A Postprint of a Fast Algorithm for Calculating Sun Outage in Inter-satellite Links of Constellation Satellites

Authors: Jingshi Tang

Date: 2026-03-03T15:53:36+00:00

Abstract

With the development and maturity of satellite technology, Inter-satellite Links (ISL) have been widely applied in satellite constellations such as navigation and communication satellites to achieve efficient inter-satellite communication and measurement. Since the establishment or maintenance of inter-satellite links is affected by solar outages, it is often necessary to calculate and predict the impact of solar outages on different satellites and links during the design and planning of satellite constellations. This allows for the assessment of the impact of solar outages on inter-satellite links and system services, facilitating better planning and design of related operations. Currently, whether using mature satellite system design tools to build scenarios or utilizing internally developed simulation systems within relevant organizations, solar outages are typically identified and predicted by judging the angle between the link and the line-of-sight direction of the sun epoch by epoch based on extrapolated ephemeris. This method is simple, intuitive, and easy to implement, but its disadvantage lies in low computational efficiency. Although solar outage calculations during the design phase do not require highly accurate ephemeris—usually, extrapolating satellite ephemeris based on a two-body model is sufficient—accurately determining the frequency, timing, and duration of potential solar outages often requires very short time intervals for the satellite ephemeris. Otherwise, short-duration solar outages may be missed, affecting the evaluation results. A fast algorithm is proposed for the determination and calculation of solar outages. By constructing an equation satisfied by the satellite' s argument of latitude u at the start and end moments of a solar outage, the possibility of a solar outage is analytically determined, and the argument of latitude at the beginning and end of the outage is calculated. This enables rapid calculation of the existence of solar outages on inter-satellite links as well as their start and end times. This algorithm also requires prior extrapolation of the ephemeris, but the ephemeris step size only needs to be no more than one orbital period. Calculations demonstrate

that this algorithm can reduce computation time by 2 to 3 orders of magnitude while stably detecting more solar outages than traditional methods. Even when the step size of the extrapolated ephemeris approaches one orbital period, this method can detect over 99.9% of solar outage arcs, and the calculation results are highly stable, with the maximum deviation in the occurrence time and duration of solar outages not exceeding 1 s.

Full Text

Preamble

Vol. 67, No. 1

January 2026

ACTA ASTRONOMICA SINICA Vol. 67 No. 1 Jan., 2026 doi: 10.15940/j.cnki.0001-5245.2026.01.008

A Fast Algorithm for Calculating Sun Outages in Inter-satellite Links of Satellite Constellations*

Tang Jingshi[1,2,3†]

([1] School of Astronomy and Space Science, Nanjing University, Nanjing 210023) ([2] Key Laboratory of Modern Astronomy and Astrophysics, Ministry of Education, Nanjing 210023) ([3] Institute of Space Environment and Astrodynamics, Nanjing University, Nanjing 210023)

Abstract

With the rapid development of large-scale satellite constellations, inter-satellite links (ISL) have become a critical technology for achieving global coverage and high-speed data transmission. However, Sun outages—a phenomenon where solar electromagnetic radiation interferes with signal reception—pose a significant threat to the stability of these links. This paper proposes a fast algorithm designed to calculate the timing and duration of Sun outage events for inter-satellite links within large constellations. By employing a hierarchical screening strategy and optimizing the geometric constraints of the Sun-satellite-satellite configuration, the algorithm significantly reduces the computational overhead compared to traditional brute-force search methods. Numerical simulations demonstrate that the proposed method maintains high precision while achieving a substantial increase in computational efficiency, making it suitable for real-time mission planning and link budget analysis in complex constellation environments.

---

1 Introduction

In recent years, the construction of low-Earth orbit (LEO) satellite constellations has entered a phase of explosive growth. Projects such as Starlink and OneWeb aim to provide global broadband internet services by deploying thousands of satellites. In these systems, inter-satellite links (ISL) are essential for reducing reliance on ground stations, minimizing signal latency, and enhancing network resilience. However, the operational environment of space subjects these links to various periodic interferences, among which Sun outages are particularly prominent.

A Sun outage occurs when the Sun, the transmitting satellite, and the receiving satellite align such that the Sun enters the field of view of the receiving antenna. Because the Sun is a powerful source of broadband electromagnetic noise, its radiation can overwhelm the communication signal, leading to a significant decrease in link quality.

With the development and maturity of satellite technology, inter-satellite links (ISL) have been widely applied in navigation and communication satellite constellations to achieve efficient inter-satellite communication and measurement. Since the establishment and maintenance of these links are affected by solar outages (Sun interference), it is often necessary to calculate and predict the impact of solar outages on different satellites and links during the design and planning of satellite constellations. This allows for the assessment of how system services are affected and enables better planning and design of related operations.

Currently, whether using mature satellite system design tools to build scenarios or utilizing internally developed simulation systems, solar outages are typically predicted by judging the angle between the link and the line-of-sight to the Sun based on extrapolated ephemerides at each epoch. While this approach is simple, intuitive, and easy to implement, it suffers from low computational efficiency. Although solar outage calculations during the design phase do not require highly accurate ephemerides—usually, extrapolation based on a two-body model is sufficient—accurately determining the frequency, timing, and duration of outages often requires very short time intervals for the satellite ephemeris. Otherwise, short-duration solar outages may be missed, compromising the evaluation results.

To address this, a fast algorithm for the determination and calculation of solar outages is proposed. By constructing equations satisfied by the satellite' s argument of latitude u at the start and end moments of a solar outage, the algorithm analytically determines whether an outage is likely to occur and calculates the corresponding arguments of latitude. This enables rapid determination of the existence of solar outages on inter-satellite links and their start and end times. Although this algorithm also requires pre-extrapolated ephemerides, the step size can be as large as one orbital period. Calculations demonstrate that this algorithm can reduce computation time by 2 to 3 orders of magnitude while detecting more solar outage events than traditional methods. Even when the

ephemeris step size approaches one orbital period, the method can detect over 99.9% of solar outage arcs with highly stable results; the maximum deviation in the onset time and duration does not exceed 1 s.

Keywords: celestial mechanics, methods: analytical, spacecraft: inter-satellite links, spacecraft: solar outage **CLC number:** P138; **Document code:** A

Navigation satellite constellations are equipped with inter-satellite links [?], which support autonomous orbit determination and operation [?] while improving accuracy through integrated space-ground orbit determination. In recent years, Low Earth Orbit (LEO) constellations have become the preferred choice for providing navigation augmentation [?] and satellite internet services (such as the Starlink constellation), and their operations are similarly dependent on links between constellation satellites. Inter-satellite links are subject to solar outages; that is, when the Sun approaches the line-of-sight direction of the satellite link, the establishment and/or maintenance of the link is affected.

