Testing the Black Hole No-Hair Theorem Using Extreme Mass-Ratio Inspirals (Postprint)

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Abstract

Extreme-mass-ratio inspirals (EMRIs) are significant wave sources for future space-based gravitational wave detectors. They can provide precise spacetime information around supermassive black holes, thereby allowing for rigorous tests of the black hole no-hair theorem. Previous simulation tests have typically employed model-dependent black hole models or used simplified simulation data. In this paper, a parameterized arbitrary axisymmetric black hole is used to calculate the EMRI gravitational wave waveforms, and time-delay interferometry is utilized to simulate space-based gravitational wave observation data. The Fisher information matrix is applied to evaluate the capability of gravitational waves to test deviations in the black hole quadrupole moment. The results indicate that future space-based gravitational wave detection can constrain the no-hair theorem to the order of 10^{-4}.

Full Text

Preamble

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Testing the Black Hole No-Hair Theorem Using Extreme Mass-Ratio Inspiral Systems Changhong Yi [1], Binglin Wang [2,3], Wenbiao Han [2,1,3]

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摘要

Extreme-mass-ratio inspirals (EMRIs) are critical wave sources for future space-based gravitational wave detectors. These systems provide precise information regarding the spacetime surrounding supermassive black holes, thereby enabling rigorous tests of the black hole no-hair theorem. Previous simulation-based tests have typically relied on model-dependent black hole frameworks or simplified simulation data.

In this paper, we employ a parameterized arbitrary axisymmetric black hole model to calculate EMRI gravitational waveforms. Furthermore, we utilize Time Delay Interferometry (TDI) to simulate space-based gravitational wave observational data. By applying the Fisher Information Matrix, we evaluate the capability of gravitational wave observations to constrain deviations in the black hole quadrupole moment. Our results demonstrate that future space-based gravitational wave detectors can constrain the no-hair theorem to an order of 10^{-4}.

关键词

Black Holes; Extreme Mass Ratio Inspirals; Parameterized Metrics; No-Hair Theorem

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Introduction

The detection of gravitational waves has opened a new window for observing the universe, providing a unique laboratory for testing general relativity in the strong-field regime. Among the various sources of gravitational waves, Extreme Mass Ratio Inspirals (EMRIs) are particularly significant. An EMRI system typically consists of a stellar-mass compact object (such as a black hole or neutron star) orbiting a supermassive black hole (SMBH) located at the center of a galaxy. Due to the large mass ratio, the smaller object undergoes millions of orbits before finally plunging into the central black hole, emitting long-lived gravitational wave signals that carry detailed information about the spacetime geometry surrounding the SMBH.

One of the fundamental predictions of general relativity is the "no-hair theorem," which states that stationary, isolated black holes are uniquely described by only three externally observable classical parameters: mass (M), spin (J), and electric charge (Q). In the astrophysical context, where charge is expected to be negligible, the Kerr metric serves as the standard description for these objects. EMRIs provide an ideal opportunity to test the validity of the Kerr hypothesis.

By mapping the spacetime metric through gravitational wave observations, we can determine whether the central object is indeed a Kerr black hole or a more exotic compact object.

To quantify potential deviations from general relativity or the Kerr geometry, researchers have developed various parameterized metrics. These frameworks introduce additional deformation parameters into the standard Kerr metric, allowing for a systematic search for non-Kerr features. If observations from future space-based gravitational wave detectors, such as LISA, Taiji, or TianQin, constrain these parameters to be zero, it would provide strong evidence in support of the no-hair theorem. Conversely, any statistically significant deviation would signal the presence of new physics beyond general relativity or the existence of non-standard compact objects.

In this paper, we investigate the dynamics of EMRIs within the context of parameterized metrics. We analyze how deviations from the Kerr geometry influence the orbital evolution and the resulting gravitational waveform. By employing a systematic parameterization, we aim to establish a robust framework for testing the no-hair theorem using future high-precision gravitational wave data. This approach not only tests the fundamental principles of black hole physics but also enhances our understanding of the complex environments at the hearts of galaxies.

1 引

Document Classification Code: A

On September 14, 2015, the Laser Interferometer Gravitational-wave Observatory (LIGO) achieved the first direct observation [?, ?] of a gravitational wave event, GW150914, produced by a binary merger. This milestone marked the beginning of a new era in gravitational-wave astronomy. Since then, the LIGO-Virgo-KAGRA collaboration has confirmed dozens of additional gravitational wave events \cite{3-6}. While ground-based gravitational wave detectors operate effectively in the $10 \sim 10^3$ Hz frequency band, space-based detectors scheduled for launch around 2030—such as the Laser Interferometer Space Antenna (LISA) \cite{7-9}, Taiji [?, ?], and TianQin—will enable the detection of gravitational waves in the millihertz band.

