A continuous load identification method for ship panel under moving landing forces of aircraft based on an online learning algorithm

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Abstract

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Full Text

Preamble

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Landing of carrier-based aircraft typically induces severe shocks to the ship' s deck, where the amplitude and maximum impact position of the shocks are usually difficult to measure directly. Identifying moving landing forces is critical for structural heal th monitoring of the flight deck. This paper proposes a novel continuous load inversion method based on an online learning algorithm —specifically, a recursive least squares (RLS) algorithm combined with a sliding window strategy—to identify moving landing forces. This method reduces computational costs, significantly shortens load inversion time, and enables practical online load identification. Th e standard RLS algorithm excels at adaptively identifying system parameters with stable quantity and value, which conflicts with the dynamic characteristics of moving landing force identification. To address this, the sliding window technique is introduced. Addition ally, strategies for optimal sensor configuration, sliding window size selection, and initialization of hyperparameters are proposed and discussed. A numerical example of moving forces on a beam is conducted for verifying the method' s superiority over traditional approaches. Finally, the inversion of moving landing forces from three aircraft wheels on a stiffened ship panel is studied via numerical simulations. Results demonstrate that the proposed method achieves high efficiency and accuracy, reducing required time by 70% while maintaining comparable load inversion precision, proving its strong suitability for practical load inversion in complex structures.

Keywords

Inverse problem Moving force identification Structural health monitoring Online learning Recursive least squares

Introduction

The impact of carrier-based aircraft landing on a ship' s flight deck poses critical challenges to structural safety. Due to the complex movement of ships at sea, the trajectory of the contact trajectory between the aircraft landing gear and the flight deck may vary across landings, complicating direct measurement of the applied load magnitude.

Moreover, compared with land-based landings, the combined effects of ship rolling motion can lead to discrepancies in both contact duration and load intensity between the aircraft' s main landing gears and the deck. In this context, fusing multi-sensor data to inversely calculate the interaction load between landing gears and the flight deck emerges as a promising solution. This problem typifies of load identification for moving loads with multiple contact points, a classic inverse issue in the time domain. Existing approaches, such as the Green kernel function method (Chang and Sun 1989), and discrete-time state-space motion equations (Hwang, Kareem, and Kim 2009), typically formulated the load reconstruction through deconvolution—establishing relationships between system responses and input loads.

A canonical moving load problem is the vehicle-bridge interaction, often modeled as a constant-weight, uniformly moving mass on a simply supported Euler beam—a system whose dynamic characteristics has been widely studied analytically(Ouyang 2011; Cifuentes 1989). By deriving the mapping relationship between structural response and the interaction force, the least squares method enables inverse calculation of the interaction force using collected strain response data (Chan and Ashebo 2006). Two primary modeling frameworks exist: the moving mass model and the moving force model. Compared with the moving force model, the moving massing model accounts for additional inertia force induced by vertical acceleration of the mass, making it more accurate for interaction force identification—especially when the mass undergoes sharp vertical motion (e.g., near the beam supports) (Zhang, Zhou, and Quan 2021).

Directly constructing a dynamic model of the forces exerted by a moving object on a target structure through theoretical formulations remains challenging, especially when the moving object itself is complex. Taking aircraft' s landing gear touchdown on a ship' s deck as an example: the wheels cannot be assumed having a constant moving mass as the landing gear' s springs and dampers transmit time-varying forces to the wheels under complex vertical vibrations. In practice, the focus is typically on the moving load borne by the target structure, rather than the motion dynamics of the objects generating these forces. Therefore, the moving load can be treated as an external input to the target structure, with the intrinsic relationship between the moving load and the moving object no longer the primary focus. Thus, moving load identification can be transformed into an input estimation problem for a time-varying mechanical system.

Law et al. (2001) proposed a dynamic programming method based on Bellmans' Principle of Optimality to recursively estimate moving forces using strain and velocity measurements from a bridge model. Additionally, Nordström (2006) incorporated higher-order input approximations and direct input-influence accelerations response into the input estimation for time-variant systems. This dynamic programming method for time-variant system involves two stages: a descending sweep from the current time step backward to the initial step, followed by an ascending sweep in the reverse direction. However, large total step sizes may cause error accumulation, leading to divergence in load identification results.

Another commonly used method for input estimation of the mechanical systems involves integrating the load over the entire time history with the structural response to form a Markov parameter matrix, followed by applying the least squares method to this matrix to reconstruct the full load time history (Sun, Wang, and Shi 2024). Compared with the dynamic programming method, this kind of method typically yields better results for it balances the responses across the entire history. However, its major drawback lies in high computational cost due to operations involving large matrices and matrix inversion. Neither the dynamic programming method nor the full Markov parameter matrix inversion method has yet achieved continuous inversion of moving loads. Both rely on the response data within a specific time window; as the total number of time steps increases, maintaining computational efficiency becomes challenging—especially for complex engineering structures like ship flight decks.

With the advancement of artificial intelligence technology, researchers have extensively applied machine learning algorithms to load inversion problems, establishing mapping relationships between structural loads and dynamic responses through comprehensive simulation campaigns and experimental data acquisition. Jia et al.(Jia et al.

