sources essential reactor start-up. elevating neutron

background

subcritical condit external start-up neutron source enables source-range detectors register counts.

These signals provide operators state information during operations loading, shutdown, approach criticality signals supply reactor protection system permissive shutdown signals Given uncertainty strength external start-up neutron source industrial demand emerged start-up without external neutron source During reactor start-up without external neutron source, source-range detectors register measurable signal, which creates detector blind during start-up.

Without external source imply absence eceived revised accepted financially supported National Natural Science Foundation China 12475181) Innovative Scientific Program (Corresponding author:

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Instead, start-up driven decay neutrons intrinsic fuel.

Therefore, process commonly referred literature weak-source start-up stochastic behavior reactor governed inherent randomness fission events, where number energies fission neutrons stochastic These effects become especially pronounced during evolution neutron field. macroscopic level, stochastic behavior manifests fluctuations neutron population spatial inhomogeneity neutron field. microscopic level, manifests itself randomness establishment individual divergent fission chains fluctuations development fission chains observed weak-source start-up experiments.

Alamos National Laboratory (LANL), weak-source ignition experiments designed performed GODIVA burst reactor, where distribution GODIVA burst waiting (BWT) measured observation these experiments burst reactor reproducible instead follows probabilistic distribution.

Subsequent studies probability ignition (POI) carried Caliban burst reactor France [10], Chinese Burst Reactor (CFBR-II) reactor operates stable regime, multiple fission chains overlap, intrinsic stochasticity fission chains largely masked statistical averaging.

During weak-source start-up however, neutron population initially small, rendering averaging inapplicable.

Consequently, weak-source start-up, stochastic neutron transport equation becomes necessary, whereas deterministic Boltzmann transport equation inadequate describing evolution neutron field during early stage. theoretical development reactor stochastic processes largely progressed parallel ignition experiments burst reactors.

Starting probability conservation, derived stochastic neutron transport equation zero-dimensional (0-D) phase space equation later studied systematically through which general formulation space energy established became known l-Bell equation.

Extensions developed incorporate delayed precursors, quantify influence neutron source, derive simplified forms together practical solution strategies.

Subsequent focused refinements l-Bell framework. particular, oz-Cobo derived master equation

probability generating function l-Bell equations China, related studies carried Institute Applied Physics Computational Mathematics (IAPCM) [16], Numerical solution approaches l-Bell equation grouped deterministic

methods

Monte Carlo methods. work, equation shown solvable using discrete ordinates

method

French Alternative Energies Atomic Energy Commission (CEA), Humbert showed delayed-supercritical systems follows gamma distribution, waiting Caliban evaluated using PANDA LANL, calculation capabilities added PARTISN compute probability distribution function (PDF) neutron population neutron fields, together probability survival (POS) IAPCM, developed discontinuous finite element formulations within SNIPER platform solve Williams developed solver l-Bell backward formalism, through which neutron population computed probability start-up under weak-source conditions evaluated [20], [21], [22], [23], recent years, increased computing capability growing techniques solving l-Bell equation. methods, equations solved implicitly simulating fission chains, through which stochastic evolution reactor represented directly.

Compared deterministic discretization, approaches allow first-principles modeling neutron tracking avoid errors induced spatial, energy angular discretization. existing studies direct Dynamic Monte Carlo (DMC) simulations implemented extending conventional transport codes.

Lawrence Livermore National Laboratory (LLNL), capability implemented Mercury LANL, Booth demonstrated rigorously variance-reduction techniques applied evaluation, which substantially improves computational efficiency Delft University Technology, Sjenitzer improved efficiency using series variance-reduction techniques, including forced decay delayed precursors implicit fission Durkee showed stochastic transport simulation conventional codes demanding, because fission chains cause substantial memory consumption Modifications MCNP6 therefore introduced allow users control truncation fission chains during runtime. addition, developed

methods

MCATK compute statistical moments neutron fission counts weak-source systems Tsinghua University, module stochastic neutron transport implemented compute observables coupling strategies between stochastic transport predictor corrector quasi-static (PCQS) dynamic

method

investigated further improve efficiency alternative route treats evolution fission chain generalized semi-Markov process (GSMP). direction, IAPCM developed GSMP-based solvers l-Bell equation simulation program, implemented directly compute prompt neutron decay constant (PNDC), [33], persistent issue theory selection time-step. time-step depends system critical state other numerical strategies, selection often relies empirical judgment. well-chosen improve efficiency, whereas poorly-chosen time-step increase runtime yield incorrect results. method, however, typically formulated model.

