Massive Star Formation at Supersolar Metallicities: Constraints on the Initial Mass Function (postprint)

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If you have a specific text to translate or a particular Aleksei Sorokin in mind (e.g., mathematician, athlete, etc.), please provide more details or the text., Zhiqiang Yan, Ziyi Guo, Tereza Jerabkova, Akram Hasani Zonoozi, Hosein Haghi

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Abstract

Metals enhance the cooling efficiency of molecular clouds and thereby promote fragmentation. As a result, increasing the metallicity may increase the formation rate of low-mass stars. Within the framework of the integrated galactic initial mass function (IGIMF) theory, this effect is empirically encoded by a linear relation between the slope of the low-mass stellar IMF, $ $1, and the metal mass fraction, Z. This linear $ $1–Z relation has been calibrated only up to 2 Z , although environments with higher metallicities are known to exist. We demonstrate that if the linear $ $1–Z relation is extrapolated to higher metallicities ([Z] 0.5), the formation of massive stars is entirely suppressed. Alternatively, the efficiency of fragmentation may saturate above some metallicity threshold if gravitational collapse proceeds sufficiently rapidly in a cascading manner. To capture this behavior, we propose a logistic function that describes the transition from a metallicity-sensitive to a metallicity-insensitive fragmentation regime. We also provide a user-friendly public code, pyIGIMF, which enables instantaneous computation of the IGIMF with both linear and logistic $ $1–Z relations.

Full Text

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Abstract

Metals enhance the cooling efficiency of molecular clouds, promoting fragmentation. Consequently, increasing the metallicity may boost the formation of low-mass stars. Within the integrated galactic initial mass function (IGIMF) theory, this effect is empirically captured by a linear relation between the slope of the low-mass stellar IMF, α_1, and the metal mass fraction, Z. This linear α_1-Z relation has been calibrated up to 2 Z , though higher metallicity environments are known to exist. We show that if the linear α_1-Z relation extends to higher metallicities ([Z]   0.5), massive star formation is suppressed entirely. Alternatively, fragmentation efficiency may saturate beyond some metallicity threshold if gravitational collapse cascades rapidly enough. To model this behavior, we propose a logistic function describing the transition from metallicity-sensitive to metallicity-insensitive fragmentation regimes. We provide a user-friendly public code, pyIGIMF, which enables the instantaneous computation of the IGIMF theory with both linear and logistic α_1-Z relations.

Key words: galaxies: star formation -stars: luminosity function, mass function -galaxies: luminosity function, mass function -methods: numerical

1. Introduction

Stars form in the coldest and densest clumps of molecular clouds (hereafter molecular cloud clumps, Bergin & Tafalla 2007; McKee & Ostriker 2007). Each clump produces an embedded cluster, constituting a single stellar population (Lada & Lada 2003; Portegies Zwart et al. 2010). Clumps that produce embedded clusters are sub-pc structures sufficiently massive to undergo collapse under self-gravitation (Kroupa et al. 2026, their Section 3.1). With their short lifetimes (usually 1 Myr), embedded clusters are the cradles of individual star-forming events (for reviews, see Kroupa et al. 2013; Hopkins 2018; Kroupa et al. 2026).

The number distribution of stellar masses, $\xi(m) = dN/dm$, for stars formed in a single star-forming event is known as the stellar initial mass function (sIMF). Over its lifetime of $\delta t \approx 10$ Myr (Lada & Lada 2003; Weidner et al. 2004), a molecular cloud will form several embedded clusters whose masses are distributed according to the embedded cluster mass function (ECMF). In galaxies with ongoing star formation, many molecular clouds coexist within each δt interval. Their combined sIMFs give rise to a galaxy-wide IMF (gwIMF). A conceptual illustration is shown in Jeřábková et al. (2018), their Figure 1 [Figure 1: see original paper]. See also Jerabkova et al. (2025).

If the sIMF were a scale-free probability distribution function (PDF, e.g., Elmegreen 1997, 2006; Kroupa & Jerabkova 2021), summing sIMFs from multiple star-forming regions would consistently produce the same composite sIMF, differing only by Poisson variations and normalizations (Yan et al. 2023). In this case, the gwIMF would have the same shape as the sIMF. This is a common approach when modeling galaxies (e.g., Borrow et al. 2023). There are, however, arguments indicating that the gwIMF does not always coincide with individual sIMFs (Kroupa et al. 2026).

One of the most mature gwIMF models grounded on the physical properties of galaxies is the "integrated galactic initial mass function" (IGIMF) theory, whose historical development and observational evidence is presented in Kroupa et al. (2013, 2026). Based on observations of the sIMF variability (Kroupa 2002), Kroupa & Weidner (2003) proposed to calculate the gwIMF as the cumulative sum of all sIMFs forming within a galaxy over a time interval of δt.

