An inflationary mechanism in compactified extra-dimensional spacetimes

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Abstract

We propose a cosmological inflation model related to compact extra-dimensional spacetime, in which the fundamental mass scale varies with either the compactification radius r_c of the extra dimensions or time. This variation induces a dynamical change in the cosmological constant, causing a period of rapid expansion in the early universe. The duration of inflation is determined by the size of the compact extra dimensions r_c, establishing a direct link between the geometry of extra dimensions and the end of inflation. Following this phase, the cosmological constant evolves further, naturally leading to a late-time accelerated expansion. The model is rigorously derived from the vacuum solution of the metric incorporating a cosmological constant within a compact extra-dimensional framework. Its ability to unify early- and late-time cosmic acceleration provides compelling support for high-dimensional cosmological theories.

Full Text

Preamble

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E-mail: huanglong20122021@163.com Abstract. We propose a cosmological inflation model related to compact extra-dimensional spacetime, in which the fundamental mass scale varies with either the compactification radius rc of the extra dimensions or time. This variation induces a dynamical change in the cosmological constant, causing a period of rapid expansion in the early universe.

The duration of inflation is determined by the size of the compact extra dimensions rc, establishing a direct link between the geometry of extra dimensions and the end of inflation. Following this phase, the cosmological constant evolves further, naturally leading to a late-time accelerated expansion. The model is rigorously derived from the vacuum solution of the metric incorporating a cosmological constant within a compact extra-dimensional framework. Its ability to unify early- and late-time cosmic acceler- ation provides compelling support for high-dimensional cosmological theories. keywords—Inflation, Compact extra-dimensional, Fundamental mass Cosmological constant scale,

1. Introduction

Spacetime may be higher-dimensional, and compactified extra dimensions represent the most plausible configuration within such frameworks. Since Kaluza and Klein first proposed a five-dimensional theory in which the extra dimension undergoes spontaneous compactification to a finite scale [1–3], extra dimensions have offered compelling explanations for fundamental problems in theoretical physics, including the hierarchy problem—why the Higgs boson mass is significantly smaller than the Planck mass—as well as the early inflation and late-time accelerated expansion of the universe [4–10].

Furthermore, various quantum gravity theories inherently require the existence of extra- dimensional spaces [11, 12], making them an essential component of modern high-energy physics.

Rubakov and Shaposhnikov in 1983 proposed an extra-dimensional model featuring an effective potential well generated by a classical scalar field, which confines particles within the well [13]. Inspired by perturbative string theory, Antoniadis introduced a An inflationary mechanism in compactified extra-dimensional spacetimes TeV-scale extra-dimensional model in 1990, proposing its connection to the scale of supersymmetry breaking [14]. Horava and Witten in 1995 and 1996 demonstrated that in M-theory, extra dimensions at the scale of (1012 GeV)−1 could lower the string theory energy scale to the grand unification (GUT) scale—approximately 1016 GeV —thereby enabling the unification of gravity with the three fundamental interactions of the Standard Model at the GUT scale [11, 12]. Also in 1995, Polchinski’ s discovery of D-brane theory provided a crucial framework for brane-world scenarios [15].

Building on the concept of a ” 3-brane,” Arkani-Hamed, Dimopoulos, and Dvali proposed a large extra-dimensional model in 1998 to address the hierarchy problem in particle physics [16, 17]. Subsequently, Randall and Sundrum introduced a curved extra- dimensional model to resolve the same hierarchy problem through geometric suppression of mass scales [18, 19], these advances have thereby motivated rigorous theoretical studies of phenomenology of the Standard Model and cosmological dynamics within extra-dimensional frameworks [20–30].

Within the framework of compactified extra-dimensional space, we rigorously derive the relationship between the fundamental mass scale (the maximum rest mass of elementary particles) and the compactification radius rc of the extra dimensions.

Furthermore, we demonstrate the necessity of a non-zero cosmological constant when the compactification radius rc exceeds the Planck length. We then propose a model to explain both the energy source of cosmic inflation and the mechanism for its termination: the fundamental mass scale varies with the compactification radius rc of the extra dimensions and cosmic time, resulting in a dynamically evolving cosmological constant within the effective four-dimensional theory, thereby driving the epoch of early cosmic inflation and facilitating its subsequent graceful exit.

In Section 2 of this paper, we derive the vacuum solutions of the metric in a spacetime with compactified extra dimensions and show that a cosmological constant must be introduced when the compactification radius rc is greater than the Planck length. In Section 3, we obtain the vacuum metric solutions incorporating a cosmological constant and establish the relationship between the fundamental mass scale and the compactification radius. Building on these results, Section 4 proposes a dynamical mechanism for cosmic inflation: a variation of the fundamental mass scale following the Big Bang induces a corresponding evolution of the cosmological constant, thereby providing the energy source for inflation and enabling its natural termination. Finally, we conclude with a summary of the key findings and their implications.