Satellite inter-satellite links (ISLs) provide high-precision inter-satellite measurements for satellite formations or constellations and facilitate inter-satellite communication. Geodetic satellite missions such as GRACE (Gravity Recovery and Climate Experiment) [?] and GRACE Follow-On [?] utilize microwave or laser links between formation satellites to achieve high-precision inter-satellite range measurements, which are used to invert high-precision global gravity field models. The establishment or maintenance of modern satellite links can be subject to interference, which in turn affects the overall service performance of the satellite constellation. Therefore, evaluating the service performance of a satellite constellation typically requires calculating and predicting the impact of solar outages on inter-satellite links for at least one year. Such analysis is essential for optimizing the design and service planning of constellation satellites.

Currently, there is limited literature specifically discussing the calculation and determination of whether inter-satellite links are affected by solar outages [?]. This may be because the impact of solar outages can be determined point-by-point in a very simple and intuitive manner based on extrapolated ephemerides. In fact, this is a common practice in current applications analyzing the impact of solar outages on constellation business performance [?]. Whether using mature satellite system design tools to build scenarios or utilizing internally developed simulation systems, the core principle of point-by-point determination based on ephemerides remains the same, despite slight differences in geometric criteria and implementation processes. While this method is simple, intuitive, and easy to implement, its primary disadvantage is the massive computational burden. To obtain accurate estimates of solar outage duration and ensure that short solar outage arcs are not missed, small-step ephemeris extrapolation is inevitable. For mega-constellations such as Starlink, which consist of thousands of satellites, the computational cost of completing a single round of solar outage analysis for all links over a one-year period is significant.

This paper proposes an alternative approach for calculating solar outages in inter-satellite links. By using the argument of latitude of the link-establishing satellites (the sum of the argument of perigee and the true anomaly, denoted as u) as the variable, we establish explicit analytical expressions to construct and solve a governing equation (hereafter referred to as the solar outage equation). This allows for the direct determination of the existence of a solar outage and the analytical calculation of the argument of latitude at its start and end points. Although this method still requires prior ephemeris extrapolation, the step size can be greatly increased—from a few seconds in traditional methods to nearly one full orbital period. Furthermore, the solar outage calculation results remain stable, showing almost no variation across different step sizes.

2.1 Prerequisites and Assumptions

This article focuses on satellite constellations. The impact of solar outages on inter-satellite links (ISLs) primarily manifests in the operational aspects of communication and navigation services. Pre-calculating the timing and duration of solar outages for different satellites and links allows for the proactive planning of system operations and service management. To meet the requirements for evaluating service interruptions caused by solar outages, current computational methods generally employ simplified dynamical models—or even two-body models—for orbit extrapolation. However, small integration steps are utilized to ensure that short-duration solar outage arcs are not overlooked.

Based on these requirements and following the aforementioned practices, the formulation of solar outage equations can be established by adopting the following assumptions regarding orbital geometry, observational geometry, and dynamical models:

1. Satellite orbit extrapolation uses a two-body model. For the Sun' s geocentric position, near-equatorial orbits utilize analytical mean element models [**?**].
2. Ignore the Sun' s geocentric parallax; that is, assume the Sun is at infinity.
3. The orientation of a constellation satellite is defined by the direction of its geocentric position vector. Satellites are assumed to have the same orbital semi-major axis a, eccentricity e, and orbital inclination i. For convenience, all subsequent derivations and analyses assume circular orbits ($e = 0$).
4. The difference between the right ascension of the ascending node $\Delta\Omega$ and the argument of latitude Δu between linked satellites is constant.

The aforementioned requirements explain the first assumption; the following provides supplementary reasons for adopting these specific assumptions. First, based on the configurations of existing communication and navigation satellite constellations, including Low Earth Orbit (LEO) internet constellations, the assumption of circular orbits implies that eccentricity e is neglected. This magnitude is comparable to the J_2 perturbation of the Earth. Under the assumption of a two-body model, this approximation does not introduce additional errors. Furthermore, an eccentricity of magnitude $O(10^{-3})$ does not affect the orbital period; it only influences the specific position of the satellite within that period, producing a positional error no greater than $O(10^{-3})$ (in units of Earth radius).

For the purposes of judging and calculating the relative phases of constellation satellites and the apparent solar angle required for Sun outage analysis, the impact is even smaller. Moreover, as can be seen from the derivation process in Section 2.2 and the Appendix, retaining $e = 0$ does not have a substantial impact on the construction and solution of the equations, other than increasing the complexity of the functional forms.

Secondly, for a satellite with a geocentric distance of 8000 km, the solar geocentric parallax is 5×10^{-5} rad; for a Medium Earth Orbit (MEO) satellite with a 12 h period, the solar geocentric parallax is 2×10^{-4} rad. This deviation is much smaller than the critical angle for determining Sun outages, the latter of which typically ranges from 3° to 5°. It is also smaller than the errors potentially generated by other factors such as actual eccentricity and dynamical models. [Figure 1: see original paper] illustrates the geometric relationship between the constellation satellites and the Sun, where the angle represents the angle between the inter-satellite link and the solar direction.

Thirdly, the assumption that satellites share the same a, e, and i is one of the defining characteristics of Walker constellations and is therefore a reasonable assumption. Even for non-Walker constellations that do not satisfy this condition, the actual derivation process of the formulas remains substantially unaffected, much like the $e = 0$ assumption. Aside from the formula expressions becoming somewhat more complex, there are no fundamental difficulties in the construction and solution of the equations.

Regarding the final assumption, it can be observed that under the first and third assumptions, the fourth assumption holds naturally. Therefore, for Walker constellations, this condition is inherently satisfied, whether they are Walker-Delta constellations like GPS and Starlink Phase I, or Walker-Star constellations like Iridium. It remains valid even for the specially designed constellations of Starlink Phase II [**?**]. This implies that the method presented in this paper is well-suited for common satellite constellations.

The symbols in the formula are shown in [Figure 1: see original paper]. The core of the fourth assumption is that, except for the satellite' s argument of latitude (u, the sum of the argument of perigee and the true anomaly), which is a fast variable, other parameters such as the satellite's remaining orbital elements, the solar orbital elements, and the $\Delta\Omega$ and Δu between the linked satellites are all slowly varying. Consequently, the Sun outage equations can be solved piecewise rather than point-by-point. Therefore, even for non-constellation satellites, as long as their orbital elements satisfy this premise, the following derivations and conclusions remain applicable.

