Quantum Econometrics: Quantum Regression Models for Cross-Sectional Data

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Abstract

To address the problem that classical cross-sectional econometric regression represents economic variables with a single scalar value and assumes a linear and homogeneous mapping—assumptions that contradict the actual characteristics of economic variables, namely uncertainty in micro-states, polymorphism of values, and heterogeneity in associations, thereby leading to estimation bias and insufficient explanatory power—this paper innovatively introduces core concepts from quantum mechanics and constructs a complete theoretical system of a cross-sectional Quantum Regression Model (QRM). The study clarifies the principles for quantizing cross-sectional data and the method for constructing density matrices, derives the theoretical framework of QRM, designs the Quantum Ordinary Least Squares (QOLS) estimation method and its analytical expressions, and conducts an empirical analysis based on real cross-sectional data of inter-provincial household consumption in China, comparing the quantum regression results with those of classical OLS and performing a decoherence-based reduction test. The results show that quantum regression, through the density matrix, accurately characterizes the triple micro-states and probabilistic features of economic variables; the quantum coefficient operator can capture the heterogeneous marginal effects of variable states; quantum goodness-of-fit entails less information loss; and when quantum effects converge to zero, the model strictly degenerates into classical OLS, confirming that classical regression is a special case of quantum regression. This paper implements the QRM methodology in practice, providing a novel methodological tool to overcome the inherent limitations of classical econometrics and demonstrating the stronger theoretical generality of quantum regression.

Full Text

Quantum Econometrics: Quantum Regression Models for Cross-Sectional Data

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Abstract: To address the problems of estimation bias and insufficient explanatory power arising from classical cross-sectional econometric regression' s use of single numerical values to represent economic variables and its assumption of linear homogeneous mapping—which contradicts the true characteristics of uncertain micro-states, polymorphic values, and heterogeneous associations of economic variables—this paper innovatively introduces core concepts from quantum mechanics to construct a complete theoretical system for cross-sectional quantum regression models (QRM). The study clarifies the principles of quantum processing for cross-sectional data and the method for constructing density matrices, derives the theoretical framework of QRM, designs the quantum least squares estimation (QOLS) method and its analytical expressions, conducts empirical analysis based on real cross-sectional data of inter-provincial household consumption in China, compares quantum regression with classical OLS results, and performs decoherence reduction tests. The results demonstrate that quantum regression accurately characterizes the triple micro-states and probabilistic features of economic variables through density matrices, that quantum coefficient operators can capture the marginal effects of variable state heterogeneity, that quantum goodness-of-fit entails less information loss, and that when quantum effects approach zero, the model strictly degenerates to classical OLS, confirming that classical regression is a special case of quantum regression. This paper implements the QRM methodology, providing a novel methodological tool to overcome the inherent defects of classical econometrics and demonstrating the stronger theoretical generality of quantum regression.

Keywords: quantum regression model; quantization processing; density matrix; quantum least squares estimation

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The core mission of econometrics lies in employing mathematical tools to precisely reveal causal relationships and evolutionary patterns among economic variables. Since its inception, the theoretical system has consistently anchored itself to the "classical numerical representation" paradigm—whether in cross-sectional, panel, or time series models, economic variables are simplified into single deterministic observed values, with mappings between variables constructed through linear or nonlinear functions. This paradigm, supported by mature estimation methods such as OLS, GLS, and MLE, has become the mainstream tool for empirical economic analysis, playing a fundamental role in macroeconomic forecasting and micro-behavioral inference. However, as uncertainty and heterogeneity in economic systems continue to increase, the inherent limitations of the classical paradigm have become increasingly prominent when characterizing real economic features, making it difficult to accommodate the essential attributes of variable polymorphism and complex associations.

In reality, economic variables are by no means static "classical deterministic values" but universally embody micro-state uncertainty and polymorphic characteristics. Taking provincial per capita disposable income as an example, an observed value of 49,800 yuan for a province represents merely a macro statistical mean. At the micro level, there exist an "optimal growth state" supported by high-income groups and high-end industries, a "bottom-line stable state" relying on basic livelihood guarantees, and an "equilibrium evolution state" fluctuating around the mean—different income states exhibit significant heterogeneity in their driving intensity and pathways for consumption behavior. Similarly, corporate R&D investment, behind its deterministic book value, constantly switches between a "high-output state of technological breakthroughs" and a "low-efficiency state of maintaining operations." Classical regression models can only estimate global average effects, completely failing to disentangle the marginal impact differences across states, thereby potentially leading to explanatory biases and policy misguidance risks. This core feature of "variable polymorphism" coexisting with "associative heterogeneity" aligns highly with the representation logic of "superposition states, probability amplitudes, and state vectors" for microscopic particles in quantum mechanics, providing a novel theoretical perspective and methodological support for econometrics to break through traditional paradigm constraints.

As a fundamental theory describing uncertainty in the microscopic world, quantum mechanics characterizes particle states through state vectors and density matrices, depicts probability distributions of particles in different basis states through probability amplitudes, and captures association strength between states through quantum inner products. Introducing these core ideas into econometrics to form quantum econometrics is by no means a simple analogy to physical theory but rather an essential expansion, logical extension, and methodological innovation of the classical econometric paradigm: classical econometrics is essentially a "deterministic value → deterministic value" unidirectional mapping, while quantum econometrics achieves a "probability distribution → probability distribution" multidimensional mapping, with the former being merely a special case of the latter under conditions where "quantum effects disappear and state uncertainty becomes negligible." This expansion enables econometric models to accommodate the polymorphic features of economic variables, moving from traditional "averaging analysis" to more realistic "state-based analysis."

The contribution of this study lies in advancing macro-econometrics toward the new dimension of "state-based analysis," achieving breakthroughs at three levels: theoretical, methodological, and empirical. Specifically: first, at the theoretical level, it pioneeringly constructs a three-dimensional quantum basis vector analysis framework, explicitly stating that quantum econometrics is a natural extension of classical econometrics and proving that classical regression is essentially a special manifestation of quantum regression under "decoherence + state collapse" conditions; second, at the methodological level, it establishes standardized conversion rules from classical variables to quantum states and constructs cross-sectional quantum regression models, achieving for the first time precise estimation of "explanatory variable state → dependent variable state" associative effects; third, at the empirical level, it successfully identifies state heterogeneity and cross-state coherence in inter-provincial consumption, transforming the traditional averaging cognition pattern of classical econometrics. As economic systems become increasingly complex, quantum econometrics, with its unique advantages in "probability amplitude representation" and "state association identification," is poised to become a key tool for cracking the "black box" of macroeconomics. This represents not only an important methodological innovation in econometrics but also provides a novel paradigm reference for understanding and regulating complex economic systems.

II. The Theoretical Core and Inherent Limitations of Classical Cross-Sectional Econometric Regression

Classical cross-sectional econometric regression serves as the core analytical tool in empirical economics, encompassing ordinary least squares (OLS), generalized least squares (GLS), maximum likelihood estimation (MLE), quantile regression, and robust regression. It fundamentally relies on the "single numerical representation + stable association assumption" paradigm to construct causal relationships between variables. Although different methods vary in error term treatment, estimation criteria, and assumption relaxation, none break through the essential constraints of the classical paradigm, creating root-level deviations from the true features of economic systems— "polymorphism, heterogeneity, and dynamic association" —and forming four types of inherent limitations that cannot be corrected through minor methodological adjustments. These limitations constitute the core motivation for quantum regression models to break through the classical framework.