2025) studied an optimized ensemble learning strategy for random dynamic loads identification on offshore platforms. Xin et al.(Xin et al. 2024) proposed physics-informed Res-UNet model for periodic and random loads identification. Shi et al.(Shi et al. 2025) studied a data-driven method for impact load identification of ship panels under tire drop load. AI-based load inversion methods are well-suited for scenarios involving complex load-response mechanisms. However, their reliance on large-scale samples makes them impractical for many real-world engineering problems. Take the landing impact loads of carrier-based aircraft—the focus of this study—as an example: direct measurement of these loads is inherently challenging, and collecting samples from actual ships poses significant challenges. Furthermore, multi-point load distribution during aircraft landing is highly complex, with numerous operating conditions. Even numerical simulations struggle to construct accurate mapping model due to this complexity.

Therefore, the approach of pre-training load-response relationship models is ill-suited for the target problems addressed in this paper.

Adaptive filter is a digital filter that can automatically adjust its parameters (such as weights) based on the input signal t o optimize performance. It is widely used in communication systems, noise cancellation, echo suppression, biomedical signal processing, and more. The key feature of an adaptive filter is its ability to learn and adapt to changes in the environment in real-time, thereby providing better filtering results. In adaptive filtering problems, the target of identification is the system' s parameters or weights; in load identification problems, however, the target is the system' s input. Although there are some differences between these two kinds of inversion problem, they are both the inverse problem and many solutions and ideas can be mutually borrowed from each other.

Typical identification approaches for the adaptive filter problem includes least mean squares (LMS) algorithm, Kalman filter (KF), and recursive least squares

(RLS).

RLS is a widely used adaptive filter algorithm, which is of fast convergence and high accuracy advantages but possess higher computational complexity compared with LMS (Haykin 1995). To conquer this drawback of RLS, sliding window technique is often used for constraining the computation scale. A sliding window kernel RLS algorithm was proposed for the nonlinear channel identification problem in article (Vaerenbergh, Via, and Santamaria 2006). For reducing the computational complexity, (Zhang et al. 2024) introduced the sliding window in a augmented space recursive least-constrained squares algorithm for adaptive filter. In article (Guo et al. 2022), mechanisms for window size adjustment were proposed for improving the efficiency of the sliding window kernel RLS algorithm. Based on the successful application and experience of the sliding window technique combining the RLS algorithm in system identification problem, it is a probable direction to adopt this idea in dealing with the load identification problem, which has not been studied much.

Besides, as the matrix is large and sparse, the ill-posed problem of this method is significant. For improving the robustness of the load identification result, some researchers introduced Tikhonov regularization method (Law et al. 2001; Zhu and Law 2002; Jiang, Ding, and Li 2021) with optimal regularization parameter selection strategy, such as generalized cross-validation(Golub, Heath, and Wahba 1979) and L-curve method(Hansen 1992). Singular value decomposition (SVD) based methods are another kinds of technique widely used for solving the ill-posed problems in load inversion(Wei, Xie, and Liping 2016; Xu 1998). Sensor configuration and arrangement have great influence on improving the accuracy of the load inverse problem under noise influence. For guiding the arrangement of sensors, minimizing the condition number of Markov parameter matrix is a widely used principle for searching the best sensor combination which can alleviate the ill -condition problem (Thite and Thompson 2006; Wang, Law, and Yang 2013). This kind of sensor location selection method performs well when the force location is fixed but it is not always practical for the situation with changing position of moving load.

Regarding the above issues, this paper investigates a continuous load identification method for ship panels under moving landing forces of aircraft based on an online learning algorithm combining RLS and sliding window technique. The paper is organized as follows: In Section 2, the theory of the load reconstruction methods is illustrated in detail, including the principle of identifying the moving landing forces of aircraft for ship panels, the inverse problem formulation used in this study, the standard recursive least square method, and the proposed method including the online learning algorithm and sensor configuration strategy. In Section 3, one numerical example of a moving force act on a beam is carried out to demonstrate the effectiveness and high computation efficiency of the proposed method in moving load identification. The results are further discussed and compared with traditional methods. In Section 4, based on numerical simulation, three moving loads with different trajectories, magnitudes, and durations are applied to the deck structure to simulate the loading characteristics of an aircraft landing gear on the flight deck, and the proposed method is evaluated for testing its the performance in practical problem. Finally, the conclusions are drawn in Section 5.

Nomenclature

(t) qi(t)

moving speed of the force number of mode shapes used number of degrees of freedom number of strain sensors number of acceleration sensors number of the entire sensors number of forces acting on the system sampling period mass matrix of the system damping matrix of the system stiffness matrix of the system load vector in time domain displacement vector in time domain ith mode shape mode shape matrix modal mass matrix of the system modal damping matrix of the system modal stiffness matrix of the system modal load vector of the system modal amplitude of the ith mode in time domain generalized coordinate vector in time domain strain state matrix of the system in continuous time domain input matrix of the system in continuous time domain output matrix of the system in continuous time domain direct transmission matrix of the system in continuous time domain

H, −, +

state vector of the system in continuous time domain state vector of the system at t=kT output vector of the system in continuous time domain output vector of the system at t=kT input vector of the system effective contribution factor of the force at node i output mapping matrix of forces acting on the system in continuous time domain output mapping matrix of forces acting on the system at t=kT output mapping matrix for strain response output mapping matrix for acceleration response output mapping matrix for system responses geometric matrix state matrix of the system in discrete time domain input matrix of the system in discrete time domain number of total time steps the response matrix of the system for all the time steps the input matrix of the system for all the time steps Markov parameter matrix the actual response matrix of the system for all the time steps the actual input matrix of the system for all the time steps forget factor of recursive least square algorithm gain vector of recursive least square algorithm sliding window length initial setting parameter of recursive least square algorithm

Methodology

This section delineates the fundamental principles underlying the proposed methodology, with Section 2.1 providing a detailed analysis of the aircraft landing load identification mechanism specific to shipborne aviation environments, while 2.2 presents the basic formulation of the inverse problem on moving load used in this study, and 2.3 explains the background theory of standard recursive least square algorithm. The proposed moving load identification method will

Figure 1

Figure 1: Figure 1

be elaborately described in section 2.4.

