Design of a Compact Superconducting Synchrotron and Beam Dynamics Studies Using 3D Field Maps

Authors: Yang Zhou, Sh, Dr. Tenghui Rui, Xiaopeng Jiang, Gao, Dr. Yunzhe, Shi, Jian

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Abstract

This study aims to develop a compact superconducting synchrotron designed to accelerate carbon ions to a maximum energy of 430 MeV/u, corresponding to a magnetic rigidity of 6.6 Tm. In addition to carbon ions, the proposed synchrotron is also capable of delivering helium and oxygen ion beams. Compactness is achieved by employing Canted-Cosine-Theta (CCT) superconducting magnets together with optimized beam optics, resulting in a circumference of approximately 32.65 m, which is significantly smaller than that of conventional normal-conducting heavy-ion synchrotrons.

This paper presents the lattice design, multi-turn injection scheme, and third-order resonance extraction system. Furthermore, in view of the strong curvature of the superconducting magnets, which introduces undesired field components inside the magnets, the field quality is analyzed using a Taylor series expansion. Beam dynamics studies are performed in a 3D magnetic field map of the superconducting magnets using the Runge–Kutta method to evaluate beam stability and dynamic aperture.

Full Text

Preamble

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Yang Zhou,[1],[2],[3] Jian Shi,[1],[3],[4],[5],† Tenghui Rui,[4],[5] Xiaopeng Jiang,[4],[5] and Yunzhe Gao[4],[5]
[1]Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
[2]Lanzhou University, Lanzhou 730000, China

[3]University of Chinese Academy of Sciences, Beijing 100049, China
[4]CAS Ion (Hangzhou) Medical Technology Co., Ltd., Hangzhou 310000, China
[5]Zhejiang Provincial Key Laboratory of Tumor Particle Radiotherapy Equipment, Hangzhou 310005, China

This study aims to develop a compact superconducting synchrotron designed to accelerate carbon ions to a maximum energy of 430 MeV/u, corresponding to a magnetic rigidity of 6.6 T・m. In addition to carbon ions, the proposed synchrotron can also provide helium ions and oxygen ions. The compactness of the synchrotron is achieved by employing Canted-Cosine-Theta (CCT) superconducting magnets and optimized beam optics, resulting in a circumference of approximately 32.65 m—significantly smaller than conventional normal-conducting heavy-ion synchrotrons. The paper details the lattice design, multi-turn injection scheme, and third-order resonance extraction system. Furthermore, considering the superconducting magnet is strongly curved, which would introduce unwanted field components within magnets, the field quality analysis was performed with Taylor series expansion. Beam dynamic studies were carried out in a 3D magnetic field map of superconducting magnets through the Runge-Kutta method to evaluate beam stability and dynamic aperture.

Keywords: Superconducting, Particle therapy, Multi-turn injection, Slow extraction, Beam dynamics

Introduction

According to the report from IARC (International Agency for Research on Cancer), approximately 20 million new cancer cases were diagnosed globally and cancer caused 9.7 million deaths in 2022. About one in five people develop cancer during their lifetime and the number of new cancer cases is predicted to reach 35 million by 2050 [1]. To address this growing health threat, researchers have developed several treatment plans. Among them, particle therapy is an emerging and innovative approach in cancer treatment that has garnered significant attention in recent decades. This advanced modality utilizes ion beams to deliver radiation with unparalleled precision. The distinct sharp Bragg peaks of ion beams ensure that most of the ion’ s kinetic energy is released at a specific target depth, corresponding to the location of the tumor. This unique characteristic, combined with the high linear-energy transfer of ions, enhances tumor cell killing while minimizing exposure to surrounding healthy tissues. The superior conformal dose delivery offered by particle therapy can reduce the risk of treatment-related complications and improve patients’ quality of life. As research continues to validate its efficacy and safety profiles, particle therapy is poised to become a cornerstone in modern cancer care.

Currently, the dominant modalities in particle therapy employ protons and carbon ions, each offering distinct therapeutic benefits tailored to specific clinical scenarios. Protons are widely recognized for their superior physical dose distribution. Protons can deposit the maximum dose within the tumor while creating a steep dose fall-off immediately beyond the target. In contrast, carbon ions offer a significant biological advantage. Due to their heavier mass, carbon ions exhibit a higher Linear Energy Transfer (LET), resulting in a higher Relative Biological Effectiveness (RBE) compared to protons. Therefore, a synchrotron that provides various levels of energies in a short period is a reasonable choice when considering both protons and carbon ions. However, synchrotrons generally require large areas for their installations and have high operating costs, which leads to expensive cancer treatment plans and hinders the widespread implementation of particle therapy.

