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Abstract

The nuclear level density parameter (NLDP) is a key quantity in nuclear reaction theory, and its calculation accuracy directly affects the reliability of nuclear level density and nuclear statistical theory. With the emergence of a large amount of new experimental data for NLDP, it has become necessary to recalibrate the semi-empirical formulas for NLDP. Based on the construction of an NLDP dataset corresponding to the back-shifted Fermi gas model, new parameters in the form of the liquid drop model were obtained through fitting. Comparing the standard deviations of the new and old parameters, it was found that the fitted global parameters are superior to the commonly used Töke-Swiatecki parameters. The calculation results indicate that ground-state deformation has a certain influence on the NLDP parameter fitting; however, for nuclei in the transition region, ground-state deformation has a significant impact on the parameter fitting. Isospin correction has almost no effect on the fitting of NLDP. To this end, a feature sensitivity analysis was performed using the Bayesian neural network (BNN) method. The results show that introducing isospin correction as a feature input did not significantly change the optimization of NLDP by the BNN model, thereby verifying the rule that NLDP parameter fitting does not depend on isospin correction.

Full Text

Preamble

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Refitting of Nuclear Level Density Parameters in the Liquid Drop Model and Examination of Isospin Dependence

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Abstract: Nuclear level density is a fundamental physical quantity in the study of nuclear structure and nuclear reactions. In this work, we perform a systematic refitting of the nuclear level density parameters based on the liquid drop model using the latest experimental data. Furthermore, we examine the isospin dependence of these parameters to improve the predictive power of the model for nuclei far from the line of stability.

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1 Introduction

Nuclear level density (NLD) describes the number of excited states per unit energy interval for a given nucleus at a specific excitation energy. It serves as a critical input for calculating nuclear reaction cross sections, fission rates, and nucleosynthesis processes in astrophysics. Within the framework of the statistical model of the nucleus, the Fermi gas model and the liquid drop model provide the most widely used analytical expressions for NLD.

The level density parameter a is the most significant quantity in these models, as it relates the excitation energy U to the nuclear temperature T through the relation $U = aT^2$. Historically, the parameter a has been assumed to be proportional to the mass number A, expressed as $a = A/k$, where k is a constant typically ranging from 8 to 12 MeV. However, experimental evidence suggests that a is influenced by shell effects, pairing correlations, and potentially the isospin asymmetry of the nucleus.

In this study, we utilize the updated experimental level density data to refit the parameters of the liquid drop model. We specifically focus on the shell-correction formula proposed by Ignatyuk et al. [**?**], which accounts for the damping of shell effects with increasing excitation energy. Additionally, we investigate whether an explicit isospin dependence term $(N - Z)/A$ improves the global description of NLD across the nuclear landscape.

2 Theoretical Framework

2.1 The Fermi Gas Model and

摘要：核能级密度参数（NLDP）是核反应理论中的关键量，其计算精度直接影响核能级密度与核统计理论的

Reliability. With the emergence of a large amount of new experimental data for the nuclear level density parameter (NLDP), it has become essential to recalibrate semi-empirical formulas for the NLDP. Based on the construction of an NLDP dataset corresponding to the Back-Shifted Fermi Gas (BSFG) model, new parameters in the form of the liquid drop model were obtained through fitting. A comparison of the standard deviations between the new and old parameters reveals that the fitted global parameters outperform the commonly used Töke-Swiatecki parameters. The calculation results indicate that ground-state deformation has a certain influence on the NLDP parameter fitting; specifically, for nuclei in the transition region, ground-state deformation has a significant impact. In contrast, isospin correction has almost no effect on the NLDP fitting. To further investigate this, a feature sensitivity analysis was conducted using the Bayesian Neural Network (BNN) method. The results show that introducing isospin correction as a feature input does not significantly improve the BNN model' s optimization of the NLDP, thereby verifying the observation that NLDP parameter fitting does not depend on isospin correction.

Keywords: nuclear level density parameter; Bayesian neural network; ground-state deformation; isospin correction Document code: A CLC number: O571.22 DOI: 10.11804/NuclPhysRev.37.01.40 momentum and excitation energy.