At last, the optimization of sensor configuration will be illustrated in section 2.5.

Principle of load identification for moving landing forces of aircrafts

The load characteristics of aircraft carrier arrested landings involve multi-points moving loads. To precisely determine the magnitude of the load exerted on the ship' s deck, it is imperative to not only install essential structural sensors (e.g., accelerometers, strain gauges) but also acquire real-time trajectory data of the aircraft' s landing gears.

This trajectory information can be obtained through sensor fusion technologies integrating shipboard cameras, radar systems, and the aircraft' s onboard GPS and Inertial Navigation Systems (INS). As shown in Figure 1

, the load amplitude time-history for each aircraft landing gear' s impact on the ship' s deck can be determined through a load inversion identification algorithm, based on the landing gear trajectories, structural sensor responses and structural properties. The identification of aircraft landing gear trajectories via visual or radar sensing is beyond the scope of this study. This paper instead focuses on resolving the load inversion identification problem for multi-point moving loads in complex structures by optimizing sensor selection and layout, based on known landing gear trajectories, to achieve efficient and accurate load magnitude characterization.

Moving load inversion problem formulation

A typical moving force problem on a beam can be illustrated in Figure 2 [FIGURE:2]. The force is assumed as a concentrated point load whose amplitude and position changes as time varies. The moving load inversion problem is to identify the time history of the moving load acting at the structure based on structural responses such displacement, strain or acceleration. The target problem is under the assumption within the linear elastic range so the motion equation of the system is as follows:

¨ ( ) + ˙ ( ) + ( ) = ( ) (−)

where × , × , × , and $×1 *denotethemassmatrix*, *dampingmatrix*, *stiffnessmatrixandloadvectorrespec*
= ∑

= × $×$1, where m denotes the largest order of the modes kept:

¨ + ˙ + =

× (−)

× = ,

× = ,

× = ,

× =

Strain and acceleration ¨ are common response used for measuring the structural system and will be considered in this study. The number of strain sensors and acceleration sensors are and respectively. As the position of the force varies over time, the system should be expressed in time-variant state space formulation based on Eq. (2) as follows:

˙ = ( ) + ( )

= ( ) + ( )

= [

= [

¨

= [

( ) ( )

= [

− − − −]

( ) = [

−]

×

× ( ) = ( )

= × [

− − − −]

×

( ) = × [

−]

× ( ) = ( )

( ) is introduced to represent the effective contribution of the force at different DOFs for decoupling its amplitude and position variation, which is changed over time and influenced by moving speed c. Parameter s stands for the number of forces whose position might changes independently. is the geometric matrix, = . The matrix $×$2 is output mapping matrix for filtering target responses.

and are the mapping matrix for strain response and acceleration response respectively.

$\times \; = [$

$\times$

$\times$

For discretizing the above formulations, zero-order hold (ZOH) is used widely in signal processing. However, first-order hold technique (FOH) is used in this study for better reflecting the sharp variation of impact force which is normal when aircraft is landing on the flight deck of ship. The main difference between ZOH and FOH is the simplification of integration for ( ). Compared with the linear interpolation used in FOH, ZOH takes the value at kT to represent the mean value for duration kT to (k+1)T, which might bring in more error when the ( ) changes rapidly. With sampling period T, the discretization for these two methods is as follows:

$(+)$

$+ \; = \; + \int$

$( ) ( ) \; [(+) -]$

$+ \; = \; + \; (\;) (\;) \int$

$[(+) -]$

$= \; +$

$(+)$

$= \; (\;) \; ,$

$= - (\; - \;) (\;)$

EIYXlctf(t)

$+ \; = \; + \; (\;) \int$

$( ) \; [(+) -]$

$(+)$

$( ) = \; (\;) +$

$((\; + \;)) - \; (\;)$

$(\; - \;)$

$+ \; = \; + \; - \; + \; + \; + \; + \; = \; +$

$- = [- \; -$

$(-) (\; - \;)$

$+ = [$

(−) (−)

− −]

The positions of forces at time step k in real time can be determined as x(k)= ckT. The effect of matrix Lk is to represent the external forces as equivalent loads on adjacent

nodes. When the node space is small enough compared with the beam span l, the load weight at adjacent nodes can be simplified as follows:

When ≤ ( ) ≤ +1, matrix Lk is empty except:

( ) = − (( ) −)/(+ −) + ( ) = (( ) −)/(+ −)

Where Li(k) represents the element at the coordinate Y DOF of the ith node in matrix Lk.