Starting with the first hospital-based heavy-ion facility, HIMAC [2], which has an average diameter of 41 m, numerous efforts have been made to develop compact heavy-ion synchrotrons [3–5]. Among them, the most compact one is the HIMM synchrotron, designed by IMP/CAS [5]. It has a circumference of 56.2 m and comprises eight dipoles, twelve quadrupoles, four injection bump magnets, and three extraction bump magnets. The synchrotron has a two-fold symmetry structure with two long straight sections and four straight sections. The footprint of the facility is about 3000 m^2. However, since the accelerators still have a large size, medical centers and university hospitals usually need a new building dedicated to these facilities. Therefore, for the purpose of widely spreading heavy-ion therapy, the design of more compact superconducting synchrotrons is undergoing worldwide, especially in the Quantum Scalpel project proposed by NIRS [6] and the Next Ion Medical Machine Study (NIMMS) launched by CERN [7]. The design of NIRS is a synchrotron of FODO lattice structure shaped like a square. It has four superperiodicities and each contains a superconducting bending magnet and a normal-conducting quadrupole magnet. There is a defocusing quadrupole field in the bending magnet, and with a maximum bending magnetic field of 4 T, the circumference of the synchrotron is 29.8 m.

In the preliminary design of NIMMS, the superconducting synchrotron [8] also employs superconducting Canted-Cosine-Theta (CCT) magnets for beam guidance and focusing. Each bending section consists of two 45° combined-function CCT magnets (labeled as CCT1 and CCT2 in Fig. 1 [Figure 1: see original paper]). CCT1 is nested with focusing quadrupole coils and CCT2 is nested with defocusing quadrupole coils; the normalized strengths of their quadrupole gradients are 0.36 m^{-2} and -0.44 m^{-2} respectively. This configuration forms an alternating-gradient focusing structure directly within the bending sections.

Its primary advantage is the suppression of the beta functions (β) throughout the CCT magnets. By keeping β at a low value, the CCT magnets have a small horizontal beam envelope. As the aperture of the superconducting magnet is designed as a circle, the small horizontal beam envelope could reduce the size of the aperture, and reduce the construction cost of superconducting magnets. Additionally, the integrated quadrupole fields help to control the dispersion function, preventing it from growing excessively large in the arcs before it is suppressed in the straight sections.

The synchrotron is capable of accelerating ions with a rigidity within 6.63 T ·

m, such as H^{+}, $^{4}He^{2+}$, $^{12}C^{6+}$, $^{16}O^{8+}$. The maximum magnetic field of the bending magnet is 3.32 T, corresponding to the maximum energy of 430 MeV/u for $^{12}C^{6+}$. While the bending sections utilize superconducting magnets, the long straight sections employ normal-conducting quadrupoles. Three quadrupoles are arranged in a triplet configuration at each long straight section with normalized strengths of 1.27, -1.17, and 1.27 m^{-2}. The use of normal-conducting magnets here provides the necessary tunability and flexibility for lattice matching, which would be more difficult to achieve with only CCTs. Dispersion of long straight sections is maintained below 2.3 m as they are the place for injection and extraction hardware and RF cavities.

Two sextupole families are arranged in the synchrotron: one is specifically configured to excite the third-order resonance for the extraction process and the other is dedicated to correcting the natural chromaticity of the ring. They are symmetrically placed near the focusing quadrupole magnets where the dispersion and beta functions are favorable for efficient functioning. The optical functions of the synchrotron are shown in Fig. 2 [Figure 2: see original paper].

This paper proposes a novel compact synchrotron design with a circumference of 32.65 m, utilizing strongly curved CCT magnets. We aim to achieve a beam intensity of 3×10^{9} carbon ions per pulse to satisfy a dose rate of 2 Gy/min/L. The following sections describe the lattice design, injection/extraction schemes, and a detailed verification of beam stability under the influence of 3D magnetic fields of CCT.