Kinetics parameters system should provided, spectral spatial effects multiplying systems cannot resolved. overcome these limitations, three-dimensional (3-D)

method

proposed solving l-Bell equation. System evolution longer discretized different time-steps.

Instead, advanced through event-driven dynamics defined model. time-step selection issue inherent removed, spectral spatial resolution

method

addressed. HEORY section presents theoretical foundations numerical realization stochastic neutron transport processes. derivation l-Bell equations first summarized using probability generating function formulation. concise overview time-meshed implementations equations provided, focusing commonly temporal discretization schemes. section further introduces solution perspectives motivate event-driven treatment equations.

Based these discussions, MC-GSMP

method

formulated solve l-Bell equations. Finally, acceleration strategies improving computational efficiency MC-GSMP

method

presented. erivation l-Bell equations Under neutron field, neutron population spatial distribution exhibit pronounced fluctuations, neutron-field evolution described probabilistic manner.

Starting probability conservation probability balance, l-Bell equations stochastic events source emission, absorption, fission, decay delayed precursors evolution equation generating function. fluctuations neutron population coupled transport process within unified theoretical framework. section, derivation reported literature summarized. l-Bell equations first presented isotropic, monoenergetic transport. equation discussed general setting.

Isotropic, Monoenergetic l-Bell Equations neutron characterized position direction velocity (supposed interaction medium specified nuclide-dependent collision cross section probability collision emits prompt neutrons, where denote probability neutron position direction

results

neutrons region quantity provides direct probabilistic statement l-Bell formulation. written following form: probability initiating neutron first collision position, times probability neutrons emerging collision neutrons probability neutron first collision leads neutrons Suppose neutron probability colliding between probability collision produces neutrons these neutrons required yield neutrons single neutron yield neutrons

$= 2$, one neutron may yield neutrons while the other yields $-$ neutrons.

formation divided where lesser Kronecker delta correction accounts probability neutron undergoes

collision either leaving the system ($=$) or before

time . If $- \geq$,

$= \delta \quad 0$ ’ $=$

If $- <$ and $+ \quad -$

neutron

If $- <$ and $+ \quad -$

neutron probability distribution generating function introduced Multiplying summing differentiating respect obtain: generating function satisfy final condition

$=$ and the boundary condition at $=$

outward direction satisfies outward normal. neutron region otherwise

$= \quad 0$. 2) Anisotropic, Energy-Dependence P á l-Bell Equations Consider the probability

neutron velocity produces neutrons region collision cross section denoted

denote probability neutron produces prompt neutrons collision velocities these neutrons within about about

2 , and similarly for the remaining emitted neutrons.

The generating function is introduced as:

=0 ∞

analogy follows that: Considering External Sources Delayed Neutrons probability prescribed final state produced neutrons considered, where neutron other generating function simply product generating functions neutrons.

Similarly, number source neutrons, generating function given product single-particle generating functions, source neutron.

Suppose birth phase-space coordinates initial neutron unknown. assumed source exists system, probability neutron appears within about about about denote probability neutrons present associated generating function relation between obtained

; =

Therefore, generating function particle known, source generating function solved straightforward substitution. delayed neutrons included, denote probability collision occurring produces neutron whose velocity within about about manner, generalized. further assumed delayed neutron emitted position first integral becomes:

Prompt emission represented time-dependent delta function, boundary final conditions those except integral steady supercritical system, continuous injection source neutrons, including neutrons spontaneous fission, external sources, delayed precursors, inevitably leads power burst.

Before burst occurs, stages traversed. first stage, first divergent fission chain established, which initiated neutron entering system second stage, divergent fission chain evolves duration until prescribed power level reached.

Because second stage characterized exponential growth, dominated sense, establishment divergent fission chain marks triggering power burst. direct connection between l-Bell equations established. burst defined triggered system neutron population reaches equivalently system fission reaches threshold distribution triggering obtained solving l-Bell equations Time-meshed

Method

solving entire fission chain evolution treated single simulation sample.

Specifically, sample corresponds complete realization initiated source event continuing until divergent fission chain established, which interpreted simulation fission chain history. contrast steady-state scheme advances physical computing neutron speed energy converting transport distance flight time.

Because system time-varying states evaluated online different instants, continuous variable discretized sequence time-steps. neutron crosses boundary time-step, stopped boundary stored time-step. length between interactions given exponential distribution therefore allowed simply neutron boundary sample

length time-step, using updated system state. overall workflow simulating single fission chain time-meshed

method

illustrated illustrated workflow simulating single fission chain divided following parts:

Sample first source event. external neutron source modeled Poisson process, interval between successive source events follows exponential distribution. therefore sampled standard inverse-transform relation where denotes external source intensity uniform random variate. subsequent source-event times generated manner.