The IGIMF is founded on the following principles, further motivated in Section 4: (i) The gwIMF can be described as the cumulative sum of sIMFs forming within a galaxy over a short ($\delta t = 10^7$ yr, Weidner et al. 2004) time interval, which is the typical lifetime of molecular clouds (Section 4.2). (ii) The sIMF, and consequently the gwIMF, are variable. Environmental properties characterizing embedded clusters, such as metallicity and clump gas density, $\rho_{\rm cl}$, affect the resulting shape of the sIMF. We discuss observational evidence for the variability of the sIMF and gwIMF in Section 4.4. (iii) For massive stars to form, a gravitational collapse of dense gas structures must occur (see Kroupa et al. 2026, their Section 3). If clumps are not sufficiently massive and dense, they cannot sustain the generation of massive stars. A tight relation arises between the most massive star, $m_{\rm max}$, formed in an embedded clump, and the total stellar mass produced in the embedded clump, $M_{\rm ecl}$ (Yan et al. 2023). This relation is encapsulated mathematically with optimal sampling (Section 4.6), which is observationally favored over stochastic approaches (Section 4.5).

In brief, optimal sampling (Equations (9) and (12) below) is a deterministic method for generating stellar populations from the sIMF. In optimal sampling, the stellar mass budget of an embedded cluster, $M_{\rm ecl}$, determines the maximum stellar mass, $m_{\rm max}$, that can form. A cloud clump would not generate higher mass stars unless lower mass stars can fully populate the sIMF (Gjergo, Zhang & Kroupa, submitted). This is consistent with low-mass stars first condensing along the molecular filaments. As the clump contracts, the filaments connect at the densest region, channeling the gas flow onto the more-massive and most-massive stars (Kroupa et al. 2026, and references therein).

Optimal sampling is particularly relevant for small embedded clusters. While, in principle, a $10^3 M_\odot$ embedded cluster could accommodate the formation of a $100 M_\odot$ star, this may only occur at the cost of leaving the lower-mass regime underpopulated. This would introduce gaps and Poisson noise that are not observed (expanded further in Section 4.6). The mathematical and physical principles of optimal sampling have been developed in Gjergo, Zhang & Kroupa (submitted).

1.1. Formalisms, Nomenclature, and Observations

Note that definitions such as $\rho_{\rm cl}$, $M_{\rm ecl}$, and most quantities in these formalisms are auxiliary states that encapsulate patterns of star formation (Kroupa et al. 2013, 2026). The very idea of an IMF is an auxiliary mathematical construct (Hilfskonstrukt in the original German, Kroupa & Jerabkova 2018): it does not represent any single instantaneous configuration observed in nature. Rather, it is an empirical pattern that serves as a statistical descriptor of stellar populations formed under similar physical conditions. They enable theoretical modeling that leads to a refined interpretation of integrated observations.

The lowercase m is used for stellar masses, while the upper case $M_{\rm ecl}$ is used for embedded cluster masses. The subscript "cl" refers to the progenitor gas strictly involved in the star formation process, while "ecl" refers to the total stellar mass of embedded clusters. The terms "top-heavy," "top-light," "bottom-heavy," and "bottom-light" have precise definitions (see Kroupa et al. 2026): relative to the solar mass (1 $M_\odot$), "top" ("bottom") refers to the high-mass (low-mass) end of the sIMF distribution. These terms are defined through comparison with the canonical sIMF (Kroupa 2001), where "heavy" ("light") denotes a number distribution higher (lower) than canonical values. Although originally derived for the solar neighborhood, the canonical sIMF happens to characterize well the typical sIMF of the Milky Way, hence the designation "canonical" (see Equations (2) and (3)).

1.2. Features and Perks of the IGIMF Theory

If a molecular cloud clump is not sufficiently massive, its gravitational potential may be too shallow to induce the dense local overdensities required for the formation of massive stars (see Pflamm-Altenburg & Kroupa 2008, 2009). Therefore, the sIMF is affected by the embedded cluster stellar mass, $M_{\rm ecl}$, or equivalently by the molecular cloud clump density, $\rho_{\rm cl}$. The sIMF is also affected by metallicity, because the presence of metals enhances cooling, which in turn increases fragmentation of a cloud (see Section 4.4). Since the sIMF varies, and the gwIMF is constructed from the ensemble of sIMFs, the gwIMF will also vary accordingly.

The IGIMF theory proves to be naturally consistent with observations. For example, the IGIMF predicts that a low-mass galaxy with a small star formation rate (SFR) will have a top-light gwIMF (Weidner & Kroupa 2005, their Figure 11 [Figure 11: see original paper]; Jeřábková et al. 2018; Yan et al. 2020; Haslbauer et al. 2024; Zonoozi et al. 2025). This is consistent with the mass-metallicity relation observed in galaxies (Köppen et al. 2007; Recchi et al. 2015; Yan et al. 2020, 2021; Haslbauer et al. 2024), and by implication, it explains the deficient Hα-over ultraviolet (UV)-luminosity observed by Lee et al. (2009) for dwarf galaxies (Pflamm-Altenburg et al. 2009). And indeed, there is direct evidence for a lack of massive stars in star-forming dwarf galaxies such as DDO 154 (Watts et al. 2018).

In the starburst environment of 30 Dor in the Large Magellanic Cloud, Schneider et al. (2018) found an excess of massive stars compared to a canonical IMF by counting the stars. In the context of star-forming dwarf galaxies, the IGIMF theory leads to a significantly higher SFR than what is derived through an invariant sIMF. This results in better agreement with the stellar mass build-up in dwarf galaxies, which may form with nearly constant SFRs in less than a Hubble time (Pflamm-Altenburg & Kroupa 2009). Observationally, the low stellar output of these systems has often been interpreted as evidence for low star formation efficiency (Shi et al. 2014), which is equivalent to an sIMF variation producing too few massive stars.

