Error Analysis of BPM Position and Phase Calibration in Linear Accelerators Using Particle Beams

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Abstract

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As a prerequisite for achieving automatic phase setting and failure compensation, superconducting hadron linacs must possess precise longitudinal alignment and radio-frequency (RF) phase calibration. Currently, several facilities have successfully aligned or synchronized linacs through beam time-of-flight (TOF) measurements; however, many studies do not explicitly present the corresponding error analysis when performing TOF measurements, nor do they consider that using the beam for BPM calibration introduces correlations among the errors of various parameters. Within the framework of beam-based linac commissioning, this study systematically analyzes the uncertainty sources of phase errors and their propagation laws in TOF energy measurement under room-temperature BPM conditions. Furthermore, for the case of cryogenic BPM calibration, the correlation between position errors and phase offset errors is investigated, revealing a resulting error cancellation effect that improves the accuracy of beam energy measurement and phase setting. The analytical results are verified through numerical simulations, and their impact on beam phase calibration experiments as well as their potential applications in the commissioning of HIAF and CiADS are discussed.

Full Text

Preamble

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Introduction

During the commissioning and operation of linear accelerators (linacs), the precise setting of the cavity operating phase is critical for achieving the designed energy distribution and ensuring longitudinal dynamical stability. Traditional phase-setting methods typically rely on cavity-by-cavity phase scanning. This process is characterized by long commissioning cycles and low efficiency, making it difficult to meet the demands of modern linacs for rapid commissioning, automated operation, and adaptation to complex changes in operating conditions. Consequently, automated phase-setting methods based on beam measurement information have gradually become mainstream. These methods, combined with key operational strategies such as cavity failure compensation, have formed a central research focus in the field of linac operation and commissioning in recent years. Such approaches generally depend on the precise measurement of beam time-of-flight (TOF) and energy variations, where TOF measurements based on Beam Position Monitors (BPMs) play a core role.

The time-of-flight method is a common technique for measuring beam energy in linear accelerators. In a non-accelerating beamline section, the signals induced by BPMs at different positions can be converted into a linear relationship between the beam arrival time and the propagation distance. This allows for the determination of the beam velocity and, subsequently, the unique determination of its energy. Due to its clear principles and simple implementation, this technology has been widely applied in the energy measurement and commissioning processes of accelerators. On one hand, TOF measurements can be used to accurately calculate beam energy to verify the correctness of relevant parameter settings; on the other hand, these measurements can be used to calibrate the effective position and reference phase of the cavities, thereby improving the accuracy of computational results. In this context, the Institute of Modern Physics (IMP) has proposed a phase-setting method based on beam calibration. The precision of the TOF measurement directly affects the reliability of automated phase-setting and cavity failure compensation schemes. In most practical applications, when using the beam for calibration, the phase and position are calculated from the same set of beam signals. Therefore, the measurement errors are not independent of each other but exhibit statistical correlations. This correlation is manifested not only in the phase error correlation and position error correlation between different BPMs but also in the cross-correlation between phase errors and position errors. However, in current analyses, while many studies utilize BPM phase differences for TOF measurements to obtain beam energy, they often focus on the measurement methods or hardware implementation without systematically discussing the propagation and correlation of measurement errors. Therefore, conducting a systematic study of measurement errors and their propagation mechanisms holds significant physical importance and engineering application value. In actual beam measurements, the BPM also serves as a beam phase monitor. To avoid terminological confusion, this paper refers to them collectively as Beam Position and Phase Monitors (BPPMs) and clarifies the physical meaning of "phase."

Therefore, under the assumption that the beam can be equivalent to a single particle (neglecting the effects of bunch length and energy spread on the average TOF), this paper establishes an analytical model for multi-BPPM time-of-flight energy measurement and systematically investigates the sources of uncertainty and their propagation laws during the measurement process. Although this assumption is simplified, it provides a foundational framework for error analysis.

Error Analysis of Beam-Based Position and Phase Calibration in Linear Accelerators

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Abstract

As a prerequisite for achieving automatic phase setting and failure compensation, superconducting hadron linacs must possess precise longitudinal alignment and Radio Frequency (RF) phase calibration. Currently, several facilities have successfully aligned or synchronized linacs through Beam Time-of-Flight (ToF) measurements. However, many studies do not explicitly present a corresponding error analysis when performing ToF measurements, nor do they consider that using the beam for ToF calibration introduces correlations between the errors of various parameters.

Under a linac commissioning framework based on beam-based calibration, this study systematically analyzes the sources of phase error uncertainty and their propagation laws in room-temperature ToF measurements. Furthermore, for cryogenic ToF calibration scenarios, this research investigates the correlation between position errors and phase offset errors. It reveals a resulting error cancellation effect, which improves the precision of beam energy measurements and phase settings. The analytical results are verified through numerical simulations, and their impact on beam phase calibration experiments, as well as their potential application in CiADS (China Initiative Accelerator Driven System) commissioning, is discussed.

Keywords

Linear Accelerator; Time-of-Flight; Error; Energy Measurement CLC number: O571.53 Document code: 10.11804/NuclPhysRev.37.01.80

Zhou Haoyu et al.: Error Analysis of Position and Phase Calibration in Linear Accelerators Using Beam-Based Methods. Longitudinal beam dynamics have been simplified to a considerable extent in this study. Subsequent analysis will demonstrate that even under idealized conditions, achieving a reliable characterization of beam energy requires the establishment of a complete measurement model and a systematic derivation of the propagation relationships for error sources such as phase jitter, velocity jitter, and calibration velocity uncertainty. The idealized case referred to here implies that the error analysis focuses primarily on the error propagation mechanisms inherent to the measurement model itself—such as position uncertainty and phase measurement errors—rather than additional disturbances introduced by specific engineering environments. Since the phase difference directly corresponds to the time-of-flight and velocity of the particles, this paper first performs error propagation analysis using velocity as an intermediate variable. The results are ultimately converted into beam energy and its associated uncertainty through the energy-velocity relationship. Furthermore, following the conventions of accelerator science, all phases in this paper are expressed in angular degrees (°). The subsequent sections are developed based on the same measurement model, with differences primarily arising from error terms introduced by varying operating conditions.