(1) The Core Paradigm of Classical Cross-Sectional Econometric Regression

The unified theoretical framework of classical cross-sectional econometric regression can be expressed as: Let sample size be n, the dependent variable observation vector be y (an $n \times 1$ column vector), and K explanatory variables constitute matrix X (with the first column being a constant term corresponding to the intercept). The core model in matrix form is:

$$y = X\beta + \varepsilon$$

where β is a $(K+1) \times 1$ vector of coefficients to be estimated, reflecting the association strength of explanatory variables on the dependent variable, and ε is an $n \times 1$ error term capturing unexplained random disturbances. Different methods solve for β through different criteria: OLS aims to minimize the sum of squared residuals, GLS corrects heteroskedasticity/autocorrelation through weighting matrices, MLE maximizes likelihood functions based on probability distribution assumptions, and quantile regression focuses on marginal effects at conditional quantiles. Model fit is evaluated through R^2, quantile fit errors, AIC/BIC, and other metrics [**?**, **?**].

The core assumptions of this paradigm are: economic variables can be simplified into single deterministic numerical values, associations between variables are stable (coefficients β do not change with observation states), and error terms satisfy "zero mean, independence, homoskedasticity" constraints (or can be corrected through transformations). This formulation significantly reduces computational complexity and has become the mainstream framework for empirical analysis. However, by ignoring micro-uncertainty and state-dependency of economic variables, it sows the seeds of theoretical disconnect from reality.

(2) Inherent Limitations of Classical Cross-Sectional Econometric Regression

(1) Single Numerical Representation Defect: Loss of Micro-State and Association Information

All classical cross-sectional methods treat variable observations as fixed scalars, extracting only macro statistical results without capturing micro polymorphic features and cross-state association characteristics. In reality, the value of an economic variable is essentially a probabilistic mixture of optimal states (development upper limits, such as consumption by high-income groups), bottom-line states (basic guarantees, such as consumption by low-income groups), and mean states (equilibrium fluctuations, such as consumption by middle-income groups), with dynamic transitions and mutual constraints between different states. Classical methods simplify this multi-state mixture into a single value, inevitably causing irreversible loss of micro-state information—even quantile regression, while capturing effect differences at conditional quantiles, remains within the "value corresponds to value" framework, unable to make explicit the probability distributions, transition mechanisms, and cross-state associations of different states, making it difficult to restore the essential attributes of economic variables.

(2) Stable Association Assumption Bias: Difficulty in Capturing State Heterogeneity Effects

Classical methods assume marginal effects of explanatory variables on dependent variables are constant (OLS/GLS) or only change in fixed gradients with quantiles (quantile regression), creating essential deviations from economic reality. From consumption theory, the impact of per capita income on consumption exhibits significant state heterogeneity: low-income groups (bottom-line state) have marginal propensity to consume close to 1, using income mainly for survival consumption; high-income groups (optimal state) have marginal propensity below 0.5, with more income going to savings and investment; middle-income groups (mean state) fall between these extremes [**?**, **?**]. Classical methods either estimate global average effects (OLS/GLS) or only capture vague differences between quantiles (quantile regression), unable to precisely identify directed differential associations of "explanatory variable state → dependent variable state," easily leading to coefficient estimation bias and misleading differentiated policy formulation.

(3) Passive Outlier Treatment: Insufficient Robustness and Anti-Interference Capability

Classical methods have essential limitations in handling extreme values, lacking systematic theoretical explanations: OLS amplifies the influence of extremes on coefficients through residual sum of squares, while GLS and Huber robust regression only passively weaken interference through weight adjustments, often treating extremes as "abnormal disturbances" to be directly eliminated while ignoring their economic connotations [**?**, **?**]. From a state perspective, extremes are essentially short-term collapses of variables to optimal or bottom-line states with sharply increased probability (e.g., a province' s income suddenly rising to optimal state due to industrial upgrading or falling to bottom-line state due to natural disasters), and such observations precisely contain critical state transition information. Passive treatment by classical methods not only loses core information but may also induce sample selection bias, significantly reducing the robustness and credibility of estimation results.

(4) Single-Dimensional Fit Evaluation: Lack of Precise State-Level Metrics

Classical goodness-of-fit evaluation metrics have obvious information coverage gaps and cannot comprehensively reflect model explanatory power: R^2 only measures the fit precision of explanatory variables on macro numerical fluctuations of dependent variables, while quantile fit errors only reflect fit effects at specific quantiles, neither capturing micro-state level fit quality. For example, two models may both have $R^2 = 0.97$, but one fits optimal and bottom-line states (key reference states for policy-making) with extremely high precision while the other only fits the mean state well—classical metrics cannot distinguish this difference, potentially masking model fit defects in critical states and leading to misjudgment of model explanatory power, making it difficult to support state-oriented precise policy-making.

In summary, the various limitations of classical cross-sectional econometric regression stem from the fundamental contradiction between the "single numerical representation" paradigm and the polymorphic, heterogeneous nature of economic systems. This contradiction cannot be resolved by adjusting estimation methods (e.g., switching from OLS to GLS or robust regression) or partially relaxing assumptions (e.g., loosening homoskedasticity or introducing instrumental variables). The paradigm must be broken, expanding variable representation from "single numerical value" to a multidimensional framework of "probability distribution + state association." Quantum mechanics' state vector and density matrix theory precisely provides suitable methodological tools for this paradigm upgrade, effectively compensating for the inherent defects of classical methods.

III. Quantum Processing Principles for Cross-Sectional Data and Density Matrix Construction

(1) Core Definition of Quantum Processing: From Classical Numerical Values to Quantum States and Density Matrices

Quantum processing is the logical starting point and theoretical cornerstone of the Quantum Regression Model (QRM). It is strictly defined as: based on the micro-polymorphic characteristics of economic variables and the basic axioms of quantum mechanics, through verifiable encoding rules, converting a single deterministic observed value x_i of a classical cross-sectional variable into a three-dimensional quantum state vector $|x_i\rangle$ satisfying normalization constraints, and generating a quantum density matrix ρ_i through the outer product of state vectors—a complete reversible process (panel data and other data types can also adopt unified quantum processing methods). The essence of quantum processing is upgrading the classical econometric paradigm of "single numerical representation" to the quantum paradigm of "probability distribution representation." This conversion is not a formal analogy to physical theory but a precise mathematical characterization of the true attributes of economic variables: the macro observed values of economic variables in reality are essentially probabilistic mixtures of micro-level optimal states (development upper limit states), bottom-line states (basic guarantee states), and mean states (equilibrium fluctuation states). Quantum processing explicitly restores this objective probability distribution. The encoding process must satisfy two core axioms that complement each other, eliminating arbitrariness in quantum processing and ensuring unity of theoretical rigor and economic rationality:

Axiom 1 (Normalization Axiom): The sum of squared moduli of quantum state vector probability amplitudes always equals 1, i.e., $\sum_{m=1}^{3} |c_{im}|^2 = 1$. Since economic variables are real-valued observations, this paper simplifies probability amplitudes to real values ($c_{im} \in \mathbb{R}$), making the constraint equivalent to $\sum_{m=1}^{3} c_{im}^2 = 1$. This axiom is a dual fundamental constraint from probability theory and quantum mechanics, ensuring no overflow or missing probability weights across quantum states, consistent with objective laws.

Axiom 2 (Monotonicity Matching Axiom): Each quantum state' s probability amplitude strictly monotonically matches the economic logic of variable values. The optimal state probability amplitude c_{i1} monotonically increases with variable values, the bottom-line state probability amplitude c_{i2} monotonically decreases with variable values, and the mean state probability amplitude c_{i3} monotonically increases with the degree of deviation from sample mean. This axiom anchors the economic connotation of quantum states, ensuring quantum processing results are not pure mathematical transformations but authentic mappings of micro-features of economic variables.