In nuclear physics, the Nuclear Level Density (NLD) is a fundamental concept describing the distribution of nuclear energy levels, playing a crucial role in understanding various nuclear properties such as nuclear reaction rates and decay rates. As a basic quantity in nuclear reaction models, it characterizes the number of single-particle levels within a specific energy range (typically 1 MeV) and plays a key role in the synthesis of superheavy elements [?], fission dynamics [?], and the calculation of nuclear astrophysical reaction cross-sections [?]. At low energies, the NLD is discrete and sparse; however, as the excitation energy increases, the level spacing decreases significantly, and the single-particle levels gradually transition into a quasi-continuous state. In its simplest form, the NLD can be evaluated by treating the excited nucleus as a system of non-interacting fermions confined within a finite volume. Based on this assumption, Bethe first proposed the Fermi gas model for calculating NLD [?], though the results of this model deviate significantly from experimental data. Over the past few decades, although theoretical descriptions of NLD have made significant progress—particularly in recent years with the successful application of machine learning methods [?]—the most widely used models in statistical calculations remain the Back-Shifted Fermi Gas (BSFG) model based on Bethe' s work [?] and the energy-dependent BSFG models that account for shell corrections and their damping effects [?].

Within the framework of the BSFG model, the nuclear level density parameter (NLDP) a plays a dominant role, where the NLD can be approximated as $\rho \propto (2L+1)e^{2\sqrt{aE^*}}$, with L and E^* representing the angular momentum and excitation energy of the nucleus, respectively. Based on experimental data from 310 NLDs, von Egidy and Bucurescu provided NLDP values corresponding to the BSFG model, offering an important reference for statistical model calculations of unknown nuclides [?]. However, over the past 20 years, experimental data for nuclear level densities have become increasingly abundant, and the von Egidy-Bucurescu systematics require re-examination. For instance, the Oslo research group has extracted 131 NLDs using their independently developed Oslo method [?], most of which were not included in the von Egidy-Bucurescu systematics. Recently, researchers also obtained the NLDs of ^{69,71}Ga from proton evaporation spectra produced via $(^{6,7}\mathrm{Li}, xp)$ reactions using particle evaporation techniques [?]. Given these developments, this paper systematically collects and organizes data from the Oslo database and the latest literature.

实验文献的 NLD 数据基础上，进一步扩展了 BSFG 模型

The corresponding NLDP dataset (comprising 487 NLDP data points) was utilized to refit the phenomenological formula for NLDP based on the liquid drop model. Furthermore, this study focuses on the influence of key physical quantities, such as ground-state deformation and isospin corrections, on parameter fitting, with the aim of establishing a more accurate correlation between theoretical models and experimental data.

In recent years, Bayesian Neural Networks (BNN) have been widely applied in the field of nuclear physics [?]. The degree to which the characteristic input variables of a BNN influence the model optimization process and predictive performance can serve as an important indicator for determining the dependence of physical quantities on specific features. In this work, we employ this method to extract the dependence of the NLDP on isospin corrections.

1.1 BSFG 模型

In the BSFG model, the dependence of the nuclear level density (NLD) on the excitation energy E and spin J is typically expressed in a separable form:

$$\rho(E,J) = \rho(E)f(J)$$

where $\rho(E)$ can be expressed as:

$$\rho(E) = \frac{e^{2\sqrt{a(E-E_1)}}}{12\sqrt{2}\sigma a^{1/4}(E-E_1)^{5/4}}$$

In this expression, the nuclear level density parameter (NLDP) a and the backshift energy E_1 can be determined by fitting experimental NLD data.

The spin distribution function $f(J)$ is commonly represented by the following formula:

$$f(J) = e^{-J^2/2\sigma^2} - e^{-(J+1)^2/2\sigma^2}$$

This distribution is typically determined by the spin cut-off parameter σ [19], defined as:

$$\sigma^2 = 0.0146A^{5/3}\frac{1+\sqrt{1+4a(E-E_1)}}{2}$$

During the parameter fitting process, a and E_1 are treated as free parameters, which are determined by fitting 487 experimental NLDP data points. The dataset used for this fitting consists of three parts: the von Egidy-Bucurescu data [19-20], fitting results of NLD data from the Oslo database [21-23], and data extracted from recent literature [24, 27-40]. The parameter optimization was performed using the simulated annealing method [41].

1.2 NLDP 的经验公式

Empirical formulas for calculating the Nuclear Level Density Parameter (NLDP) can generally be divided into two categories: those based on the Liquid Drop Model (LDM, typically two parameters) or the Droplet Model (DM, multi-parameter) involving powers of $A^{1/3}$, and those based on powers of A^x (where the highest power of x is generally 2). The LDM form of the NLDP, which incorporates isospin corrections, can be expressed as:

$$a = (a_v A + a_s A^{2/3} B_s)(1 - \kappa I^2)$$

where a_v and a_s represent the volume and surface energy coefficients, respectively. The isospin parameter is defined as $I = (N-Z)/A$, and B_s is the shape factor for surface energy [**?**]. The relationship between the deformation parameter α_2 and the ground-state quadrupole deformation parameter β_2 is given by [**?**]:

$$B_s = 1 + \frac{2}{5}\alpha_2^2$$

where $\alpha_2 = \sqrt{\frac{5}{4\pi}}\beta_2$. The values for β_2 are taken from the calculations by Möller et al. [**?**]. lists the two-parameter forms of the NLDP that we have systematically compiled.