2.2 Orbital and Observational Geometry

The occurrence of solar outages depends on the geometric relationship between the constellation satellites and the Sun. For a communication link established between satellites S_i and S_j, the geometric relationship between the satellites and the Sun is illustrated in [Figure 1: see original paper].

[Figure 1: see original paper] Geometry of constellation satellites and the Sun, where S_i and S_j are two constellation satellites, E and S are the Earth and the Sun, respectively. Symbols $\mathbf{r}_i$, $\mathbf{r}_j$, and $\mathbf{r}_s$ are the geocentric position vectors of S_i, S_j, and the Sun, respectively. $\mathbf{r}_{si}$ and $\mathbf{r}_{ji}$ are the position vectors of the Sun and satellite S_j relative to satellite S_i, respectively. $\theta_{si,ji}$ and $\theta_{i,j}$ represent the angular separation between the Sun and S_j relative to S_i, and the geocentric angle between satellites S_i and S_j, respectively.

Whether the link originating from S_i and pointing toward S_j (denoted as $L_{i\to j}$, hereafter the same) is affected by a solar outage can be determined by the angle $\theta_{si,ji}$ between $\mathbf{r}_{si}$ and $\mathbf{r}_{ji}$. Specifically, by substituting $\theta_{si,ji}$ into (1), we obtain:

$$\cos\theta_{si,ji} = \frac{\mathbf{r}_{si}\cdot\mathbf{r}_{ji}}{|\mathbf{r}_{si}||\mathbf{r}_{ji}|}$$

By neglecting the geocentric parallax and the eccentricity of the satellite orbits, (3) can be simplified. The right-hand side of the resulting equation depends on the positions of the satellites. Since the dynamic model employs a two-body model and assumes that $\mathbf{r}_s$ remains constant, the change in satellite position can be reduced to the argument of latitude u. If we define θ_{max} as the critical angle at the boundary of the solar outage, the solar outage equation is given by (5). The next step is to provide an algorithm for the analytical calculation of u at the solar outage boundary based on (5).

2.3 Sun Outage Equation: Solution and Application

By rearranging the solar outage equation (5), the argument of latitude u can be explicitly expressed. To simplify the main text, the detailed derivation is provided in the Appendix. Through a series of transformations, the solar outage equation (5) can be expressed as:

$$K\cos(2u+q) = M_u \tag{6}$$

where K, q, and M_u are parameters derived from a combination of invariants and slowly varying variables in the orbits of the satellite and the Sun; specific definitions can be found in equations (24) and (26) of the Appendix. Given existing extrapolated ephemerides, these parameters are known and can be considered constant over the interval between ephemeris nodes.

Equation (6) is not entirely equivalent to equation (5). It contains not only the solar outage solutions for the link defined in (5) but also the solutions for the link in the opposite direction. Since the latter is also required when evaluating the impact of solar outages on operations, we will proceed by discussing equation (6) directly.

The solutions to this equation correspond explicitly to the argument of latitude u. Therefore, in the following description of the solutions to (6), the subscript "1" will be omitted where no ambiguity arises. There are several possibilities for the solutions to equation (6), where the superscripts "(1)" and "(2)" represent the first and second pairs of solutions, respectively (there are typically two pairs), and the subscripts "a" and "b" denote the start and end of the solar outage.

If $|M_u/K| > 1$, the equation has no solution, indicating that no solar outage occurs for the link. The case where $|M_u/K| = 1$ corresponds to a critical condition and is also treated as having no solar outage. If $|M_u/K| < 1$, the equation has four solutions, which can be divided into two pairs. Let the principal value of the inverse cosine be $\psi_0 = \arccos(M_u/K)$. Then the two pairs of solutions are determined based on the orbital parameters.

Under the assumption of a circular orbit, and using the orbital elements of the specific arc from the ephemeris as a reference, the time corresponding to the aforementioned argument of latitude can be calculated directly using the following formula. Taking the calculation of time t corresponding to u as an example:

$$t = t_0 + \sqrt{\frac{a^3}{\mu}}(u - u_0)$$

where μ is the gravitational constant, and a and u_0 are the semi-major axis and the argument of latitude at time t_0, respectively. If eccentricity is considered, the "transition time" can also be solved using Lambert' s equation. Lambert' s equation is extensively discussed in the literature with mature solution methods [**?**, **?**], and will not be repeated here.

The results of the solar outage equations (5) or (6) correspond to the argument of latitude of the satellite at the beginning and end of the solar outage, assuming other parameters in the equation are constant. In practical applications, because the orbital parameters of the satellite and the Sun change slowly, the solar outage equation must be constructed, evaluated, and solved segment by segment. A workflow suitable for practical reference is shown in [Figure 2: see original paper], and the meaning of each step is introduced below.

Depending on the sign of the parameters, the quadrant of the principal value differs, as does the physical meaning of the solution. Therefore, we discuss the cases separately: (a) When $K > 0$ and $M_u > 0$, the two pairs of solutions are: (b) When $K > 0$ and $M_u < 0$, the two pairs of solutions are:

(c) When $K < 0$ and $M_u > 0$, the principal value is within a specific range, and the two pairs of solutions are:
(d) When $K < 0$ and $M_u < 0$, the returned value is: In equations (7)–(9), the two sets of solutions for each equation correspond to the solar outages

of two different links. Using the notation defined in equations (16)-(18) of the Appendix, these represent the solar outage for Link 1 and Link 2, respectively.

After calculating the argument of latitude u at the start and end of the solar outage, the process follows the circular orbit approximation or Lambert' s method as previously described. [Figure 2: see original paper] illustrates the flowchart for using the solar outage equation in practice. Steps beginning with numerical identifiers are explained in the text.

[Figure 2: see original paper] A flowchart of applying equation of transit in practice. Numbered steps are explained in the text.

The geometric relationship between the satellites S_1 and S_2 determines the conditions for solar outage. When the ratio $|M_u/K_q| \geq 1$, the equation has no real solutions, indicating that no solar outage occurs during that period. However, when $0 \leq |M_u/K_q| < 1$, we define $\theta = \cos^{-1}(M_u/K_q)$, where $\theta \in (0, \pi)$. The solutions for the argument of latitude u can be categorized based on the value of M_u/K_q and the phase angle q.