Potential wave sources observable by space-based gravitational wave detectors include massive black hole binary (MBHB) mergers, the inspiral of stellar-origin black hole binaries (SOBHB), and extreme-mass-ratio inspirals (EMRI). This paper focuses primarily on EMRI systems. An EMRI typically consists of a massive black hole (MBH) with a mass of $10^4 M_\odot \sim 10^7 M_\odot$ and a stellar-mass compact object (SCO) with a mass of $1 M_\odot \sim 10^2 M_\odot$ that continuously inspirals toward the MBH. In a typical EMRI system, the mass ratio between the secondary and primary body is generally between $10^{-7} \sim 10^{-4}$. Due to this significant mass disparity, the SCO inspirals slowly; during the final year before the SCO falls into the MBH, the system undergoes approximately $10^4 \sim 10^5$

orbital cycles. For the majority of these cycles, the SCO remains within the strong gravitational field near the MBH' s event horizon.

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EMRI signals contain rich information regarding the spacetime geometry surrounding a central massive black hole. Consequently, the observation and analysis of EMRI signals will provide a novel pathway for studying the laws of gravitational physics in extreme astrophysical environments and the fundamental nature of black holes. A common formation channel for EMRIs involves an SCO being captured from a nuclear star cluster into a high-eccentricity orbit around an MBH \cite{16-20}. During the LISA mission, it is expected that anywhere from several to several thousand EMRI systems formed via this channel will be observed \cite{15, 21-23}.

[24–29]

According to the no-hair theorem, a classical black hole is uniquely determined by three parameters: mass, spin, and charge. Given that black holes in realistic astrophysical environments rapidly lose their charge, they can be treated as electrically neutral. Consequently, the spacetime geometry surrounding a black hole is fully described by the Kerr metric. Therefore, testing the no-hair theorem reduces to verifying whether a black hole is indeed a Kerr black hole. The background spacetime of a massive black hole (MBH) can be described via a multipole expansion. For a Kerr black hole, higher-order multipole moments are entirely determined by the black hole' s mass and spin. Since the multipole structure varies across different theories of gravity, the Kerr metric can be tested by measuring these higher-order moments. Beyond mass and angular momentum, the quadrupole moment is the dominant term in the multipole expansion; thus, this paper aims to verify the Kerr metric by measuring the quadrupole moment. Previous studies have indicated that the observation and analysis of Extreme Mass Ratio Inspiral (EMRI) signals can achieve this goal with high precision. Using an analytical kludge (AK) EMRI waveform template modified by a dimensionless quadrupole parameter, Barack and Cutler found that for a given central MBH mass, LISA could constrain this dimensionless parameter to the order of 10^{-4}. Furthermore, Babak et al. and Fan et al. have respectively investigated the capabilities of LISA and TianQin to constrain non-Kerr quadrupole moments.

To test the Kerr metric in a model-independent manner, researchers have proposed several parameterized metrics. The stationary, axisymmetric metric proposed by Konoplya, Rezzolla, and Zhidenko—hereafter referred to as the KRZ metric—provides an effective and general parameterized method for describing the spacetime around a black hole. By utilizing a power series expansion in the polar angle and a Padé approximation in the radial direction, the KRZ metric can accurately reproduce metrics such as the Kerr metric throughout all space using only low-order expansions. Consequently, it exhibits superior convergence near the black hole horizon. Moreover, metrics from other gravitational theories can be obtained by adjusting the deformation parameters within the KRZ framework. These advantages make the KRZ metric an excellent tool for testing the Kerr hypothesis.

In this paper, we utilize the KRZ metric to introduce an additional quadrupole moment into the EMRI waveform. We simulate the observational data of EMRI waveforms for LISA and Taiji and test the no-hair theorem by measuring this additional quadrupole moment. The structure of this paper is as follows: Section 2 introduces the transformation from the KRZ metric to the Bumpy metric and calculates the EMRI waveforms derived from the KRZ metric using the numerical kludge (NK) method. Section 3 applies the Fisher information matrix to estimate the measurement precision of the quadrupole moment. Throughout this work, geometric units ($G = c = 1$) are adopted.