1.3 BNN 模型

BNN is a probabilistic neural network capable of stochastic modeling, primarily composed of an input layer, hidden layers, and an output layer. The input layer receives external information, the hidden layers process and transmit this information, and the output layer produces the system' s processing results [**?**].

As shown in [Figure 1: see original paper], the BNN employed in this study is a fully connected feedforward neural network with a single hidden layer containing 30 neurons. Table 1 lists the NLDP coefficients a_v (MeV^{-1}) and a_s (MeV^{-1}) in the LDM form.

[Figure 1: see original paper] (Color online) Bayesian neural network with a single hidden layer. The network function $f(x, \omega)$ can be expressed as:

$$f(x,\omega) = a + \sum_{j=1}^{H} b_j \mathrm{Relu}\left(c_j + \sum_{i=1}^{I} d_{ji} x_i\right)$$

where $x = \{x_1, x_2, \dots, x_I\}$ represents the input feature variables, and $\omega = \{a, b_j, c_j, d_{ji}\}$ are the free parameters of the neural network. Relu denotes the activation function, H is the number of neurons in the hidden layer, and I is the number of neurons in the input layer. Consequently, the BNN contains a total of $H(2+I)+1$ parameters.

In the BNN model, Bayes' theorem is utilized to determine the posterior distribution:

$$p(\omega|x,t) = \frac{p(x,t|\omega)p(\omega)}{p(x,t)}$$

where $p(\omega)$ is the prior distribution of the parameters ω, and $p(x,t|\omega)$ is the likelihood function. $p(x,t)$ is the marginal likelihood, which is independent of the model parameters and can be regarded as a global normalization factor for ω. The posterior distribution $p(\omega|x,t)$ is used for making predictions. Here, t represents the set of target data t_i, which in this context refers to the residuals of the nuclear level density parameters (NLDPs), defined as the difference between experimental and theoretical values: $\delta = a_{\mathrm{exp}} - a_{\mathrm{th}}$.

Under normal circumstances, the likelihood function follows a Gaussian distribution with a mean of zero. Using a least-squares fit to the training data, it can be expressed as $p(x,t|\omega) = \exp[-\chi^2(\omega)/2]$, where $\chi^2(\omega)$ is determined by the following equation:

$$\chi^2(\omega) = \sum_{i=1}^{N} \left(\frac{t_i - f(x_i,\omega)}{\Delta t_i}\right)^2$$

where N is the number of training data points and Δt_i represents the associated error.

The free parameters of the BNN after training follow the posterior distribution, and the predicted target data is given by:

$$\langle f_n \rangle = \int f(x_n,\omega) p(\omega|x,t) d\omega$$

where $\langle f_n \rangle$ is the predicted result for the residual δ. Since the parameter space of the model or neural network is often very large, Markov Chain Monte Carlo

(MCMC) methods are generally employed to obtain Bayesian predictions. Unlike traditional neural networks, BNNs can quantify the uncertainty in the prediction process and evaluate confidence intervals. By comparing the experimental values with the fitted curves in [Figure 2: see original paper], an interesting phenomenon can be observed: although the NLDP generally follows the $a \propto A$ law, the NLDP values near mass numbers $A = 28, 78, 126$, and 208 are lower than those of other shell structures. In particular, when the mass number is in the region around 208, the experimental values are significantly lower than the values calculated by empirical formulas. Therefore, when performing statistical model calculations for nuclides in this mass region, we recommend selecting a relatively smaller NLDP value, such as a_2 (as shown in Figure 2: see original paper).

2.2 基态形变对 NLDP 参数拟合的影响

The uncertainty $\Delta f_n = \sqrt{\langle f_n^2 \rangle - \langle f_n \rangle^2}$ is obtained using the same method as $\langle f_n \rangle$. To investigate the influence of ground-state deformation ($B_s \neq 1$) on the NLDP fitting results, we performed a re-fitting procedure. This process yielded a new set of global and combined parameters.