When $M_u/K_q = 0$, which implies $\theta = \pi/2$, the solutions are:

$$\begin{aligned} u_a^{(1)} &= -\pi/4 - q/2; \quad u_b^{(1)} = \pi/4 - q/2 \\ u_a^{(2)} &= 3\pi/4 - q/2; \quad u_b^{(2)} = 5\pi/4 - q/2 \end{aligned} \tag{7}$$

For the case where $0 < M_u/K_q < 1$, and consequently $\theta \in (0, \pi/2)$, the solutions are given by:

$$\begin{aligned} u_a^{(1)} &= \frac{1}{2}(-\theta - q); \quad u_b^{(1)} = \frac{1}{2}(\theta - q) \\ u_a^{(2)} &= \pi + \frac{1}{2}(-\theta - q); \quad u_b^{(2)} = \pi + \frac{1}{2}(\theta - q) \end{aligned} \tag{8}$$

Conversely, when $-1 < M_u/K_q < 0$, such that $\theta \in (\pi/2, \pi)$, the solutions are:

$$\begin{aligned} u_a^{(1)} &= \frac{1}{2}(\theta - q); \quad u_b^{(1)} = \frac{1}{2}(-\theta - q) \\ u_a^{(2)} &= \pi + \frac{1}{2}(\theta - q); \quad u_b^{(2)} = \pi + \frac{1}{2}(-\theta - q) \end{aligned} \tag{9}$$

In these scenarios, the sign of the term $K\cos(u_1 + \phi)$ determines the specific link affected.

1. Extrapolate satellite and Sun orbits to the next epoch. Although this paper uses a two-body model and mean orbital models, it is entirely feasible to utilize other applicable extrapolation methods at this stage, such as numerical methods that account for complex perturbations.

2. Solve the Sun outage equation. This step corresponds to the analytical or semi-analytical methods mentioned earlier. Compared to the traditional small-step extrapolation and point-by-point determination, the method presented in this paper allows for the rapid assessment of Sun interference conditions near a specific epoch, thereby enabling segmented calculations. This implies that the step size for orbit extrapolation does not need to be restricted to a few seconds; instead, it can be relaxed to the order of minutes or even longer. In fact, since Sun interference occurs continuously over a period of time, the ephemeris interval can be extended to nearly one full orbital period, provided it does not lead to confusion with adjacent orbital cycles.
3. The solution of Equation (6) yields the argument of latitude u at the start and end of the Sun outage. Since the equation yields solar transit events occurring within a specific timeframe (during which the other "constant" parameters of the equation remain applicable), multiple time nodes may correspond to the same set of solar transits if the ephemeris interval from the previous step is sufficiently small. Because the "constant"parameters in Equation (6) constructed at different epochs will inevitably exhibit slight variations, the resulting solar transit solutions will differ marginally. One fundamental approach to resolving these discrepancies is through iteration, which allows solutions derived from different epochs to converge toward a single consistent moment. However, these parameters evolve slowly. Even for the relatively faster-changing parameters, such as the right ascension of the ascending node and the argument of perigee, the variation within a single orbital period does not exceed 10^{-3} rad. For practical applications, since a specific solar transit event can be detected at nodes both before and after its actual occurrence, it is sufficient to average these sets of proximate solar transit solutions. In the numerical examples presented in the following section, the solar transit times are obtained solely through averaging, without iterating Equation (6).
4. Post-processing of Sun outage solutions. As mentioned in Step 2, when the ephemeris interval is relatively small, multiple epochs of solar interference equations may correspond to the same solar interference event. Therefore, it is necessary to aggregate and process all solar interference solutions after completing the cycle shown in [Figure 2: see original paper] to obtain the final information, such as the total number of occurrences and the specific time intervals.

It is worth noting that steps 2 through 4 in [Figure 2: see original paper] are specific to the analytical solution of solar interference equations. In terms of operational details, these steps are not necessarily simpler than traditional step-by-step determination methods. Consequently, this method does not hold an advantage when the ephemeris step size is very small. However, the underlying principle demonstrates that this approach can extrapolate satellite ephemerides using step sizes far exceeding those of traditional methods without missing any solar interference arcs. This capability provides a significant advantage in computational efficiency, which will be demonstrated in the simulation examples in the following section.

3.1 Experimental Design

This section compares the traditional algorithm with the proposed fast algorithm based on the solar transit equation. For convenience, the traditional step-by-step determination method is referred to as the "numerical method," while the method based on the analytical solution of the solar transit equation is referred to as the "analytical method."

First, simulation examples are used to compare the differences in computational performance and results between the numerical and analytical methods. The test cases utilize two inter-linked satellites from a Walker constellation as listed in . These satellites have a simulated altitude of approximately 1122 km in circular orbits, with an orbital plane spacing and a phase difference between adjacent satellites in different orbits of 10°. The orbital elements a, e, i, Ω, ω, and M represent the semi-major axis, eccentricity, inclination, right ascension of the ascending node, argument of perigee, and mean anomaly, respectively. Taking the bidirectional link between S_1 and S_2 as the research object, we compare the numerical and analytical methods regarding the existence of solar transit, the number of transit events, start times, transit durations, and computational efficiency.

Initial orbital elements of the constellation satellites used in the test cases, corresponding to the epoch 2025-01-01 00:00:00 (UTC).

The orbital extrapolation for the satellites in these test cases adopts a two-body model, which is a common strategy in current solar transit calculations and meets the operational requirements of relevant satellite constellations. The study also examines the impact of different ephemeris intervals (extrapolation step sizes) on the calculation results and efficiency. Six step sizes were selected: 0.1 min, 1 min, 10 min, 45 min, 60 min, and 90 min. The total extrapolation duration is 365 days.

The simulation examples are used to compare the differences between the two methods under various ephemeris step sizes and to evaluate the stability of the analytical method' s results across these steps, as detailed in Sections 3.2 and 3.3. Furthermore, using the Starlink satellite predicted ephemeris published by SpaceX on space-track.org at 21:21 (UTC) on April 10, 2025, solar transit calculations are performed on discrete ephemerides using both the numerical and analytical methods. Starlink' s predicted ephemerides are primarily used by other satellite operators to calculate and assess collision risks with Starlink satellites. The underlying dynamical model includes a relatively complete perturbation model and accounts for potential continuous low-thrust maneuvers implemented by Starlink satellites [?]. Each predicted ephemeris file typically spans 72 hours with a time interval of 1 min.

Currently, the Starlink satellites in orbit consist of two phases of constellations, each composed of multiple shells [?, ?, ?], where each shell in the first-phase constellation is a Walker constellation. Since SpaceX has not publicly released the actual deployed constellation/shell structures or the specific mapping between ephemeris files and their respective shells, this study refers to the filing information for the Starlink Phase I constellation submitted to the U.S. Federal Communications Commission (FCC). Based on actual orbital data of Starlink satellites from Two-Line Elements (TLE) in recent years, this section selects two orbital planes from the fourth shell of the Starlink Phase I constellation, separated by approximately 220° in right ascension of the ascending node, designated as Plane A and Plane B. Each orbital plane contains 18 satellites; their respective NORAD (North American Air Defense Command) ID numbers are listed in .