On the higher end of the SFR spectrum, Gunawardhana et al. (2011) noted with Hα and UV observations that massive star-forming disk galaxies have top-heavy gwIMFs. For extreme cases of dusty gas-rich starburst conditions, Zhang et al. (2018a) found that only a top-heavy gwIMF can explain the observed isotopic abundance ratios of ^{13}CO and C^{18}O. This can be further extended to the so-called "main-sequence" galaxies in the early Universe, which have relatively high SFRs (Guo et al. 2024). These are accounted for naturally by the IGIMF theory (Weidner et al. 2013b; Yan et al. 2017; Jeřábková et al. 2018). Simulations of star-forming tidal-dwarf galaxies produce larger stellar masses when the IGIMF is applied, since low SFRs permit substantially more stellar mass build-up than under an invariant gwIMF (Ploeckinger et al. 2014). In the latter case, massive stars would form even at low SFRs, potentially disrupting the dwarf galaxy through feedback given its shallow gravitational potential. Along similar lines, hydrodynamical simulations of star-forming dwarf galaxies by Steyrleithner & Hensler (2023) show that the gwIMF calculated from the IGIMF leads to oscillations in the SFR with periods comparable to the free fall time of the cloud. The IGIMF-based SFRs are, on average, higher than those obtained using an invariant gwIMF that permits a constant fraction of massive stars. The model galaxy also experiences distinct chemical evolution patterns in the two scenarios.

The IGIMF is also consistent with properties of axisymmetric disk galaxies, whose extended UV disk is larger than the Hα-emitting disk (Pflamm-Altenburg & Kroupa 2008). While all stars emit in the UV, only massive stars contribute to Hα emission. In an exponential galactic disk, only the inner regions reach surface gas densities high enough to sustain the formation of massive stars. Low-mass star formation can still occur at larger galactocentric radii in smaller embedded clusters. This has also been seen from maser studies in our Milky Way, where Sun et al. (2018) found a lack of masers associated with massive stars in the outer Galactic disk compared to that in the inner disk. In these outer regions, however, star formation is Hα-dark, meaning that O-type stars are not produced (other SFR tracers may be missing as well, see also Section 3.2).

The reported top-heavy gwIMFs in starburst galaxies at redshifts $2 < z < 3$ are also consistent with the theoretical expectations of the IGIMF (Zhang et al. 2018b). It therefore comes as no surprise that the IGIMF theory, which implies an SFR- and Z-dependent gwIMF shape, accounts well for the properties of elliptical galaxies which needed to have formed with top-heavy gwIMFs (Yan et al. 2021): the metallicity and photometric properties of the stellar populations in massive elliptical galaxies considered together imply that said stellar populations must have formed very rapidly (within 1 Gyr, Eappen et al. 2022, and references therein) with SFRs between 10^3 and $10^4 M_\odot$ yr^{-1} and very early in cosmic history, with negligible star formation at later ages (e.g., Salvador-Rusiñol et al. 2020) and with major implications for the cosmic microwave background (Gjergo & Kroupa 2025). Furthermore, under the IGIMF theory, the gwIMF of an elliptical galaxy at supersolar metallicity will be very bottom-heavy. Strikingly, the masses of supermassive black holes (SMBHs) and their scaling with host galaxy mass emerge naturally within the IGIMF theory (Kroupa et al. 2020).

1.3. This Work

In this work, we extend the analysis to supersolar metallicities up to 10 Z . We compare the behavior of the sIMF and gwIMF within the IGIMF framework in this regime. We find that the production rate of low-mass stars increases with metallicity. In other words, the sIMF becomes increasingly bottom-heavy.

The low-mass (bottom) end of the sIMF is highly sensitive to metallicity. We consider the possibility that the current linear dependence of the low-mass sIMF slope, α_1(Z), can be extrapolated to extreme values. With the existing literature, estimates of the α_1(Z) slope can be inferred for systems of up to Z = 2 Z (i.e., for metal mass fractions of Z 0.03 and metallicities [Z] 0.3, see Appendix

C.1 for definitions). At these metallicities, the dependence of fragmentation on metal content may break down. A transition could then occur to a metallicity-insensitive regime in which α_1(Z) no longer increases with metallicity.

We provide an easy-to-use open-source Python package that quickly calculates stellar populations of galaxies. The new pyIGIMF package is applied here to predict the $m_{\rm max}$-$M_{\rm ecl}$ relation for galaxies of different metallicity. At present, this relation is constrained only near solar metallicity. But since metal-poor and metal-rich gas of the same density exhibit different cooling rates and fragmentation behavior, and because stellar feedback depends on Z, the relation is expected to shift with metallicity. This shift can be computed under the hypothesis that the sIMF is a ρcl- and Z-dependent optimally sampled distribution function (ODF). At subsolar metallicity, the sIMF has been found to be top-heavy (e.g., Banerjee & Kroupa 2012; Kalari et al. 2018; Schneider et al. 2018; Yasui et al. 2023) such that the $m_{\rm max}$-$M_{\rm ecl}$ relation should shift to smaller $M_{\rm ecl}$ for a given $m_{\rm max}$. The currently known metal-dependence of the sIMF suggests that the inner regions of massive elliptical galaxies that are at super-solar abundance ought to have very bottom-heavy gwIMFs, as has been argued from spectroscopic analysis (e.g., van Dokkum & Conroy 2010; Salvador-Rusiñol et al. 2021; van Dokkum & Conroy 2021). Such a significantly bottom-heavy sIMF will lead to a significant shift in the $m_{\rm max}$-$M_{\rm ecl}$ relation for metal-rich very young star clusters toward large $M_{\rm ecl}$ values that can be observationally tested. We emphasize that if and only if the sIMF is a systematically variable ODF, described above and quantified below, then the $m_{\rm max}$-$M_{\rm ecl}$ relation will shift as proposed.