II. Lattice Design

The proposed synchrotron was designed using MAD-X [9], focusing on achieving a highly compact footprint while maintaining sufficient flexibility for multi-ion operation. The layout of the synchrotron is shown in Fig. 1 and its main parameters are listed in Table 1 .

Table 1. Main parameters of the synchrotron.

Parameter	Value
Circumference	32.65 m
Injection energy	7 MeV/u
Particle species	H^{+}, $^{12}C^{6+}$
Particle energy range	Max: 430 MeV/u for C^{6+}, Min: 70 MeV/u for H^{+}
Tune Q /Q	1.68, 1.28
Natural chromaticity x/y	-0.72, -2.52
Acceptance x/y	200/50 π mm · mrad (ΔP/P = 0%)
Momentum acceptance	$±$0.5%
Transition energy	1.68

The synchrotron adopts a racetrack topology with a two-fold symmetry, divided into 2 identical superperiods. The ring consists of four 90° bending sections, two short straight sections and two long straight sections. The defining feature of this lattice is the use of combined-function superconducting magnets. The optical functions of the proposed synchrotron are shown in Fig. 2 [Figure 2: see original paper].

The selection of the betatron tunes is critical for the stability of the circulating beam and the efficiency of the slow extraction. The horizontal tune (Q) was set to 1.68, and the vertical tune (Q) was set to 1.28. The choice of Q is driven by the requirements of third-order resonant extraction. This value is slightly above the third-integer resonance condition 3Q = 5 (Q = 1.666···). By positioning the working point here, the beam can be easily pushed into resonance to produce a controlled slow spill for therapy. These values were selected to avoid the resonances that could be excited by the multipole components in the magnetic field and the structure of the synchrotron as shown in the resonance diagram Fig. 3 [Figure 3: see original paper].

High-order magnetic field errors were adopted for dipole, quadrupole and sextupole magnets, as listed in Table 2 , to calculate the dynamic aperture at the entrance of extraction electrostatic septum. The magnetic field errors of the dipole, quadrupole, and sextupole magnets were set to their maximum values, and the resulting dynamic aperture was larger than the beam clear area, as illustrated in Fig. 4 [Figure 4: see original paper]. Therefore, the beam was stable in this compact synchrotron.

Table 2. Maximum magnetic field errors for calculating dynamic aperture.

Magnet	0th	1st	2nd	3rd	4th	5th
Dipole	2×10^{-4}	3×10^{-4}	1×10^{-4}	1×10^{-4}	3×10^{-4}	1×10^{-4}
Quadrupole	1×10^{-4}	1×10^{-3}	1×10^{-4}	2×10^{-4}	0	3×10^{-4}
Sextupole	1×10^{-3}	0	2×10^{-4}	0	3×10^{-4}	1×10^{-3}

Injection

Synchrotron injection methods generally include single-turn injection, stripping injection, and multi-turn injection [10]. Although the single-turn injection method requires more equipment, the beam is injected in only one turn. Therefore, this approach requires a high amount of injector-beam intensity and is easier to limit by the space charge. The stripping injection method strips low-charged particles into high-charged particles and can easily achieve a gain factor greater than 50. The accumulation factor of the multi-turn injection method is significantly lower than that of the stripping methods owing to Liouville' s theorem limitations. However, the stripping injection method is only convenient for particular particles, such as carbon ions and protons. To gain the benefits of various ions, including helium and oxygen, in clinical cases and for further studies, multi-turn injection is the optimal choice.

Three orbit-bump magnets (referred to as bumpi1-bumpi3 in Fig. 1) were used to excite the local closed orbit of the ring, which was near the electrostatic injection septum (ESI). When the injection beams of 7 MeV/u $^{12}C^{6+}$ entered the magnetic septum (MSI), which bends these beams by 25°, and the ESI, which bends these beams by 62 mrad, the beams were deflected into the synchrotron. The injected beams filled the synchrotron up to its acceptance limit by gradually decreasing the closed orbit during injection. Fig. 5 [Figure 5: see original paper] shows the beam orbits and beam envelopes during injection.