The three-dimensional quantum state vector $|x_i\rangle$ is the basic pure state representation of variable micro-states, capable only of depicting single-dimensional state probability distributions. The quantum density matrix $\rho_i = |x_i\rangle\langle x_i|$ is the complete ultimate representation of variable micro-states, containing all probability information from state vectors while additionally capturing associative interference effects between different quantum states through off-diagonal elements, adapting to the essential features of economic variables' "multi-state mixing and mutual linkage." The two are progressive rather than contradictory—the density matrix, as a higher-order extension of the state vector, serves as the optimal core carrier for subsequent quantum regression model derivation and estimation.

(2) Step-by-Step Implementation of Cross-Sectional Data Quantum Processing (Classical Numerical Value → State Vector → Density Matrix)

For any classical cross-sectional variable x (with sample size n), denote sample maximum as $x_{\max}$, minimum as $x_{\min}$, and sample mean as $\bar{x}$. The quantum processing algorithm constructed in this paper is a strict reversible encoding process without information loss, satisfying the two axioms above throughout, and completing the full conversion from classical numerical values to density matrices in three steps:

Step 1: Calculate Core Anchor Statistics for Quantum Basis Vectors Perform descriptive statistics on classical variable x (first removing outliers to ensure statistic validity) to compute three uniquely determined core anchor values serving as economic anchors for the three-dimensional basis vectors $\{|1\rangle, |2\rangle, |3\rangle\}$ (corresponding to optimal, bottom-line, and mean states):

$$x_{\max} = \max\{x_1, x_2, \dots, x_n\}, \quad x_{\min} = \min\{x_1, x_2, \dots, x_n\}, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

These statistics have unique economic connotations: the optimal state anchors the development upper limit, the bottom-line state anchors the basic lower limit, and the mean state anchors the central tendency, ensuring quantum processing results always align with economic reality rather than detached mathematical forms. Using 2024 Chinese provincial per capita disposable income as an example, $x_{\max} = 88,366$ yuan (Shanghai), $x_{\min} = 26,612$ yuan (Gansu), and

$\bar{x} = 39,682.19$ yuan—the three anchor values precisely characterize the gradient distribution features of inter-provincial income, providing clear economic references for quantum processing. If a variable has no variation ($x_{\max} = x_{\min}$), the quantum state defaults to a pure mean state collapse with probability amplitudes $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, ensuring satisfaction of the normalization axiom.

Step 2: Compute Unnormalized Raw Probability Amplitudes
Based on the monotonicity matching axiom, construct initial probability amplitudes for three dimensions corresponding to optimal, bottom-line, and mean states (additional basis dimensions can be added according to actual needs, and unified quantum processing methods can also be applied to panel data and other data types). The values directly reflect the relative degree to which the i-th sample belongs to a particular quantum state:

$$c_{i1} = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}, \quad c_{i2} = \frac{x_{\max} - x_i}{x_{\max} - x_{\min}}, \quad c_{i3} = \frac{|x_i - \bar{x}|}{x_{\max} - x_{\min}}$$

Example Calculation: For a province with 2024 per capita disposable income $x_i = 49,800$ yuan, raw probability amplitudes are computed as:

$$c_{i1} = \frac{49,800 - 26,612}{88,366 - 26,612} = \frac{23,188}{61,754} \approx 0.375$$

$$c_{i2} = \frac{88,366 - 49,800}{61,754} = \frac{38,566}{61,754} \approx 0.624$$

$$c_{i3} = \frac{|49,800 - 39,682.19|}{61,754} = \frac{10,117.81}{61,754} \approx 0.164$$

The sum of squared raw probability amplitudes $\sum_{m=1}^{3} c_{im}^2 = 0.375^2 + 0.624^2 + 0.164^2 = 0.141 + 0.389 + 0.027 = 0.557 \neq 1$, requiring further normalization.

Precise Economic Interpretation of Probability Amplitudes:
$c_{i1} \in [0,1]$ characterizes the sample' s relative degree of approaching optimal development state (taking 1 when $x_i = x_{\max}$ and 0 when $x_i = x_{\min}$); $c_{i2} \in [0,1]$ characterizes the sample's degree of being in bottom-line guarantee state (taking 1 when $x_i = x_{\min}$ and 0 when $x_i = x_{\max}$); $c_{i3} \in [0, 0.5]$ (maximum under symmetric distribution) increases with deviation from the mean, characterizing the sample' s dispersion degree from industry/regional average level.

Step 3: Probability Amplitude Normalization and Construction of Quantum State Vectors and Density Matrices
To satisfy the normalization axiom, raw probability amplitudes must be standardized through square-root-of-sum-of-squares correction to obtain final normalized probability amplitudes:

$$c'_{im} = \frac{c_{im}}{\sqrt{\sum_{m=1}^{3} c_{im}^2}}, \quad (m = 1, 2, 3)$$

where $\sum_{m=1}^{3} c_{im}^2$ is the normalization factor. After correction, $\sum_{m=1}^{3} {c'}_{im}^2 = 1$, fully conforming to quantum mechanical normalization requirements.

Continued Example: Using the above provincial income data, normalized probability amplitudes are:

$$c'_{i1} = \frac{0.375}{\sqrt{0.557}} \approx \frac{0.375}{0.746} \approx 0.503$$

$$c'_{i2} = \frac{0.624}{0.746} \approx 0.836$$

$$c'_{i3} = \frac{0.164}{0.746} \approx 0.220$$

Now $\sum_{m=1}^{3} {c'}_{im}^2 \approx 0.253 + 0.699 + 0.048 = 1.0$, satisfying the normalization axiom. The three-dimensional quantum state vector (column vector form) for sample i is uniquely determined:

$$|x_i\rangle = \begin{pmatrix} c'_{i1} \\ c'_{i2} \\ c'_{i3} \end{pmatrix}$$

Through outer product operation of the quantum state vector ($\rho_i = |x_i\rangle\langle x_i|$), we obtain the 3×3 quantum density matrix for sample i, which is also the core basic unit of quantum regression:

$$\rho_i = \begin{pmatrix} {c'}_{i1}^2 & c'_{i1}c'_{i2} & c'_{i1}c'_{i3} \\ c'_{i2}c'_{i1} & {c'}_{i2}^2 & c'_{i2}c'_{i3} \\ c'_{i3}c'_{i1} & c'_{i3}c'_{i2} & {c'}_{i3}^2 \end{pmatrix}$$

Dual Core Connotations of Density Matrix:
Diagonal elements ${c'}_{im}^2$ are the absolute probability weights of sample being in the m-th quantum state, 刻画 "intra-state pure effects" —in the above matrix, diagonal elements 0.253, 0.699, and 0.048 respectively represent the province' s absolute probabilities of being in income optimal, bottom-line, and mean states. Off-diagonal elements $c'_{im}c'_{ik}$ ($m \neq k$) are inter-state associative interference terms, 刻画 "inter-state linkage effects" —for example, off-diagonal element 0.421 reflects the linkage intensity between the province's income optimal and bottom-line states, embodying the dynamic association characteristics of its income level between "development enhancement" and "basic guarantee." This matrix satisfies core quantum mechanical density matrix constraints: real symmetry, semi-positive definiteness, and trace equal to 1 ($\mathrm{Tr}(\rho_i) = 0.253+0.699+0.048 = 1.0$).

(3) Key Desirable Properties of Quantum Processing

The cross-sectional data quantum processing algorithm constructed in this paper, from classical numerical values to density matrices, satisfies four key desirable properties, ensuring the rationality and validity of quantum processing results and providing a solid foundation for subsequent quantum regression model derivation:

(1) Uniqueness: Given core statistics of classical variables ($x_{\max}, x_{\min}, \bar{x}$) are uniquely determined and encoding rules are strictly fixed, quantum state vectors and density matrices are uniquely determined without multiple solutions or arbitrariness, avoiding subjective bias in quantum processing.

(2) Complete Reversibility: Quantum processing is a bidirectional "encoding-decoding" reversible process. Probability amplitudes can be extracted from density matrix diagonal elements ($c'_{im} = \sqrt{\rho_i(m,m)}$), and combined with core anchor statistics to reverse-engineer classical numerical values. The process involves no information loss, solving the pain point of information loss in traditional econometric transformations.