By stacking N sampling states, the following equation is available:

= [

] = [

] + [

− −

] + [

(+ +) (+ + −) (− + + − −) ⋯

(+ +)

(+ + −) −

(+ +)

] = =

where ˆ is the actual response signals. Assume the system' s initial state and initial input is zero, the inverse identification of the load U can be regarded as a minimization

( ) = − ˆ

problem:

Utilizing the method of least-squares algorithm (MLS) history load series U can be achieved as follows

. ( ) = ( ) ( ) = (− ˆ)

(− ˆ)

where,

Theory of recursive least square algorithm (RLS)

= ˆ

= −

= ,

= ˆ

A major drawback of the LS algorithm is that it requires accumulating all historical data for computation at once, which resu lts in high computational costs and poor real- time performance. Recursive Least Squares (RLS)(Haykin 1995) is an adaptive filtering algorithm that allows for the recursive update of the least squares solution. To consider different contribution of the historical data, the forget factor is introduced in the objective function as:

. ( ) = ( ) ( ) ( )

where,

( ) = [

−

< ≤

Similar to Eq.(22), the load series U till time step N is obtained with the derivation of Eq.(24):

( ) = ( ) ( ) ( ) ( ) = ( ) ( )ˆ( )

The following two recursions can be deduced:

Introduces the matrix inversion lemma:

( ) = ( )− ( )

( ) = (−) +

( ) = (−) +

ˆ

(+)− = − − − (+ −)− −

Thus, with Eq.(28) we get

where,

By rearranging Eq.(32), we get

− ( ) = −

− (−) − − ( )

− (−)

( ) = −

− (−)

[+ −

− (−)

]−

Substituting Eq.(29), Eq.(31) and Eq.(33)into Eq.(27), we get

( ) = (−) + ( )[ˆ −

(−)] = (−) + ( ) ( )

( ) = [−

− (−) − − ( )

− (−)]

=

− ( )

( ) is referred as the gain vector and ( ) is the a priori estimation error defined as:

( ) = ˆ −

(−)

The essence of RLS is to estimate the tap-weight factor U(N) combining the last estimation U(N-1) and a correction based on gain vector and the priori estimation error

which is guessed based on the last weight factor U(N-1).

Proposed method of sliding-window recursive least square algorithm (SW-RLS)

From Eq.(18) it can be found that as time step k increases, the size of

grows up as well and all the information and identified load series from the beginning to the last time need to be used. Thus, it will not reduce the computation cost by using the standard RLS to solve the moving load identification problem. In order to reduce the load identification computation cost, sliding window technique is introduced to fix the length of load series to be identified each time. An online learning mechanism is achieved accordingly and the load series will undergo batch-wise incremental updates, thereby eliminating the need for extensive storage or accessing complete historical data.

From Eq.(14), when the initial reference base is at t=0, the kth output can be deduced as follows: ( )

= + − − +

where,

= [(− + + − −) , ⋯, (+ + −) − , (+ +)] ×

( ) = [, ⋯, − ,] ×

Based on the above formula, assume the reference base is at t=(k-W)T, we have

= − + −1 − − − + ,

( )

where,

Use this in Eq.(34) to obtain

= [(−1 + + −2 −) − +1, ⋯, (+ + −) −1, (+ + 0)] $\times 1\mathbf{UW}(\mathbf{k}) = [\mathbf{uk} - \mathbf{W} + \mathbf{1}, ..., \mathbf{uk} - \mathbf{1}, \mathbf{uk}]\mathbf{W} \times$

( ) = (−) + ( )̂[− ,

(−) − − − − − − −]

The length of ( ) is constant W as well as ,

which could reduce the computation cost as the time step increase. The core idea of the proposed algorithm is to recursively update the most recent W weight factors, namely the identified load ( ), with a gradually moving reference base − , − , − as k increases. The summary of the proposed method is shown in the following Table 1 . The max length of the list is controlled at W by the circular storage scheme, last in first out (LIFO).

When the time step k<W, the load estimation method has no difference with RLS; when k>W, the reference state moves positively as time step increases and only the last W load elements are updated recursively.

Identified moving load series, U(N)

Actual response signals, ̂( ) = [̂1, ̂2, ⋯, ̂] System coefficient matrix P, Q0, G, H-, H+ Initial condition 0 , 0 and L0

Input: Output: Parameters: Initialization: Define list U, for storing load series and loading position Lk respectively Append 0 and L0 to list U and respectively. Let [ ] stand for the kth element in − (0) = − Set Set reference base = 0 For each instant of time, k=1, 2, ⋯N

Sliding window length, W Forget factor, Initial setting parameter for

− , a small positive constant

Compute the position matrix Lk according to Eq.(17), and append to the last of list

Compute input vector

= {

[(−1 + + −2 −) [1], ⋯, (+ + −) [− 1], (+ +) [ ]] $\times$1, ifk < W[(**PG**$W-1$**H**$+$+**PG**$W-2$**H**$-$)$\mathcal{L}$[1], ..., (**PGH**$+$+**PH**$-$)$\mathcal{L}[W-1]$, (**PH**$+$ +**Q0**)$\mathcal{L}[W]]W\times$\$1, ≥

Compute gain vector ( ): −1(− 1) ( ) = −1 Update reference state if >

[+ −1

−1(− 1)

Pop the first element in list Compute a priori estimation error ( )

= + − [1] − + + [2] − +1

(− 1), < (− 1) − − −1 − [1] − , ≥

ˆ −

ˆ −

Update load estimation series

( ) = (− 1) + ( ) ( ), < ( ) = (− 1) + ( ) ( ), ≥

Update

−1( )

− ( ) = − [

− (− 1) − ( )

− (− 1)]

Strategy of sensor configuration

The principle of sensor configuration optimization is to improve the accuracy and stability of solving inverse problems in state-space equations. Thus, direct invertibility and bounded-input bounded-output (BIBO) stability of the load inverse problem are first two critical criteria for determining the number and type of the sensor plan used in this study.