2. Vacuum solutions of the metric in compactified extra-dimensional spaces

without a cosmological constant The metric form in a compactified extra-dimensional spacetime can be written as ds2 = A(r) dt2 − B(r)−1dr2 − r2dΩ2 2 − 2C(r)dtdr − rc (cid:80) where D represents the total spacetime dimension, rc is the compactification radius of An inflationary mechanism in compactified extra-dimensional spacetimes the extra dimensions, ϕi denotes the coordinate of the extra dimension. We take the units c = 1 throughout the next work for convenience.

Substituting equation 1 into the field equations yields the metric vacuum solution satisfying A(r) = 1 − 2G M , B(r) = 1 − 2G M r)D−4, C(r) = (cid:113) [2GM (1 − (rc r)D−4)]/[1 − 2G M r)D−4], to ensure that the component C(r) is a real number, From formula 2, compactification radius must satisfy a specific condition rc ≤ lp, where lp is the Planck length, under which the fundamental mass scale is equal to the Planck mass. The cosmological constant must be introduced when rc > lp. We now consider the vacuum metric solution in the presence of a cosmological constant.

3. Vacuum solutions of the metric in compactified extra-dimensional spaces

with a cosmological constant Substituting Equation 1 into the field equation yields the vacuum solution corresponding to the cosmological constant metric r − Λr2 3 , B(r) = 1 − 2GM A(r) = 1 − 2GM r)D−4, 3 − 2GM where Λ is the cosmological constant.

C(r) = [Λr2 (cid:113) ((rc r)D−4 − 1)]/[1 − 2GM r)D−4], From Equation 3, it can be observed that the component C(r) becomes a real number when rc > lp and the cosmological constant Λ exceeds a critical value.

The metric components B(r), when combined with the quantum uncertainty relation (ΔpΔx ≥ 2), yield the fundamental mass scale relation M D−2 = MP l)D−4, where MP l is the Planck mass. The value of the cosmological constant Λ under these conditions will be specified in Section 4.

We next explore a model that accounts for the energy source of cosmic inflation and its termination mechanism.

4. The scale-dependent variation of the fundamental mass scale

With the beginning of the Big Bang, we assume that the fundamental mass scale varies with either the compactification radius rc of the extra dimensions or time, according to (MP l)D−4) t < tc t ≥ tc An inflationary mechanism in compactified extra-dimensional spacetimes where tc is the timescale obtained by dividing rc by the speed of light.

Based on Equation 3 and 5, the requirement that the metric tensor component C(r) be real implies that the time variation of the cosmological constant must also satisfy this condition 4 [(rc)D−4 − 1] t < tc t ≥ tc where rM is the wavelength associated with the fundamental mass scale derived from the quantum uncertainty relation.

From Equation 6, it is evident that the universe underwent inflation following the Big Bang, which ended at time tc. We consider that rc 10−32 m in a five-dimensional spacetime, and the duration of inflation can be derived from equation 5 and 6 as 10−42 10−40 s.

After the termination of inflation at tc, the cosmological constant can be interpreted as the source of dark energy, with a magnitude Λ 10−47GeV 4 consistent with the observed late-time accelerated expansion of the universe.

5. Summary

Compact high-dimensional spacetime provides a viable explanation for both the origin of inflationary energy in the early universe and the mechanism governing the end of inflation. We first derive the form of the metric vacuum solution in compact extra- dimensional spacetime in the absence of a cosmological constant. Based on the metric vacuum solution, we found that when the radius rc of the compact extra dimension is less than the Planck length, the cosmological constant does not need to be introduced.

However, when the radius rc of the compact extra dimension is greater than the Planck length, the cosmological constant must be introduced.

We then obtain the corresponding metric vacuum solution in the presence of a cosmological constant. By incorporating the quantum uncertainty principle, we further establish a quantitative relationship between the fundamental mass scale M and the radius rc of the compact extra dimensions. We investigate how variations in the fundamental mass scale—driven by changes in either the extra-dimensional radius or time—induce dynamical evolution of the cosmological constant, thereby causing the early inflationary process of the universe and its end, as well as the late-time accelerated expansion of the universe.

The successful realization of such an inflationary model constitutes strong support for high-dimensional cosmological models. However, compared to the well-established physical theories in four-dimensional spacetime, further development of consistent and predictive high-dimensional theories is necessary to robustly establish the existence of extra dimensions. Ultimately, definitive confirmation will require a combination of theoretical predictions and experimental observations.

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