Under room-temperature conditions, the longitudinal spacing between components can be regarded as known and stable; thus, measurement uncertainty is primarily determined by phase errors and system jitter. In cryogenic superconducting modules, however, the effective spacing may change due to thermal contraction effects, thereby introducing uncertainty in the flight distance.

Based on these differences, Section 3 introduces position uncertainty into the aforementioned model to extend the analysis of error propagation and error coupling. Consequently, the entire paper can be viewed as a step-by-step study of the error structures corresponding to different operating conditions under a unified measurement model.

Specifically, Section 1 presents the basic definitions and primary results of energy measurement using the Beam Position and Phase Monitor (BPPM) Time-of-Flight (TOF) method. Section 2 analyzes the impact of system jitter on phase measurement. Section 3 introduces the beam-based phase calibration method. Section 4 combines calibration and jitter analysis to derive expressions for measurement uncertainty under multi-beam conditions.

Section 5 further considers the issue of position errors under cryogenic conditions. Section 5.1 analyzes the cancellation mechanism between position errors and phase errors under single-beam calibration conditions. Section 5.2 extends this to the multi-beam case, discussing the effects of velocity uncertainty and error correlation. Section 6 verifies the theoretical results through numerical simulations. Finally, Section 7 provides a summary of the entire paper.

It should be noted that this work focuses on establishing an analytical framework for error propagation and error cancellation effects from the perspective of physical mechanisms, with principle verification conducted via numerical simulations. The relevant analysis is based on a single-particle dynamics approximation, assuming the beam is in the commissioning phase or under low-current conditions. In these scenarios, space charge effects and other collective effects on time-of-flight measurements are negligible. Under high-intensity beam conditions, these effects may alter error propagation characteristics and introduce additional systemic complexity. The engineering implementation and experimental validation for specific accelerator facilities usually involve parameter optimization and system-level debugging, which are beyond the scope of this paper and will be addressed in future research.

1 飞行时间能量测量

This section provides the fundamental equations for measurements using phase monitors. The derivations of some of these equations will be discussed in detail in the following sections.

1.1 BPPM

The measured phase only possesses physical significance when defined relative to a reference, which simultaneously comprises a position (denoted as z_{ref}) and the phase at that position (denoted as ϕ_{ref}). It should be noted that while the selection of the reference is arbitrary, its existence is mandatory.

The true phase of ϕ is defined as the phase that should be measured when the ϕ is perfectly synchronized with the reference. In an ideal accelerator system, for a particle with velocity v, this can be expressed as:

$$\varphi_i = \varphi_0 + \frac{z_i - z_0}{v} 360^\circ f + 360^\circ k_i \tag{1}$$

installation position, where f_{rf} represents the radio frequency (RF) of the phase detection system. The term n is an integer used to represent the number of RF cycles spanned by the phase, thereby constraining the true phase ϕ within the interval $(-180^\circ, 180^\circ]$ according to standard convention. If the system is not synchronized with the reference signal, the actual measured phase is denoted as ϕ_m, and its expression is given by:

$$\tilde{\varphi}_i = \varphi_i + \Delta\varphi_i \tag{2}$$

···relative to the reference phase offset. In practical applications, the reference is typically selected in the following manner:

$$z_0 = z_1 \tag{3}$$

$$\varphi_0 = 0 \tag{4}$$

The reference position is taken as the location of the first cavity, and the phase of the beam upon reaching this first cavity is defined as zero. By adopting this convention, all subsequent phases are measured relative to the first cavity, establishing a unified reference frame that simplifies the mathematical derivation. It should be noted that the quantities and are retained in the subsequent equations to clearly distinguish between variables that depend on the choice of reference and those that are reference-independent. Ultimately, the final physical results must remain invariant regardless of the chosen reference position. Quantities that depend on the reference selection serve only as intermediate variables in the derivation and do not possess independent physical significance.

1.2 使用

Suppose we have obtained the phase offset $\phi_{0,i}$ for each cavity through a beam-based calibration method. This method utilizes a beam with a known velocity v_0 to synchronize the phases of the cavities (see Section 3). If we define the true phase of the i-th cavity as:

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$$\varphi_i = \tilde{\varphi}_i - \Delta\varphi_i \tag{5}$$

is the measured phase, and represents the integer number of cycles corresponding to the distance between the reference position and the i-th position. When performing Time-of-Flight (TOF) energy measurements, it is necessary to have prior knowledge of the approximate range of the beam energy. Measuring the energy is equivalent to determining the slope in a time-distance relationship diagram, which is achieved by solving the following equation:

represent the position and the cumulative phase corrected by the offset ϕ_0 for the i-th position, respectively. The beam energy obtained from the Time-of-Flight (TOF) measurement can be directly expressed as:

$$v = a^{-1} \tag{7}$$

The error associated with this process will be analyzed in detail in the following subsection. Another parameter obtained from the solution of Eq. (6), denoted as τ, represents the arrival time of the beam at the reference position z_0. The magnitude of this parameter should be consistent with the time jitter analyzed in Section 2. In practical applications, Eq. (6) can be solved using the ordinary least squares (OLS) method, under the assumption that all phases possess the same degree of uncertainty. The uncertainty of the beam velocity v can be expressed as:

$$\frac{\sigma_v}{v^2} = \frac{\sigma_{v_c}}{v_c^2} \tag{8}$$

represents the beam velocity during the beam-based phase calibration process. It should be noted that the contribution of phase measurement uncertainty is not considered here; however, the validity of this approach will be verified in Section 3.2 through a detailed discussion on uncertainty quantification. This quantitative analysis is built upon the analysis of jitter error propagation laws presented in Section 2, as well as the derivation of the beam-based phase calibration measurement model and its associated error propagation relationships in Section 3.

2 抖动分析

This section analyzes the scenario where the positions of the normal-conducting modules are assumed to be accurate and error-free. We will describe how to systematically characterize the "noise" of the system through jitter analysis. The magnitude of the jitter term not only provides critical information regarding the accelerator system but also directly impacts the precision of energy measurements.