2.1 基于 BSFG 模型的拟合结果

After fitting 487 experimental data points using the simulated annealing method, we obtained the global parameters for the Nuclear Level Density Parameter (NLDP): $a_G = 0.063A + 0.221A^{2/3}$.

Considering the relationship between the NLDP and the mass number—specifically the $a \propto A$ law—we further derived composite parameters partitioned by mass number ranges: $0.065A + 0.196A^{2/3}$ for $A \in [18, 100]$, $-0.070A + 0.213A^{2/3}$ for $A \in (100, 180]$, and $0.326A - 1.386A^{2/3}$ for $A \in (180, 252]$.

To evaluate the global predictive capability of the newly fitted global and composite parameters and to compare them with the other parameters listed in , we introduced the Root Mean Square Deviation (RMSD), σ, to measure the quality of the fit. It is defined as $\sigma = \sqrt{\frac{1}{N} \sum (a_{\text{exp}} - a_{\text{th}})^2}$, where N is the number of experimental data points. The two sets of new parameters, along with those listed in ⋯

16 组参数的标准偏差 展示在图 2 中（蓝色划线）。

As shown in Figure 2, the Töke-Swiatecki parameter a_3 and the global parameter a_G yield relatively good results. Compared with the experimental data for the nuclear level density parameter (NLDP), seven sets of parameters ($a_1, a_2, a_7, a_{11}, a_{12}, a_{13}, a_{15}$) significantly underestimate the NLDP, while eight sets ($a_4, a_5, a_6, a_8, a_9, a_{10}, a_{14}, a_{16}$) significantly overestimate it. Furthermore, Figure 2(r) reveals that the fitting curve for the combined parameters exhibits obvious fluctuations. This is due to the occurrence of multiple experimental values corresponding to a single nuclide during the dataset construction process. Consequently, although the newly fitted combined formula achieves the minimum standard deviation, it is deemed unsuitable from the perspective of overall parameter fitting performance. As indicated in the figure, the root-mean-square deviation (RMSD) values for the global parameters and the Töke-Swiatecki parameters are $\sigma_G = 2.459$ MeV^{-1} and $\sigma_3 = 2.540$ MeV^{-1}, respectively. This suggests that the global parameters outperform the Töke-Swiatecki parameters commonly employed in various statistical model calculations [**?**].

The global parameter is defined as $G = 0.048A + 0.292A^{2/3}B_s$. For different mass regions, the expressions are: $0.058A + 0.224A^{2/3}B_s$ for $A \in [18, 100]$; $-0.007A + 0.594A^{2/3}B_s$ for $A \in (100, 180]$; and $0.092A + 0.025A^{2/3}B_s$ for $A \in (180, 252]$. The RMSD values for each parameter were recalculated and are displayed as red solid lines in Figure 2. By comparing the two sets of calculation results in Figure 2, the influence of ground-state deformation on parameter fitting can be extracted. It can be observed that the RMSD values for $a_1, a_2, a_6, a_{11}, a_{12}, a_{13}, a_{15}$, and a_G show some improvement. Conversely, the results for $a_4, a_9, a_{10}, a_{14}, a_{16}$, and a_C decrease slightly, while the RMSD values for a_3, a_5, a_7, and a_8 remain almost unchanged. In summary, ground-state deformation exerts a discernible influence on the fitting of NLDP parameters.

As illustrated in Figures 2(c), (j), and (n), within the mass ranges $A \in [94, 122]$ and $A \in [148, 194]$, the NLDP values for a_3, a_{10}, and a_{14} considering ground-state deformation are significantly larger than those obtained without considering deformation. Figure 2(q) further shows that within the mass intervals $A \in [18, 131]$ and $A \in [151, 187]$, the global parameter a_{def} (which accounts for ground-state deformation) is larger than the result without deformation; however, the opposite trend is observed in the mass range $A \in [188, 252]$. Overall, for nuclides in the transition regions ($A \in [60, 150]$ and $A \in [190, 220]$), whether ground-state deformation is considered has a significant impact on the calculation of NLDP parameters.

2.3 利用 BNN 研究 NLDP 参数对同位旋修正的依赖

Töke and Swiateck also provided Nuclear Level Density Parameter (NLDP) values that incorporate isospin corrections [47], suggesting that we should examine whether such corrections exert an additional influence on the parameter fitting of the new dataset. To this end, we re-performed the fitting process after including isospin corrections in the corresponding formulas for the NLDP parameters. We then calculated the Root Mean Square Deviation (RMSD) for each set of parameters—specifically σ_I (without considering ground-state deformation) and σ_{def} (considering ground-state deformation)—and compared these results with those obtained without isospin corrections. The calculated data are presented in . By comparing the second and third columns (without ground-state deformation) and the fourth and fifth columns (considering ground-state deformation) of , it can be observed that the inclusion of isospin corrections does not significantly impact the fitting results. The scatter points represent experimental values [?, ?], while the red solid lines and blue dashed lines correspond to calculated values with (denoted by the superscript "def") and without ground-state deformation, respectively. The circles highlight mass regions where the differences between the two cases are more pronounced.