NORAD IDs of Starlink satellites used for solar transit calculations, listed sequentially from phase slot 1 to 18 of the Walker configuration. In this case study, the solar transit conditions for the inter-satellite links between adjacent satellites within each orbital plane are calculated (i.e., satellite 01 to 02, 02 to 03, ⋯, 18 to 01). The calculation results are presented in Section 3.4.

In all test cases in this section, the critical angle for solar transit (denoted as θ_{max} in Eq. (5)) is set to 1.5°.

3.2 Results of the Two Methods Under Different Step Sizes

First, we compare the computational efficiency of the two methods. To eliminate bias from single-run timing measurements, both methods were executed 10 consecutive times for the given test case, and the average execution time per run was recorded. The computational environment consisted of a standard laptop, and the test code was implemented in MATLAB. For the numerical method, the recorded time includes orbit extrapolation, Sun-outage determination at each node, and the identification of the start and end points for each Sun-outage interval. For the analytical method, the timing covers the entire workflow illustrated in Figure 2. The timing results are presented in Table 3 .

Average per-run time consumption of the given test using the two methods.

At the same step size, the execution time of the analytical method is approximately 3 to 6 times that of the numerical method. This confirms the discussion in Section 2.3: the overhead of solving the Sun-outage equations at each step and the final statistical processing of Sun-outage arcs in the analytical method is no less than the time required for the numerical method to perform Sun-outage determination via dot products (as in Eq. (1)) at each step. Another reason the timing gap widens as the step size increases is that the numerical method detects fewer Sun-outage events with larger steps, thereby reducing the additional overhead required for processing and cataloging the Sun-outage intervals.

The fact that the analytical method consumes more time at the same step size does not affect the overall efficiency advantage of the analytical method in Sun outage detection. lists the number of solar transit arcs detected by the numerical and analytical methods under different step sizes. It can be observed from the listed simulation tests that when the step size exceeds 1 min, the results provided by the numerical method become essentially unusable.

In contrast, the number of solar transit arcs provided by the analytical method remains remarkably stable. Even when the step size is increased to 90 min (noting that the satellite orbital period in this case is 107.7 min), the detection results decrease by only two arcs compared to the results obtained with a 0.1 min step size. This indicates that the analytical method can still detect nearly all (over 99.9%) solar transit arcs.

Counts of transit arcs detected using the two methods.

To further analyze the results calculated by both approaches, the solar transit epochs and durations obtained with a 0.1 min step size are plotted in [Figure 3: see original paper]. As shown in the figure, the results obtained by the two methods are consistent. However, the results from the analytical method are "smoother," whereas the numerical method produces results that exhibit a distinct "staircase" pattern. By magnifying the results of the first 10 days, we obtain [Figure 4: see original paper].

Observation of [Figure 4: see original paper] reveals that the "staircase" in the numerical method' s results originates from the intervals of its extrapolated ephemeris. Specifically, the start and end points of a solar transit detected by the numerical method can only fall on the nodes of the ephemeris. In the example shown in [Figure 4: see original paper], a step size of 0.1 min was used to calculate the extrapolated ephemeris, resulting in a staircase interval of 6 s. In comparison, the start and end points calculated by the analytical method are largely independent of the ephemeris interval. The ephemeris only serves to provide reference points for constructing and solving the solar transit equations; the actual solved start and end points are not restricted to ephemeris nodes. Consequently, the overall result is "smooth" and can accurately depict the actual boundaries of the solar transit.

[Figure 3: see original paper] Solar transit results obtained by the numerical and analytical methods. The horizontal and vertical axes represent the starting epoch of the transit (days elapsed since 2025-01-01 00:00:00 UTC) and the duration of the transit (s), respectively. Circles ($\circ$) and squares ($\square$) represent the results for the $S_1 \rightarrow S_2$ and $S_2 \rightarrow S_1$ links obtained by the numerical and analytical methods, while crosses ($\times$) and plus signs ($+$) represent the corresponding results for the other links.

[Figure 4: see original paper] Zoom-in image of the first 10 days of results from [Figure 3: see original paper], utilizing the same symbols and legend.

Since the direct result of the solar transit equation is the argument of latitude for the start and end points, there can be an infinite number of solutions satisfying this phase condition if no constraints are applied. In practice, considering the slow variation of parameters within the equation, the epochs can be calculated within one orbital period around the ephemeris nodes. This is also why the maximum step size in the numerical examples of this paper is set not to exceed the orbital period.

Finally, we discuss the differences in the number of solar transit arcs detected by the two methods. Even when the step size is reduced to 0.1 min, the numerical method still detects four fewer solar transits per link compared to the analytical method. It is easy to understand that the transits missed by the numerical method must be those with shorter durations at the edges—specifically, those near the four zero points shown in [Figure 3: see original paper]. Magnifying the zero point near day 300 yields [Figure 5: see original paper].

[Figure 5: see original paper] Zoom-in image of the zero point near the 300th day from [Figure 3: see original paper], using the same symbols and legend.

In [Figure 5: see original paper], the results from the analytical method exhibit a smooth and continuous distribution, while the solutions detected by the numerical method still follow a "staircase" distribution. In the figure, the duration of the first solar transit detected by the numerical method is zero, indicating that this point is the only one in the extrapolated ephemeris that falls within that specific transit interval; thus, it is simultaneously identified as both the start and end point. It is evident that near this zero point, the first segment of the solar transit in the link detected by the analytical method, as well as the first two segments in another link, were not detected by the numerical method. Similar situations occur near the other three zero points, ultimately leading to the discrepancies between the numerical and analytical results shown in .

3.3 Differences in Analytical Method Results Under Different Step Sizes

Table 4 demonstrates that the analytical method yields highly consistent counts of solar transit arcs across different step sizes. This subsection further compares the solar transit results under various step sizes to evaluate the stability of the analytical approach. [Figure 6: see original paper] illustrates the differences between the results obtained using a 1-minute step size ephemeris and a reference 0.1-minute step size ephemeris, both calculated using the same analytical method. The top panel of the figure displays the results for the S_1S_2 link, while the bottom panel displays the results for the S_2S_1 link.

It is observed that although the time difference between ephemeris nodes for these two step sizes can be as large as 54 s, the resulting differences in the calculated start times of solar transits are mostly within 0.005 s, with a maximum not exceeding 0.02 s. Similarly, the differences in transit durations are generally within 0.02 s, with a maximum of 0.05 s. The points with larger discrepancies occur primarily at the edges of the transit start and end times. This indicates that the choice of step size has no significant impact on the analytical method' s ability to solve for solar transits, and the results remain highly consistent.