When performing continuous multiple measurements of the phase, the measurement results exhibit fluctuations originating from three primary sources: timing jitter (fluctuations in the arrival time of the bunch at the reference position), velocity jitter (fluctuations in the velocity of the bunch pulse itself), and phase jitter (the inherent measurement phase jitter of the instrument). This last term most closely corresponds to what is commonly referred to as "noise." Both timing jitter and velocity jitter contribute to the variance of the measured phase. Consequently, if the variance of multiple measurements is simply treated as "noise," timing jitter—which is not an intrinsic characteristic of the instrument—would be incorrectly included. Furthermore, because downstream components are more susceptible to velocity jitter, such a simplification would lead to the erroneous conclusion that downstream noise levels are higher. In the following sections, we present a rigorous analytical method to distinguish and decouple these distinct effects.

In an accelerator system, if the three aforementioned types of jitter are temporarily disregarded, the phase measured at the bunch can be considered a deterministic quantity. According to Eq. (1), when the bunch velocity is v, the phase is given by:

$$\tilde{\varphi}_i = \varphi_0 + \Delta\varphi_i + \frac{z_i - z_0}{v} \cdot 360^\circ f + 360^\circ k_i \tag{9}$$

When the three aforementioned jitter terms are introduced into the measurement, the phase in the i-th measurement becomes:

$$\tilde{\varphi}_i^{(k)} = \varphi_0 + \Delta\varphi_i + \frac{z_i - z_0}{v} \cdot 360^\circ f + \delta\varphi_0^{(k)} + \delta\tilde{\varphi}_i^{(k)} \tag{10}$$

The timing jitter is denoted as Δt, the velocity jitter as Δv, and the phase jitter as $\Delta\phi$. For any physical quantity x corresponding to N measurements, we introduce the following notation for the average value of x:

$$\langle x\rangle \equiv \frac{1}{N}\sum_{k=1}^{N} x^{(k)} \tag{11}$$

The total number of measurements is denoted as N. The covariance of x and y is defined as:

$$\langle xy\rangle \equiv \frac{1}{N}\sum_{k=1}^{N}(x^{(k)}-\langle x\rangle)(y^{(k)}-\langle y\rangle) \tag{12}$$

The sample covariance estimation employed is statistically unbiased; however, its variance increases significantly as the number of measurements M decreases. These finite-sample statistical fluctuations propagate into the uncertainty evaluation of subsequent physical quantities. Consequently, in practical applications, a sufficient number of measurements must be ensured to obtain a stable covariance estimate. Generally, at least M repeated measurements are required to reliably extract the covariance or correlation terms. The right-hand side of the equation utilizes the mean value defined in Eq. (11). Following the same definition, from Eq. (12):

Zhou Haoyu et al.: Error Analysis of Position and Phase Calibration in Linear Accelerators Using Beam Measurements $\langle x^2\rangle \equiv$

$$\frac{1}{N}\sum_{k=1}^{N}(x^{(k)}-\langle x\rangle)^2 \tag{13}$$

We aim to determine the variance that characterizes the magnitude of the jitter term. By analyzing the jitter term introduced in $\langle \delta v^2\rangle$, we obtain:

$$\tilde{\varphi}_i^{(k)}-\langle\tilde{\varphi}_i\rangle = (\delta\varphi_0^{(k)}-\langle\delta\varphi_0\rangle)+(\delta\tilde{\varphi}_i^{(k)}-\langle\delta\tilde{\varphi}_i\rangle) \tag{14}$$

The variance of the measured phase, denoted as $-\langle\delta v\rangle$, is given by:

$$\langle\tilde{\varphi}_i\tilde{\varphi}_i\rangle = \langle\delta\varphi_0\delta\varphi_0\rangle+\langle\delta\tilde{\varphi}_i\delta\tilde{\varphi}_i\rangle+2\langle\delta\varphi_0\delta\tilde{\varphi}_i\rangle \tag{15}$$

We assume that phase jitter is uncorrelated with both time jitter and velocity jitter. Under this assumption, the cross-correlation terms between these variables vanish, allowing for a simplified analysis of the system' s noise characteristics.

Furthermore, the relationship between the velocity fluctuations $\langle \delta v \delta v \rangle$ and the phase fluctuations $f\langle \delta\tilde{\varphi} \rangle$ can be characterized by their respective spectral densities. In the context of the proposed model, these fluctuations are treated as independent stochastic processes, which facilitates the derivation of the overall timing error budget.

$$\langle \delta\varphi_0 \delta\tilde{\varphi}_i \rangle = \langle \delta\tilde{\varphi}_i \delta v \rangle = 0 \tag{16}$$

Furthermore, the phase jitters among different signal components are mutually uncorrelated, which can be expressed as:

$$\langle \delta\tilde{\varphi}_i \delta\tilde{\varphi}_j \rangle = 0 \quad (i \neq j) \tag{17}$$

However, we contend that:

$$\langle \delta\varphi_0 \delta v \rangle \neq 0 \tag{18}$$

In general, there is an inherent correlation between the arrival time jitter and the velocity jitter of a bunch. Within a drift space, the magnitude of this correlation depends on the choice of the reference position. By selecting an appropriate reference position, it is possible to decouple the position error from the phase error at that specific location; however, the two remain correlated at all other positions. Under the assumption of Eq. (16), Eq. (15) simplifies to:

$$\langle \tilde{\varphi}_i \tilde{\varphi}_i \rangle = \langle \delta\varphi_0 \delta\varphi_0 \rangle + \langle \delta\tilde{\varphi}_i \delta\tilde{\varphi}_i \rangle \tag{19}$$

There are N linear equations, but they contain $N+3$ unknowns: N phase jitter variances $\langle \delta\phi_i \delta\phi_i \rangle$, the time jitter variance $\langle \delta\tau \delta\tau \rangle$, the velocity jitter variance $\langle \delta v \delta v \rangle$, and the covariance between time and velocity jitter $\langle \delta\tau \delta v \rangle$. Consequently, it is impossible to solve for all individual terms using these equations alone.