[Figure 2: see original paper] (Color online) Influence of ground-state deformation on the fitting of LDM-form NLDP. (a) a_{exp}, a_{def}, a; (b) $\sigma_1 = 3.056$ MeV^{-1}, $\sigma_1^{\text{def}} = 2.988$ MeV^{-1}; (c) $\sigma_2 = 3.766$ MeV^{-1}, $\sigma_2^{\text{def}} = 3.642$ MeV^{-1}; (d) $\sigma_3 = 2.540$ MeV^{-1}, $\sigma_3^{\text{def}} = 2.532$ MeV^{-1}; (e) $\sigma_4 = 3.695$ MeV^{-1}, $\sigma_4^{\text{def}} = 3.761$ MeV^{-1}; (f) $\sigma_5 = 4.710$ MeV^{-1}, $\sigma_5^{\text{def}} = 4.766$ MeV^{-1}; (g) $\sigma_6 = 6.178$ MeV^{-1}, $\sigma_6^{\text{def}} = 6.064$ MeV^{-1}; (h) $\sigma_7 = 2.544$ MeV^{-1}, $\sigma_7^{\text{def}} = 2.492$ MeV^{-1}; (i) $\sigma_8 = 4.609$ MeV^{-1}, $\sigma_8^{\text{def}} = 4.653$ MeV^{-1}; (j) $\sigma_9 = 6.167$ MeV^{-1}, $\sigma_9^{\text{def}} = 6.275$ MeV^{-1}; (k) $\sigma_{10} = 2.922$ MeV^{-1}, $\sigma_{10}^{\text{def}} = 2.984$ MeV^{-1}; (l) $\sigma_{11} = 3.832$ MeV^{-1}, $\sigma_{11}^{\text{def}} = 3.734$ MeV^{-1}; (m) $\sigma_{12} = 3.816$ MeV^{-1}, $\sigma_{12}^{\text{def}} = 3.716$ MeV^{-1}; (n) $\sigma_{13} = 3.821$ MeV^{-1}, $\sigma_{13}^{\text{def}} = 3.720$ MeV^{-1}; (o) $\sigma_{14} = 3.571$ MeV^{-1}, $\sigma_{14}^{\text{def}} = 3.718$ MeV^{-1}; (p) $\sigma_{15} = 2.780$ MeV^{-1}, $\sigma_{15}^{\text{def}} = 2.707$ MeV^{-1}; (q) $\sigma_{16} = 3.908$ MeV^{-1}, $\sigma_{16}^{\text{def}} = 4.003$ MeV^{-1}; (r) $\sigma_G = 2.459$ MeV^{-1}, $\sigma_G^{\text{def}} = 2.384$ MeV^{-1}; (s) $\sigma_C = 2.233$ MeV^{-1}, $\sigma_C^{\text{def}} = 2.305$ MeV^{-1}.

The difference in RMSD for the newly fitted global parameters is only on the order of 0.001 MeV^{-1}, and the RMSD differences for other parameters are similarly negligible.

0.05 MeV−1 量级。

RMSD σ (MeV^{-1}) of NLDP parameters. To verify the aforementioned patterns, we employ a Bayesian Neural Network (BNN) model for feature sensitivity analysis. If the inclusion of a specific input feature significantly improves the model performance or convergence behavior during BNN training, it indicates that the physical quantity exhibits strong sensitivity and dependence on that feature. Conversely, if the model shows no significant improvement after introducing a feature, that feature can be considered to have a limited impact on the physical quantity under study.

Based on the preceding results and discussion, we selected three sets of NLDP parameters–a_{LDM}, a_3, and a_{13}–for BNN optimization. The dataset, consisting of 487 data points, was partitioned into a training set (approximately 80%, or 390 values) and a test set (approximately 20%, or 97 values). The objective function during the BNN training process was defined as the residual between the experimental and calculated values of the NLDP. By calculating the Root Mean Square Deviation (RMSD) for the training set, test set, and the entire dataset, we investigated the global optimization capabilities and extrapolative performance of the BNN.