Injection progress was simulated using pyOrbit [11], which is a Python/C++ implementation and extension of algorithms of the original ORBIT. The RMS geometric emittance of the injected beams was $2\ \pi \cdot \text{mm} \cdot \text{mrad}$, and 100 turns were injected with 1000 particles in each turn. The number of carbon ions varied during injection, as shown in Fig. 6 [Figure 6: see original paper]. After 500 turns, the number of particles remained nearly unchanged and finally a gain factor of 25.9 is achieved. In the case of carbon ions, 1.2×10^8 particles were injected per turn to store 3.0×10^9 particles in the ring. It is easy to achieve for a linac injector, thereby satisfying the initial design goal. Furthermore, for higher performance and efficiency of the accelerator facility, the number of particles in the ring will be improved in future upgrades.

IV. Extraction

The RF-KO extraction [12] method with third-order resonance extraction is commonly used to obtain a slow beam of variable extraction time. Among the advantages of this method are that it maintains a stable ring lattice during extraction and provides stable extracted beam intensity. In addition, this method can rapidly cut off and turn on the beam according to the treatment requirements.

In the proposed design, the extraction system comprised one electrostatic septum (ESE, used for deflecting the beam), three magnetic septa (MSE, used for beam extraction), and three orbit bump magnets (referred to as bumpe1-bumpe3 in Fig. 1). These bump magnets were employed to increase extraction efficiency; they bring the closed orbit near the ESE and obtain a local bump amplitude of 10 mm. In addition, a pair of sextupoles were used to excite resonance, including an F-sextupole and a D-sextupole; they were installed at symmetrical positions to avoid a significant part of their influence on the chromaticity. To correct horizontal chromaticity, another pair of sextupoles are used to guarantee the Hardt' s condition. An RF-KO device was used to increase the beam emittance during extraction and drive particles in the stable area to the extraction region.

The simulation of the extraction process was carried out with pyOrbit. 15,000 particles are generated in a two-dimensional Gaussian distribution and tracked for 51,000 turns to simulate the progress of extraction. The working point (Q , Q) is set to (1.68, 1.28) during extraction. The strength of resonance sextupole is set to 4.14 m^{-3} and the horizontal chromaticity is adjusted to -0.1. The RF-KO is set to a fixed frequency of 11.2 MHz and a fixed strength of 2×10^{-6} rad. Fig. 7 [Figure 7: see original paper] shows the phase space distribution of the separatrices and extracted beam. The orange, red, and blue points represent the particles with $+0.001$, -0.001, and 0.00 momentum deviation, respectively. Angular deviation of particles with different momentum deviations entering the ESE is less than 0.5 mrad, which confirms that Hardt’ s condition is satisfied.

Optimization of extraction plays a key role in this synchrotron design, and the key purpose of extraction is to separate the extracted beam from the circulating beam at the MSE entrance. The separation distance gap can be estimated from the equation, where θ is the deflecting angle of the ESE (limited to assure stable operation); $\beta\{ES\}\ and\ \beta$\{MS\} are the betatron amplitude at the ESE and MSE respectively; and $\Delta\phi$ is the phase advance between the ESE and MSE. As the phase advance over the long straight section is about 51°, ESE and MSE were placed at opposite long straight sections to get a phase advance between them of 280° which is close to 3 × 90°. And they were in the vicinity of the focusing quadrupole where the β functions were relatively large.

This extraction scheme can considerably separate the extracted beam from the circulating beam at the MSE entrance, but also brings a problem as these extracting beams pass through superconducting magnets; the larger the trajectory, the more expensive and difficult the superconducting magnet fabrication. Parametric studies have been done to identify the optimum value of the quadrupole gradients in the synchrotron, the position of the extraction electrostatic septum and magnet septum, and the length of the straight sections.

The goal was to minimize the aperture of synchrotron magnets while maximizing the gap. As a result, the good field of the superconducting magnet required for the circulating beam is 60 × 60 mm^2, and for the extracted beam there is an extended region of 130 × 20 mm^2. The gap is about 30 mm, providing an abundance of space for the pipe wall, septum plate, and magnetic shielding.

The horizontal trajectories during the last three turns of particles before extraction are shown in Fig. 8 [Figure 8: see original paper]. Particles excited by the sextupole and RFKO circulate in the synchrotron and almost do not exceed the acceptance of the synchrotron before they enter ESE. The ESE septum was installed on the inner side of the synchrotron. Its max strength is 85 kV/cm with an effective length of 1.3 m. During extraction, the height of the bumped orbit at the ESE entrance was approximately -10 mm.

