The Heterogeneous Tiebout-Oates Collapse Theorem: Thresholds, Location Selection, and Spatial Fiscal Fragility

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Abstract

This paper extends the Tiebout-Oates Trilemma (Jiang, 2026) from local spectral instability to global nonlinear collapse. We analyze a parabolic-elliptic system on a closed Riemannian surface in which forward Brownian diffusion competes with a nonlocal fiscal aggregation field generated by heterogeneous fundamentals. The main results are twofold. First, a perturbative threshold theorem establishes a sharp regime map: when total population falls below a critical mass determined by the peak of the heterogeneous fiscal amplifier, the spatial equilibrium persists globally; when total population exceeds this threshold, the system undergoes finite-time measure concentration for suitably localized initial data. Second, a location-selection conjecture, supported by a detailed proof strategy based on concentration-compactness and inner-outer matched asymptotics, identifies the effective fiscal potential that governs where collapse occurs: the blow-up point is selected by local productivity weighted by global geographic centrality in the spillover network. The critical mass is inversely proportional to the peak fiscal amplifier, so that spatial inequality lowers the collapse threshold and makes the system more fragile.

Full Text

Preamble

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This paper extends the Tiebout-Oates Trilemma (Jiang, 2026) from local spectral instability to global nonlinear collapse. We analyze a parabolic-elliptic

system on a closed Riemannian surface in which forward Brownian diffusion competes with a nonlocal fiscal aggregation field generated by heterogeneous fundamentals. The main results are twofold. First, a perturbative threshold theorem establishes a sharp regime map: when total population falls below a critical mass determined by the peak of the heterogeneous fiscal amplifier, the spatial equilibrium persists globally; when total population exceeds this threshold, the system undergoes finite-time measure concentration for suitably localized initial data. Second, a location-selection conjecture, supported by a detailed proof strategy based on concentration-compactness and inner-outer matched asymptotics, identifies the effective fiscal potential that governs where collapse occurs: the blow-up point is selected by local productivity weighted by global geographic centrality in the spillover network. The critical mass is inversely proportional to the peak fiscal amplifier, so that spatial inequality lowers the collapse threshold and makes the system more fragile.

Keywords

fiscal federalism, spatial collapse, finite-time blow-up, Keller–Segel, critical mass, location selection, Robin function.

JEL Classification: H71, H73, R12, R50.

- Theorem 3.1: Perturbative heterogeneous threshold (subcritical global existence and supercritical finite-time blow-up). Proved modulo standard lemmas.
- Conjecture 5.1: Location selection by the weighted Robin landscape. Conjecture with detailed proof strategy.
- Remark 8.1: Collapse-mechanism invariance. Priority defense.

1 Introduction

In a companion paper (Jiang, 2026), we establish the Tiebout–Oates Trilemma: under potential-driven factor mobility (C1), superlinear territorial revenue (C2), and local Lyapunov stability of the uniform allocation (C3), any two of these conditions are mutually compatible, but all three are not. The proof identifies a sign mechanism—superlinear per-capita revenue combined with utility-gradient mobility forces the effective principal coefficient to be negative—and derives a sharp spectral instability threshold in a homogeneous benchmark.

That analysis is inherently local: it characterizes the onset of instability near the uniform steady state but does not describe what happens after instability sets in. The present paper addresses the global nonlinear question: When does instability escalate to finite-time measure concentration, and where does collapse occur in heterogeneous space?

We work with the nonlocal parabolic–elliptic system introduced in Section 4.4 of Jiang (2026) as the "screened Keller–Segel fiscal analogue." In this system, forward Brownian diffusion ($\nu > 0$) competes with a nonlocal fiscal attraction field generated by a Poisson equation with spatially varying coefficient $A(x)$. This is the mathematically correct framework for blow-up analysis: the local equation with $\ell = 0$ is backward parabolic (Hadamard ill-posed), and standard energy/entropy methods cannot be applied to it. The nonlocal system preserves forward parabolicity while encoding the destabilizing fiscal force through the drift term.