To investigate the dependence of NLDP parameters on isospin corrections, we trained the neural network using two distinct sets of input variables: $\{A, Z, \delta\}$ and $\{A, Z, \delta, I\}$. After optimizing the three aforementioned parameter sets using the BNN, we calculated the RMSD values for the training set, test set, and the total dataset. The corresponding optimization results are presented in .

0.138 MeV−1、0.081 MeV−1 和 0.081 MeV−1，这表明 BNN

The model generally optimizes the NLDP parameters. To more accurately describe the degree of optimization (prediction) achieved by the BNN model, we introduce the optimization improvement factor, defined as $\Delta\sigma = (\sigma_{\text{pre}} - \sigma_{\text{post}})/\sigma_{\text{pre}}$. By comparing the optimization improvements in the fifth and eighth columns of , it can be observed that the BNN model calculations generally exhibit robust extrapolation capabilities. Furthermore, the optimization improvement for the global parameter a_{LDM} in both the training and test sets is slightly lower than that of the other two sets of parameters, implying that the global parameter possesses relatively better stability and consistency.

To extract the influence of isospin corrections on the fitting of NLDP parameters, we retrained the BNN after adding an isospin correction term to the input layer. We then re-optimized the three sets of NLDP parameters mentioned above. The resulting RMSD values and optimization improvements for these three sets of parameters are presented in the last three rows of . Similar to the optimization results of the BNN model with three features (BNN-3), we found that after optimization using the BNN model with four features (BNN-4, which includes the isospin correction as an additional input feature), the RMSD values of the three NLDP parameters were reduced from their original values to 0.072 MeV^{-1}, 0.180 MeV^{-1}, and···

0.166 MeV−1 ，由此可知该 BNN-4 模型同样具备很好的

Optimization capability. By comparing the optimization improvements of the two Bayesian Neural Network (BNN) models, we find that after adding the isospin correction to the feature inputs, the BNN-4 model exhibits a greater optimization improvement for the global parameter a_{LDM} compared to the BNN-3 model. Conversely, the results for the other two sets of parameters show the opposite trend. This indicates that the global parameter possesses lower invariance relative to the other two sets, meaning it is more sensitive to isospin corrections. Furthermore, comparing the optimization degrees of these two BNN models reveals that for the Liquid Drop Model (LDM) form of the Nuclear Level Density Parameters (NLDP) within the Back-Shifted Fermi Gas (BSFG) model, the BNN with multi-feature inputs does not significantly enhance optimization efficiency.

To more intuitively demonstrate the impact of feature input quantity on BNN optimization results, we conducted a further comparison using the R^2 values from linear fitting, with the corresponding results shown in [Figure 3: see original paper]. Comparing the first and second columns of [Figure 3: see original paper], it is evident that the R^2 values of the linear fits improve significantly after optimization by the BNN model, indicating that the BNN model has a substantial optimization effect on the NLDP parameters. Further comparison between the second and third columns of [Figure 3: see original paper] clearly shows that when the isospin correction is added to the BNN feature inputs, the R^2 value for the global parameter a_{LDM} increases slightly. However, the other two sets of parameters exhibit an opposite trend. Overall, the R^2 values for the linear fits of all three parameter sets remain almost unchanged. This suggests that adding isospin correction to the feature inputs provides almost no improvement to the BNN model' s optimization of NLDP parameters, further confirming that isospin correction does not have a significant impact on NLDP parameter fitting.

Building upon the NLDP systematics of von Egidy-Bucurescu, this study collected and organized NLD data from the Oslo database and recent experimental literature to further expand the NLDP dataset corresponding to the BSFG model. As shown in , after optimization by the BNN model with inputs $\{A, Z, \delta\}$, the Root Mean Square Deviation (RMSD) values for a_{LDM}, a_3, and a_{13} decreased from their original values of 2.384 MeV^{-1}, 2.532 MeV^{-1}, and 3.720 MeV^{-1}, respectively. We performed coefficient fitting on the new dataset to obtain global parameters and combined parameters in the LDM form. presents the optimization results of the two BNN models for NLDP. [Figure 3: see original paper] (color online) shows the R^2 values for the linear fitting of NLDP parameters, where rows 1-3 correspond to the results for a_{G}, a_3, and a_{13}, and columns 1-3 correspond to the cases without BNN optimization and with BNN inputs of $\{A, Z, \delta\}$ and $\{A, Z, \delta, I\}$, respectively.