2 任意轴对称黑洞 EMRI 波形

The differences between steady-state axisymmetric black hole solutions in alternative theories of gravity and the Kerr black hole solution can be characterized by variations in their quadrupole moments. By introducing an additional quadrupole moment into the Kerr black hole EMRI (Extreme Mass Ratio Inspiral) waveform and measuring this deviation, one can test for potential violations of the no-hair theorem. This chapter will describe how to adjust the parameters within the KRZ metric based on this additional quadrupole moment to transform it into the corresponding "Bumpy" black hole metric, as well as the methodology for calculating EMRI waveforms using the KRZ metric.

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2.1 从 KRZ 度规到 Bumpy 度规

The KRZ metric, proposed by Konoplya, Rezzolla, and Zhidenko, provides an effective parametrization method for describing the spacetime of general stationary, axisymmetric black holes. This framework incorporates a set of deformation parameters used to measure deviations from the Kerr metric. By selecting appropriate deformation parameters, the KRZ metric can be reduced to the Kerr metric, the Bumpy black hole metric, or other common metrics found in various gravitational theories. In Boyer-Lindquist coordinates, the line element of the KRZ metric is given by:

$$
\begin{aligned}
ds^2 = & -\frac{N^2(r,\theta) - W^2(r,\theta)\sin^2\theta}{K^2(r,\theta)}dt^2 - 2W(r,\theta)r\sin^2\theta dt d\phi \\
& + \frac{\Sigma(r,\theta)}{N^2(r,\theta)}dr^2 + \Sigma(r,\theta)r^2 d\theta^2 + \frac{B^2(r,\theta)K^2(r,\theta)r^2\sin^2\theta}{\Sigma(r,\theta)}d\phi^2
\end{aligned}
$$

d 2 = −

The functions $K(r,\tilde{\theta})$, $N(r,\tilde{\theta})$, $W(r,\tilde{\theta})$, and $B(r,\tilde{\theta})$ in the above equation can be expanded as series in $\cos\theta$. Their lowest-order expressions are given as follows:

$$N^2 = \left(1 - \frac{r_0}{r}\right)\left[1 - \epsilon_0\frac{r_0}{r} + (k_{00} - \epsilon_0)\frac{r_0^2}{r^2} + \delta_1\frac{r_0^3}{r^3} + ...\right]$$

$$B = 1 + \delta_4\frac{r_0^2}{r^2} + \delta_5\frac{r_0^2}{r^2}\cos^2\theta$$

$$W = \frac{w_{00}\frac{r_0^2}{r^2} + \delta_2\frac{r_0^3}{r^3} + \delta_3\frac{r_0^3}{r^3}\cos^2\theta}{\Sigma}$$

$$K = 1 + \frac{aW}{r} + k_{00}\frac{r_0}{r} + k_{21}\frac{r_0}{r}\left[1 + \frac{k_{22}(1 - r_0/r)}{1 + k_{23}(1 - r_0/r)}\right]\cos\theta/\Sigma$$

$$+\left[a_{20}\frac{r_0}{r} + a_{21}\frac{r_0}{r} + k_{21}\frac{r_0}{r}\left(1 + \frac{k_{22}(1 - r_0/r)}{1 + k_{23}(1 - r_0/r)}\right)\right](1 + \cos^2\theta)$$

˜ = / , 20 = 2 ˜ 2 / 03 ,

˜ = / ,

21 = − 20 − 21 ,

= 1 + 2 cos2 / ˜2 ,

0 = (2 − 0)/ 0 ,

00 = ˜ 2 / 02 ,

00 = 2 / ˜ 02 .

This paper follows the conventions established by Li and Han for the Konoplya-Rezzolla-Zhidenko (KRZ) metric parameters: $a_{01} \leftrightarrow \delta_1$ and $\omega \leftrightarrow \delta_2$.

21 3 , 01 4 ,

21 5 21 ˜ 4 / 04 − 2 ˜ 2 / 03 − 6 22 − ˜ 2 / 02 + 7

23 ˜ 2 / 02 + 8

When all parameters δ_i are set to zero ($\delta_i = 0$), the KRZ metric reduces to the Kerr metric. The multipole moment structure of a bumpy black hole exhibits partial deviations from the predictions of general relativity for classical black holes. When these deviations are set to zero, the bumpy black hole degenerates into a standard black hole, such as a Schwarzschild or Kerr black hole.