The paper makes two main contributions. First, Theorem 3.1 provides a perturbative threshold theorem, proved modulo standard compact-manifold adaptations of the Keller–Segel free-energy method: for small ϵ, subcritical total mass guarantees global existence, while supercritical total mass near a productivity peak triggers finite-time blow-up. The critical mass $L^*(x) = \frac{8\pi\nu\ell^2}{\mu\kappa A(x)}$ is inversely proportional to the local fiscal amplifier $A(x)$, so that a peak in productivity lowers the collapse threshold.

Second, Conjecture 5.1 identifies the effective fiscal potential that governs blow-up location. We define $\Theta(x) = A(x)\exp(+4\pi\ell^2 H_\ell(x,x))$, where $H_\ell(x,x)$ is the diagonal regular part (Robin function) of the screened Green’ s function. The conjecture states that single-point blow-up must concentrate at the global maximizer of Θ. A detailed proof strategy based on concentration-compactness, inner rescaling, and reduced-functional analysis is provided. The sign $\exp(+4\pi\ell^2 H_\ell)$ is confirmed by independent matched-asymptotic derivations: geography enters the collapse selection as an amplifier, not a penalty.

The economic interpretation is direct. Collapse does not occur at an arbitrary location. It is selected by the interplay of local fiscal productivity $A(x)$ and global geographic centrality encoded in the Robin function $H_\ell(x,x)$. The city that collapses first is the one that is simultaneously most productive and most central in the spillover network.

2 Setup and Notation

Let (M, g) be a closed (compact, without boundary), connected, smooth Riemannian surface. We write Δ_g, ∇_g, div_g, $d_g(x,y)$, and dV_g for the Laplace-Beltrami operator, gradient, divergence, geodesic distance, and Riemannian volume element, respectively.

Assumption 2.1 (Heterogeneous fiscal system) Fix parameters $\nu, \mu, \kappa, \ell > 0$ and let $A(x)$ satisfy $A(x) > 0$ for all $x \in M$. Write $A(x) = \bar{A}(1 + \epsilon a(x))$ with $\int_M a\, dV_g = 0$. The density $\rho(x,t)$ and fiscal potential $\phi(x,t)$ satisfy the parabolic–elliptic system:

$$\partial_t \rho = \nu\Delta_g\rho - \frac{\mu\kappa}{\ell^2}\mathrm{div}_g(\rho\nabla_g\phi)$$

$$-\Delta_g\phi + \frac{1}{\ell^2}\phi = A(x)\rho$$

with smooth strictly positive initial data ρ_0, and total mass $\bar{L} := \int_M \rho_0\, dV_g$.

Definition 2.2 (Screened Green' s function) The screened Green' s function $G_\ell(x,y) : M \times M \to \mathbb{R} \cup \{+\infty\}$ is the distributional solution of $(-\Delta_g + \frac{1}{\ell^2}\mathrm{Id})G_\ell(x,\cdot) = \delta_x$. On a two-dimensional closed surface, for x fixed, one has the diagonal singularity expansion $G_\ell(x,y) = \frac{1}{2\pi}\ln\frac{1}{d_g(x,y)} + H_\ell(x,y)$, where H_ℓ extends smoothly to a neighborhood of the diagonal in $M \times M$. The Robin function is the diagonal restriction $R_\ell(x) = H_\ell(x,x)$.

Definition 2.3 (Effective fiscal potential) The effective fiscal potential is $\Theta(x) = A(x)\exp(4\pi\ell^2 R_\ell(x))$.

3 The Heterogeneous Threshold Theorem

Theorem 3.1 (Perturbative heterogeneous threshold on a closed surface) Under Assumption 2.1, let $\epsilon \geq 0$. Then, for sufficiently small ϵ, the following hold:

(i) **Subcritical global existence.** There exists $m_c(\epsilon)$ such that if $\bar{L} < m_c(\epsilon)$, then the corresponding smooth local solution extends for all $t > 0$, and no finite-time concentration can occur.

(ii) **Supercritical finite-time blow-up.** Let x^* be a nondegenerate strict local maximizer of $A(x)$, and write $A^* := A(x^*)$, $m_{c,*} := \frac{8\pi\nu\ell^2}{\mu\kappa A^*}$. If $\bar{L} > m_{c,*}$, then there exists a smooth positive initial datum of mass $\bar{L}$, concentrated in a sufficiently small geodesic ball around x^*, such that the corresponding solution blows up in finite time.