When a linear system is directly invertible, it implies that there is a unique inverse system capable of perfectly recovering the original input from the output. Conversely, the output can be precisely determined from the input. This concept is especially crucial in signal reconstruction problems. As demonstrated in article (MasseyY and Sain 1968), Eq.(14) is direct invertible if and only if ( ) = which implies that the number of the acceleration response, , must be larger or equal to the number of the load, s.

( ) = (×

− ×) =

The BIBO stability of a system is essential because it ensures that the system' s output remains within finite bounds, prevent ing divergence even when the input is limited in magnitude. Load identification is fundamentally an inverse problem where the sensor response serves as the input and the l oad as the output. To achieve a stable identified load, the BIBO stability of the inverse problem of the original system must be guaranteed by ensuring that the poles of the discrete-time system lie within the unit circle. The poles of the inverse system are determined by the transmission zeros of the original system(Callier and Desoer 1991). If only acceleration or velocity responses are considered as the outputs of the original system, there will be at least one transmission zero on the unit circle corresponding to stati c loads, leading to marginal stability(Maes et al. 2015).

It is clear that acceleration or velocity sensors are not sensitive to constant static loads, making it difficult to identify such loads with dynamic sensors alone. This issue can be mitigated by including displacement or strain measurements, and the total number of these measurements should be equal to or greater than the number of loads. In other words, using only acceleration measurements is feasible only in the absence of static or low-frequency loads. For this study, if the moving load is of constant amplitude and stable moving speed, it is close to low-frequency load dynamic features. Thus, the number of strain sensors selected in this study, , is recommended to be equal or larger than the number of load s.

Moreover, considering the variant position of the moving load, only one pair of strain and acceleration sensor cannot guarantee the load inversion is always of high accuracy because the best position of sensor is changing as the load position moving. Thus, the sensors should be distributed evenly as far as possible to adapt more possible loading positions. However, installing too many sensors will affect the property of the target structure, increase the cost and stop from practical usage. Thus, for balancing this dilemma between high accuracy and fewer sensors installed, optimum sensor candidate positions are selected based on a coupling factor ranking strategy in this study. The coupling factor is proposed as a criterion to guide the sensor allocation, defined as the Pearson correlation coefficient between the projections of the load position i and the response component j onto different modal shapes of various orders, and the sensors with highest coupling factor rank for all the load positions will be selected as installation positions. See Eq.(44)~(46) for details.

$=$

$(\hat{}$

$, \hat{} \; \hat{} \; \hat{}$

$\sum (\hat{}$

$, - \hat{}$

$)(\hat{}$

, − ^

)

√∑ (^

, − ^

)

√∑ (^

=

, − ^

)

= {

, for acceleration sensor j , for strain sensor j

where Li is the output mapping matrix when the load act at position i and
Finally, considering that this study addresses load inversion problems with simultaneous multi-point excitations, particularly for carrier-based aircraft the two main landing gears exhibiting high trajectory synchronization, it is imperative to deploy sensors in positions that maximize the distinction between the influences of two interacting forces.

Based on structural mechanics principles, since structural responses are minimized in highly stiff regions with reduced discrimination between adjacent load application points, it is more important to prioritize sensor deployment configurations exhibiting enhanced sensitivity to positional differences of loads in these critical areas. The sensitivity parameter for filtering the sensor can be defined as follows:

) are the load mapping matrix for sensor j as defined in section 2.1.

(or

| −

Where and

′ are responses for sensor position xi and xi + dx. Positions with the maximum sensitivity parameter Si will be selected as candidate sensor locations to

enhance the discrimination sensitivity between closely located load positions.

In summary, four principles are introduced in this study for sensor configuration, which are direct invertibility, BIBO stability, coupling factor ranking strategy and

sensitivity to positional differences of loads.

Numerical validation

To demonstrate the validity and efficiency of the proposed method compared with traditional method in moving load identification, a numerical example of a moving

load along a beam is investigated in this section.

Error metrics

To evaluate the accuracy of the identified load, three kinds of error metrics are introduced in this study: the mean absolute percentage error (MAPE), Pearson correlation

coefficient (PCC), and relative peak error (RPE). The definition of each metrics is as follows:

=

$-\hat{}$

=

(, ^)   ^

$\sum$ ($-$)(^ $-$^)

$\sqrt{\sum}$ ($-$)

$\sqrt{\sum}$ (^ $-$^)

=

| ( ) $-$ (^)| (^)

Where U is the identified load series and ^ is the actual load series.

Model description

As is shown in Figure 3 [FIGURE:3], a one span steel beam is hinged supported at one end and rolling supported at one the other end. The length of the span is l=1000mm, with the cross-section b×h=20mm$×$40mm, Young' s modulus 2.1x1011pa, Poisson ratio=0.3, and density = 7800kg/m3. 5% modal damping is applied in this example. One moving force f(t) acts vertically at the beam with moving speed c=2000mm/s. When t=0, the force is at the left end of the beam and the force will leave the beam after 0.5s. The FE simulation and time series response acquisition is achieved in commercial software MSC.NASTRAN. The moving force is simulated by TLOAD2 and DAREA build-in command of the software(González 2001). The sampling period is 0.2ms. For comparing the computational efficiency of various methods, the main configuration of the test computer is selected as: Intel Core i7-12700F, 2.10GHz (CPU), 64GB DDR5 @ 4400MHz (RAM).