Initialize neutron time-step. neutrons transported within time-step initialized stored neutron bank. first time-step, neutron contains neutrons emitted spontaneous fission accumulated delayed precursors. spontaneous fission, number emitted neutrons sampled spontaneous fission multiplicity later time-steps, additionally include delayed neutrons emitted within time-step survival neutrons. improve computational efficiency, importance-weight combing technique employed control, respectively, number delayed precursors survival neutrons.

Transport neutrons competition within time-step. neutron taken transported sampling competing distances collision, surface crossing, (iii) time-step boundary. event determined selecting minimum these distances. collision event, collision between neutron nucleus processed absorption reaction, neutron history terminated. fission reaction, number emitted neutrons sampled induced fission multiplicity emitted neutron classified prompt delayed according effective fraction delayed neutron. delayed neutron, emission sampled using decay constant precursor.

Store neutrons reach time-step. Neutrons survive time-step stored neutron time-step. secondary particles produced, transported until either terminated reach time-step. procedure continues until neutrons time-step processed.

Advance time-step. Return initialize simulate neutrons subsequent time-step.

Terminate history fission chain. simulation current fission chain realization stopped prescribed termination criterion satisfied.

Sample the time for the first source event of spontaneous fission accumulated delayed precursors source neutrons and delayed neutrons to the bank of next time a neutron from the bank of current time Absorption Fission Scattering the neutron Determine the type reaction End the history of Treat scattering of emitted neutron a secondary neutron Transport the neutron to the end of the time step and add it to the bank of next time Neutrons in the bank of current time step?

Divergence of the fission chain? chart simulating single fission chain time-meshed method. advantage time-meshed naturally fixed-source workflow conventional steady-state methods. characteristic allows existing codes reused minimal modification, leading straightforward implementation development overhead. principal limitation time-step specified advance. overly large time-step, divergent fission chain generate large number sub-generations within time-step, advancing particles boundary time-step becomes computationally expensive. overly small time-step, transport workload time-step decreases, total runtime increase substantially. because system-state updates, fission chain checks, parallel communication performed frequently.

Time-meshed often applies fixed-threshold truncation time-step.

Population control survival neutrons delayed precursors typically performed time-step. fission chain becomes divergent within time-step, survival neutrons their sub-generations rapidly, sharply reducing simulation efficiency.

Method

multiplying system, distribution neutron population delayed precursors treated system state.

Source emission, collision, precursor decay formulated state-transition events. state, occurrence probability event determines transition probability, while these probabilities defines total transition intensity. interval current state state governed intensity. state depends current state, which implies Markov property. simulation fission chain illustrated tialize running system parameters. running parameters include truncation criterion control algorithm, other parameters. system parameters include intensity equivalent fundamental-mode source reactivity, effective fraction group delayed neutrons, transition probabilities different events.

Sample transition event total transition intensity: transition intensity system reaction intensity system neutrons, decay intensity delayed precursors decay intensity external sources. where neutron population average neutron lifetime.

Sample event conditioned current state. probability neutron-removal (history-terminated) probability external source

event is N =

probability decay event

event is s =

for delayed precursor group is p =

being population precursor group being decay constant.

Initialize GSMP simulation and system parameters Source event Precursor event Neutron event Source neutron Neutron removal event Delayed neutron emission

emission Update system parameters Divergence of the fission chain? chart simulating single fission chain GSMP.

Update system state according realized event. event precursor decay occurs, neutron population increases event spontaneous fission source occurs, neutron population increases where number emitted neutrons spontaneous fission sampled fission multiplicity. neutron-loss event occurs, neutron population changes where number descendant neutrons produced associated reaction. induced fission reaction realized, number emitted neutrons sampled induced fission multiplicity.

Return continue until truncation criterion Compared time-meshed truncation criterion principled because event-driven representation chain growth.

Specifically, suppose fission exhibits clear exponential growth trend. simulation terminated interval determined standard deviation (STD) estimated growth parameter which required smaller prescribed tolerance parameter fitted fission history multiple instants within which yields statistically controlled truncation rule. justified because formulation propagates system using lumped

variables, fission evolution reflects average behavior entire neutron population. setting, randomness single neutron dominate global fluctuation level, which stabilizes fission estimation underpins truncation. advantage model constructs transition intensity whole system, which enables strictly event-driven simulation stable truncation criterion compact algorithm computationally efficient. limitation depends equivalent kinetics parameters, which makes difficult capture spatial spectral effects shape establishment fission chain.