In Section 2 we overview the theoretical framework of the IGIMF, which includes both the distribution of the ECMF and of the sIMF. We also present an alternative prescription for the low-mass end of the sIMF. In Section 3 we present variations of the sIMF over a wide range of physical conditions, and we also quantify the shift of the $m_{\rm max}$-$M_{\rm ecl}$ relation with metallicity and cloud core density. Other approaches for formulating the IMF on the galaxy-wide scale have recently been proposed, and we discuss these in comparison to the IGIMF theory in Section 4.1. A historical overview of the development of the IGIMF theory, along with its observational support, is presented in Section 4.

2. Methods

In this section, we first start from the concept of the canonical IMF (Section 2.1). We present the mathematical formulation of the IGIMF theory, which is given by the cumulative sum of sIMFs (Section 2.2) relative to the embedded cluster mass distribution (Section 2.3). To derive the gwIMF, we integrate the sIMFs over the ECMF distribution (as explained in Section 2.4). We then present an alternative prescription (Section 2.5) for the low-mass end of the sIMF slope, α_1. Interestingly, the α_1 prescription affects the $m_{\rm max}$-$M_{\rm ecl}$ relation for high metallicities.

2.1. The Canonical IMF

The concept of the IMF as the distribution of initial stellar masses was introduced by Salpeter (1955, S55) (for an overview, see Kroupa et al. 2026, their Section 1), where N is the number of stars, m is the stellar mass, k is a normalization constant, and $\alpha_{\rm S55} = 2.35$ is the Salpeter slope (see also Kroupa & Jerabkova 2019 for a historical overview of Salpeter' s contribution). The sIMF is typically normalized as a PDF, such that the integral of $\xi_{\rm S55}(m)$ over the full mass range is unity.

Kroupa (2001) drew on detailed studies of the solar neighborhood, including nearby star-forming regions and open clusters, and anchored theoretical IMF models in resolved stellar populations rather than integrated-light assumptions, as in some earlier works. The empirical features of the resulting IMF are well described by a broken power law where the slopes, α_i, are defined according to the mass interval:

$$\xi_{\rm can}(m) = \begin{cases} k_1 m^{-\alpha_{1,\rm can}} & \text{for } 0.08 \le m/M_\odot < 0.5 \\ k_2 m^{-\alpha_{2,\rm can}} & \text{for } 0.5 \le m/M_\odot < 1.0 \\ k_3 m^{-\alpha_{3,\rm can}} & \text{for } 1.0 \le m/M_\odot \le 150 \end{cases}$$

with $\alpha_{1,\rm can} = 1.3$, $\alpha_{2,\rm can} = 2.3$, and $\alpha_{3,\rm can} = 2.3$. The turnover point at 0.5 solar masses reflects the distribution of the stellar masses, as shown in Kroupa (2001). It is mathematically convenient, implying a fixed relationship of the normalization constants: $k_1 = k_2 = k_3$, without requiring additional continuity conditions.

The above IMF (Equation (2) along with (3)) is also known as the "canonical" IMF because, while being derived in the solar neighborhood, it also applies to the whole Galaxy, as well as to most Milky-Way-class spiral galaxies. Given that spiral galaxies constitute the most commonly observed morphology, works involving large cosmic volumes may use the canonical IMF as a "universal" IMF. However, the earlier notion of an invariant IMF has long been superseded. Andrew Hopkins remarked at the ESO/GALCROSS conference (2024, Brno, Czech Republic) that the question is no longer whether the IMF varies, but rather by how much.

Using the detailed analysis by Kroupa (2001), Chabrier (2003) replaced the segmented power-law slopes α_1 and α_2 of the low-mass regimes with a smooth lognormal function. This modification yields results that are observationally indistinguishable from Kroupa (2001), and both formulations are suitable and equivalent representations of the canonical IMF.

Some authors, when referring to the "Kroupa IMF" actually intend the Kroupa et al. (1993) formulation, where the high-mass slope, $\alpha_3 = 2.7$, was based on the Scalo (1986) analysis of massive stars in the nearby Galactic field, out to a few kpc.

Kroupa (2005), Kroupa et al. (2013) and Hopkins (2018) emphasized the importance of distinguishing between the stellar initial mass function, sIMF, defined for individual star-forming regions (i.e., embedded clusters) and the galaxy-wide IMF, gwIMF, that characterizes an entire galaxy. Throughout this manuscript, we explicitly differentiate between these two forms and avoid using the ambiguous acronym "IMF." Figure 1 summarizes the features of the canonical sIMF as it was formulated in Kroupa (2001). It fits well Milky Way-like spiral galaxies because of the prevalence of moderate SFRs (a few $M_{\odot}$ yr^{-1}) at around solar metallicities in this morphology.