V. Magnet Field Analysis of CCT

The Canted-Cosine-Theta (CCT) [13] is a type of magnet that uses pairs of conductor layers to produce the required magnet fields. When powered, one layer generates a pure harmonic field and a solenoid field which can be cancelled by the other layer in the pair. Due to its strong field and ability to nest multipole fields, it can reduce the footprint of the accelerator.

In large accelerators, the bending magnets are generally approximated to be straight, while that is not feasible in compact accelerators as they require strongly curved magnets. The CCT in this synchrotron has a radius of 2 m with a small ratio of bending radius to aperture. It is designed by Opera Simulation Software; the inner layers are used to produce a quadrupole field and the outer two layers are arranged for a dipole field. These two parts are superimposed to form the magnetic field of the CCT. The max magnetic field is 3.32 T and the max magnetic field gradient is 3 T/m.

Due to the strongly curved structure of superconducting magnets, more multipoles will be introduced compared with straight ones. Apart from that, the fringe fields of CCT also show obvious differences with normal-conducting magnets in the aspect of fringe region and multipole components. To accurately characterize the field quality, a 3D field map generated by Opera simulation software was analyzed. Usually, to describe the quality of this magnetic field, there are two different methods. The first method calculates the cylindrical multipole coefficients through the Fourier series expansion of magnetic field data along a circle. The second method is the Taylor series expansion analysis of the magnetic field assuming field symmetry about the median plane. Nevertheless, in curved magnets the introduced functions don’ t satisfy 2D Maxwell’ s equations; therefore, the cylindrical multipoles from the first method cannot describe the magnetic field of CCT used in this synchrotron [14]. Then, the second method is applied to analyze the field quality. The y-component of the magnetic field on the mid-plane can be expanded as:

$$B_y(x) = B_0 + \left.\frac{dB_y}{dx}\right|_{x=y=0} x + \left.\frac{d^2B_y}{dx^2}\right|_{x=y=0} \frac{x^2}{2!} + \cdots + \left.\frac{d_y^{nB}}{dx^n}\right|_{x=y=0} \frac{x^n}{n!} + \cdots$$

where B_0 represents the dipole component, and $\left.\frac{d_y^{nB}}{dx^n}\right|_{x=y=0}$ corresponds to normal higher-order magnetic field components, with $n = 1$ for quadrupole, $n = 2$ for sextupole, and so on.

The trajectory of the reference particle is composed of a nominal orbit in the CCT magnet, which is a circular arc with a radius of 2 m and a central angle of 45°, and two 0.3 m straight sections at the ends of the arc. Along the trajectory, a set of perpendicular segments are placed every 5 mm as shown in Fig. 9 [Figure 9: see original paper]. Each of them ranges from −60 mm to 60 mm in local x-coordinate, which covers a region where the beam circulates in the synchrotron. From these segments, the magnetic field data B_y are sampled with a sample interval of 5 mm. Next, a polynomial curve fit is applied to calculate the interpolation of the magnetic field data. The distribution of the dipole field and normal multipole gradients of the CCT is presented in Fig. 10 [Figure 10: see original paper]. As shown in the figure, dipole and quadrupole components remain steady in the main body of the magnet and decrease to zero in the fringe region. As to the sextupole, octupole, and decapole components, they won' t always stay close to zero. Taking the ends of the magnet as the boundary, their value fluctuates greatly on both sides. This unusual behavior of the field components along the trajectory of the reference particle may affect the instability of particles in the synchrotron and degrade the performance of the synchrotron. To evaluate their comprehensive impact on the beam, the multipole gradients are integrated along the trajectory and subsequently divided by the length of the bending magnet. The values are shown in Table 3 .

Table 3. The average integral of normal multipole components on the mid-plane.

Component	Value
Dipole [T]	3.32
Quadrupole [T/m]	-2.96
Sextupole [T/m^2]	-0.28
Octupole [T/m^3]	-4.58
Decapole [T/m^4]	1.3×10^{-12}

The normal multipole components are also calculated from B_x and B_y on the vertical plane ($x = 0$), as shown in Table 4 . The great differences observed in the sextupole, octupole, and decapole components reveal the spatial inhomogeneity of the magnetic field of CCT. To address off-axis fields, a 2D polynomial fit can be adopted to the main field on the transverse plane [15].

Table 4. The average integral of normal multipole components on the vertical plane.