MC-GSMP

Method

basis neutron transport geometry, MC-GSMP

method

established. event-driven characteristics incorporated equations treated explicitly. overall implementation workflow illustrated Divergence of the fission chain? ontrol the number of delayed precursors and surviving neutrons Find the earliest event in the system Collision Source event Precursor event emission Source neutron Treat collision between Delayed neutron emission neutron and nucleus Sample next collision event of the neutron Analyze the system parameters chart simulating single fission chain MC-GSMP.

The detailed procedure for simulating a single fission chain with the 3-D MC-GSMP

method

is as follows. 1) Check for divergence of the fission chain .

Whether the system has become divergent is determined by examining whether the fission rate exhibits exponential growth.

The estimation for the parameter of exponential growth is specified in Step 5. 2) Identification of the earliest system event.

The earliest

event system located through subroutine illustrated First, earliest external-source event determined.

Next, earliest decay event selected delayed precursors bank.

Finally, earliest neutron-collision event identified survival-neutron bank. competition among these three candidates performed, through which earliest system event selected.

Find the earliest external source event Find the earliest decay event Find the earliest neutron collision event Compete and select the earliest event in the system chart finding earliest event system.

Execution system event. chosen event processed according type. external-source event, source neutrons generated. event precursor decay, delayed neutron emitted. event neutron collision, collision reaction processed. handling collision reaction follows procedure time-meshed simulation. neutrons produced collision event sampled. subroutine shown event first sampled neutron, sampling repeated until collision event obtained. neutron stored neutron scheduled collision time.

Events surface crossing treated system events because change system parameters. events therefore handled directly during sampling neutron collision event.

Analysis

system-level parameters. After every fission events, system parameters analyzed. exponential-growth parameter computed where which cumulative number fission events system reaches fission system After samples collected, computed After independent estimates obtained, relative standard deviation (RSD) calculated. criterion divergent fission chain specified

prescribing target value First, MC-GSMP

method

eliminates time-meshing required time-meshed method. event-driven formulation adopted advance entire evolution, through which equation coupled system-level events.

Sample next event of neutron Collision Surface crossing Transport the neutron Save the neutron to the surface to the bank The neutron alive chart finding collision event survival neutrons.

Second, MC-GSMP

method

addresses uncontrollable calculation within time-step arises time-meshed during evaluation divergence. time-meshed method, exponential growth fission checked boundary time-step.

However, large number sub-generations generated within single time-step. truncation become disproportionately expensive within time-step, which degrades overall simulation efficiency.

MC-GSMP, explicit fission-count threshold introduced. exponential-growth triggered cumulative number fission events reaches workload therefore transformed fluctuating number neutron histories within time-step directly bounded stable computational predictable runtime consequently obtained.

Finally,

method

enables neutron transport modeling, which provided original method. applicability transient modeling reactor spatial description fission chain growth therefore substantially improved.

Acceleration Strategies MC-GSMP

method

MC-GSMP method, transport process reformulated ordered sequence events, which entails certain compromises computational efficiency. algorithm implementation, however, simulation performance improved through dedicated acceleration strategies. method, population control neutrons precursors adopted enhance efficiency. section focuses additional acceleration strategies specific MC-GSMP method.

Neutron Sorting Algorithm operation MC-GSMP

method

identification earliest system event which requires earliest-collision neutron earliest-decay precursor located efficiently. min-heap structure maintain banks survival neutrons delayed precursors. min-heap special complete binary widely implement priority queues. defining property stored equal children, always contains minimum pseudo-code inserting element min-heap given Algorithm

Here, denotes array stores either survival neutrons delayed precursors, current element denotes neutron record inserted. event attribute, which corresponds scheduled collision neutron decay precursor. routine exchanges contents entries.