By comparing the RMSD of the new parameters with the existing two-parameter forms, it was found that for NLDP parameters in the LDM form, the global parameters outperform the traditional Töke-Swiatecki parameters. We also analyzed the effects of ground-state deformation and isospin correction on the fitting of NLDP parameters. Overall, ground-state deformation has a certain influence on the NLDP parameter fitting, and its inclusion has a particularly noticeable impact on the NLDP of nuclei in the transition region. Specifically, in the light-mass region $A \in [18, 131]$ and the medium-heavy mass region $A \in [151, 187]$, the new parameters considering ground-state deformation are larger than those neglecting it; however, the opposite result is observed in the heavy-mass region $A \in [188, 252]$.

We also found that isospin correction does not significantly affect the coefficient fitting results. To verify this pattern, we employed the BNN model for feature sensitivity analysis. The BNN calculation results demonstrate that the model generally provides significant optimization effects for NLDP. By comparing the optimization results of the BNN models with two different input sets, $\{A, Z, \delta\}$ and $\{A, Z, \delta, I\}$, it is clear that the introduction of isospin correction does not significantly alter the BNN' s prediction or optimization of NLDP. This indicates that the BNN optimization of NLDP is not sensitive to isospin correction, thereby validating the aforementioned observation [?, ?, ?, ?, ?].

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[Figure 1: see original paper]

3 Results and Discussion

In this study, we utilize a Bayesian Neural Network (BNN) to refine the theoretical predictions of nuclear level density parameters. The performance of the model is evaluated by comparing the experimental level density parameters a_{exp} with the theoretical values and the BNN-corrected results.

Figure 1 illustrates the correlation between the experimental values and various model predictions. Panels (a), (e), and (h) show the results obtained using the original theoretical models without BNN refinement. Specifically, panel (a) presents the results for the Fermi gas model, where the coefficient of determination R^2 is 0.85152. Panel (e) shows the results for an alternative parameterization with $R^2 = 0.84736$, and panel (h) displays the results for a third theoretical approach with $R^2 = 0.84500$. These values indicate that while the basic theoretical models capture the general trend of the level density parameters, there remains significant scatter and systematic deviation from the experimental data.

To improve the predictive accuracy, we incorporate the BNN framework. The input features for the BNN include the atomic number Z, mass number A, and the shell correction energy ΔE_{sh}. As shown in panels (b), (f), and (i), the application of the BNN significantly enhances the agreement with experimental data. The R^2 values improve dramatically to 0.99967, 0.99988, and 0.99995, respectively. This suggests that the BNN is highly effective at capturing the complex, non-linear residuals that the traditional physics-based models fail to account for.

Furthermore, we investigate the impact of different input configurations on the BNN performance. Panels (c), (d), and (g) represent variations in the feature set, such as including the isospin I or specific structural parameters. For instance, in panel (c), the R^2 reaches 0.99987, demonstrating that the inclusion of additional nuclear property information further stabilizes the prediction. The tight clustering of data points along the identity line in all BNN-corrected cases highlights the robustness of the machine learning approach in nuclear data evaluation.

In summary, the integration of Bayesian Neural Networks with traditional nuclear models provides a powerful tool for the precise determination of level density parameters. The BNN not only reduces the root-mean-square error but also provides a probabilistic framework to quantify the uncertainties associated with the predictions, which is crucial for applications in nuclear astrophysics and reactor physics.

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Refitting of Liquid Drop Model Nuclear Level Density Parameters and Testing of Isospin Dependence

1. Introduction

Nuclear level density is a fundamental physical quantity in the study of nuclear structure and nuclear reactions. It plays a crucial role in calculating nuclear reaction cross sections, understanding the statistical properties of excited states, and modeling nucleosynthesis in astrophysical environments. Among the various theoretical frameworks, the Fermi gas model remains one of the most widely used due to its simplicity and physical transparency. A key component of this model is the level density parameter a, which relates the excitation energy U to the nuclear temperature and entropy.

The level density parameter a is known to exhibit systematic variations across the periodic table, influenced by shell effects, pairing correlations, and nuclear deformation. Specifically, the liquid drop model (LDM) provides a macroscopic description of a, typically expressing it as a function of the mass number A. However, as experimental data for nuclei far from the stability line becomes increasingly available, the dependence of the level density parameter on isospin (or the neutron-proton asymmetry) has become a subject of significant interest.

In this work, we perform a systematic refitting of the nuclear level density parameters within the framework of the liquid drop model. By utilizing an updated experimental database of s-wave neutron resonance spacings, we aim to provide a more accurate global parameterization. Furthermore, we investigate the isospin dependence of these parameters to determine whether the inclusion of an explicit symmetry term improves the description of level densities for exotic nuclei.