The sIMF is defined from 0.08 $M_{\odot}$ up to a theoretical upper limit of 150 $M_{\odot}$. The true upper stellar mass in a given embedded cluster may fall below the theoretical maximum mass. Objects less massive than 0.07-0.08 $M_{\odot}$ are brown dwarfs. They have a distinct origin and are characterized by their own IMF (Thies et al. 2015 and Kroupa et al. 2026, see their Figure 7 [Figure 7: see original paper]).

2.2. The Stellar Initial Mass Function

The sIMF for individual embedded clusters, $\xi(m|M_{\rm ecl}, Z) = dN/dm$, is a function of stellar mass, m, and will vary in shape according to the embedded cluster mass, $M_{\rm ecl}$, as well as metal mass fraction, Z. $\xi(m|M_{\rm ecl}, Z)$ is defined as (Kroupa 2001; Kroupa et al. 2026)

$$\xi(m|M_{\rm ecl}, Z) = \begin{cases} k_1 m^{-\alpha_1(Z)} & \text{for } m_{\rm min} \le m/M_{\odot} < 0.5 \\ k_2 m^{-\alpha_2(Z)} & \text{for } 0.5 \le m/M_{\odot} < 1.0 \\ k_3 m^{-\alpha_3(\rho_{\rm cl}, Z)} & \text{for } 1.0 \le m/M_{\odot} \le m_{\rm max} \end{cases}$$

where the lower mass limit is set to $m_{\rm min} = 0.08 M_{\odot}$, k_1 is the normalization constant, and k_2, k_3 ensure continuity of the power law segments.

A functional dependence of the sIMF power-law indices on the iron abundance, [Fe/H] (Equation (C3)), was first introduced by Kroupa (2002) and then applied by Marks et al. (2012). The modern formulation by Yan et al. (2021), which uses the metal mass fraction, Z (Equation (C1)), is encapsulated as follows:

$$\alpha_1(Z) = \alpha_{1,\rm canon} + \Delta\alpha \cdot Z$$

where $Z_{\odot} = 0.0142$ refers to the solar metal mass fraction, from Asplund et al. (2009). Yan et al. (2021) found the fit for $\Delta\alpha$ to be $\Delta\alpha = 63$.

The high-mass sIMF slope, α_3, depends weakly on the metallicity, and more strongly on the gas clump density, $\rho_{\rm cl}$. Since $\rho_{\rm cl}$ characterizes the conditions during the protostellar phase, it effectively encapsulates both the initial gas mass and the resulting stellar mass, $M_{\rm ecl}$, of the embedded cluster. Marks et al. (2012) and then Yan et al. (2021) obtain the following best fits:

$$\alpha_3(\rho_{\rm cl}, Z) = \begin{cases} 2.3 & \text{if } x < 0.14 \\ 2.3 + 0.87x + 0.41x^2 & \text{if } x \geq 0.14 \end{cases}$$

where x is described by

$$x = -0.99 \log_{10}\left(\frac{\rho_{\rm cl}}{10^6 M_\odot {\rm pc}^{-3}}\right) + 0.63[Z]$$

The -6 comes from normalizing $\rho_{\rm cl}$ inside of the log by 10^6. These relations make the sensible assumption (Kroupa et al. 2026 and references therein) that the star formation efficiency of a clump is 33% (1/3 of the gas clump mass is condensed into stars), leading the embedded clusters to follow a radius-mass relation

$$\rho_{\rm cl} = 0.61 M_{\rm ecl}^{-0.11} \times 10^{2.85}$$

The above equation assumes astronomical units, i.e., $M_\odot$ for $M_{\rm ecl}$ and $M_\odot$ pc^{-3} for $\rho_{\rm cl}$ (see also Section 4.4 and Section 1.1).

Figure 2 [Figure 2: see original paper] shows the range of α_3 values as a function of its physical parameters, namely metallicity and $M_{\rm ecl}$. Note that there is no clear evidence for sIMFs with $\alpha_3 > 2.3$ (Kroupa et al. 2026). The steeper Galactic-field or solar neighborhood IMFs for stars ($m > 1 M_\odot$, with $\alpha_3 \approx 2.7$, Scalo 1986; Kroupa et al. 1993, 2026 and references therein) result from composite IMFs, i.e., field stars originate from many embedded clusters. The region with $M_{\rm ecl} > 10^8 M_\odot$ and $[Z] < -3$ is relevant for the formation of SMBHs (Kroupa et al. 2020).