After Algorithm applied, guaranteed store neutron associated earliest event time. gorithm nsert Element Min-heap Require: min-heap array (1-based); current size; insert; event Ensure: updated min-heap; updated while floor break return pseudo-code extracting given Algorithm element minNode denotes min-heap guaranteed smallest After minNode removed, property restored global ordering implied min-heap structure maintained. pseudo-code rebuilding min-heap provided Algorithm importance-weighted combing

method

selects neutrons simulated scanning neutron matching particle weights. invocation

importance weight combing procedure disrupts min-heap organization bank. routine Algorithm restores property, through which global ordering event times within min-heap re-established. gorithm Extract Min-node Min-heap Ensure: minNode extracted minimum (heap top); updated minNode break break return minNode gorithm Rebuild Min-heap floor(break break return Parallel Strategy Based Dynamic Allocation Fission Chains Because length fission chains varies substantially stochasticity, parallel strategy based uniform allocation particle histories typically history-based simulations severe imbalance. mitigate issue, parallel strategy based dynamic allocation fission chains introduced. framework illustrated scheme, master process responsible distributing computational tasks collecting

results

worker processes. After worker process simulates fission chain history returns master process.

result

worker process immediately dispatched worker process Through continuous, on-demand assignment, workload balanced dynamically across worker processes, which improves parallel efficiency simulations.

Schematic diagram parallel strategy based dynamic allocation fission chains.

UMERICAL

RESULTS

NECP-MCX Monte-Carlo particle-transport developed Nuclear Engineering Computational Physics (NECP) Jiaotong University time-meshed

method

MC-GSMP

method

implemented NECP-MCX calculate BWTs. Three highly enriched uranium (HEU) fast-burst-reactor benchmarks selected verify time-meshed MC-GSMP methods.

These GODIVA-I GODIVA-II United States CALIBAN France. parameters three reactors shown Table RAMETERS EACTORS arameters GODIVA-I GODIVA-II CALIBAN Reactivity

235 U

Enrichment Lifetime prompt neutrons Number bursts GODIVA-II CALIBAN experiments conducted prompt-supercritical regime, delayed neutrons negligible impact contrast, GODIVA-I experiments conducted delayed-supercritical regime, delayed precursors fully accounted delayed-neutron parameters taken evaluated nuclear files (ENDF) GODIVA-II GODIVA-II unmoderated burst-irradiation reactor, cylindrical geometry. 1960, Wimett conducted bursts reactor

experiment

began device initially maintained delayed-subcritical state seconds, control inserted (approximately seconds) bring reactivity above prompt-supercritical According literature system undergone period precursor accumulation, intrinsic source strength increased (spontaneous fission) total. calculations started maximum reactivity state equilibrium delayed-neutron spectra fission obtained literature time-meshed method, calculation time-step transporting neutrons within time-step, number survival neutrons combed whenever exceeded fission burst explicitly given.

Therefore,

results

different prescribed fission rates shown fission chain histories simulated using cores every case.

GODIVA-II gradually converges cutoff fission increases. behavior occurs because, fission enters exponential-growth regime, burst triggered within short time.

Dependence GODIVA-II cutoff fission rate.

For the 3-D MC-GSMP method, f = 100000 , =

= 0.05 adopted determine divergence fission chains. fission chain histories simulated using cores.

Table shows

results

GODIVA-II together experiments, time-meshed MC-GSMP method.

ESULTS GODIVA-II

Method

Experiments 3.1/2.4 ECP-MCX time-meshed

method

.18/2.38

method

3.08/2.41 ECP-MCX MC-GSMP

method

3.11/2.36 reduce computational cost, cutoff fission fission/s fission chains simulated.

Calculation fission chains simulated. Calculation GODIVA-II obtained time-meshed MC-GSMP methods, gamma fitting curves. histograms simulated samples, while solid dashed curves represent gamma fittings; curve denotes gamma fitting experimental data.

Distribution GODIVA-II gamma fitting curves. GODIVA-II,

methods

agreement experimental results, indicating predicted accurately NECP-MCX time-meshed MC-GSMP methods.

Since time-meshed approach requires multiple calculations different cutoff criteria MC-GSMP

method

computationally efficient. CALIBAN CALIBAN burst reactor constructed 93.5% enriched Ratel summarized burst experiments performed under weak-source conditions these tests, reactor preconditioned = 0.9 minutes, reached delayed-supercritical state after assembly = 1.00129 then, after about control inserted rapidly reach burst state = 1.00719 According literature intrinsic source strength increased (spontaneous fission) total. calculations started maximum reactivity state time-meshed

method, calculation time-step reduce computational cost, number survival neutrons combed whenever exceeded

Results

different prescribed fission rates shown fission chain histories simulated using cores.

Dependence CALIBAN cutoff fission rate. MC-GSMP method, cutoff criterion adopted CALIBAN GODIVA-II fission chain histories simulated using cores.

Table shows

results

CALIBAN together experiments, time-meshed (NECP-MCX codes MC-GSMP method.