[Figure 6: see original paper] Difference of analytically solved transits using 1-minute-step ephemerides, compared with those using 0.1-minute-step ephemerides. Top is for S_1S_2 Link and bottom is for S_2S_1 Link. In both frames, circles (∘) represent the difference of durations of transits, corresponding to the left axis, while dots (·) represent the difference of starting epochs of transits, corresponding to the right axis. "diff" in the legends is short for "difference."

Regarding the influence of other step sizes on the analytical method, [Figure 7: see original paper] illustrates the differences between results obtained using ephemeris step sizes ranging from 10 min to 90 min compared to the 0.1-minute step size reference. To conserve space, the figure only displays the results for the S_1S_2 link; the results for the S_2S_1 link are highly similar and are not shown separately.

[Figure 7: see original paper] Differences of analytically solved transits using ephemerides of various stepsizes, compared with those using 0.1-minute-step ephemerides. In all frames, circles (∘) represent difference of durations of transits, corresponding to the left axis, while dots (·) difference of starting epochs of transits and the right axis. Crosses (×) in the three frames indicate missing transits in the solutions. "diff" in the legends is short for "difference" .

By observing these images in conjunction with [Figure 6: see original paper], it can be observed that the solar conjunction solution deviations basically increase with the step size. But even so, the differences between using an ephemeris with a 90-minute step size and one with a 0.1-minute step size are minimal. Specifically, the deviation in the duration of the solar outage is generally less than 1 s, with the maximum deviation not exceeding 1 s. For the start time of the solar outage, the majority of deviations are within 1 s, with the maximum deviation being approximately 1 s.

When using a large-step ephemeris, results with larger calculation deviations occur primarily at the beginning and end of the solar outage period; similarly, any missed detection points are also concentrated at these start and end positions. Overall, these cases and figures demonstrate that for the analytical method of solving solar outages, the calculated results remain remarkably stable and reliable even when the ephemeris step size is increased from 0.1 minutes to 90 minutes. The few instances of larger deviations or missed solar outage arcs are both controllable and predictable.

3.4 Sun Outage Calculation Based on Starlink Satellite Ephemeris

This subsection presents a numerical case study focusing on two orbital planes within the fourth shell of the Starlink Phase I satellite constellation. We calculate the solar transit events for 18 links over a 72-hour period using both numerical and analytical methods. For the 18 links in orbital plane A, [Figure 8: see original paper] illustrates the number of solar transit occurrences and the cumulative duration over the 72-hour window as determined by both methods. The results are consistent with the simulations in Table 4; specifically, at a 1-minute sampling interval, the numerical method yields slightly lower values than the analytical method. This discrepancy arises because the numerical approach tends to miss solar transit arcs whose durations are shorter than the ephemeris step size. Furthermore, the total duration for some links is recorded as zero by the numerical method, indicating that only a single node in the ephemeris fell within those specific solar transit arcs.

A closer examination of specific links further validates the aforementioned conclusions. [Figure 9: see original paper] displays the solar transit start times and durations for links 07-08 and 09-10. These results exhibit the same patterns observed in previous simulations: the analytical method produces continuous and smooth results, whereas the numerical method yields a step-like pattern consistent with the ephemeris step size. Another noteworthy phenomenon is that the analytical method, which uses ephemeris nodes only as reference points to solve for nearby transit arcs, is capable of backtracking to transit start times that occur before the beginning of the ephemeris. In the analytical results for the two links shown in [Figure 9: see original paper], the first solar transit arc in each case begins prior to the initial epoch of the ephemeris.

Further observation of links 01-02 and 07-08, as shown in [Figure 11: see original paper], reveals a clear distinction between the two methods. Due to the limitations imposed by the 1-minute step size of the Starlink satellite ephemeris, the total duration of solar transits detected by the numerical method is consistently shorter than that calculated by the analytical method.

[Figure 9: see original paper] Transits solved using the numerical and analytical methods, where the x and y axes respectively mean the starting epochs of the transit (t_0 = 2025-04-10 19:02:42 (UTC)) and the duration of the transit (in seconds). Cross ($\times$) and plus ($+$) signs respectively stand for transits of links solved using the numerical and analytical methods. In the titles are the NORAD ID of the satellites of the links.

We evaluate the computational efficiency of the numerical and analytical methods for solar transit calculations using predicted ephemerides. The solar transit calculations for 18 inter-satellite links (ISLs) in orbital planes A and B were repeated 10 times each. The average execution time for a single run is presented in . The results indicate that when using external ephemerides with a 1-minute step size—excluding the time required for orbit prediction—the analytical method is more computationally efficient than the numerical method.

For the scale of the fourth shell of the Starlink constellation (consisting of 72 orbital planes with 18 satellites per plane), the analytical method demonstrates clear advantages in both calculation accuracy and computational efficiency. This holds true provided that the predicted ephemerides are generated in advance according to Starlink' s autonomous avoidance and maintenance strategies, and subsequently used for independent solar transit calculations.

[Figure 8: see original paper] illustrates the solar transits affecting the ISLs within orbital plane A of the fourth shell of the Starlink Stage 1 constellation. The link identifiers correspond to the phase indices of the satellites at both ends. The upper panel shows the total number of solar transit intervals for each link over a 72-hour period, while the lower panel shows the total duration of these transits (in minutes). For each link, the left bar represents the results of the analytical method, and the right bar represents the numerical method. For a link denoted as $S_1 - S_2$, the figure only displays the transits experienced by S_1 due to S_2, excluding the reverse case.

Applying similar procedures to orbital plane B yields the results shown in [Figure 10: see original paper]. It is evident that the ISLs between satellites in this orbital plane are significantly affected by solar transits during this period, with each link experiencing 35–36 transits every 72 hours. Although the number of transit intervals detected by the numerical method is close to that of the analytical method, the total duration calculated by the numerical method is significantly shorter—approximately only half that of the analytical result.

Even if the conditions are only partially met, it merely introduces slight complexities in the construction and solution of the equations without fundamentally undermining the applicability of the analytical method. Furthermore, considering that variations in the orbital parameters arise from perturbations and are thus slowly varying, we can assume these parameters remain constant within specific intervals even if they do change in practice. By reasonably controlling the ephemeris step size, the continued applicability of the analytical method can be ensured.

[Figure 10: see original paper] Solar transits affecting the ISLs within orbital plane B of the fourth shell of the Starlink Stage 1 constellation. The link identifiers correspond to the phase indices of the satellites at both ends. The upper panel shows the total number of solar transit intervals for each link over a 72-hour period, while the lower panel shows the total duration of these transits (in minutes). For each link, the left bar represents the results of the analytical method, and the right bar represents the numerical method.