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The line element in Boyer-Lindquist coordinates is given by:

$$
\begin{aligned}
ds^2 = & -e^{2\psi_1}\left(1-\frac{4a^2Mr\sin^2\theta}{\Delta^2\Sigma}\right)dt^2 + e^{2\psi_1-\gamma_1}(1-e^{\gamma_1})dtdr - e^{2\gamma_1-2\psi_1}\frac{4aMr\sin\theta}{\Sigma}dtd\phi \\
& + e^{\gamma_1}\left[\frac{\Sigma}{\Delta}+\frac{a^2\sin^2\theta}{\Delta\Sigma}(1-2e^{4\psi_1-4\gamma_1})(1-e^{\gamma_1})\right]dr^2 - e^{-2\psi_1}\left(1-\frac{4a^2M^2r^2\sin^2\theta}{\Delta^2\Sigma}\right)(1-e^{\gamma_1})a\sin^2\theta drd\phi \\
& + \Sigma d\theta^2 + \frac{e^{2\gamma_1-2\psi_1}(\Sigma-2Mr)}{\Delta\Sigma}\sin^2\theta d\phi^2
\end{aligned}
$$

[45, 47]

In this context, $\Delta \equiv r^2 - 2Mr + a^2$, while γ_1 and ψ_1 represent the perturbation potentials induced by mass moment and spin moment perturbations, respectively.

In the absence of perturbations ($\gamma_1 = 0, \psi_1 = 0$), the Bumpy black hole metric reduces to the Kerr metric. Vigeland and Hughes provided the quadrupole perturbation potentials ($l = 2$) for a Bumpy black hole in Boyer-Lindquist coordinates:

$$
\psi_1(r,\theta) = \frac{3L(r,\theta,a)}{d(r,\theta,a)^3}\frac{2\cos^2\theta}{d(r,\theta,a)^2}\frac{B_2M^3}{5}
$$

$$
\gamma_1(r,\theta) = \frac{B_2}{d(r,\theta,a)^5}\left[L(r,\theta,a)c_{20}(r,a) + c_{22}(r,a)\cos^2\theta + c_{24}(r,a)\cos^4\theta\right]
$$

(, ,) = 2 − 2 + (2 + 2) cos2 , (, ,) = (−) 2 + 2 cos2 , 20 (,) = 2(−) 4 − 5 2 (−) 2 + 3 4 , 22 (,) = 5 2 (−) 2 − 3 4 + 2 4(−) 2 − 5 2 , 24 (,) = 2 2 2 + 5 2 ,

B_2 is determined by the Bumpy quadrupole formula: $Q_B = -Ma^2 - \frac{B_2M^3}{\sqrt{5/4\pi}} = Q_K + \Delta Q$.

We selected the KRZ metric parameters provided by Li and Han and reduced them to the Bumpy black hole metric:

$$
\begin{cases}
\delta_1 = (2\psi_1+1)\left[1-\frac{3}{r}\left(1-\sqrt{1-\delta_6}\right)\right]\frac{3\cos^2\theta-1}{2} - \frac{1}{3}(1-2\psi_1) \\
\delta_6 = 3\tan^2\theta \\
\delta_2 = \delta_3 = \delta_4 = \delta_5 = 0
\end{cases}
$$

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2.2 波形计算方法

The extreme mass ratio of EMRI systems implies that high-precision EMRI waveforms can be calculated using black hole perturbation theory. To further improve waveform accuracy, the influence of the orbiting stellar-mass compact object’ s own gravitational field on the orbit—commonly referred to as the gravitational self-force effect—has been effectively integrated into this framework. However, the solution process for this method is complex and requires significant computational resources, making it unsuitable for scenarios that require rapid and efficient waveform generation.

In practical applications, approximate templates such as AK, NK, and AAK (augmented analytic kludge) are also widely adopted. These approximate waveform templates satisfy the requirements for efficient waveform generation while retaining the primary characteristics of the signal.

This paper adopts the NK method introduced in references [**?**, **?**]. Its specific implementation is as follows: the compact object inspiraling toward the supermassive black hole is treated as a test particle. Given the metric surrounding the supermassive black hole, the trajectory of the test particle can be obtained by numerically integrating the geodesic equations. Subsequently, the gravitational waves are derived from the test particle’ s geodesic using the quadrupole formula. Due to gravitational wave radiation, orbital parameters such as energy and angular momentum are no longer conserved. After calculating the radiation reaction of the gravitational waves, the orbit is corrected through the evolution of energy, angular momentum, and the Carter constant. In this section, Greek letters $(\mu, \nu, \sigma, ...)$ denote indices from 0 to 3, and the Einstein summation convention is assumed by default.