To normalize the sIMF so that its integral over the full stellar mass range correctly identifies the total number of stars produced, we invoke optimal sampling (Kroupa et al. 2013; Schulz et al. 2015, see also Section 4.6). The time interval $\delta t = 10$ Myr (Yan et al. 2021) is given by

$$\int_{m_{\rm min}}^{m_{\rm max}} m\xi(m|M_{\rm ecl}, Z) dm = M_{\rm ecl}$$

2.3. The Embedded Cluster Mass Function

The ECMF ($\xi_{\rm ECMF}$) can be represented by a power law (see Kroupa et al. 2013 and references therein)

$$\xi_{\rm ECMF}(M_{\rm ecl}) = \frac{dN}{dM_{\rm ecl}} = k_{\rm ecl} M_{\rm ecl}^{-\beta_{\rm ecl}}$$

which is valid in the range $M_{\mathrm{ecl,min}} < M_{\mathrm{ecl}} < M_{\mathrm{ecl,max}}$ (Weidner et al. 2004; Kirk & Myers 2011; Joncour et al. 2018), where the power law index β_{ecl} will depend on the SFR. The empirical dependence of the index of the power law and the average SFR of a galaxy over δt is given by (Yan et al. 2021)

$$\beta_{\mathrm{ecl}}(\overline{\mathrm{SFR}}) = 1.9 - 0.15 \log_{10}\left(\frac{\overline{\mathrm{SFR}}}{M_{\odot}\mathrm{yr}^{-1}}\right)$$

The left panel of Figure 3 [Figure 3: see original paper] shows the range spanned by β_{ecl} for an average SFR in the range between $10^{-5.5}$ and $10^{4} M_{\odot}$ yr^{-1}. The right panel shows the normalized ECMF.

While the mass function of very young embedded clusters follows a power law at a given location in a galaxy, the galaxy-wide ECMF takes the form of a Schechter-like function in spiral galaxies. This transition arises naturally from the spatial distribution of mass, and hence the local SFR surface density, across the disk (Lieberz & Kroupa 2017). We defer detailed analysis of the spatially distributed ECMF to future works.

Embedded clusters span a wide range of masses. In the extreme starburst cases (Banerjee & Kroupa 2018), an embedded cluster may produce tens of millions of stars. Low-mass embedded clusters will produce \$ \$50-100 $M_{\odot}$ of stars, M_{ecl}, but will be deprived of massive stars (Hsu et al. 2012). As for the low-mass end, Kirk & Myers (2011) and Joncour et al. (2018) observed embedded clusters as small as a few low-mass binaries, leading to a low-mass estimate of $M_{\mathrm{ecl}} \approx 5 M_{\odot}$.

To return the correct number of embedded clusters once integrated, the embedded cluster mass from Equation (10) needs to be normalized. This is achieved applying the system of equations from optimal sampling (for a detailed discussion of the method, see Section 4.6)

$$\int_{M_{\mathrm{ecl,min}}}^{M_{\mathrm{ecl,max}}} M_{\mathrm{ecl}} \xi_{\mathrm{ECMF}}(M_{\mathrm{ecl}}) dM_{\mathrm{ecl}} = M_{\mathrm{tot}}$$

where $M_{\mathrm{tot}} = \overline{\mathrm{SFR}} \cdot \delta t$ is the total stellar mass formed in the galaxy over the time interval δt.

2.4. The Integrated Galactic Initial Mass Function

The gwIMF, just like the sIMF, returns the number distribution of stars over their stellar masses: $\xi_{\mathrm{IGIMF}}(m) = dN/dm$. In the IGIMF formulation, the number distribution of stars is integrated across all the galaxy, and across all embedded clusters generated during the time interval t to $t + \delta t$ and is given by

$$\xi_{\mathrm{IGIMF}}(m|\overline{\mathrm{SFR}}(t), Z) = \int_{M_{\mathrm{ecl,min}}}^{M_{\mathrm{ecl,max}}} \xi(m|M_{\mathrm{ecl}}, Z) \xi_{\mathrm{ECMF}}(M_{\mathrm{ecl}}) dM_{\mathrm{ecl}}$$

where $\overline{\mathrm{SFR}}(t)$ is the average SFR sampled over a time interval $\delta t = 10^7$ yr around time t, and ξ_{ecl} is the ECMF for embedded clusters of mass M (for compactness, we use $M = M_{\mathrm{ecl}}$). $M_{\mathrm{ecl,max}}$ is the theoretical upper mass limit for any embedded cluster and it is taken to be $10^{10} M_{\odot}$. This corresponds to the birth mass of an ultra-compact dwarf galaxy (Dabringhausen et al. 2009, 2012; Zonoozi et al. 2016; Haghi et al. 2017; Jeřábková et al. 2018; Mahani et al. 2021).

In practice, we find $M_{\mathrm{ecl,max}}$ and k_{ecl} by taking the ratio between the two equations in the system above (Equation (12)), which cancels out the presence of the ECMF normalization constant, k_{ecl}, and we find the root of

$$\int_{M_{\mathrm{ecl,min}}}^{M_{\mathrm{ecl,max}}} M_{\mathrm{ecl}} \xi_{\mathrm{ECMF}}(M_{\mathrm{ecl}}) dM_{\mathrm{ecl}} = \overline{\mathrm{SFR}} \cdot \delta t$$

which returns the best fit for the real most massive embedded cluster formed in a galaxy at time t. k_{ecl} is then obtained by substituting back into Equation (12). This procedure is equivalent to optimal sampling used for the sIMF (Equation (9)).