To solve for the terms other than phase jitter, we consider the correlations between different i. Intuitively, velocity jitter or arrival time jitter should exert a consistent influence on the measured phases of all i; this consistency is reflected in the covariance. Under the same assumptions used in Eq. (16), we obtain:

$$\langle \tilde{\varphi}_i \tilde{\varphi}_j \rangle = \langle \delta\varphi_0 \delta\varphi_0 \rangle \tag{20}$$

By solving these equations, we obtain $n(n-1)/2$ linear equations for $\langle \delta v \delta v \rangle$. These quantities can then be easily determined using the least squares method. If the resulting residuals are small, it indicates that the results are consistent with our initial assumptions, thereby validating the rationality of the hypothesis.

Once the aforementioned terms are obtained, they can be substituted back into each equation in Eq. (19) to derive the phase jitter variance for each component. These results provide the necessary foundation for the uncertainty analysis of time-of-flight energy measurements discussed in the subsequent sections.

3 基于束流的

In the previous section, we noted that the measured phase may exhibit a phase offset relative to the true phase. The factors contributing to a non-zero phase offset can be categorized into two primary groups:

The choice of reference point. The phase offset is dependent on the selection of the reference point, which can be arbitrary. Consequently, even if all components are perfectly synchronized, a uniform phase shift is typically required to maintain consistency with the chosen reference point. Additionally, other factors can lead to varying phase offsets across different components; potential causes include discrepancies in signal cable lengths as well as variations in signal propagation delays within the electronic systems.

3.1 相位标定

One method for obtaining the phase offset involves calibration using a known velocity. A natural question arises: if the bunch velocity can already be determined through other methods, what is the significance of the calibration? The fundamental reason is that while alternative methods for measuring bunch velocity exist, they are typically more cumbersome or more restricted than the method. In contrast, measurements are non-invasive and can be performed continuously online. Consequently, other energy measurement methods are only employed on specific occasions.

Nuclear Physics Review. All subsequent measurements can then be conducted via the bunch-based calibration process, which is described as follows. Given a bunch velocity v, and according to Eq. (1), the phase measured at position n (ignoring errors) is:

$$\tilde{\phi}_i = \varphi_0 + \Delta\varphi_i + \frac{z_i - z_0}{v_c} 360^\circ f + 360^\circ k_i \tag{21}$$

Here, we use the symbol ϕ to represent the phase measured during the calibration process. When we perform phase calibration based on energy for N measurements, the phase offset is given by the following expression:

$$\Delta\varphi_i = \langle\tilde{\phi}_i\rangle - \varphi_0 - \left(\frac{z_i - z_0}{v_c} 360^\circ f\right) - 360^\circ k_i \tag{22}$$

3.2 不确定度分析

The two terms on the right side of the equation introduce errors into the phase offset uncertainty originating from noise. Depending on the specific energy measurement method employed, the bunch velocity during the calibration process also possesses an inherent uncertainty. Assuming these two components are uncorrelated, the square of the uncertainty in the phase offset can be expressed as:

$$\sigma^2_{\Delta\varphi_i} = \sigma^2_{\langle\tilde{\phi}i\rangle} + \left(\frac{z_i - z_0}{v_c} 360^\circ f\right)^2 \tag{23}$$

We refer to the two terms on the right-hand side of Eq. (23) as the noise term and the velocity uncertainty term, respectively. The remainder of this section will provide a detailed discussion of these two components.

3.2.1 噪声项

is the error in the average measured phase, namely the standard deviation of the mean:

$$\sigma_{\langle\tilde{\phi}i\rangle} = \frac{1}{\sqrt{N}}\sigma_{\tilde{\phi}i} = \frac{1}{\sqrt{N}}\sigma_{\phi_i} \tag{24}$$

The standard deviation of the measured phase, denoted as the phase jitter standard deviation, can be calculated according to the method described in Section 2. In the analysis presented in this section, time jitter is not considered, as it induces a uniform shift across all phases and thus has no impact on the relative phase differences. In principle, velocity jitter can be resolved through energy measurement methods. However, in the time-of-flight (TOF) based beam energy measurement system discussed in this paper, the average bunch velocity remains substantially stable between adjacent measurements. Consequently, the velocity jitter introduced by fluctuations in the accelerator' s operating state is typically much smaller than the velocity uncertainty of the bunch itself—generally by an order of magnitude. Therefore, during the calibration process, the velocity jitter is effectively absorbed into the overall uncertainty of the bunch velocity.

3.2.2 速度项

This uncertainty originates from the uncertainty of the bunch velocity during the calibration process. Due to the presence of the factor η, the magnitude of the phase offset uncertainty depends on the choice of the reference position z_0. While this may appear counterintuitive, we note that the phase offset itself is also dependent on the choice of the reference position; therefore, such a dependency is not unreasonable.

Another way to understand this dependency is that the uncertainty in the bunch velocity inevitably affects the uncertainty of the phase offset. Considering an extreme case, if the uncertainty of the bunch velocity is sufficiently large as to be virtually unknown, the phase offset obtained using that bunch would be effectively random. Consequently, the uncertainty must include a term related to the velocity uncertainty.

We will demonstrate in Section 4 that the same results are consistently obtained in energy measurements, regardless of how the reference position is chosen.

4 TOF

Uncertainty in Energy Measurement

This section provides a detailed analysis of the uncertainties involved in energy measurement using bunch-based calibration. We first reiterate the distinction between the two types of bunch velocities: v_0 represents the known bunch velocity used during the calibration process, while v denotes the unknown bunch velocity that is to be determined through the calibration.

The conventional least-squares method is not suitable for solving Eq. (1), as the uncertainty in bunch velocity during the calibration process introduces correlations between the phase errors of different cavities. According to Eq. (2), the measured phase is subject to three primary sources of uncertainty:

$$\delta\varphi_i = \delta\tilde{\varphi}_i + \delta\langle\phi_i\rangle - 360^\circ f(z_i - z_0) \tag{25}$$

The uncertainty in the average measured phase caused by phase jitter during the self-measurement process is defined as the uncertainty of the bunch velocity during the calibration process. The first two terms originate from phase jitter, under the assumption that different ϕ values are uncorrelated. The third term stems from velocity uncertainty, which exerts an identical effect on the phases of all ϕ; consequently, a correlation exists between the phase errors of different ϕ.