The geodesic equations for the particle are as follows:

$$\dot{u}^{\mu} = -\Gamma^{\mu}_{\rho\sigma} u^{\rho} u^{\sigma},$$

¤ = ,

In this context, x^{μ} represents the Boyer-Lindquist coordinates of the particle, u^{μ} denotes the 4-velocity, and $\Gamma^{\rho}_{\mu\sigma}$ refers to the Christoffel symbols. During the integration process, we monitor numerical errors by leveraging the conservation of mass, energy, and angular momentum. At each integration step, we verify the following conserved quantities: the magnitude of the 4-velocity $|u|$, the energy E, and the z-component of the angular momentum L_z:

$$|u| = -1 = g_{\mu\nu} u^{\mu} u^{\nu}$$

= − = − − ,

= = + .

In the Kerr case, the Carter constant Q must also be examined: $Q = g_{\theta\theta}u^\theta u^\theta + \cos^2\theta[a^2(1-E^2) + L_z^2/\sin^2\theta]$.

- cos −

sin

During the calculation process, we maintain the relative offset of the aforementioned conserved quantities within 10^{-7}. This ensures that the waveform precision meets the requirement of 10^{-5} (for a typical EMRI, the waveform precision generally requires a phase error of less than 1 radian after 10^5 cycles).

Stable bounded geodesics in Kerr spacetime can be characterized using three parameters: eccentricity e, semi-latus rectum p, and inclination angle ι. They are defined as follows:

$$e = \frac{r_a - r_p}{r_a + r_p}, \quad p = \frac{2r_a r_p}{r_a + r_p}, \quad \iota = \frac{\pi}{2} - \theta_{\min}$$

where r_a is the apastron, r_p is the periastron, and $\theta_{\min}$ is the minimum value of the θ coordinate along the geodesic. In Kerr spacetime, the three orbital parameters (e, p, ι) can be determined from the three conserved quantities (E, L_z, Q), and vice versa. In KRZ spacetime, we

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continue to employ these definitions; specifically, for numerically generated trajectories, we still define (e, p, ι) via the aforementioned relationships using $(r_a, r_p, \theta_{\min})$.

If the influence of radiation reaction on the waveform is considered, Eq. (13) is modified as:

$$\frac{Du^\mu}{d\tau} = -\Gamma^\mu_{\rho\sigma}u^\rho u^\sigma + \mathcal{F}^\mu$$

From which we obtain $\dot{E}, \dot{L}_z, \dot{Q}$:

The radiation force $\mathcal{F}^\mu$ can be determined by the radiation fluxes $\dot{E}$,

¤ = − F − F ¤ = F + F

¤ = 2 2 F + 2 cos2 2 ¤ + 2 cos2 sin2 F = 0

After generating the particle trajectories, we transform the coordinates from the spherical coordinate system definition (rather than the Boyer-Lindquist coordinate system) into Cartesian coordinates (x, y, z) as follows:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

= sin sin ,

= cos .

Finally, to calculate the gravitational waveform, we continue to employ the quadrupole formula derived from the Einstein field equations. This approach is justified by our assumption that even if the black hole deviates from the no-hair theorem, such deviations remain small. Consequently, any resulting corrections to the waveform amplitude are higher-order small quantities. Furthermore, considering that gravitational wave detection is primarily sensitive to the phase rather than the amplitude, we adopt the following quadrupole formula for the approximation:

$$h_{jk} = \frac{2}{r}\ddot{I}_{jk}(t_0)$$

where $t_0 = t - r$ represents the retarded time, and the quadrupole moment is defined as $I_{jk} = \mu x_j x_k$.

¯ (,) =

Here, $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\rho\sigma}h_{\rho\sigma}$ represents the trace-reversed metric perturbation. By transforming the waveform into the transverse-traceless (TT) gauge, we obtain the "+" and "×" polarization components at the polar angle Θ and azimuthal angle Φ:

$$\begin{aligned} h_+ &= h_{\Theta\Theta} - h_{\Phi\Phi} \\ &= \frac{2}{r}\{h_{xx}\cos^2\Theta\cos^2\Phi + h_{xy}\cos^2\Theta\sin 2\Phi + h_{yy}\cos^2\Theta\sin^2\Phi + h_{zz}\sin^2\Theta \\ &\quad - \sin 2\Theta[h_{xz}\cos\Phi + h_{yz}\sin\Phi]\} - [h_{xx}\sin^2\Phi - h_{xy}\sin 2\Phi + h_{yy}\cos^2\Phi] \end{aligned}$$

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3.1 时间延迟干涉技术

A typical space-based gravitational wave detector consists of three satellites that maintain a nearly equilateral triangular configuration while operating in orbit. These detectors observe disturbances caused by gravitational waves by measuring the laser Doppler frequency shifts exchanged between the satellites.