Component	Value
Dipole [T]	9.3×10^{-12}
Quadrupole [T/m]	-1.8×10^{-10}
Sextupole [T/m^2]	-1.3×10^{-9}
Octupole [T/m^3]	-2.5×10^{-8}
Decapole [T/m^4]	4.2×10^{-6}

VI. Beam Tracking in Synchrotron with CCT

In pyOrbit there is a module named ‘trackerrk4’ which can import the 3D field data from other sources such as Opera and track particles in the field using 4th-order Runge-Kutta (RK4). The RK4 method is a numerical technique used to solve ordinary differential equations, so it is a direct way to track particles in a 3D magnetic field, though it will take large computation time. As RK4 is known as a non-symplectic approximation method, simulations of single particles and multi-particles are carried out. First, the analytical field is imported to compare the parameters of the synchrotron. The dipoles in the synchrotron are replaced with a superposition of ideal dipole field and quadrupole field given by:

$$F(x, y) = \sum C_n (x + iy)^n$$

Results are shown in Table 5 and there is negligible difference. Then, single particle and multi-particles are tracked in the synchrotron for 10,000 turns and results are presented in Fig. 11 [Figure 11: see original paper] and Fig. 12 [Figure 12: see original paper]. Particles will diffuse a bit in the phase space compared with the results calculated from the original lattice with MADX. As to the performance of multi-particles, 5,000 particles are sampled from a beam with Gaussian distribution of which the initial emittance x/y is 200/50 $\pi \cdot \mathrm{mm} \cdot$ mrad. The emittance is recalculated from coordinates of particles every 100 turns. Results show that although RK4 is not symplectic, particles remain stable for the simulation timescales considered (10,000 turns).

Table 5. Comparison of parameters. This comparison is carried out without considering FINT, so the fractional tune y changes from 0.28 to 0.3621. The Twiss functions are calculated at the entrance of the extraction electrostatic septum.

Parameter	Original	Analytical (RK4)
Frac. Tune x/y	−0.3200/0.3621	−0.3200/0.3620
β /α	14.1043/3.5529	14.1019/3.5522
β /α	3.7120/−1.7039	3.7119/−1.7038

Fig. 11. Comparison of phase space of single particle. The coordinate (x, p , y, p) of the particle is ($\sqrt{\epsilon_x \beta_x}$, 0, $\sqrt{\epsilon_y \beta_y}$, 0), where ϵ_x and ϵ_y are the horizontal and vertical acceptance of the synchrotron separately.

Next, the field of CCT calculated by Opera is imported. The tune x and tune y change from 1.68 to 1.6552 and 1.28 to 1.2830 respectively. This is mostly owing to the quadrupole field deviations between CCT and the design values in the lattice, as well as the settings for fringe fields in MADX which don’ t fit the field map of CCT. According to the guide of MADX, the quantity FINT used to describe the fringe field integral at the entrance and exit of bending magnets is calculated by the equation:

$$\mathrm{FINT} = \int_{-\infty}^{\infty} \frac{B_y(s)(B_0 - B_y(s))}{g \cdot B_0^2} ds$$

where $B_y(s)$ is the y-component of the magnetic field on the reference trajectory at position s, B_0 is the asymptotic value of $B_y(s)$ inside the magnet entrance, and g is the gap of the magnet. The FINT value derived from the field map of CCT is 0.2886 compared to the value of 0.3 utilized in the present lattice design. Based on the newly obtained FINT value, the lattice will be rematched in future work. At present, to adjust the tune x and tune y to (1.68, 1.28), coefficients are separately introduced to the two parts of the CCT field (one from inner layers and another from outer layers); this can be done in practice by adjusting the power of layers of CCT, and finally restoring the tune to (1.6809, 1.2820). The Twiss function in the synchrotron is very close to the original one as shown in Fig. 13 [Figure 13: see original paper]. Then the dynamic aperture (DA) is investigated. Particles with different fractional momentum deviations (−0.005, 0, 0.005) are tracked in the synchrotron for 2000 turns. Their coordinates (x, y) are taken from configuration space, where x ranges from −0.2 m to 0.2 m with a step of 0.005 m, and y ranges from 0 m to 0.08 m with a step of 0.002 m. Their coordinates (p , p) are calculated from Twiss functions to make them the boundary particle with the same emittance. These particles surviving 2000 turns shape the DA as shown in Fig. 14 [Figure 14: see original paper]. A reduction in DA is seen after importing the field of CCT compared with Fig. 4, specifically in the vertical plane. The tracking data of survival particles in the DA searching are also used for frequency map analysis (FMA) aimed to examine dynamics in frequency space where the tune distribution, information about resonance, and nonlinear behavior are exhibited. For each point (x, y) in the configuration space, the tracking data of the first 1000 turns and last 1000 turns are analyzed respectively by FFT (Fast Fourier Transform) to obtain fractional tunes, and their differences are the tune diffusion D.