(3) Quantum Compatibility: Density matrices naturally satisfy core quantum mechanical constraints—real symmetry (property of outer product operation), semi-positive definiteness (verified through eigenvalue decomposition as all non-negative), and trace equal to 1 (derived from normalization axiom). All results conform to original quantum theoretical norms.

(4) Classical Collapsibility: When variable values are constant ($x_i = \bar{x}$), the density matrix degenerates into a pure-state diagonal matrix of identity matrix form (e.g., $\rho_i = \text{diag}(1,0,0)$), quantum states collapse to single classical numerical values, achieving complete equivalence with classical econometric numerical representation and seamless connection between quantum and classical paradigms.

(4) Construction of Overall Quantum Density Matrix (Sample Level)

For cross-sectional data with sample size n, each sample corresponds to a 3×3 quantum density matrix. Since density matrices are real symmetric with identical row and column information, to simplify subsequent matrix operations while retaining core information, we extract the first row of each sample' s density matrix (corresponding to optimal state as baseline for state distribution and association terms) and stack them row-wise to form the overall quantum density matrix.

Taking dependent variable y as an example, the overall quantum dependent variable matrix Y_q has the form:

$$Y_q = \begin{pmatrix} \rho_{1y}(1,1) & \rho_{1y}(1,2) & \rho_{1y}(1,3) \\ \rho_{2y}(1,1) & \rho_{2y}(1,2) & \rho_{2y}(1,3) \\ \vdots & \vdots & \vdots \\ \rho_{ny}(1,1) & \rho_{ny}(1,2) & \rho_{ny}(1,3) \end{pmatrix}_{n\times 3}$$

where $\rho_{iy}(1,m)$ respectively correspond to the absolute probability of optimal state, association strength between optimal and bottom-line states, and association strength between optimal and mean states for sample i' s dependent variable density matrix.

Similarly, the overall quantum explanatory variable matrix X_q is $n \times 3(K + 1)$ (where K is the number of explanatory variables): the constant term' s quantum density matrix is I_3 (I_3 being the 3×3 identity matrix satisfying trace-1 normalization constraint), which after stacking forms an $n \times 3$ constant term matrix; each explanatory variable's quantum density matrix after row extraction forms an $n \times 3$ matrix, ultimately stacked column-wise with the constant term matrix to obtain X_q, providing core data foundation for subsequent quantum regression models.

IV. Theoretical Derivation and Estimation Methods for Cross-Sectional Quantum Regression Models (QRM)

(1) Rationale for Density Matrix as Optimal Core Carrier

In the theoretical construction of cross-sectional quantum regression models (QRM), using quantum density matrices as the core basis for derivation and estimation represents the optimal choice that balances quantum mechanical original logic with econometric practical adaptability, possessing essential advantages over using only three-dimensional quantum state vectors. The comparative advantages and selection rationale can be systematically demonstrated across three dimensions: theoretical compatibility, information completeness, and econometric applicability.

(1) Inherent Limitations of Using Only "Three-Dimensional Quantum State Vector"

Quantum state vectors $|x_i\rangle$ can only precisely characterize pure-state quantum systems (i.e., systems deterministically in superposition of a single basis state without state uncertainty). However, micro-states of economic variables in reality are all mixed superposition states of optimal, bottom-line, and mean states—their state distributions are influenced not only by their own characteristics but also by implicit linkages with other variables, absolutely not characterizable by single pure states. Relying solely on state vectors causes two core problems: first, information loss, unable to capture associative interference effects between different states, leading to model specification bias; second, theoretical incompleteness, difficult to adapt to the essential features of economic systems' "multi-state coexistence and dynamic linkage." For instance, a province' s per capita income exhibits both "optimal state characteristics" driven by high-income groups and "bottom-line state characteristics" dominated by low-income groups—state vectors can only provide probability proportions of the two states but cannot capture their mutual checks and dynamic transition relationships, whereas off-diagonal elements of density matrices can precisely capture these core associations.

(2) Core Advantages of Using "Quantum Density Matrix"

Theoretical Completeness: Density matrix $\rho_i = |x_i\rangle\langle x_i|$ is the ultimate complete representation form of quantum states, compatible with both pure states (off-diagonal elements all zero) and precisely characterizing mixed states (non-zero off-diagonal elements), perfectly adapting to the multi-state mixed features of economic variables without model specification bias, fully conforming to quantum mechanical description rules for uncertain systems.

Information Integrity: The diagonal and off-diagonal elements of density matrices form complementary information structures—diagonal elements $\rho_i(m,m) = {c'}_{im}^2$ 刻画 "intra-state heterogeneity pure effects," reflecting absolute probabilities and marginal impacts of samples being in single quantum states; off-diagonal elements $\rho_i(m,k) = c'_{im}c'_{ik}$ $(m \neq k)$ 刻画 "inter-state associative interference effects," capturing linkage strength and interaction mechanisms between different states. This structure can completely cover implicit information entirely lost in classical econometrics, constituting the core innovation of quantum regression.

Economic Adaptability: The binary structure of density matrices can precisely characterize economic variables' dual effects of "intra-state independent influence + inter-state linkage spillover," enabling identification of heterogeneous marginal impacts across different development states while capturing cross-state linkage mechanisms between variables, perfectly solving the heterogeneity and linkage pain points that classical econometric "averaging estimation" cannot address.

Density matrices and quantum state vectors are not contradictory but rather progressive and inclusive mathematical relationships—density matrices are directly generated from state vector outer products, with elements completely determined by probability amplitudes c'_{im}, thus containing all probability information of state vectors while expanding to include state association information through off-diagonal elements, achieving an information upgrade from "single probability distribution" to "probability distribution + association structure." The quantum regression model constructed in this study uses density matrices as the core basis and state vectors as foundational components, balancing theoretical rigor with practical adaptability, achieving deep integration of quantum econometric logic and economic analysis needs.

(2) Strict Theoretical Form of Cross-Sectional Quantum Regression Model

Based on the complete representation of density matrices and combined with sample characteristics of cross-sectional data, this paper constructs the core theoretical form of the cross-sectional quantum regression model (QRM) as follows:

$$Y_q = X_q\beta_q + \varepsilon_q \tag{4.1}$$

The dimensions, definitions, and "quantum-economic" dual connotations of all core matrices have been strictly calibrated, with specific explanations as follows:

(1) Quantum Dependent Variable Matrix Y_q
With dimension $n \times 3$ (n being sample size), each row corresponds to the first-row elements of a sample' s dependent variable density matrix (based on density matrix real symmetry, row and column information are completely identical; extracting the first row simplifies operations while retaining core information), respectively 刻画 the absolute probability of dependent variable' s optimal state, association strength between optimal and bottom-line states, and association strength between optimal and mean states, completely reflecting the micro-state distribution and cross-state linkage features of the dependent variable.

(2) Quantum Explanatory Variable Composite Density Matrix X_q
With dimension $n \times 3(K+1)$ (K being number of explanatory variables), formed by stacking column-wise the constant term quantum density matrix and all explanatory variable quantum density matrices. The constant term' s quantum density matrix is I_3 (the 3×3 identity matrix satisfying quantum normalization constraint), which after stacking forms an $n \times 3$ constant term matrix; each explanatory variable' s quantum density matrix after row extraction forms an $n \times 3$ matrix, ultimately stacked column-wise to obtain X_q.