If the covariance between the phases of any two ϕ is calculated, we obtain:

$$\sigma^2_{\varphi_i\varphi_j} = (z_i - z_0)(z_j - z_0)\left(\frac{360^\circ f}{v_c^2}\sigma_{v_c}\right)^2 \tag{26}$$

To eliminate the correlation between different phase errors and satisfy the independence conditions required for the least squares method, a linear transformation can be employed to ensure that all off-diagonal covariance terms vanish in the new coordinate system. However, this generalized least squares approach is relatively cumbersome and lacks physical intuition.

Instead, we define the new coordinates and slopes as follows:

Zhou Haoyu et al.: Error Analysis of Position and Phase Calibration in Linear Accelerators Using Beam Measurements $\hat{a} \equiv a + 1$. After the aforementioned transformation, the system equation becomes:

The error of is:

$$\delta\hat{\varphi}_i = \delta\tilde{\varphi}_i + \delta\langle\phi_i\rangle \tag{30}$$

Among these, the two terms on the right side both originate from the phase jitter of ϕ. When there is no correlation between them, it follows that:

$$\sigma^2_{\hat{\varphi}i} = \sigma^2_{\tilde{\varphi}i} + \sigma^2_{\langle\phi_i\rangle} \tag{31}$$

$$\sigma_{\hat{\varphi}i\hat{\varphi}_j} = 0 \quad (i \neq j) \tag{32}$$

It should be noted that $\sigma_{\Delta\phi}$ consists of two distinct contributions: one part arises from the phase jitter of ϕ during the measurement process (represented in σ_ϕ), while the other part stems from the phase jitter of ϕ during the calibration process (represented in σ_{ϕ_0}). Generally, these two components are not identical. This discrepancy occurs because the magnitude of the jitter may fluctuate over time, and furthermore, the standard deviation of the mean during the calibration process decreases as the number of measurements N increases. By applying a simple least-squares method to solve (1), one can obtain ϕ and its standard deviation σ_ϕ, which further yields:

$$a = \hat{a} - 1 \tag{33}$$

$$\sigma^2_a = \sigma^2_{\hat{a}} + \left(\frac{\sigma_{v_c}}{v_c^2}\right)^2 \tag{34}$$

The measured bunch velocity

$$\sigma_v = \frac{\sigma_a}{a^2} \tag{35}$$

The results of ... can be obtained through direct substitution. The uncertainty in the energy measurement consists of two primary contributions: the phase jitter during the measurement process and the uncertainty of the bunch velocity during the calibration process.

The aforementioned results are consistent with the physical intuition that measurement errors should be independent of the choice of the reference position z_0. Although the selection of z_0 may cause the variance of the phase shift (see

Eq. (23)) to become arbitrarily large, the analytical approach described above demonstrates that the error components introduced by the choice of z_0 do not propagate into the final energy measurement error.

5 误差抵消关系的理论分析

Analysis of Position Variations in Cryomodules

This section analyzes scenarios in the cryomodule where positions may shift during the cooling process. Currently, the Institute of Modern Physics of the Chinese Academy of Sciences is employing a beam-based calibration method. This approach involves placing a pair of calibrated BPPMs downstream of the beamline and using a beam with known energy to calibrate the relative positions or phase offsets between the reference BPPMs. However, a certain degree of cancellation may occur between different error sources. The specific mechanism of this error cancellation and its applicable conditions currently lack rigorous theoretical analysis.

Next, we will analyze the mutual cancellation mechanism between position errors and phase offset errors when using a pair of BPPMs for calibration in the presence of both systematic and random errors.

We select a relatively simple model of a pair of BPPMs, as shown in [Figure 1: see original paper]. Calibration is performed using a bunch with velocity v; specifically, the measured distance between the two BPPMs and the phase deviation readings are denoted as L_{meas} and ϕ_{meas}, respectively. We define the difference in phase deviation as $\Delta\phi$. By converting the phase deviation into a time deviation, we define the time difference of the offsets as Δt_{offset}. In this section, "systematic error" refers specifically to deterministic errors introduced by structural deviations, such as the actual positional shift of accelerator components or fixed time biases, which remain constant or change slowly during repeated measurements. In contrast, "random error" originates from measurement noise and pulse-to-pulse fluctuations in beam parameters (such as statistical fluctuations in beam velocity or measurement readings), manifesting as stochastic variations between events.

5.1 存在系统误差

As shown in [FIGURE:N], there is a discrepancy between the actual distance and the calibrated distance. We aim to investigate how this error affects the precision of the final bunch velocity measurement.

In the calibration model, this distance error ΔL causes a shift in the calibration of the time offset. Specifically, if the actual distance between the components becomes $L + \Delta L$, the perceived time offset t_{think} will deviate from the true time offset t_{real}.

$$\tau_{think} = t_d - \frac{L}{v_0} \tag{36}$$

Nuclear Physics Review

Abstract

This paper provides a comprehensive review of recent developments and significant breakthroughs in the field of nuclear physics. We examine the evolution of theoretical frameworks, from the shell model to modern ab initio calculations, and discuss their implications for understanding nuclear structure and dynamics. Furthermore, we analyze the role of high-energy particle accelerators and advanced detection technologies in probing the fundamental constituents of matter. The review also covers the intersection of nuclear physics with astrophysics, particularly in the context of nucleosynthesis and neutron star properties. Finally, we outline future research directions and the potential impact of emerging technologies, such as quantum computing and machine learning, on the field.

1. Introduction

Nuclear physics remains a cornerstone of modern science, providing essential insights into the fundamental forces that govern the universe. Since the discovery of the atomic nucleus, the field has transitioned from basic structural studies to complex investigations of many-body systems and extreme states of matter. The primary objective of contemporary nuclear research is to develop a predictive understanding of nuclei across the entire Segrè chart, including those far from the line of stability.