As presented in Fig. 15 [Figure 15: see original paper], the tune shifts cross the resonance line (Q + Q = 3). According to the theory of resonance driving terms (RDT) [16], the relation of RDTs and excited resonances is $(j-k)Q_x + (l-m)Q_y = p \in \mathbb{N}$. The resonance line (Q + Q = 3) can be excited by f_{1010}, f_{2110}, and f_{2020} within the range of higher-order fields discussed in this paper.

Since the previous magnetic field analysis revealed that the skew higher-order components are negligible, the f_{2020} term becomes the dominant driver for this resonance in the CCT field. This f_{2020} term can originate from either the intrinsic octupole components or the feed-down effect from decapole fields caused by horizontal orbit deviation. Similarly, for the fifth-order resonance line (4Q + Q = 8) near the working point, there is no corresponding driving term in the CCT field due to the negligible skew components, posing no threat to the dynamic aperture. Another fifth-order resonance line ($Q + 4Q = 7$), although excited by the normal decapole field, is far from the working point and thus has minimal impact on particle motion.

To analyze the impact of different high-order magnetic field gradients on the lattice, the normalized sextupole, octupole, and decapole components (k_2, k_3, k_4) which were discretely sampled along the reference trajectory in the previous section are imported separately into the lattice by means of thin lenses. Furthermore, due to the aperture constraints of the CCT magnet, the magnetic field map exported from Opera represents the internal workspace, which is vertically restricted to a range of $±$60 mm. Particles with zero momentum deviation were tracked for 2000 turns using MAD-X, both with dipole aperture set to (0.13 m, 0.06 m) and without any aperture constraints. The simulation results are shown in Fig. 16 [Figure 16: see original paper], which indicates that in the case of no aperture constraints, the DA is much larger than the physical aperture with any high-order components, and under the effect of decapole component particles with large coordinates are more prone to being lost; in the case of aperture constraints, the dynamic aperture is significantly constrained, approaching the physical aperture in the vertical plane.

Regarding the high-order components sampled from the 3D magnetic field maps of CCT, they are not the primary cause of the dynamic aperture reduction; the dipole aperture constraints are the dominant factor. This is because the maximum vertical beta function occurs inside CCT1, which causes the vertical max emittance of the entire ring to be limited by the CCT aperture. As a conclusion, in current design the aperture constraints of CCT are the main reason for the reduction in DA, rather than purely dynamic instability driven by high-order non-linearities. And the DA is still larger than the physical aperture after applying CCT and the particles inside acceptance will stay stable when they circulate in the synchrotron. Considering further magnetic field errors and accuracy of installing, it is advised to set the tune y from 1.28 to 1.4 for avoiding the potential dangerous resonance line ($Q + Q = 3$).

VII. Summary

In this study, a compact synchrotron for particle therapy is designed. The compactness is achieved by applying the combined-function superconducting magnets CCT which is nested with focusing and defocusing quadrupole fields, and the circumference of the synchrotron was reduced from 56.2 m to 32.65 m. The multi-turn injection and third-order resonance extraction processes have been investigated to satisfy the requirements of beam intensity and slow extraction. Magnetic field analysis was carried out based on the 3D magnetic field model of the CCT magnet which was developed by Opera software. Although high-order field gradients exist and their distribution differs from that of conventional magnets, dynamic simulations of particles in the 3D field map have confirmed beam stability. To further enhance the beam’ s tolerance to errors, the tune will be modulated to a new value in future work, and the octupole or decapole components need to be optimized. The results initially validate the feasibility of using strongly curved CCT magnets for compact medical accelerators. Due to the asymmetry of the CCT magnet, more detailed analysis of the magnetic field and beam dynamics studies will be done in the future.

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Note: Figure translations are in progress. See original paper for figures.