However, due to the relative motion between the satellites, space-based gravitational wave detectors cannot maintain equal arm lengths like ground-based detectors. When two laser beams converge on a photodetector, the laser frequency noise cannot be effectively canceled because the beams have undergone different time delays. The magnitude of this laser frequency noise can be several orders of magnitude stronger than the target gravitational wave signal, completely obstructing signal observation. To suppress laser frequency noise, Time-Delay Interferometry (TDI) was developed \cite{55-57}. The operating principle of TDI is to construct a virtual equal-arm interferometer by introducing appropriate time delays into the signals and linearly combining multiple interference signals.

The first-generation TDI combinations can effectively eliminate laser frequency noise in a stable unequal-arm array (where the distances between satellites remain constant). The Michelson combination X_1 channel is expressed as follows:

$$X_1 = y_{13} + D_{13}y_{31} + D_{131}y_{12} + D_{1312}y_{21} - [y_{12} + D_{12}y_{21} + D_{121}y_{13} + D_{1213}y_{31}]$$

D ( ) = − ( ) , D 1 , 2 ,···, ( ) = −

+1 ( ) ,

In this context, y_{ij} represents the relative frequency shift of the laser after traveling along link ij, and D denotes the time delay operator. The term $L_{ij}(t)$ refers to the propagation time along link ij at the reception time t, while $x(t)$ represents an arbitrary data stream. The other two Michelson channels, Y_1 and Z_1, can be obtained through the cyclic permutation of the satellite indices.

First-generation TDI is only applicable to the ideal case where the connections between satellites are rigid. In practice, the relative motion between satellites cannot be ignored; therefore, second-generation TDI was developed. Within this framework, the Michelson combination for the X_2 channel is defined as:

$$X_2 = X_1+D_{13121}y_{12}+D_{131212}y_{21}+D_{1312121}y_{13}+D_{13121213}y_{31}-[D_{12131}y_{13}+D_{121313}y_{31}+D_{1213131}y_{12}+D_{12131312}y$$

Similarly, the other two channels, Y_2 and Z_2, can be derived by cyclically permuting the satellite indices. Since the Michelson channels contain correlated noise, a set of decorrelated TDI combinations (A, E, T) can be obtained through a linear combination of (X, Y, Z) as follows:

$$A = \frac{1}{\sqrt{2}}(Z - X)$$
$$E = \frac{1}{\sqrt{6}}(X - 2Y + Z)$$
$$T = \frac{1}{\sqrt{3}}(X + Y + Z)$$

In this paper, we will perform simulations based on the observational data from the A channel.

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3.2 Fisher 信息矩阵

Following the detection of a gravitational wave signal, the subsequent step involves determining the physical parameters of the gravitational wave source. This is typically achieved using Markov Chain Monte Carlo (MCMC) sampling methods to obtain the posterior probability distribution of the parameters as determined by Bayesian inference. However, the parameter space for gravitational waves is high-dimensional, and MCMC methods require the collection of a vast number of sample points, which significantly increases the total computational time. In contrast, the Fisher information matrix does not search the entire parameter space; it can only predict the measurement errors of a gravitational wave signal for a given set of parameters. Nevertheless, it consumes far fewer computational resources and can serve as an effective preprocessing step prior to more precise calculations.

The likelihood L is defined as follows:

(|) exp − h − ( ) | − ( )i ,

Here, $x = n + s$ represents the measured data, which is the sum of the actual gravitational wave signal s and the noise n. The term $h(\theta)$ denotes the gravitational wave signal template used, where θ represents the various parameters of the signal $\{\theta_1, \theta_2, \ldots, \theta_i, \ldots\}$. The inner product of two signals, $\langle h_1 | h_2 \rangle$, is defined as:

$$\langle h_1 | h_2 \rangle = 2 \int_{-\infty}^{\infty} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df = 4\mathrm{Re} \int_0^{\infty} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df$$

where $S_n(f)$ is the noise power spectral density of the detector. Given a known waveform template, the Fisher information matrix can be obtained as follows:

$$\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle_{\theta=\theta_0}$$

The inverse of the Fisher matrix Γ_{ij} is the covariance matrix of the parameter errors. The error for each parameter σ_i and the correlation coefficient c_{ij} between two parameters are given by:

$$\sigma_i = \sqrt{(\Gamma^{-1})_{ii}}, \quad c_{ij} = \frac{(\Gamma^{-1})_{ij}}{\sqrt{(\Gamma^{-1})_{ii}(\Gamma^{-1})_{jj}}}$$