(3) Quantum Regression Coefficient Operator Matrix β_q
With dimension $3(K+1) \times 3$, this is the core parameter to be estimated in the model, representing the quantum-complete expansion of classical explanatory variable matrices. The economic meaning of element $\beta_q(m, k)$ is the "heterogeneous marginal effect of explanatory variable' s k-th quantum state on dependent variable' s m-th quantum state" —where $m, k = 1, 2, 3$ respectively correspond to optimal, bottom-line, and mean states. Unlike classical OLS' s single coefficient, this matrix can completely characterize differential impacts and cross-state linkage effects of explanatory variables' different states on dependent variables' different states, achieving an upgrade from "global average effects" to "state-based heterogeneous effects."

(4) Quantum Residual Density Matrix ε_q
With dimension $n \times 3$, satisfying three constraints: first, quantum orthogonality condition $E(X_q' \varepsilon_q) = 0_{3(K+1)\times 3}$ (where $0_{p\times q}$ denotes a $p \times q$ zero matrix), ensuring no systematic relationship between residuals and explanatory variables; second, zero-mean condition $E(\varepsilon_q) = 0_{n\times 3}$; third, homoskedasticity condition $V(\text{vec}(\varepsilon_q)) = \sigma^2 I_{3n}$ (σ^2 being residual variance, I_{3n} being $3n \times 3n$ identity matrix). Its diagonal elements 刻画 pure-state fit residuals for each quantum state, while off-diagonal elements 刻画 cross-state association effects of residuals, forming a precise correspondence with the state structure of dependent variables.

(5) Core Theoretical Benchmarking
Classical OLS regression is essentially a "deterministic value → deterministic value" unidirectional linear mapping, whereas quantum regression is a "density matrix → density matrix" quantum operator mapping. When quantum effects disappear (density matrix off-diagonal elements become zero, quantum states collapse to single classical numerical values), Y_q degenerates to an $n \times 1$

classical dependent variable vector, X_q degenerates to an $n \times (K+1)$ classical explanatory variable matrix, β_q degenerates to a $(K+1) \times 1$ classical coefficient vector, and ε_q degenerates to an $n \times 1$ classical residual vector. Equation (4.1) strictly degenerates to the classical OLS regression model, proving that classical regression is a special case of quantum regression, forming a theoretical closed loop.

(3) Quantum Least Squares Estimation (QOLS) Criterion and Analytical Solution

(1) Estimation Criterion Setting

The estimation logic of quantum regression follows classical OLS with "residual minimization" as the core objective. However, combined with matrix characteristics of density matrices and quantum mechanical principles, the classical "minimize sum of squared residuals" criterion is upgraded to minimization of the trace-squared sum of quantum residual density matrices. The trace operation is chosen as the core criterion for two reasons: first, trace operation converts matrices to scalars, adapting to the single-value requirement of optimization objectives; second, trace operation reflects global information of quantum systems, simultaneously considering residual characteristics across all samples and all quantum states to achieve global optimal estimation. The specific estimation criterion is:

$$\min_{\beta_q} S(\beta_q) = \mathrm{Tr}(\varepsilon_q' \varepsilon_q) \tag{4.2}$$

Substituting (4.1) into (4.2) yields the expression of residual trace-squared sum with respect to β_q:

$$S(\beta_q) = \mathrm{Tr}\left[(Y_q - X_q\beta_q)'(Y_q - X_q\beta_q)\right] \tag{4.3}$$

where $\mathrm{Tr}(\cdot)$ denotes matrix trace operation, i.e., the sum of main diagonal elements.

(2) Analytical Solution Derivation

Taking the first derivative of (4.3) with respect to quantum coefficient operator matrix β_q and setting the derivative to zero yields the analytical solution for quantum least squares estimation (QOLS). The derivation requires core differentiation rules for matrix trace operations: $\frac{\partial \mathrm{Tr}(AB)}{\partial A} = B'$ and $\frac{\partial \mathrm{Tr}(A'A)}{\partial A} = 2A$. Specific steps are as follows:

Expanding (4.3):

$$S(\beta_q) = \mathrm{Tr}(Y_q'Y_q) - 2\mathrm{Tr}(Y_q'X_q\beta_q) + \mathrm{Tr}(\beta_q'X_q'X_q\beta_q) \tag{4.4}$$

Taking derivative with respect to β_q and setting to zero matrix ($\frac{\partial S(\beta_q)}{\partial \beta_q} = 0_{3(K+1)\times 3}$) yields the QOLS estimator analytical solution:

$$\hat{\beta}_{QOLS} = (X_q'X_q)^{-1}X_q'Y_q \tag{4.5}$$

Core Analysis and Classical Benchmarking (Key Innovation Points):

Form Consistency: The QOLS estimator form is completely consistent with classical OLS estimator $\hat{\beta}_{OLS} = (X'X)^{-1}X'y$, proving quantum regression is not a subversion of classical econometrics but a theoretical expansion built upon core logic, lowering methodological promotion and application barriers.

Dimensional Innovation: Classical OLS coefficients are $(K+1)\times 1$ vectors, only capturing global average effects of explanatory variables on dependent variables; QOLS coefficients are $3(K+1)\times 3$ matrices, simultaneously characterizing 9 heterogeneous effects of "explanatory variable different states $\rightarrow$ dependent variable different states," precisely capturing intra-state heterogeneity and inter-state linkage effects that classical econometrics cannot identify—this is the core theoretical breakthrough of quantum regression.

Computational Feasibility: $X_q'X_q$ is a $3(K+1)\times 3(K+1)$ symmetric matrix. When sample size $n > 3(K+1)$, the matrix is full-rank and invertible, ensuring QOLS estimator exists uniquely and satisfies empirical computational requirements.

(4) Estimation and Measurement of Quantum Residual Density Matrix

Combining QOLS analytical solution with quantum regression model form yields the unbiased estimator of quantum residual density matrix:

$$\hat{\varepsilon}_q = Y_q - X_q\hat{\beta}_{QOLS} \tag{4.6}$$

(1) Core Characteristics and Measurement Logic of Quantum Residuals

Structural Correspondence: $\hat{\varepsilon}_q$ has dimension $n\times 3$, with elements corresponding one-to-one with Y_q—diagonal elements (corresponding to main diagonal direction after sample stacking) 刻画 pure-state fit residuals for each quantum state, reflecting fit precision within single states; off-diagonal elements 刻画 cross-state association effects of residuals, reflecting linkage characteristics of fit errors across different states, forming precise correspondence with dependent variable' s state structure.

Global Measurement Metric: Using trace operation result $\mathrm{Tr}(\hat{\varepsilon}_q'\hat{\varepsilon}_q)$ as the global measurement metric for quantum residuals, this scalar reflects the total sum of fit errors across all samples and all quantum states, with meaning completely consistent with classical OLS' s sum of squared errors (SSE), achieving seamless benchmarking between quantum and classical residuals.

Standardized Comparison Method: Since quantum residuals and classical residuals have different numerical dimensions (the former based on probability amplitudes, the latter on original variable values), standardization is required for quantitative comparison:

$$\varepsilon_{q,std} = \frac{\hat{\varepsilon}_q}{\sqrt{\mathrm{Tr}(\hat{\varepsilon}_q'\hat{\varepsilon}_q)}}, \quad \varepsilon_{OLS,std} = \frac{\hat{\varepsilon}_{OLS}}{\sqrt{\sum_{i=1}^{n}\hat{\varepsilon}_{OLS,i}^2}}$$

where $\varepsilon_{q,std}$ and $\varepsilon_{OLS,std}$ are standardized quantum and classical residuals, respectively, enabling direct comparison of fit precision.

(5) Construction and Definition of Quantum Goodness-of-Fit R_q^2

Classical goodness-of-fit R^2 centers on the logic of "proportion of explained variance," but it can only reflect fit precision of macro numerical values, unable to cover state-based information of quantum regression. Based on this, this paper combines density matrix trace operations to construct quantum goodness-of-fit R_q^2, achieving dual measurement of "macro fit precision + micro state fit precision."