4 Discussion on the Application of the Method

Based on the numerical examples presented in Section 3, it is evident that the analytical method for calculating solar outages in constellation inter-satellite links offers significant advantages over traditional point-by-point search methods. The primary benefit is its ability to produce stable and reliable results—including the frequency, occurrence time, and duration of solar outages—with a computational speed advantage of two to three orders of magnitude. Considering the premises and derivation process of the analytical formulas used in this approach, several points regarding its practical application are discussed below.

1. Constellation satellites are considered because they effectively satisfy the premises and assumptions outlined in Section 2.1. Furthermore, in practical operations, constellation satellites necessitate the prediction and analysis of solar transit interference for inter-satellite links (ISLs). However, among the four assumptions described in Section 2.1, only the fourth condition (constant β) significantly impacts the validity of the solar transit equations and the applicability of their solutions. The other conditions, even if not strictly met, do not fundamentally compromise the model.

[Figure 11: see original paper] Transits solved using the numerical and analytical methods, where the x and y axes respectively mean the starting epochs of the transit (t_0 = 2025-04-10 18:34:42 (UTC)) and the duration of the transit (in seconds). Cross ($\times$) and plus ($+$) signs respectively stand for transits of links solved using the numerical and analytical methods. In the titles are the NORAD ID of the satellites of the links.

Average per-run time consumption of the Starlink test case using the two methods.

2. From the examples in Section 3, it is not difficult to see that in practice, one can fully utilize analytical methods to calculate Sun outages and evaluate their impact on operations. If higher precision is required for predicting the timing and duration of Sun outages, one can first employ analytical methods to determine the general time window in which the outage occurs. Subsequently, numerical methods can be used to perform a localized, accurate search to calculate the precise moments and durations of the Sun outage. This hybrid approach leverages the significant speed advantages of analytical methods alongside their inherently reliable estimation results. Even if a large step size causes the analytical method to miss individual, short-duration Sun outage arcs at the boundaries, the search range for the numerical method can be appropriately extended by one orbital period forward or backward based on the analytical results to ensure complete coverage.

By utilizing the analytical framework to narrow down the search space, the computational overhead is significantly reduced. This is particularly critical for large-scale constellations like Starlink, where calculating potential Sun outages for thousands of links using purely numerical integration would be computationally prohibitive. The results indicate that the analytical method provides a robust foundation for operational planning, with the numerical refinement serving to finalize high-precision mission schedules.

3. The orbital model (two-body model for satellites and mean elements for the Sun) is designed to facilitate the formulation and solution of solar conjunction equations. In practice, high-precision numerical methods can be employed to extrapolate ephemerides, allowing the solar conjunction equations to be solved specifically at the ephemeris nodes. Assuming a two-body model between two consecutive ephemeris nodes does not sig-

nificantly impact accuracy. This approach simultaneously improves the precision of the ephemeris while retaining the advantages of analytically solving the solar conjunction equations. Furthermore, the high-precision numerical methods can utilize standard numerical extrapolation step sizes (for example, 20–30 seconds for Low Earth Orbit satellites). This eliminates the need to constrain the ephemeris step size as required by point-by-point search methods, which otherwise severely limits computational efficiency.

5 Conclusion

This paper introduces an algorithm based on an analytical method for calculating Sun interference in inter-satellite links (ISLs) within satellite constellations. By constructing equations that represent the conditions satisfied when link satellites enter and exit Sun interference, the proposed method can analytically solve for Sun interference events for any given link over a specified period. This allows for the determination of the total number of interference arcs as well as the precise start and end times of each event.

Numerical examples demonstrate that, compared to traditional point-by-point search methods based on ephemeris data, this analytical approach imposes almost no restrictions on the extrapolation step size of the ephemeris. Furthermore, the stability and accuracy of the calculated Sun interference results are excellent. Provided the step size does not exceed the orbital period, the analytical method can reliably capture nearly all Sun interference events, with highly stable results for both the onset time and duration.

Using the simulation examples in this paper as a reference, we compared the shortest step size (0.1 min) with the longest step size (90 min). Taking the former as the benchmark, the latter was able to capture over 99.9% (2881/2883) of the Sun interference events over a one-year period. The deviations in both the start time and duration were less than 1 s, with the vast majority remaining stable within 10^{-3} s. In test cases using Starlink satellite ephemeris as input, the analytical method outperformed traditional methods in terms of both calculation results and computational efficiency.

Due to its significant speed advantages and stable, reliable precision, this method can be directly applied to determine and calculate Sun interference for inter-satellite links in constellations. It also supports derivative applications, such as: (1) using the analytical method to rapidly search for time windows affected by Sun interference, followed by a refined numerical search with restricted step sizes; and (2) using numerical methods to extrapolate high-precision ephemeris (without step size restrictions) and subsequently employing the analytical method to search for and calculate Sun interference events.

Acknowledgements

The author would like to thank the reviewers for their valuable suggestions on this paper.

References

Tapley, B. D., Bettadpur, S., Watkins, M., et al. *Geophysical Research Letters*, 2004, 31: L09607.

Landerer, F. W., Flechtner, F. M., Save, H., et al. *Geophysical Research Letters*, 2020, 47: e2020GL088306.

Chen, Z. G., Shuai, P., Qu, G. J. *Scientia Sinica (Technologica)*, 2009, 39.

A Fast Algorithm to Calculate Transits of Inter-satellite Links between Constellation Satellites

TANG Jing-shi[1,2,3] ([1] School of Astronomy and Space Science, Nanjing University, Nanjing 210023) ([2] Key Laboratory of Modern Astronomy and Astrophysics, Ministry of Education, Nanjing 210023) ([3] Institute of Space Environment and Astrodynamics, Nanjing University, Nanjing 210023)

ABSTRACT

With the development and readiness of inter-satellite links (ISL), they have been widely used in satellite constellations for navigation, communication, and other purposes. As establishing or maintaining an ISL can be affected by transits (also known as Sun outage), these transits must be calculated and predicted so that system service can be properly assessed and scheduled. Currently, the standard practice to determine and predict transits—whether by building scenarios using proven commercial satellite system toolkits or by developing in-house simulation tools—is to calculate the elongation between the ISL and the line-of-sight of the Sun based on propagated ephemerides. While this approach is simple, straightforward, and easily implemented, it bears a significant disadvantage: the computation is extremely CPU-intensive and therefore slow. Although predicted transits do not require extreme precision during the design phase and a two-body model may suffice, ephemerides are always limited to small stepsizes to properly detect transit events. Otherwise, transits with short durations may be overlooked, compromising the service assessment.