3.3 结果分析

Using the waveform where all δ values are 0 as a baseline, we varied a specific δ while keeping all other δ values at 0. It can be observed that δ_1, δ_2, and δ_4 (corresponding to values below 0.97) have a significant impact on the matching factor. In contrast, δ_6, δ_7, and δ_8 (represented by the brown, pink, and gray lines in [Figure 1: see original paper], respectively) have a relatively weaker effect. [Figure 2: see original paper] and [Figure 3: see original paper] further illustrate this difference. [Figure 2: see original paper] shows the EMRI waveforms generated using the calculation method introduced in Section 2.2, where panels (a) and (b) represent the "+" and "×" polarization components, respectively. The yellow, black, and blue lines correspond to three scenarios: all δ set to 0, only δ_1 changed to 0.001, and only δ_6 changed to 0.001. It is evident that the waveform with the modified δ_1 exhibits a phase shift relative to the control group much more rapidly. [Figure 3: see original paper] displays the simulation results of the waveforms from Figures 1 and 2 within the LISA TDI-A channel, showing that the impact of changing δ_1 is even more pronounced.

We simulated the response of the Taiji and LISA TDI-A channels to EMRI waveforms incorporating quadrupole moment deviations. We then estimated the error of the quadrupole moment using the Fisher Information Matrix, with a signal-to-noise ratio (SNR) of approximately 50. The results are shown in [Figure 4: see original paper].

Regarding the dependency relationships, it can be observed that within our selected parameter range, both Taiji and LISA can constrain the precision of ΔQ to the order of

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Note: The parameters for the baseline waveform are: primary mass $M = 2.0 \times 10^6 M_\odot$, mass ratio $\mu = 1.0 \times 10^{-5}$, primary spin $S = 0.9$, initial orbital eccentricity $e = 0.5$, and semi-latus rectum $p = 10$.

The effect of varying a single deformation parameter δ on the signal matching factor of the LISA TDI-A channel.

Note: (a) and (b) represent the "+" and "×" polarization components of the waveform, respectively. The waveform parameters are: primary mass $M = 2.0 \times 10^6 M_\odot$, mass ratio $\mu = 1.0 \times 10^{-5}$, primary spin $S = 0.9$, initial orbital eccentricity $e = 0.5$, and semi-latus rectum $p = 10$. The yellow line represents the case where all deformation parameters δ in the KRZ metric are 0 (Kerr), while the black and blue lines represent waveforms obtained by changing only δ_1 and δ_6 to 0.001, respectively.

EMRI waveforms generated by the calculation method introduced in Section 2.2.

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Note: Waveform parameters are consistent with those in Figure 2. Panel (a) simulates the signal and noise in the A channel over a duration of 30 days; panel (b) excludes noise and shows the TDI-A signal corresponding to the waveforms in Figure 1.

Note: Panels (a) and (b) show the dependence of the quadrupole moment estimation precision $\sigma(\Delta Q)/\Delta Q$ on the relative quadrupole deviation $\Delta Q/Q_{\mathrm{Kerr}}$ and the MBH spin, respectively.

The blue and orange lines represent the estimation results for LISA and Taiji, respectively.

Estimation results of ΔQ using the Fisher Matrix.

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10^{-4}. The measurement precisions of Taiji and LISA are quite similar; in the future, even higher detection precision could be achieved by combining multiple detectors such as Taiji, LISA, and TianQin. Furthermore, it is noted that the estimation precision of ΔQ improves slightly at lower spins, which may be due to the relatively simpler waveform structure at lower spins enhancing observational precision. Finally, to investigate the correlations between mass, spin, and ΔQ, we utilized the Fisher Matrix results to obtain an approximate posterior probability distribution. The specific implementation is as follows:

A multivariate Gaussian distribution of the parameters was derived from the Fisher Matrix, and samples from this distribution were used as starting points for MCMC sampling. For relatively simple probability distributions, this initialization can accelerate the convergence of the Markov chain and reduce the number of sampling steps. The results are shown in [Figure 5: see original paper], where a clear negative correlation between spin and ΔQ can be observed.