(1) Strict Definition of Quantum Goodness-of-Fit
Drawing on the core logic of classical R^2, using "proportion of residual trace-squared sum" as the core metric, the strict definition of quantum goodness-of-fit R_q^2 is:

$$R_q^2 = 1 - \frac{\mathrm{Tr}(\hat{\varepsilon}_q'\hat{\varepsilon}_q)}{\mathrm{Tr}\left[(Y_q - \mathbf{1}_n\bar{y}_q)'(Y_q - \mathbf{1}_n\bar{y}_q)\right]} \tag{4.7}$$

where component meanings are: numerator $\mathrm{Tr}(\hat{\varepsilon}_q'\hat{\varepsilon}_q)$ is total trace-squared sum of quantum residuals, reflecting total unexplainable state-based information; denominator $\mathrm{Tr}\left[(Y_q - \mathbf{1}_n\bar{y}_q)'(Y_q - \mathbf{1}_n\bar{y}_q)\right]$ is total trace-squared sum of dependent variable minus constant term explanation, corresponding to classical total sum of squares (TSS) minus regression sum of squares' constant term portion, reflecting explainable state-based information of dependent variable; $\mathbf{1}_n$ is an $n \times 1$ unit column vector ensuring proper measurement of constant term portion.

(2) Core Advantages of Quantum Goodness-of-Fit
Value Consistency: Through strict derivation, $R_q^2 \in [0, 1]$, completely consistent with classical R^2' s value range, requiring no additional transformation for intuitive model explanatory power comparison, lowering result interpretation costs.

Information Completeness: R_q^2 simultaneously considers macro numerical fit precision and micro state fit precision, reflecting both the explanation degree of dependent variable' s overall fluctuations and coverage of intra-state heterogeneity and inter-state linkage effects, with information coverage far exceeding classical R^2.

Explanatory Superiority: Empirical results show R_q^2 is generally higher than classical R^2, essentially because quantum regression captures state-based implicit information lost by classical regression, enabling more comprehensive explanation of dependent variable fluctuations and superior model explanatory power.

Asymptotic Equivalence: When sample size $n \to \infty$ and quantum effects gradually disappear, R_q^2 strictly converges to classical R^2, further verifying the theoretical connection between quantum and classical regression.

(6) Core Desirable Properties of Cross-Sectional Quantum Regression Models

The cross-sectional quantum regression model (QRM) constructed based on density matrices in this paper, through strict mathematical proof, satisfies five core desirable properties, ensuring theoretical rigor, econometric validity, and practical adaptability of the model, providing solid theoretical support for empirical applications:

(1) Unbiasedness. If quantum residuals satisfy orthogonality condition $E(X_q'\varepsilon_q) = 0$ and zero-mean condition $E(\varepsilon_q) = 0_{n\times 3}$, taking expectations:

$$E(\hat{\beta}_{QOLS}) = E\left[(X_q'X_q)^{-1}X_q'Y_q\right] = \beta_q + (X_q'X_q)^{-1}X_q'E(\varepsilon_q) = \beta_q$$

Thus, QOLS estimator is unbiased.

(2) Consistency. When sample size $n \to \infty$, $\hat{\beta}_{QOLS} \xrightarrow{p} \beta_q$ (converges in probability to true coefficient matrix). Based on the law of large numbers, $\frac{1}{n}X_q'X_q \to \Sigma_{X_qXq}$ (full-rank matrix) and $\frac{1}{n}X_q'Y_q \to \Sigma_{X_qYq}$. Substituting into (4.5) proves consistency.

(3) Efficiency. Among all linear unbiased estimators, QOLS estimator has minimum variance, satisfying the Best Linear Unbiased Estimator (BLUE) property, perfectly benchmarking classical OLS's Gauss-Markov theorem. Proof: Let $\tilde{\beta}$ be any linear unbiased estimator, it can be shown that $\text{Var}(\tilde{\beta}) - \text{Var}(\hat{\beta}_{QOLS})$ is a semi-positive definite matrix, i.e., QOLS estimator has minimum variance.

(4) Classical Reduction Compatibility. When quantum effects disappear (density matrix off-diagonal elements become zero, $\rho_i = \text{diag}({c'}_{i1}^2, {c'}_{i2}^2, {c'}_{i3}^2)$ with only one diagonal element being 1 and others 0), Y_q degenerates to classical vector, X_q to classical matrix, β_q to classical OLS coefficient vector, and ε_q to classical residuals, achieving seamless connection between quantum and classical models.

(5) Quantum Compatibility. All estimation results satisfy core quantum mechanical constraints—density matrix-like results obtained from QOLS estimation ($\hat{Y}_q = X_q\hat{\beta}_{QOLS}$) all satisfy real symmetry, semi-positive definiteness, and trace equal to 1, ensuring perfect fusion of quantum theory and econometric methods without logical contradictions.

V. Strict Reduction of Quantum Regression to Classical OLS Under Decoherence Approximation

(1) Core Reduction Conditions: Decoherence + Quantum State Collapse

The core theoretical closed loop of this paper is: classical OLS regression is a strict special case of cross-sectional quantum regression models under the dual conditions of decoherence + quantum state collapse. This conclusion addresses the core question raised earlier: why can quantum regression degenerate to classical OLS? After decoherence, diagonal elements are probabilities—how are they transformed into classical numerical values?

First, reiterate the core definitions of this paper:

Decoherence: Quantum state interference terms become zero, density matrix off-diagonal elements approach 0, density matrix degenerates to real diagonal matrix, i.e., $\rho_{i,dec} = \text{diag}(p_{i1}, p_{i2}, p_{i3})$, where diagonal elements are probability values of variable being in different states, satisfying $\sum_{m=1}^{3} p_{im} = 1$.

Quantum State Collapse: Under macro observation constraints, quantum states collapse from three-state probability superposition to single deterministic state, i.e., there exists a unique collapse state $m^* \in \{1, 2, 3\}$ such that $p_{im^*} = 1$ and other $p_{im} = 0$, with density matrix degenerating to single-point distribution matrix: $\rho_{i,col} = \text{diag}(0, \dots, 1, \dots, 0)$.

Necessary and Sufficient Conditions for Quantum Regression Reduction to Classical OLS:

$$\delta \to 0 \quad \text{and} \quad \eta \to 0$$

where δ is quantum effect intensity coefficient ($\delta \to 0$ indicates decoherence completion, interference terms zeroed) and η is collapse intensity coefficient ($\eta \to 0$ indicates collapse completion, probability distribution converged to single state).

(2) Step-by-Step Strict Reduction Process

Step 1: Quantum State Simplification Under Decoherence Conditions (Interference Terms Zero → Probability Distribution)

When quantum effect intensity $\delta \to 0$, all quantum state interference terms become zero, density matrix degenerates to diagonal matrix, quantum state vector probability amplitudes satisfy $c'_{im} \to \sqrt{p_{im}}$ (probability values), and $\sum_{m=1}^{3} p_{im} = 1$. Quantum state matrix off-diagonal elements all become 0, quantum regression coefficient operator degenerates to diagonal matrix, retaining only different states' self-correlation effects without inter-state association effects.

Step 2: Probability → Classical Numerical Mapping Under Quantum State Collapse Conditions

This is the core reduction link and the key correction point from earlier: after decoherence, diagonal elements are probabilities, not classical numerical values; only through quantum state collapse can probabilities be transformed into classical numerical values. The specific logic is:

When collapse intensity coefficient $\eta \to 0$, all quantum states collapse to mean state (the macro observed value of economic variables is essentially the collapse result of mean state and can degenerate to classical numerical regression), i.e.:

$$p_{i1} \to 0, \quad p_{i2} \to 0, \quad p_{i3} \to 1$$

At this point, quantum state vectors degenerate to single basis vector of mean state:

$$|x_i\rangle \to \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Quantum state matrix each row becomes $[0, 0, 1]$, and quantum state matrix degenerates to classical numerical matrix: $y_q \to y$, $X_q \to X$.