Recent experimental advancements at facilities worldwide have allowed researchers to explore the "islands of inversion" and the properties of superheavy elements. These discoveries challenge existing theoretical models and necessitate the refinement of nuclear force descriptions. As we push the boundaries of the known nuclear landscape, the synergy between experimental observation and theoretical innovation becomes increasingly critical.

2. Theoretical Frameworks and Computational Methods

The theoretical description of the nucleus has seen a paradigm shift with the advent of high-performance computing. While the traditional shell model and liquid drop model provided the initial foundation, they are now being supplemented or replaced by more fundamental approaches.

2.1 Ab Initio Calculations Ab initio methods aim to solve the nuclear many-body problem starting from the basic interactions between nucleons. These approaches, such as the No-Core Shell Model (NCSM) and Coupled-Cluster (CC) theory, utilize chiral effective field theory (χEFT) to derive nuclear forces.

The mathematical formulation often involves solving the Schrödinger equation for A nucleons:

$$H\Psi(r_1, r_2, \dots, r_A) = E\Psi(r_1, r_2, \dots, r_A)$$

where the Hamiltonian H includes two-body (NN) and three-body ($3N$) forces. The inclusion of $3N$ forces has been shown to

$$\tau_{true} = t_d - \frac{L + \Delta L}{v_0} \tag{37}$$

When measuring a bunch with an unknown velocity v, the actual drift time and the perceived drift time for the two cases are respectively:

$$t_{true} = \frac{L + \Delta L}{v} \tag{38}$$

$$V_0 = \frac{V_0(L + \Delta L) - V\Delta L}{L} \tag{39}$$

$$t_{think} = \frac{L + \Delta L}{v} \tag{40}$$

The resulting measured bunch velocity is given by:

$$V_{think} = \frac{L}{t_{think}} = \frac{VV_0L}{V_0(L + \Delta L) - V\Delta L} \tag{41}$$

Using $V_{\text{dev}} = \Delta V/V$ and $L_{\text{dev}} = \Delta L/L$ respectively represent the relative deviations between the measured values and the true values, from which the following relationship can be derived:

$$\frac{1}{V_{dev}} = \frac{V_0}{V - V_0} \tag{42}$$

To measure the ratio between the measured bunch velocity and the calibrated bunch velocity, and to plot the measurement error of the bunch velocity under different conditions, an analysis was conducted as shown in [FIGURE:N]. It can be observed that when the deviation between the measured bunch velocity and the calibrated bunch velocity is within $\pm 10\%$, the error in the measured bunch velocity is more than ten times smaller than the error induced by position discrepancies alone.

This phenomenon indicates a significant error cancellation relationship between position errors and phase errors, which substantially reduces the final measured bunch velocity error. This error cancellation effect is further enhanced as the measured bunch velocity approaches the calibrated bunch velocity. The underlying principle of this mechanism is that a positive length error corresponds to a positive time error, thereby producing a compensatory effect during the velocity calculation. The principles governing the error cancellation effects discussed in subsequent sections are consistent with this mechanism.

5.2 存在随机误差

If we further consider the existence of deviations in the calibrated bunch velocity, the expression for the measured bunch velocity can be derived using the same methodology:

$$V_{dev} = \frac{-\Delta L V_0 A + V_0 B - VAL}{V_0 AB - V_0 B + VAL} \tag{43}$$

Where $A = V_0 + \Delta V_0$ and $B = L + \Delta L$.

By plotting the three-dimensional relationship (see [FIGURE:N]), the complex trends in the error cancellation relationship can be observed. In general, the calibration bunch velocity deviation and the measured bunch velocity deviation typically have opposite signs, while they exhibit a positive correlation. When the signs are opposite, an error cancellation effect occurs. When the signs are the same, the error cancellation effect persists, but the fluctuation amplitude of the error is greater. Furthermore, when the value is negative, the fluctuation amplitude of the error becomes significantly larger.

Schematic diagram of the relationship between the velocity ratio and the distance error ratio, illustrating the influence of the distance error ratio on the measured bunch velocity deviation. In the figure, layers are plotted at intervals (all coordinate axes are dimensionless). Left: $= -0.1$ and $= 0.1$. Blue region:

6 多束流标定的不确定度分析

In the previous section, we discussed how systematic errors in the calibration beam velocity and the distance between components can lead to a significant error cancellation effect when calibrating phase deviations. In this section, we further analyze whether a similar error cancellation effect exists during the calibration of inter-component distances and phase deviations when using multiple beams characterized by random errors. Unlike Section 5.1, we no longer assume the beamline length is a known quantity; instead, it is treated as a parameter to be estimated within the error analysis. This section focuses on analyzing the dominant physical mechanisms of error propagation by establishing analytical models under controlled assumptions. Engineering factors such as thermal deformation in multi-cavity cryomodules, temperature gradients, and time-varying cable phase drifts are not explicitly included here; their impacts will be addressed in subsequent system-level studies.

Zhou Haoyu et al.: Error Analysis of Position and Phase Calibration in Linear Accelerators Using Beams. We employ the model shown in [Figure 1: see original paper] and use n beams with random errors for calibration to derive the expression for the uncertainty of the beam velocity v under measurement. The expression for the velocity v is given by:

$$\frac{1}{v} = \frac{\tau}{L} \tag{44}$$

Rewrite it into the linear form $y = a + bx$. From this, the expression for the variance of the measured beam velocity can be derived as follows:

$$\langle \delta v \delta v \rangle = L^4 \tag{45}$$

To derive the expressions for $\langle \delta a \delta a \rangle$, $\langle \delta b \delta b \rangle$, and $\langle \delta a \delta b \rangle$ at time $t+\tau$, we employ the least squares method for error analysis. Assuming that the standard deviation of the random error added to each beamline is σ, the final expressions are given as follows:

$$\langle \delta a \delta a \rangle = \frac{\sigma^2}{\Delta'} \sum x_i^2 \tag{46}$$

$$\langle \delta b \delta b \rangle = \frac{N\sigma^2}{\Delta'} \tag{47}$$

$$\langle \delta a \delta b \rangle = -\frac{\sigma^2 \sum x_i}{\Delta'} \tag{48}$$

$$\Delta' = N \sum x_i^2 - (\sum x_i)^2 \tag{49}$$

It can be observed that the expression for $\langle \delta a \delta b \rangle$ is consistently negative. This indicates that when considering the potential correlation between random errors added to multiple beams in engineering applications, the correlation can be expressed as:

σ^2 when $j = k$

λ when $j \neq k$ (50)

The aforementioned expression can be decomposed into the superposition of two parts:

$$\langle \delta y_j \delta y_k \rangle = \lambda + \langle \delta y'_j \delta y'_k \rangle \tag{51}$$

$\sigma^2 - \lambda$ when $j = k$

0 当

By performing the calculations, we can obtain the values for $\langle \delta a \delta a \rangle$, $\langle \delta b \delta b \rangle$, and $\langle \delta a \delta b \rangle$.

$$\langle \delta a \delta a \rangle = \frac{\sigma^2 - \lambda}{\Delta'} \sum x_i^2 + \lambda \tag{52}$$

$$\langle \delta b \delta b \rangle = \frac{N(\sigma^2 - \lambda)}{\Delta'} \tag{53}$$

$$\langle \delta a \delta b \rangle = -\frac{\sigma^2 - \lambda}{\Delta'} \sum x_i \tag{54}$$

By substituting the obtained values into the equation, it can be demonstrated that the measurement of $\langle \delta v \delta v \rangle$ directly corresponds to the uncertainty in the particle velocity. To determine the uncertainty of the beam energy, error propagation can be performed using the relationship between energy and velocity. The relativistic energy of a particle is given by:

$$E = (\gamma - 1)mc^2 \tag{55}$$

By performing a perturbative expansion of the velocity, we obtain $\delta E = dE$.

$$dv\delta v = m\gamma^3 v\delta v \tag{56}$$

Therefore, the relationship between the variance of energy and the variance of velocity satisfies:

$$\langle \delta E \delta E \rangle = m^2 \gamma^6 v^2 \langle \delta v \delta v \rangle \tag{57}$$

By substituting the value of $\langle \delta v \delta v \rangle$ into the expression for $\langle \delta E \delta E \rangle$ and taking the square root of the result, the uncertainty in the measurement of the beam energy under test can be obtained.

In practical facilities, the measurement resolution typically varies across different beam energies. The model presented in this paper can be directly generalized by introducing an energy-dependent weight matrix. The corresponding optimal energy configuration problem will depend on specific device parameters and operational modes, which constitutes a subject for subsequent engineering optimization research.

7 多束流标定的模拟验证

In the previous section, we derived the expression for the energy uncertainty when measuring a target beam after calibration using multiple beams with random errors. Furthermore, based on engineering experience, we conducted a more detailed analysis by incorporating correlations into the random errors of each beam. In this section, we perform a simulation analysis based on a specific set of parameters to verify the error cancellation effect and the uncertainty calculations derived in our theoretical analysis.

7.1 模拟参数

We adopt the model shown in [Figure 1: see original paper], where the distance between the two gaps and the difference in phase offset are unknown. For calibration, we utilize beam lines at various velocities ranging from $0.95v$ to $1.05v$. A random error of $0.01v$ is added to each calibration beam line. Within this framework, the random error for each beam line is independent, while the true value of the distance remains constant across all measurements.

25 MeV

The velocity corresponding to a 25 MeV proton. The number of sampling iterations is

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7.2 模拟结果

By performing sub-sampling, one can obtain the distribution of the calculated values for $\hat{E}$, as well as measure the true energy value based on the $\hat{E}$ derived from each individual calculation.

25 MeV

The following figures illustrate the particle energy distribution derived from the proton beam. All three figures exhibit a Gaussian distribution profile. By performing a Gaussian fit on these computational results, the standard deviation (σ) for each distribution can be determined. The fitting results are presented in [FIGURE:N]. [FIGURE:N] displays the time deviation calculated over multiple sampling iterations, while [FIGURE:N] represents the energy distribution when the true energy value is set to 25 MeV.

25 MeV

The measurement results for the proton beam are presented. The uncertainties shown in the figure are calculated based on the uncertainty formula derived in the previous section. Subsequently, the uncertainty of τ can be utilized to calculate the uncertainty of the energy distribution map under the assumption that no error cancellation effects occur—that is, assuming the errors are mutually independent—based on the propagation of uncertainty formula.

We transform the energy measurement of protons with unknown energy into a measurement of their Lorentz factor τ. Consequently, the uncertainty of τ, denoted as $\delta\tau$, is related to the uncertainty of the measured parameters. This relationship can be expressed as $\delta\tau = \frac{\partial\tau}{\partial x}\delta x$.

$$(\delta\gamma)^2 = \left(\frac{\partial\gamma}{\partial L}\delta L\right)^2 \tag{58}$$

Since the velocity of the measured particle can be expressed as:

$$v = \frac{L}{t-\tau} \tag{59}$$

The value can be expressed as

$$\beta = \frac{L}{c(t-\tau)} \tag{60}$$

Based on the formula for the particle Lorentz factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, the following conclusions can be derived:

$$\frac{\partial\gamma}{\partial\beta} = \gamma^3\beta \tag{61}$$

$$\frac{\partial\beta}{\partial L} = \frac{\beta}{L} \tag{62}$$

$$\frac{\partial\beta}{\partial\tau} = \frac{\beta}{t-\tau} \tag{63}$$

By combining the above equation with Eq. (58), we obtain:

$$\frac{\partial\gamma}{\partial L} = \frac{\gamma^3\beta^2}{L} \tag{64}$$

$$\frac{\partial\gamma}{\partial\tau} = \frac{\gamma^3\beta^2}{t-\tau} \tag{65}$$

By substituting the above equation into Eq. (58), the expression for the uncertainty in measuring the energy of an unknown proton is obtained as:

$$\delta\gamma = \gamma(\gamma^2 - 1)\sqrt{\left(\frac{\delta L}{L}\right)^2} \tag{66}$$

Based on the relationship between particle energy and the Lorentz factor, $E = \gamma m$, the expression for the energy uncertainty when measuring a proton of unknown energy can be derived as:

$$\delta E = m_0 c^2 \gamma(\gamma^2 - 1)\sqrt{\left(\frac{\delta L}{L}\right)^2} \tag{67}$$

It can be observed that the uncertainty is related to the energy of the proton.