Regarding the determination and calculation of transits, a fast algorithm is proposed in this paper. By utilizing the equation of the satellite’ s argument of latitude u at the transit boundary, it can be analytically determined whether a transit may occur and precisely when it starts or ends, allowing for a rapid assessment of how transits affect service. This method is also based on propagated ephemerides, but it can accept a much larger stepsize. Tests show that this analytical method outperforms standard practices by detecting more transits while reducing calculation time by two to three orders of magnitude. Even when the

stepsize of the ephemerides is close to one orbital period, this method is still able to detect more than 99.9% of transits. The results are robust against varying stepsizes, with maximum deviations of calculated onset epochs and durations not exceeding 1 second.

Appendix: Derivation of the Solar Transit Equation

Section 2.2 of the main text provides Equation (5), which defines the satellite position at the boundary of a solar transit event. This equation is referred to as the solar transit equation. To facilitate its solution, we perform a series of transformations to express it as an explicit function of the argument of latitude u.

In the above equations, i, i', and u' represent the orbital inclination of S_1, the orbital inclination of S_2, and the argument of latitude (the sum of the argument of perigee and the true anomaly) of S_2 in the geocentric mean equatorial coordinate system, respectively. Let the unit geocentric vectors of the satellite and the Sun be $\hat{r}$ and $\hat{r}'$, respectively. According to [**?**], we have:

$$\begin{aligned} \cos\psi &= \hat{r}' \cdot \hat{r} \\ \hat{r} &= A_1 \cos u_1 + B_1 \sin u_1 \end{aligned} \tag{11}$$

where the coefficients are defined as:

$$\begin{aligned} A_1 &= \cos\Delta\Omega\cos u' + \cos i' \sin\Delta\Omega\sin u' \\ B_1 &= \cos i(-\sin\Delta\Omega\cos u' + \cos i'\cos\Delta\Omega\sin u') + \sin i\sin i'\sin u' \\ &= \cos i B_{11} + \sin i \sin i' \sin u' \end{aligned} \tag{12}$$

In these expressions, i also denotes the inclination of the ecliptic (i.e., the obliquity of the ecliptic) when considering the Sun' s position, and $\Delta\Omega$ represents the difference between the right ascension of the ascending nodes of the satellite and the Sun in the geocentric mean equatorial coordinate system.

Using the same notation, an explicit expression for θ_{ij} can be derived for the satellites. In Equation (5), θ_{ij} represents the geocentric angle between the linked satellites i and j. According to the spherical trigonometric formula, we have $\cos\theta_{ij} = \mathbf{r}_i \cdot \mathbf{r}_j / (|\mathbf{r}_i||\mathbf{r}_j|)$. Consequently, Equation (15) can be rewritten as:

$$\theta_{ij} = \text{atan2}\left(\sqrt{1-\cos^2\theta_{ij}}, \cos\theta_{ij}\right)$$

where atan2 denotes the four-quadrant inverse tangent function that determines the radian angle within the interval $[0, \pi]$ based on the sine and cosine values.

Let α and δ represent the right ascension and declination, respectively, of the satellite in the Geocentric Equatorial Inertial (GEI) coordinate system. From the orbital geometry of the satellite in the GEI system shown in [Figure 12: see original paper], we can derive the relationship for δ.

In the spherical triangle NQS_1 shown in Figure 12, the following relationships can be derived using the spherical trigonometric sine formula, cosine formula, and the five-element formula [**?**]:

$$\begin{cases} \cos\phi\cos\vartheta = \cos u \\ \cos\phi\sin\vartheta = \sin u\cos i \\ \sin\phi = \sin u\sin i \end{cases} \tag{20}$$

By expanding equation (19) and substituting equation (20) into it, we obtain:

$$\cos\varphi = \frac{1}{2}(\cos\Delta\Omega - 1)\sin^2 i\cos(2u_1+\Delta u) + \frac{1}{2}[\cos\Delta\Omega(1+\cos^2 i)+\sin^2 i]\cos\Delta u - \sin\Delta\Omega\sin\Delta u\cos i \tag{21}$$

By squaring both sides of the Sun outage equation (5) and substituting equations (17) and (21), the expression can be rearranged. If we consider the Sun outage for a specific link, it can be observed that if a value is a solution to equation (22), then its conjugate is also a solution. As illustrated in Figure 1, if one solution corresponds to the Sun outage of a specific link, the other corresponds to the Sun outage of the reverse link.

This implies that although squaring both sides of equation (22) introduces an "extraneous root" compared to the original link' s Sun outage equation, this additional root actually represents the Sun outage of the reverse link. Therefore, by solving equation (22) directly, we can simultaneously obtain the Sun outage periods for both directional links.

Further rearranging Eq. (22), we obtain the substituted expression. After minor simplification, it follows that Eq. (23) can be rewritten as:

$$\begin{aligned} K^2\cos(2u_1+2\phi) - 2\cos^2\theta\sin^2(\Delta\Omega/2)\sin^2 i\cos(2u_1+\Delta u) = \\ -K^2/2 + 2\cos^2\theta[1-\cos\Delta u\cos^2(\Delta\Omega/2)+\cos^2 i\cos\Delta u\sin^2(\Delta\Omega/2)+\sin\Delta\Omega\sin\Delta u\cos i] \end{aligned} \tag{23}$$

At this point, through these transformations, the solar transit outage equation is simplified into the following explicit form:

$$C_u\cos 2u_1 - S_u\sin 2u_1 = M_u \tag{25}$$

where the auxiliary variables are defined as:

$$
\begin{aligned}
C_u &= \frac{K^2}{2}\cos 2\phi - 2\cos^2\theta \sin^2(\Delta\Omega/2)\sin^2 i \cos\Delta u \\
S_u &= \frac{K^2}{2}\sin 2\phi - 2\cos^2\theta \sin^2(\Delta\Omega/2)\sin^2 i \sin\Delta u \\
M_u &= -\frac{K^2}{2} + 2\cos^2\theta[1 - \cos\Delta u \cos^2(\Delta\Omega/2) + \cos^2 i \cos\Delta u \sin^2(\Delta\Omega/2) + \sin\Delta\Omega \sin\Delta u \cos i]
\end{aligned} \tag{24}
$$

This leads to:

$$K_q \cos(2u_1 + q) = M_u \tag{27}$$

where $K_q = \sqrt{C_u^2 + S_u^2}$ and $q = \text{atan2}(S_u/K_q, C_u/K_q)$.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.