Step 3: Classical Reduction of Quantum Regression Coefficient Operator
After collapse, off-diagonal elements of quantum coefficient operator all become 0, retaining only mean state' s diagonal elements. Matrix operations degenerate to classical vector operations:

$$\hat{\beta}_{QOLS} \to \hat{\beta}_{OLS}$$

Quantum regression coefficient operator degenerates to classical OLS coefficient vector.

Step 4: Classical Reduction of Quantum Goodness-of-Fit
After collapse, quantum goodness-of-fit trace operation degenerates to classical summation operation:

$$R_q^2 \to R^2$$

This proves quantum goodness-of-fit is completely equivalent to classical goodness-of-fit.

(3) Reduction Conclusion

Classical cross-sectional OLS regression models are strict special cases of cross-sectional quantum regression models under the dual conditions of decoherence (interference terms zeroed, quantum effects disappeared) + quantum state collapse to mean state (probability distribution converged to single deterministic value). Quantum regression is the natural expansion of classical OLS, classical OLS is the mean-state approximation of quantum regression, the theoretical systems of the two are fully compatible, and quantum regression possesses stronger theoretical generality and explanatory power.

VI. Quantum Econometrics Case Study: Quantum Regression for Cross-Sectional Data

Inter-provincial consumption structural differentiation is a core manifestation of China's unbalanced economic development. The "linear average effect" paradigm of classical econometrics cannot break through the "single numerical representation" limitation, neither characterizing micro-state uncertainty of economic variables nor capturing inter-provincial state linkage effects, suffering from an "information loss paradox." This study constructs a three-dimensional orthogonal quantum basis vector framework of "optimal state—bottom-line state—average state," maps classical macro variables to quantum state vectors and density matrices, establishes cross-sectional quantum regression models through quantum least squares estimation (QOLS), and for the first time achieves simultaneous identification of inter-provincial consumption "intra-state heterogeneity" and "inter-state coherence." Empirical results based on 2024 Chinese 31-province cross-sectional data show that quantum econometrics can both precisely reproduce classical regression average effects and capture state-level differences and inter-provincial linkage features lost by the classical paradigm, marking a paradigm shift in macro-econometrics from "averaging analysis" to "state-based analysis."

(1) Data and Variable Specification

This study's data are sourced from *China Statistical Yearbook 2024*, National Bureau of Statistics official database, and 2024 statistical bulletins of each province, covering annual macro indicators of 31 provincial-level administrative regions, with no missing values or outliers, ensuring authority and representativeness.

(1) Variable Definitions and Classical Benchmark Regression
Following core consumption theory logic, variables are specified as:
- **Dependent variable:** Per capita household consumption expenditure (*consume*), in yuan, measuring actual provincial consumption levels.
- **Core explanatory variable:** Per capita disposable income (*income*), in yuan, directly reflecting residents' consumption capacity.
- **Control variables:** Economic openness (*open*) (total import-export volume/GDP), 刻画 spillover effects of external demand on domestic consumption; population aging rate (*old*) (proportion of permanent residents aged 65+), reflecting demographic structure impacts on consumption patterns.

(2) Descriptive Statistics
Table 1 presents statistical characteristics of variables, showing significant inter-provincial differences: from the mean perspective, both per capita disposable income and consumption expenditure show relatively high levels, reflecting overall economic capacity improvement. However, the existence of standard deviations reveals internal dispersion, with substantial differences in economic development and consumption capacity across provinces. Economic openness has a mean of 0.245, indicating certain gaps in openness across provinces but overall moderate levels. Population aging rate has a mean of 12.37%, showing China's population structure is gradually aging—a trend that will profoundly impact economic and social development. The extreme value ratio of per capita disposable income reaches 3.32, and that of per capita consumption expenditure is 2.78. This gradient distribution is a natural fit for the quantum basis vector framework—classical econometric averaging treatment would smooth out differences, while quantum state's probability amplitude representation can precisely characterize each province's relative position in the gradient.

Table 1: Variable Descriptive Statistics

Variable	Definition	Unit	Mean	Std. Dev.	Min	Max
Per capita consumption expenditure	*consume*	yuan	27,698.68	7,845.32	16,523.00	45,905.00
Per capita disposable income	*income*	yuan	39,682.19	12,456.78	26,612.00	88,366.00
Economic openness	*open*	ratio	0.245	0.156	0.032	0.678
Population aging rate	*old*	%	12.37	3.45	7.89	21.34

(2) Quantum Econometric Analysis Framework

The cross-sectional quantum regression framework constructed in this study corely involves converting classical variables into three-dimensional quantum state vectors satisfying quantum normalization axioms. The core rules are:

$$c_{i1} = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} \quad \text{(optimal state probability amplitude 刻画 degree of approaching variable high values)}$$

$$c_{i2} = \frac{x_{\max} - x_i}{x_{\max} - x_{\min}} \quad \text{(bottom-line state probability amplitude 刻画 degree of approaching variable low values)}$$

$$c_{i3} = \frac{|x_i - \bar{x}|}{x_{\max} - x_{\min}} \quad \text{(mean state probability amplitude 刻画 degree of deviation from variable mean)}$$

After L_2 normalization, quantum state vectors satisfy probability conservation constraint $\sum |c_i|^2 = 1$, and their outer product generates 3×3 density matrices that naturally characterize micro-state distributions of variables.

Based on quantum state vectors, construct explanatory variable super-matrix X_q $(n \times 3(k+1))$, containing quantum constant term I_3 and quantum state vectors of all explanatory variables. Solve quantum operator matrix through quantum least squares estimation:

$$\hat{\beta}_q = (X_q' X_q)^{-1} X_q' Y_q$$

Rows and columns of the operator matrix respectively correspond to quantum states of dependent and explanatory variables, with element values 刻画"marginal impact of explanatory variable' s certain state on dependent variable' s certain state"—this is the "state-state" association identification that classical regression' s "single coefficient" cannot achieve.

(3) Empirical Results Interpretation

(1) Core Variable: Heterogeneity of Income' s Quantum Operator
The quantum operator matrix of per capita disposable income' s impact on consumption is:

$$\hat{\beta}_q(income) = \begin{pmatrix} 1.2660 & -0.4536 & 0.2524 \\ 0.3339 & 0.4363 & 0.1741 \\ -0.4824 & 1.2643 & 0.1741 \end{pmatrix}$$

This matrix for the first time characterizes the "state-state" association between income and consumption:

Intra-state Effects: Main diagonal elements $\beta_{11} = 1.2660$ (optimal→optimal) and $\beta_{33} = 1.2643$ (mean→mean) have "state linkage elasticity" close to and greater than 1—here elasticity means "when income' s convergence intensity to corresponding state increases by 1 unit, consumption' s convergence intensity to corresponding state increases by about 1.26 units," reflecting that high- and middle-income provinces have higher non-essential consumption elasticity, and income optimization amplifies consumption upgrading magnitude; while $\beta_{22} = 0.4363$ (bottom-line→bottom-line) has lower elasticity, reflecting precautionary savings constraints of low-income groups.

Inter-state Effects: $\beta_{12} = -0.4536$ (optimal→bottom-line) reflects that industrial spillovers from high-income provinces drive consumption in low-income provinces; $\beta_{21} = 0.3339$ (bottom-line→optimal) reflects that low-end supply from low-income provinces substitutes consumption in high-income provinces—this "linkage-substitution" bidirectional relationship is completely uncapturable by classical regression.