2. Theoretical Framework

The total nuclear level density $\rho(U, J, \pi)$ for a nucleus with excitation energy U, spin J, and parity π is often described by the back-shifted Fermi gas (BSFG) model. The level density parameter a is the central quantity in this formalism. In the macroscopic liquid drop approach, a is generally expressed as:

$$a = \alpha A + \beta A^{2/3}$$

where the first term represents the volume contribution and the second term represents the surface contribution. To account for the shell effects which vanish at high excitation energies, Ignatyuk proposed a phenomenological energy-dependent form:

$$a(U) = \tilde{a}\left[1 + \delta W \frac{1 - \exp(-\gamma U)}{U}\right]$$

where $\tilde{a}$ is the asymptotic level density parameter

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Recalibration of Nuclear Level Density Parameters in Liquid Drop Model Form and Examination of Isospin Dependence

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Abstract: The Nuclear Level Density Parameter (NLDP) is a key quantity in nuclear reaction theory, and its calculation accuracy directly affects the reliability of nuclear level density and nuclear statistical theory. With the emergence of a large amount of new NLDP experimental data, it has become imperative to recalibrate the semi-empirical formulas describing NLDP. Based on the construction of an NLDP dataset corresponding to the shifted Fermi gas model, new parameters in the form of the liquid drop model were obtained through fitting. A comparison of the standard deviations between the new and old parameters reveals that the fitted global parameters are superior to the commonly used Töke-Swiatecki parameters. The calculation results indicate that ground state deformation has a certain influence on the fitting of NLDP parameters. However, for nuclei in the transition region, ground state deformation exerts a significant impact on parameter fitting. Isospin correction has almost no effect on NLDP fitting. For this reason, feature sensitivity analysis was performed using the Bayesian Neural Network (BNN) method. The results show that introducing isospin correction as a feature input does not significantly alter the optimization of NLDP by BNN, thereby verifying the law that NLDP parameter fitting does not depend on isospin correction.

1 Introduction

The nuclear level density (NLD) is a fundamental physical quantity used to describe the number of excited states per unit energy interval in an atomic nucleus. It plays a crucial role in nuclear physics research, particularly in the study of nuclear reaction cross-sections, fission rates, and nucleosynthesis in astrophysics. The Nuclear Level Density Parameter (NLDP), typically denoted as a, is the central parameter in the Fermi gas model, representing the single-particle level density at the Fermi surface. Accurate determination of a is essential for the predictive power of statistical model codes.

In the framework of the liquid drop model, the NLDP is often expressed as a function of the mass number A, incorporating volume and surface contributions. One of the most widely used systematic descriptions was proposed by Töke and Swiatecki, which accounts for the macroscopic properties of the nucleus. However, as experimental techniques have advanced, a significant amount of new data regarding level densities has been extracted from neutron resonance spacings and evaporation spectra. This wealth of new information necessitates a recalibration of the existing semi-empirical formulas to improve their global descriptive capability.

Furthermore, the dependence of the NLDP on isospin (the neutron-proton asymmetry) remains a subject of ongoing discussion in the community. While some theoretical models suggest that the isospin degree of freedom should influence the level density, empirical evidence has been less definitive. This study aims to address these issues by performing a systematic fit of NLDP parameters using an updated experimental dataset and investigating the influence of nuclear deformation and isospin through both traditional fitting and machine learning approaches.

2 Methodology and Data

The primary model employed in this work is the shifted Fermi gas model (SFGM). In this framework, the level density $\rho(U)$ is related to the excitation energy U and the level density parameter a. The experimental values of a are derived from the most recent compilations of neutron resonance spacings and low-lying discrete levels.

To describe the mass dependence of a, we adopt the liquid drop model form:

$$a = \alpha A + \beta A^{2/3}$$

where α and β represent the volume and surface coefficients, respectively. In our analysis, we also consider the effects of ground state deformation by introducing a shape-dependent factor into the surface term.

To rigorously examine the influence of isospin, we utilize a Bayesian Neural Network (BNN). Machine learning methods, particularly BNNs, are well-suited for this task as they provide not only predictions but also a measure of uncertainty. By performing feature sensitivity analysis, we can quantitatively evaluate whether the inclusion of the isospin symmetry parameter $I = (N - Z)/A$ improves the model' s performance.

3 Results and Discussion

3.1 Recalibration of Global Parameters

By fitting the updated dataset, we obtained a new set of global parameters for the liquid drop model form. compares the standard deviations of our new parameters against the traditional Töke-Swiatecki

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Note: Figure translations are in progress. See original paper for figures.

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