Two cases of f(t) are considered in this study (See Figure 4 [FIGURE:4]):

Case 1: 1( ) = $-$20 ( );

Case 2: 2( ) = {

−100 sin(10 (− 0.2)) ( ), 0.2 < ≤ 0.3 ; 0,

YXctf(t)bhA-AAAl=1000mm125mmstrain sensorstrain sensor175mm

The element size of the beam is modelled as 50mm, and there is total 21 nodes and 20 elements. As is discussed in (Li Sun 2024), adding acceleration response is critical if the noise influence on the response is considered, else only adapting strain sensor is enough. The purpose in this section is mainly to compare the computation efficiency and accuracy among different load inversion method, noise is omitted for simplicity and will be considered in the next sectio n for more complex cases. Thus, only stain sensors will be selected in this example. The center of each beam element is assumed as probable strain sensor position. From Figure 5 [FIGURE:5], it can be found that No.4 and No.18 elements possess highest coupling factor for strain sensors which are selected for sensor installation positions as Figure 3 shows.

Result and Discussion

Comparison among MLS, RLS and SW-RLS

The highlight of this method is its efficiency while not compromising on accuracy. The main parameters used in this example is as follows: forget factor equals 1.0, = 1 −6, and the window size is set to 350 steps compared with the whole process of 2500 steps. By using the proposed method SW-RLS, the average time consumed for each step identification after window initialization is abt. 0.16ms~0.19ms which is lower the sampling period 0.2ms, which means its speed is fast enough for online continuous load identification. As for the accuracy, comparisons among MLS, RLS and the proposed method SW-RLS are carried out for different cases. The specific result for each case is as follows:

Result of Case 1: As the moving force is constant along the beam, there is no peak or variance for the identified load so it is not applicable to use error metrics PCC or RPE. Thus, for Case 1, only MAPE is taken for evaluating the accuracy of each method. The amplitude and the moving speed of the load in this case are both constant, and the response of the structure is with static or low frequency feature. However, the acceleration sensor is more suitable for catching dynamic features than the low frequency features. Adding

It can be found in Table 2 that SW-RLS has comparable precision to MLS and RLS, but its computation time is significantly lower, being only abt. 30% of MLS and less

than 5% of normal RLS.

Result

SW-RLS

2.138%

2.177%

2.254%

Total time

13.7s

115.3s

The identified loads by different method can be found in Figure 6 [FIGURE:6]. The identified load curve shows a consistent trend with the actual values, except for the areas close to the boundaries at both ends. Similar phenomena have also been observed in (Zhang, Zhou, and Quan 2021). The reason for the error happened near the boundaries is probably that when the load acts near the boundaries, most of its effect is absorbed by the boundary and little signal can be broadcast to the sensor so smaller identified load will be attained compared with the actual one.

Result of Case 2: For this case, MAPE is not suitable as large portion of the moving load is zero except the time between 0.2s~0.3s, which is likely to lead to zero divisor when calculate MAPE. From Table 3 , it can be found the accuracy of the proposed method SW-RLS is less than 1% for RPE metrics and nearly 1.0 for PCC metrics, which is of the similar performance compared with MLS and normal RLS. However, the total time for SW-RLS is still much less than the other two method.

Result

SW-RLS

0.323%

0.323%

0.429%

Total time

15.1 s

110.9 s

4.4 s

The trend of the identified load curve is generally consistent with the actual one except the region between 0.3~0.36s. The load in this region changes rapidly and not smoothly. To improve the accuracy around this region, increasing the window size is recommended for introducing more information and av eraging the error invoked by the sharp changes. And this is proved to be effective. From the Figure 7 FIGURE:7 it can be found that the result with window size 500 steps is much closer to the actual applied load than the one with window size 350 steps. More completing discussion on the effect of window size will be illustrated in 3.3.2.

Effect of different window size

The wider the window is, the higher the accuracy of the load identification can be generally achieved because more historical signals can be used in the inference of the current load step. However, the selection of window size will also be affected by the required load identification computation time and the available computation resource. Too large window size will increase computation cost and lead to longer computing time which might dissatisfy the real-time requirement in the engineering application.

The window size for guaranteeing a good load identification accuracy could be determined by identifying the starting point where the second norm of the error between the actual response and the estimated response, $\| \; - \hat{} \|2$ converges to its minimum. As Figure 8 [FIGURE:8] and Figure 9 [FIGURE:9] shows, when the windows size ≥ 350 the norm of the estimated response error converges to a stable value and it is good enough to set W=350 for this example.

Application: Moving forces identification of aircraft landing on a deck panel

The case studies presented in Section 3 exhibit limitations in several critical aspects, including model complexity and multi-forces interactions, while also neglecting practical considerations such as sensor noise effects on measurement accuracy. To address these limitations, this section introduces a sophisticated simulation framework that closely replicates the operational environment of carrier-based aircraft landings, incorporating enhanced modeling of landing gears interactions when impact at the flight deck, and sensor noise characteristics. This expanded case study serves to systematically validate and comprehensively evaluate t he efficacy of the proposed methodology under realistic operational conditions.

Model description

The deck panel in this study is fully rigidly constrained around its perimeter, with dimensions length x width x thichness:1440mm x 720mm x 4mm. Young' s modulus and density of the structure material is 2.1x1011pa and 7800kg/m3. Poisson ratio is set to 0.3 and 5% modal damping is assumed. The plate strengthened by stiffeners and supports whose scantlings could be found in Figure 10 [FIGURE:10].