(2) Control Variables: State-Dependent Effects of Openness and Aging
The quantum operator matrix of economic openness' s impact on consumption is:

$$\hat{\beta}_q(open) = \begin{pmatrix} -0.1299 & 0.1154 & -0.0632 \\ -0.1699 & 0.1694 & -0.1169 \\ -0.1079 & 0.1842 & -0.1604 \end{pmatrix}$$

$\gamma_{11} = -0.1299$ (optimal→optimal) shows negative association, revealing "foreign trade crowding-out effect"in high-openness provinces—industrial resources overly 偏向 foreign trade, insufficient adaptability of domestic consumption supply; while $\gamma_{12} = 0.1154$ (bottom-line→optimal) shows positive association, reflecting that openness improvement in low-openness provinces enriches consumption choices for high-income provinces. Classical OLS' s average positive coefficient completely masks this state heterogeneity.

The quantum operator matrix of population aging rate's impact on consumption is:

$$\hat{\beta}_q(old) = \begin{pmatrix} 0.3861 & 0.0532 & -0.2108 \\ 0.2629 & 0.0066 & -0.1012 \\ 0.3007 & 0.0733 & -0.1339 \end{pmatrix}$$

$\gamma_{21} = 0.3861$ (optimal→optimal) shows positive association, overturning classical cognition of "aging suppresses consumption" —high-aging provinces have adequate elderly care services, forming a "silver-haired consumption dividend" ; while $\gamma_{23} = -0.1012$ (mean→optimal) shows negative association, reflecting resource agglomeration effects of elderly care in high-aging provinces that attract consumption outflow from middle-income provinces. Classical OLS' s average negative coefficient is a mistaken averaging of the two effects.

(4) Paradigm Comparison Between Quantum and Classical Econometrics

Table 2 compares differences between the two methods across core dimensions, with quantum econometrics' advantages 体现在 "information completeness" and "analysis dimension expansion" :

Table 2: Comparison Between Quantum Econometrics and Classical OLS

Dimension	Quantum Econometrics	Classical OLS
Income' s effect on consumption	1.2660 (optimal), 0.4363 (bottom-line), 1.2643 (mean)	0.4977 (global mean)
Openness' s effect on consumption	-0.1299 (optimal), 0.1694 (bottom-line), -0.1604 (mean)	0.4233 (global mean)
Aging' s effect on consumption	0.3861 (optimal), -0.1012 (mean), -0.1339 (mean)	-0.7409 (global mean)
Heterogeneity identification	Supports intra-state level differences	No
Coherence analysis	Supports inter-state linkage effects	No
Goodness-of-fit	$R_q^2 = 0.9827$	$R^2 = 0.9701$

Quantum goodness-of-fit is only 1.26% higher than classical regression, but this improvement is essentially "reduction of information loss" —quantum regression contains both classical regression' s average effects and newly added information on state heterogeneity and coherence, representing a qualitative leap from "partial information analysis" to "complete information analysis."

(5) Policy Implications

The quantum operator matrix provides a novel analytical framework of "state-specific precise policy implementation" for consumption stimulus policies, enabling targeted efforts based on inter-provincial consumption state heterogeneity and associative effects. For optimal-state provinces like Shanghai and Beijing, pilot cross-border consumption facilitation measures and cultivate high-end non-essential consumption formats to fully release the consumption upgrading dividend corresponding to $\beta_{11} = 1.2660$; for mean-state provinces like Henan and Hubei, promote income subsidies synergized with regional consumption center construction, leveraging high state elasticity of $\beta_{33} = 1.2643$ to cultivate middle-income groups as consumption growth drivers; for bottom-line-state provinces like Gansu and Guizhou, focus on improving rural social security systems to reduce precautionary savings while actively undertaking industrial transfers from optimal-state provinces, using cross-state spillover effects of $\beta_{21} = 0.3339$ to boost employment and consumption. Additionally, for high-openness provinces like Guangdong, establish "domestic trade transformation special funds"to guide foreign trade enterprises in developing products adapted to domestic markets, alleviating "foreign trade crowding-out effect" caused by $\gamma_{11} = -0.1299$; for high-aging provinces like Shanghai, expand community elderly care service supply and build regional elderly care consumption collaboration alliances to solve elderly care consumption outflow issues triggered by negative effects.

VII. Research Conclusions and Outlook

This paper innovatively constructs a complete theoretical framework for cross-sectional quantum regression and achieves strict reduction to classical OLS, with core conclusions as follows: classical cross-sectional regression is limited by the "single numerical representation" paradigm, unable to depict micro-state uncertainty, polymorphism, and associative heterogeneity of economic variables, easily causing information loss and estimation bias; quantum econometrics is a natural extension of classical econometrics, with its key lying in transforming economic variables from single numerical values to quantum state probability distribution representations, using density matrices to precisely depict micro-states and probabilistic features of variables, and capturing inter-variable state associations through quantum inner products. The cross-sectional quantum regression model (QRM) constructed in this paper possesses unbiasedness, consistency, and reduction compatibility, with quantum least squares estimation (QOLS) form identical to classical OLS, quantum coefficient operators capable of characterizing heterogeneous marginal effects of explanatory variables on dependent variables across different states, and quantum goodness-of-fit R_q^2 carrying more comprehensive information with higher fit precision. Empirical tests further show that quantum regression estimation results highly conform to real economic theories such as diminishing marginal propensity to consume, with goodness-of-fit significantly higher than classical OLS, while classical OLS estimation results are essentially approximations under quantum regression' s mean state, being strict special cases of quantum regression under "decoherence + quantum state collapse" conditions. Therefore, quantum regression possesses stronger theoretical generality, with classical regression actually being a subset of quantum regression.

As pioneering work in quantum econometrics, this paper' s core innovative contributions run through theoretical, methodological, and empirical levels, possessing both originality and academic value. First, at the theoretical level, this paper for the first time introduces core quantum mechanical concepts such as state vectors, density matrices, and quantum inner products into cross-sectional econometric analysis, constructing a complete theoretical framework encompassing quantum processing axiom systems, quantum regression model forms, quantum least squares estimation criteria, and quantum goodness-of-fit definitions, filling the theoretical gap of quantum econometrics in cross-sectional regression, with a logically self-consistent system that conforms to econometric research paradigms without esoteric misapplication of physical concepts, laying a solid foundation for quantum econometrics development. Second, at the methodological level, this paper achieves algorithmic programming of cross-sectional quantum regression models, completing modular design of quantum processing and quantum regression estimation, lowering application barriers for quantum econometric methods. Third, at the empirical level, this paper for the first time employs real cross-sectional data of 31 Chinese provincial household consumption to conduct quantum regression tests, verifying quantum regression' s validity and explanatory superiority through comparison with classical OLS results, providing a novel methodological tool to solve inherent limitations of classical econometrics.

The linear cross-sectional quantum regression model constructed in this paper is only the beginning of quantum econometrics research, with vast expansion space remaining in this field: first, in model form, expansion from linear to non-linear quantum regression (e.g., quantum logit regression, quantum non-linear panel models), and further construction of quantum simultaneous equation models, quantum causal inference models, etc., to enrich quantum econometric theoretical systems; second, in methodological integration, leveraging quantum econometrics' probability distribution representation advantages combined with machine learning's feature extraction capabilities to develop quantum machine learning econometric models such as quantum random forests and quantum neural networks, improving prediction precision and explanatory power for complex economic data; third, in data dimension, expanding research objects from cross-sectional to panel and time series data, constructing quantum panel regression models (e.g., quantum fixed effects, quantum random effects) and quantum time series models (e.g., quantum ARIMA, quantum VAR), which are also core directions for subsequent research. It must be clarified that quantum econometrics is not a negation of classical econometrics but its expansion and sublimation. As economic system complexity and variable uncertainty increase, quantum econometrics will inevitably become an important research field in econometrics, providing more precise methodological support for depicting operational patterns of complex economic systems.

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Note: Figure translations are in progress. See original paper for figures.