25 MeV

The value corresponding to the proton

25 MeV

The standard deviation of the fitted values is 0.68 ns, which corresponds to a distance of 0.046 m given the timing resolution of 7.37 ns over a distance of 0.5 m. By substituting these parameters into the relevant equations, it is possible to calculate the measurement results under the assumption that the errors are mutually independent.

25 MeV

The energy uncertainty of the proton is approximately $\delta E \approx 7.34$ MeV. As shown in the figure, the energy uncertainty obtained through Gaussian fitting is significantly lower than the value calculated using the theoretical formula. This discrepancy indicates the presence of a strong error cancellation effect. Furthermore, it can be observed from the figure that the maximum probability value of the energy uncertainty, calculated based on the velocity uncertainty formula derived in the previous section, is 0.37. In contrast, the value obtained through multiple sampling fits is···

25 MeV

The beam energy value is close to the energy uncertainty derived from multi-sampling fitting, with a deviation within . This demonstrates that the uncertainty analysis of the multi-beam calibration presented in the previous section is relatively accurate.

8 总结与展望

This report first systematically summarizes the methods used for measurements. When employing room-temperature measurements that require phase determination, we successfully quantified the uncertainty of energy measurements through a beam-based phase monitor calibration technique and a rigorous treatment of accelerator system jitter analysis. We further demonstrated that the feasibility of jitter analysis is fundamentally due to the adoption of multiple monitors, which fully reflects the added value of utilizing more than two such devices. Research indicates that during the beam-based phase calibration process, the total uncertainty in energy measurement is primarily determined by the uncertainty in the calibration beam velocity, whereas the contributions from phase jitter and position errors are relatively small.

Zhou Haoyu et al.: Error Analysis of Position and Phase Calibration in Linear Accelerators Using Beam-Based Methods. The key to achieving this lies in the use of multiple monitors, highlighting the significant advantages of employing more than two units. Our study shows that in the process of beam-based phase calibration, the total uncertainty of the energy measurement is dominated by the uncertainty of the calibration beam velocity, while the contributions of phase jitter and position errors remain minor.

When using cryogenic measurements to calibrate position and phase, it is essential to understand the intrinsic correlation and mutual cancellation effects between position and phase errors. Research has found that when the calibration beam velocity is close to the target beam velocity, the errors between the two exhibit a significant cancellation phenomenon. Consequently, the precision of the final measured beam velocity is much higher than the calibration error itself. This discovery provides an important theoretical basis for automated phase setting and high-precision energy measurement.

In the future, we will further apply the aforementioned research results to the beam commissioning and energy measurement tasks of large-scale facilities, such as the High Intensity heavy-ion Accelerator Facility (HIAF) and the China Initiative Accelerator Driven Subcritical System (CiADS).

By combining further experimental validation with engineering practice, we aim to refine the error cancellation models and explore their applicability under higher energy levels and more complex beam conditions. This work will provide more reliable phase and energy diagnostic technical support for next-generation high-power accelerator systems.

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Error Analysis of Beam-Based BPM Position and Phase Calibration in Linear Accelerators

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Abstract

In modern linear accelerators, the precise calibration of Beam Position Monitors (BPMs) is essential for high-intensity beam transport and effective machine protection. This paper presents a comprehensive error analysis of beam-based calibration techniques used for determining BPM offsets and phase responses. By leveraging beam dynamics simulations and statistical modeling, we evaluate the impact of various error sources—including magnet misalignment, power supply ripples, and sensor noise—on the accuracy of the calibration process. Our results provide a theoretical framework for optimizing calibration procedures and improving the overall reliability of beam diagnostics in high-power accelerators.

Introduction

The performance of high-intensity linear accelerators (linacs) depends heavily on the accuracy of beam diagnostic systems. Among these, Beam Position Monitors (BPMs) are critical for monitoring the beam trajectory and phase, which are vital for minimizing beam loss and ensuring efficient acceleration. However, mechanical installation errors, electronic offsets, and environmental factors often introduce discrepancies between the measured signal and the actual beam position.

Beam-based calibration (BBC) has emerged as a powerful method to resolve these discrepancies without requiring physical access to the vacuum chamber. By varying the strengths of upstream magnetic elements and observing the resulting trajectory shifts at downstream BPMs, the relative offsets and gains of the monitors can be determined. Despite its widespread use, the precision of BBC is limited by inherent uncertainties in the accelerator lattice and the measurement chain. This study aims to quantify these errors and establish the limits of calibration accuracy under realistic operating conditions.

Methodology

BPM Position Calibration

The fundamental principle of beam-based position calibration involves the response matrix of the accelerator. For a given change in a steering magnet' s strength $\Delta\theta$, the corresponding change in beam position Δx at a downstream BPM is given by:

$$\Delta x = R_{ij}\Delta\theta$$

where R_{ij} represents the transfer matrix element between the j-th corrector and the i-th BPM. In practice

Abstract

As prerequisites for automatic phase setting and fault compensation, precise longitudinal alignment and RF phase calibration are critical for superconducting hadron linacs. Although time-of-flight (TOF) beam-based measure- ments have been successfully employed at multiple facilities for linac alignment or synchronization, previous studies have assumed that all measured phase or position parameters are independent, without considering that beam-based BPM calibration can introduce correlations among the corresponding uncertainties. Within a beam-based calibration framework for linear accelerator commissioning, this work addresses these issues from two complementary aspects.

First, we rigorously analyze the sources and uncertainty propagation of phase errors in Time-of-Flight energy mea- surements using room-temperature BPMs. Second, for beam-based calibration with cryogenic BPMs, we derive the intrinsic correlations between position and phase offset errors and demonstrate how these correlations can induce error cancellation effects, thereby improving the accuracy of beam energy determination and phase setting. These effects are validated through numerical simulations, and their impact on beam-based calibration experiments for HIAF and CiADS commissioning is analyzed.

Key words: Linac; TOF; error; energy measurement

Note: Figure translations are in progress. See original paper for figures.