Proof of Theorem 3.1 (Sketch). Part (i): We use the free energy functional $F_\epsilon[\rho] = \nu\int\rho\ln\rho\,dV_g - \frac{\mu\kappa}{2\ell^2}\int\rho\phi\,dV_g$. Using the logarithmic Hardy-Littlewood-Sobolev inequality on manifolds, we show that for $\bar{L} < \frac{8\pi\nu\ell^2}{\mu\kappa\|A\|_\infty}$, the functional is bounded from below. Perturbative control for small ϵ ensures the dissipation inequality holds, preventing blow-up.

Part (ii): We employ a localized virial-type argument. By choosing initial data sufficiently concentrated near the productivity peak x^*, the superlinear aggregation term dominates the diffusion term in the evolution of the local second moment, forcing it to reach zero in finite time.

4 Economic Interpretation of the Threshold

The critical mass $L^*(x) = \frac{8\pi\nu\ell^2}{\mu\kappa A(x)}$ is the local fiscal carrying capacity at location x. It balances centrifugal forces (idiosyncratic preference shocks ν and the spatial spillover radius ℓ) against centripetal forces (migration responsiveness μ, fiscal extraction efficiency κ, and local productivity $A(x)$).

The most counterintuitive implication is that a peak in local productivity $A(x)$ —which is an asset in static analysis—lowers the aggregate collapse threshold. Because territorial revenue is superlinear, a productivity advantage generates a disproportionate spike in local public goods, creating a potent utility gradient that attracts mobile factors. When total population exceeds the threshold set by the productivity peak location, Tiebout sorting transforms from an equilibrating mechanism into a self-accelerating agglomerative process.

5 Location Selection

Conjecture 5.1 (Location selection by the weighted Robin landscape) Under Assumption 2.1, assume that $\Theta(x) = A(x)\exp(4\pi\ell^2 R_\ell(x))$ has a unique strict nondegenerate maximizer x_0. Let $m^* := \frac{8\pi\nu\ell^2}{\mu\kappa A(x_0)}$. If $\bar{L} = m^* + \eta$ for $0 < \eta \ll 1$, then there exist initial data such that the solution blows up at finite time T, and the blow-up point $x_{blow} = x_0$.

Detailed proof strategy. 1. **Concentration-compactness:** Show that any blow-up sequence must concentrate into a finite set of atoms. 2. **Inner rescaling:** Use the Euclidean critical Keller–Segel theory to show that the mass at an isolated blow-up point must be $m(x) = \frac{8\pi\nu\ell^2}{\mu\kappa A(x)}$. 3. **Reduced functional:** Expand the free energy near the concentration profile. The renormalized energy is minimized when $\Theta(x)$ is maximized.

6 Economic Interpretation of Location Selection

The effective fiscal potential $\Theta(x) = A(x)\exp(4\pi\ell^2 R_\ell(x))$ synthesizes two forces. The first factor, $A(x)$, is local productivity. The second factor, $\exp(4\pi\ell^2 R_\ell(x))$, is global: the Robin function encodes how the fiscal field is filtered by the spatial network. A city that collapses first is one that is simultaneously most productive and most central in the spillover geometry.

7 Extensions and Robustness

Conjecture 7.1 (Full heterogeneous collapse threshold) For arbitrary ϵ, the leading-order blow-up existence threshold is $L^* = \frac{8\pi\nu\ell^2}{\mu\kappa \max_M A(x)}$.

Remark 8.1 (Collapse-mechanism invariance) The collapse mechanism studied here inherits the sign logic of the Tiebout–Oates Trilemma (Jiang, 2026). The combination of forward diffusion with an endogenous fiscal-attraction field whose local singular part dominates entropy at the mass-critical scale is the universal driver of spatial fragility.

9 Conclusion

This paper develops a nonlinear collapse theory for decentralized public finance. The perturbative threshold theorem establishes that spatial inequality lowers the fiscal carrying capacity. The location-selection conjecture identifies the effective fiscal potential as the selector of collapse points, combining local productivity with global geographic centrality. Future work should focus on removing the perturbative restriction and completing the inner–outer gluing construction for the location-selection theorem.