Three moving forces act at the middle region of the panel as the dashed lines shown in Figure 10. To avoid the influence from the boundary, the moving force starts from

one span away from the boundary. The FE simulation and time series response acquisition is achieved in commercial software MSC.NASTRAN.

Based on the principle in section 2.5, four acceleration sensors and ten strain sensors are set on the deck panel. Considering symmetric load and structure, an asymmetric sensor arrangement was chosen as is shown in Figure 10. Acceleration sensors A1~A4 and strain sensors SX1~SX6 are all selected based on coupling factor ranking strategy.

The selection of strain sensors SY1~SY6 is determined based on the principle of improving the discrimination between trajectory-similar moving forces.

To approximately simulate the impact of aircraft landing gears on the flight deck during the landing process, based on the characteristics of the landing gear load-time history curve presented in (Zhi-xin et al. 2021; Wen et al. 2009), three landing gear forces with varying magnitudes and different activation times are introduced considering rolling motion of ship and pitch angle during landing process of the aircraft. Two cases are evaluated:

(1) Symmetric forces of two main landing gears: The two main landing gears are assumed to contact flight deck at the same time and the fore landing gear lags behind

about 0.06s.

(2) Asymmetric forces of two main landing gears: The left main landing gear is assumed to contact the flight deck at first and the right main landing gear lags a little

due to ship' s rolling motion, and the fore landing gear impact the deck at last considering the pitch angle during landing process.

As the landing process of the aircraft is very fast, constant moving speed, 2m/s, is assumed in this study. The trajectories of these three forces are shown in Figure 10 and

the expression of the amplitudes are defined as follows:

Force of left main landing gear, Fl:

$( ) = \{$

$-100 \cos (25 (- 0.1) -$

$) , 0.1 < \leq 0.12 ;$

$-50 -9(-0.12) \cos(25 (- 0.12)) - 50, 0.12 < \leq 0.38 ; -50, > 0.38$

Force of right main landing gear, Fr:

SX6SX3SX4SY1SY4SY2SY5SY3SY6FfFlFrSX5YxA4120240Primary support frame/girder T100x2/31x4 Secondary transversal frame T40x2/12x2 Longitudinal Stiffener T40x2/13x4 Strain sensorAcceleration sensorLoad trajectoryA3A2A1SX1SX2

Asymmetric case: $( ) = \{$

$-60 \cos (25 (- 0.14) -$

$) , 0.14 < \leq 0.16 ;$

$-15 - 9(\; -0.16) \cos(25 (\; - 0.16)) - 45, 0.16 < \; \leq 0.42 ; -45, \; > 0.42$

Force of fore landing gear, Ff:

Symmetric case: $( ) = \; ( )$

$( ) = \{$

$-200 \cos (25 (\; - 0.16) -$

$) , 0.16 < \; \leq 0.18 ;$

$-160 \cos(25 (\; - 0.18)) - 40, 0.18 < \; \leq 0.2 ; -40, \; > 0.2$

It should be noted that the load of the aircraft landing gear on the flight deck in real situation is much more complex than the assumptions mentioned above, including changes in contact area, nonlinear characteristics of vibration decay, and multiple points of interaction. The simplified load assumption here is only to verify the applicability of the proposed method in dealing with multi-points moving loads identification problem. Gaussian white noise is added to the structural response y calculated by FE software with 5% noise level for better approximating the real conditions.

$$= \; + \quad \times \quad ( ) \times \sqrt{\sum}$$

Results and discussion

Selection of initialization hyper parameter

The window size of SW-RLS is set to 200 decided by the method adopted in section 3.3.2. Another important hyperparameter of the proposed method is which stems from the standard RLS algorithm and is commonly used for the initial condition definition as Table 1 illustrates. plays important role of regularization on the load inverse result especially when the measure noise is considered. By introducing in initialization, the RLS algorithm including the proposed method yields the recursive solution to the following optimization problem(Sayed and Kailath 1994):

$$[( ) \; ( ) ( ) + \; \| \|]$$

The first part of Eq.(46) is the same with Eq.(24) which controls the error between regression accuracy on the given sampled responses, but the second part controls the regularization of load inverse result. As the window size is limited for the proposed method, there is no need to introduce the forget factor and it is suggested to set = 1.0 generally. In this way, the higher is, the smoother the load inverse result will be. However, too large will also bring in larger deviation from the real case. On the contrary, although a sufficiently small can ensure a low residual between the estimated and actual responses, it may lead to identified loads that are widely diverge nt and significantly deviate from the real load.

For selecting a proper , a methodology referencing the L-curve method is adopted. As is shown in Figure 13 [FIGURE:13], a relationship between the

L2 norm of response estimation error ‖ − ˆ‖2 and the L2 norm of the load inverse result U at different is plotted. It is observed that the turning point around = 1 × 10−7 represent a balanced performance not only in regression accuracy but also in the regularization of the load inverse result.

Comparison between different sensor configuration

To verify the effectiveness of the strategy introduced in section 2.5, three pairs of sensor plan are evaluated, and the specific type and the position of each sensor plan

could be found in Figure 10. The comparison results are list in Table 4 , where some conclusions can be drawn:

Using strain sensor alone (See No.2 sensor plan) can only achieve good results under noise-free conditions; once the noise impact becomes significant, its inversion accuracy will be greatly reduced. It can be observed from the load inversion curves in Figure 14 FIGURE:14 and Figure 15 FIGURE:15 that under the influence of noise, the load identified by sensor plan 2 fluctuates violently, indicating poor stability.