Note: The parameters of the injected signal are $(M, S, \Delta Q) = (2 \times 10^6 M_\odot, 0.9, 0.01)$.

Approximate posterior probability distributions of mass M, spin S, and additional quadrupole moment ΔQ obtained from the Fisher Matrix.

4 总结与展望

This paper primarily explores the potential of Extreme Mass Ratio Inspiral (EMRI) signals for testing the no-hair theorem. EMRI signals serve as unique probes for revealing the spacetime properties surrounding supermassive black holes. During the inspiral phase, these signals undergo tens of thousands of cycles within the sensitive frequency bands of space-based gravitational wave detectors, making it highly probable that they can be identified with a high signal-to-noise ratio through matched filtering. In an astrophysical context, testing the no-hair theorem can be reduced to a test of the Kerr metric. To this end, we employ the Konoplya-Rezzolla-Zhidenko (KRZ) parameterized metric to perform a model-independent test of the Kerr metric.

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We present the parameter transformation relationship between the KRZ metric and the Bumpy metric, and subsequently demonstrate a method for introducing an additional quadrupole moment into EMRI waveform templates using the KRZ metric. We simulated the response of the Time Delay Interferometry (TDI) A-channel of space-based gravitational wave detectors, such as Taiji and LISA, to EMRI signals. Using the Fisher Information Matrix, we evaluated the capability of EMRI signals to constrain deviations in the quadrupole moment. Our results indicate that future space-based gravitational wave detectors will be capable of constraining the no-hair theorem to the order of 10^{-4}.

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1. Introduction

In recent years, the rapid development of observational technology and the continuous accumulation of massive astronomical data have driven astronomy into the era of Big Data. From the study of the early evolution of the universe to the precise measurement of exoplanet atmospheres, modern astronomical research increasingly relies on high-performance computing and advanced data processing algorithms. Machine learning and deep learning, as core technologies in the field of artificial intelligence, are gradually becoming indispensable tools for astronomers to handle complex data and extract physical information.

2. Current Status of Astronomical Observations

Current large-scale sky surveys, such as the Large Synoptic Survey Telescope (LSST) and the Square Kilometre Array (SKA), generate data at the petabyte scale. Traditional manual analysis methods are no longer sufficient to meet the demands of modern research. Consequently, automated pipelines for data reduction, source detection, and classification have become a primary focus of development. These systems must not only handle high-volume data streams in real-time but also maintain high precision and recall rates under low signal-to-noise ratio conditions.

[Figure 1: see original paper]

3. Application of Machine Learning in Astronomy

Machine learning algorithms have demonstrated significant advantages in various subfields of astronomy. For instance, in galaxy morphology classification, convolutional neural networks (CNNs) can achieve accuracy levels comparable to or even exceeding those of human experts. In the search for exoplanets, researchers utilize random forests and support vector machines to identify subtle transit signals from complex light curves.

Furthermore, the application of deep learning in gravitational wave detection has significantly improved the speed of signal identification, enabling rapid follow-up observations of electromagnetic counterparts. The integration of these algorithms allows for the efficient processing of multi-messenger data, providing a more comprehensive understanding of extreme astrophysical phenomena.

4. Challenges and Future Prospects

Despite the remarkable achievements of machine learning in astronomy, several challenges remain. First, the "black box" nature of deep learning models makes it difficult to interpret the underlying physical mechanisms. Second, the performance of models often depends on the quality and representativeness of the training sets; however, obtaining high-quality labeled data in astronomy is often costly.

Future research will likely focus on developing "physics-informed" machine learning models that incorporate known physical laws into the neural network architecture. This approach aims to improve the interpretability and generalization capabilities of the models. Additionally, the cross-disciplinary collaboration between astronomers and computer scientists

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Abstract

Extreme Mass Ratio Inspirals (EMRIs) are important sources for future space-based gravitational wave detectors, as they provide precise spacetime information around supermassive black holes (SMBHs), allowing for rigorous tests of the black hole no-hair theorem. However, previous simulations have often relied on model-dependent black hole models or simplified simulation data.

In this study, we used a parametrized, axisymmetric black hole metric to compute the gravitational waveforms of EMRIs. We then simulated space-based gravitational wave observation data using the Time Delay Interferometry (TDI) technique. Utilizing the Fisher information matrix, we assessed the ability to test deviations in the black hole' s quadrupole moment through gravitational waves. The results show that future space-based gravitational wave detectors can constrain the no-hair theorem to the level of 10−4 .

Key words: black hole; EMRI; parametrized metric; no-hair theorem