Figure 3 [Figure 3: see original paper] displays the ECMF. On the left-hand side, the normalization constant is set to $k_{\mathrm{ecl}} = 1$. On the right-hand side, the ECMF is normalized according to optimal sampling. The vertical lines on the right-hand side represent the effective upper mass limit for each cluster at a given SFR. Notice that the slopes between the two figures are the same, only the k_{ecl} varies. Figure 4 [Figure 4: see original paper] depicts the three parameters involved in the SFR-dependent power-law ECMF: the slope, β_{ecl}, and the two quantities derived from solving the system of equations under optimal sampling (Equation (12)), i.e., $M_{\mathrm{ecl,max}}$ and the normalization, k_{ecl}.

The parameters of the IGIMF theory are summarized in Table 1 .

The integral in Equation (14) yields a function of stellar mass, m, which depends on both the metallicity and the average SFR of a galaxy over a time interval δt. At any given time t, a galaxy is characterized by an average SFR(t), $\overline{\mathrm{SFR}}$, and metal mass fraction, Z, and each interval t_i has an associated ξ_{IGIMF}.

2.5. Alternative Prescription of the Low-mass (Bottom) Slope

Equation (5) describes the linear increase of α_1 with metallicity. This trend highlights the role of metals in enhancing cooling and accelerating gas fragmentation. However, this enhancement may saturate above a critical (yet unknown) metallicity. In this scenario, there is a transition from a regime sensitive to metallicity variation to a metallicity-insensitive regime. A logistic function is the appropriate formulation to capture the transition between these two regimes. We therefore propose that α_1(Z) can be

$$\alpha_1(Z) = \alpha_{1,\text{canon}} + \frac{\alpha_{1,\max} - \alpha_{1,\text{canon}}}{1 + e^{-k(Z-Z_0)}}$$

where Z_0 is the x-axis midpoint of the logistic function, $\alpha_{1,\max}$ is the curve' s maximum value, and k is the growth rate.

We choose the following parameters, which represent the extreme example where α_1 does not increase beyond the values tested to date. In this case $\alpha_{1,\max} = 2.2$, where $\alpha_{1,\text{canon}}$ is defined in Equation (3).

3. Results

The results are presented in three steps. First we examine the variability of the sIMF as a function of metallicity and embedded cluster mass, using both the linear and logistic α_1 prescriptions. It also considers the potential dependence of the $m_{\max}$-M_{ecl} relation and the sIMF slopes on metallicity. Next, we address the variability of the gwIMF and its dependence on metallicity and SFR. Lastly, we compare these results with other models of a variable gwIMF.

3.1. The sIMF

Figure 5 [Figure 5: see original paper] provides a comparison of α_1(Z) for the linear (Equation (5)) and logistic (Equation (15)) formulations and with observational constraints. A systematic survey investigating the α_1 slope with consistent assumptions and methodology does not exist. From existing literature adopting disparate assumptions, Yan et al. (2024) (e.g., their Figure 4) compiled estimates of the α_1 slope. We applied a minimization algorithm, specifically Brent' s method (Atkinson 1989, their Section 2.8), to solve for α_1 under the assumption that $\alpha_2 = \alpha_1 + 1$ holds at all times. A detailed compilation of the results is provided by Yan et al. (2024). IMF slope estimates are available up until a metal mass fraction (Equation (C1)) of Z 0.03, which is approximately two times the solar value Z (see Table 1). However, stars and environments with higher supersolar metallicities are known to exist. For example, the Andromeda bulge may contain stars with [Z] 0.5 (Saglia et al. 2010). Cinquegrana et al. (2022) is among the first theoretical works tackling extreme supersolar metallicities. They computed supersolar asymptotic giant branch (AGB) yields for metal mass fractions 0.04 < Z < 0.1. Their paper mentions known stellar populations with supersolar metallicities, but emphasizes the scarcity of direct observations due to their rarity in the Galaxy, as well as practical limitations. Therefore there are yet no observational counterparts for AGBs in this metallicity range. However, Cohen et al. (2003) find that the giant elliptical M49 hosts globular clusters whose metallicities are as high as Z 7Z .

Recently, Floris et al. (2024) measured the metallicity in the broad-line regions of local active galactic nuclei (AGNs), adopting the definition in Equation (C4), where [Z/H] denotes the number density ratio of all metals to hydrogen. They report among the highest metallicities in the literature, reaching values at least several dozen times solar and up to truly extreme levels of [Z/H]   3. It is to be expected that, prior to condensing into molecular clouds, such gas will be diluted by the surrounding metal-poor medium. While these observations do not directly confirm the existence of stellar populations with metallicities as high as Z = 10Z = 0.142, they suggest such values may be possible.

Figure 6 [Figure 6: see original paper] shows how the sIMF varies as a function of the embedded cluster mass $M_{\rm ecl}$ for different metallicities. [Z] = 0 is on the bottom left panel. This is the metallicity where the low-mass ($m < 1 M_{\odot}$) α_1 and α_2 slopes are the closest to canonical values (Equation (3)). Lower metallicities are bottom-light while higher metallicities are bottom-heavy. The high-mass slope, α_3, remains canonical as long as the most massive star in the embedded cluster, $m_{\rm max}$, does not approach the theoretical upper limit, $m^*_{\rm max}$. Only as $m_{\rm max} \to m^*_{\rm max}$ does the high-mass slope become top-heavy.