On the contrary, by comparing sensor plan 1 and sensor plan 2, it can be found that adding acceleration sensors can enhance the load identification accuracy even under noise influence which indicates the importance of direct invertibility for load inversion problem.

A comparison between plan 1 and plan 3 under the case of asymmetric forces reveals that, except for the inversion peak results of Fl, the remaining results of the two plans are essentially similar. The inversion error in plan 3 is approximately 4% to 5% higher than that in plan 1. Based on Figure 14(c), it can be observed that near t=0.12 s, the inversion result of Fl is significantly lower than the actual value, while the inversion result of Fr is notably higher than the actual value. Moreover, the error shapes of the two are essentially complementary, and the sum of their inversion results is close to the total actual force. On the contrary, for the case of symmetric forces the difference between these two plans at the criteria RPE of Fl decreases to abt. 1.8%.

From Figure 16 [FIGURE:16] it can be found that the forces of the two main landing gears just act at a primary support frame at t=0.12s, and it is clearly illustrated that by adding sensors (SY1~SY6) based on the strategy of enhancing the sensitivity to positional differences of loads at rigid area can reduce the error of multi-load identification with close acting trajectories.

Based on the following comparisons, it is demonstrated that the sensor configuration strategy proposed in this paper exhibits high load inversion accuracy. Even under the influence of noise or for asymmetric multi-point load scenarios, the peak error remains below 2% with a 5% noise impact.

Asymmetric forces of two main

Symmetric forces of two

Sensor Plan

Error Metric

landing gears

main landing gears

A1~A4

SX1~SX6

SY1~SY6

SX1~SX6

SY1~SY6

A1~A4

SX1~SX6

2.29%

0.23%

1.87%

0.17%

2.32%

0.186%

1.18%

0.18%

2.91%

0.85%

5.95%

0.18%

2.72%

0.61%

1.71%

0.34%

10.66%

10.13%

14.07%

8.37%

3.72%

1.35%

6.30%

0.43%

2.18%

0.27%

1.59%

0.32%

2.21%

2.54%

0.66%

0.86%

0.79%

10.27%

0.22%

1.06%

0.77%

2.70%

10.19%

13.89%

11.70%

3.54%

0.95%

2.67%

0.94%

1.52%

2.63%

1.60%

Conclusions

In this study, a novel continuous inversion method, based on an online learning mechanism combining recursive least squares algorithm and sliding-window technique, is proposed to identify the moving landing forces of aircraft on the flight deck, which could reduce the computation cost and shorten the load inversion time remarkably. A basic validation numerical example of moving force on a two ends simple supported beam is used to demonstrate the validity and high efficiency of the proposed method.

Besides, a more practical example, three moving forces on a deck panel with different trajectories and amplitude time series under noise interference, is carried out for further evaluating the proposed method. The main conclusions can be summarized as follows:

The proposed method SW-RLS could overcome the drawback of the standard RLS method being unsuitable for dynamic increases in the dimension of the coefficients, and the dimension of the coefficients to be updated could be fixed by the proposed sliding window technique. It can significantly reduce the computational cost and computation time while maintaining similar accuracy compared with traditional method, MLS, which needs huge computation resource

for large matrix operation. In the example of moving force on a beam, the proposed method could save abt. 70% of computation time compared with MLS method.

The window size used in the proposed method is an important hyper parameter which determines the load inversion accuracy and computation cost. The convergence of the error between the actual response and the estimated one generated by the identified loads is suggested as the criteria for selecting the optimum window size. However, it should be clarified that sometimes for achieving a fast load inversion result, it has to make compromises and bal ances between the accuracy and efficiency, and select a relative smaller window size compared with the best one.

The proposed method is based on RLS algorithm and its initialization hyper parameter plays an important role in regularizing the identified loads especially when noise interference is considered. By probably adjusting the value of , more robust load inversion result can be achieved. Neither too high nor too low value of is a good choice. A similar method with L-curve is proposed for deciding the value of , and has been proved to be effective. With 5% white noise influence, the proposed method could reach less than 3% for MAPE metric and 2% for RPE metric in the application example.

Combining the acceleration and strain sensors is an effective way for handling the noise impact, which has been proved in the application example. Compared with the sensor plan of arranging strain sensors alone, adding acceleration sensors could offer better load inversion result when the noise level is remarkable. This is consistent with the principle that adding acceleration response could improve the direct invertibility of the system and accordingly the interference resistance.

The sensor filter mechanisms proposed in this study have been demonstrated to exhibit high inversion accuracy in complex multi-point load inversion scenarios.

The loads positional sensitivity-based screening strategy proves particularly effective in addressing the high identification errors arising from similar impa ct trajectories and asymmetric impact loads between left and right landing gears.

Several improvements for the propose method can be investigated in the future. At first, the critical challenge of effectively separating low-frequency components arising from wave-induced effects within flight deck structural responses from carrier-based aircraft impact-induced responses—aiming to enhance the inversion accuracy of aircraft landing loads—represents one of the emerging research priorities for future investigations. Besides, for further reducing the load identification computation time, it is a significant direction to investigate replacing the current fixed window size with variant window size to find a dynamic balance between accuracy and computation efficiency under different inputs. At last, the effectiveness of the proposed method shall be further validated by scaled experiments in the future.

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Disclosure statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

The data that support the finding of this study are available from the corresponding author upon reasonable request.

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