ESULTS CALIBAN

Method

Experiments 0.742/0.70 ECP-MCX time-meshed

method

0.706/0.53 time-meshed

method

0.680/0.53

method

0.795/0.76 ECP-MCX MC-GSMP

method

0.738/0.50 Cutoff fission fission/s. Calculation Calculation CALIBAN distributions obtained time-meshed method, MC-GSMP method, experimental measurements MC-GSMP calculation gives agreement experimental CALIBAN, although predicted slightly lower measured value. codes using time-meshed

method

NECP-MCX) yield similar results, showing consistency contrast,

method

overpredicts relative experimental data. Moreover, MC-GSMP

method

offers higher computational efficiency time-meshed method.

Distribution CALIBAN gamma fitting curves. GODIVA-I GODIVA-I bare, nearly spherical, unreflected metal reactor. first operated delayed critical classic GODIVA-I burst experiments, system taken through prompt critical sudden reactivity increase essentially initial power external neutron source. final reactivity reached under controlled conditions about intrinsic source strength spontaneous fission calculations started maximum reactivity state time-meshed method, calculation time-step Under delayed-supercritical conditions, exponential growth fission chain develops slowly, stricter cutoff criterion required.

MC-GSMP method, = 500000 = 100 = 0.01 adopted determine divergence fission chains.

NECP-MCX methods, fission-chain histories simulated using cores. shows distribution GODIVA-I obtained time-meshed method, MC-GSMP method, code.

Distribution GODIVA-I gamma fitting curves.

Results

GODIVA experiments, time-meshed (NECP-MCX codes MC-GSMP

method

shown Table NECP-MCX, GODIVA-I predicted time-meshed MC-GSMP

methods

agrees results. However, obtained NECP-MCX larger obtained discrepancy mainly attributed limited number simulated samples differences treatment initial source distribution. delayed-supercritical reactors,

time-meshed

method

computationally efficient MC-GSMP

method

because stricter cutoff criterion adopted. calculated

results

lower experiments, which related delayed-neutron parameters.

Previous point-kinetics studies shown delayed-neutron parameters significantly affect predictions particular, calculations using delayed-neutron parameters recommended Tuttle exhibit better agreement experimental measurements.

ESULTS GODIVA-I NEUTRON ARAMETERS

Method

Experiments 31.8/4.6 ECP-MCX time-meshed

method

22.2/5.9 time-meshed

method

21.5/4.3 ECP-MCX MC-GSMP

method

21.2/6.0 Cutoff fission 2.7E11 fission/s. Calculation Calculation Table lists delayed-neutron parameters different data, including Tuttle model ENDF.

ENDF, group largest decay constant higher fraction higher decay constant those Tuttle model. result, calculated noticeably shorter. calculations repeated using Tuttle parameters, updated

results

reported Table

results

obtained time-meshed MC-GSMP

methods

NECP-MCX agree calculations. ELAYED NEUTRON PARAMETERS UTTLE uttle raction constant constant raction ESULTS GODIVA-I NEUTRON ARAMETERS UTTLE

Method

Experiments 31.8/4.6 ECP-MCX time-meshed

method

28.7/6.8 time-meshed

method

29.8/4.7 ECP-MCX MC-GSMP

method

8.1/6.8 ONCLUSION

method

solving l-Bell equations, namely MC-GSMP method, proposed implemented NECP-MCX code.

method

combines event-driven characteristics neutron transport. removes time-step dependence time-meshed

method

overcomes limited spatial description formulation. addition, acceleration strategies, including min-heap-based neutron sorting dynamic parallel allocation fission chains, developed improve computational performance method. time-meshed

method

MC-GSMP

method

implemented NECP-MCX calculations burst reactors. Benchmark calculations carried prompt-supercritical reactors GODIVA-II CALIBAN delayed-supercritical reactor GODIVA-I.

GODIVA-II CALIBAN, NECP-MCX

results

agree experimental measurements reference-code results, which validates capability NECP-MCX prediction demonstrates correctness MC-GSMP method. prompt-supercritical problems, MC-GSMP

method

shows computational-efficiency advantage because avoids repeated calculations different cutoff criteria delayed-supercritical GODIVA-I, predicted agrees reference-code results, while predicted strongly influenced delayed-neutron parameters.

Calculations Tuttle delayed-neutron parameters yield better agreement experimental those data.

These

results

demonstrate MC-GSMP

method

provides effective framework solving l-Bell equations simulating weak-source start-up problems realistic reactor models.

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Note: Figure translations are in progress. See original paper for figures.