We stress again that top-light sIMFs are not possible (see the discussion for Figure 2 in Section 2.2). In Figure 6, we show embedded clusters ranging from 10 to $10^9 M_{\rm ecl}$. This wide range of values is supported by observations (see Sections 4.2 and 2.3). In the Galactic center, however, the most massive observed embedded clusters reach masses of $10^4\ M_{\odot}$ as in the case of Westerlund 2 (Zeidler et al. 2021), corresponding to the bright green curves in Figure 6.

Figure 7 [Figure 7: see original paper] represents the same sIMFs from Figure 6, but allows a direct comparison between the linear and the logistic formulation at supersolar metallicities. Up until [Z] = 0.3, the two formulations are equivalent. In fact, the top and bottom left panels of Figure 6 also apply to the linear case. At [Z] = 0.5, which is approximately Z = 3Z , the two formulations begin to deviate. In this plot it is more clearly evident that an increase in metallicity causes the sIMF to become bottom-heavy. For lightest-to-intermediate embedded cluster masses, the low gas mass available in the embedded cluster poses a hard limit on the most massive stellar mass generated by that given stellar population.

Figure 8 [Figure 8: see original paper] depicts the "$m_{\rm max}$-$M_{\rm ecl}$" relation as a function of metallicity for the linear and logistic α_1(Z) prescriptions. The data points were compiled by Yan et al. (2023), their Table A.1, with the following literature sources: Kirk & Myers (2011), Stephens et al. (2017), Weidner et al. (2013a). Data deviations from the nominal solar-metallicity "$m_{\rm max}$-$M_{\rm ecl}$" relation as computed here are affected by the metallicity of the embedded clusters, mergers and ejections of massive stars and stellar-feedback self-regulation of the star formation process as discussed in detail by Yan et al. (2023) and Zhou et al. (2024). In particular, the frequency of mergers in and ejections from an embedded cluster is subject to the largely unknown properties of massive multiple systems at birth. In some cases, embedded clusters with masses of a few $10^3 M_{\odot}$ eject all of their massive stars (Oh & Kroupa 2012, 2016, 2018; Kroupa 2025).

In Figure 9 [Figure 9: see original paper] we show contours of the most massive stellar masses across the parameter space of embedded clusters and supersolar metallicities. In these panels it becomes clear that for supersolar metallicities, the most massive star that an embedded cluster may form is strongly dependent on the embedded cluster stellar mass, $M_{\rm ecl}$, but it is weakly dependent on metallicity in the logistic case.

Figure 10 [Figure 10: see original paper] shows sIMF variation described by Equations (5)-(7), namely that at solar metallicity the sIMF remains canonical in molecular cloud clumps that spawn embedded clusters with masses such that $m_{\rm max} < m^*_{\rm max}$. Only when $M_{\rm ecl} \gtrsim 10^4 M_\odot$, i.e., when $m_{\rm max} \to m^*_{\rm max}$ (Figure 8), does the sIMF become top-heavy. This can be interpreted in terms of the fragmentation into proto-stellar cores proceeding in a “normal/canonical”process up until the clump mass becomes so large that the density ($> 10^4 M_\odot$ pc^{-3}, or $> 2 \times 10^5$ cm^{-3} in H_2, Bergin & Tafalla 2007; McKee & Ostriker 2007) leads to the coalescence of the cores before they can collapse to protostars, implying a significantly top-heavy sIMF (Kroupa et al. 2026). We note for the first time that the sIMF becomes top-heavy ($\alpha_3 < 2.3$) only when the most massive star in the cluster approaches the theoretical upper limit, i.e., $m_{\rm max} \to m^*_{\rm max}$.

3.2. The IGIMF Theory Approach to the gwIMF

Figures 11 and 12 display the gwIMF as described by the IGIMF theory for the linear and logistic α_1(Z) prescription, respectively. In these figures, each panel corresponds to a different galaxy-wide SFR, averaged over the δt time interval. The various lines span a wide range of metallicities, with a focus on the supersolar range. In the logistic case (Figure 12 [Figure 12: see original paper]), the gwIMF becomes progressively top-heavier with increasing SFR. The increasing metallicity predominantly affects the low-mass end of the sIMF, which becomes progressively bottom-heavy with increasing metallicity, with only a secondary decrease in massive star production for the highest metallicities. This is a direct consequence of the bottom-heaviness of these gwIMFs: a higher fraction of stellar mass is locked into low-mass stars due to enhanced fragmentation. However, for smaller SFRs, the impact of increasing metallicity becomes progressively more significant. Galaxies with the smallest SFRs will be unable to produce any massive star resulting in core-collapse supernovae ($m \gtrsim 13 M_\odot$, Limongi 2017), regardless of the α_1(Z) prescription.

In the case of the linear prescription (Figure 11), however, the bottom-heaviness induced by metallicity-driven fragmentation becomes so pronounced that it affects the ability to produce massive stars. Interestingly, for low to average SFRs ($\lesssim 1 M_\odot$ yr^{-1}), metallicities above [Z] > 0.5 begin to dramatically reduce star formation of $\approx 1 M_\odot$ stars.

Figure 13 [Figure 13: see original paper] for the gwIMF, similarly to Figure 9 for the sIMF, shows the mass of the most massive star as a function of its parameter space. In this case, this is the metallicity and SFR. In the logistic α_1(Z) prescription, the value of the most massive star that may form in a galaxy is fairly independent of metallicity. Meanwhile in the linear case, for metallicities [Z]   0.5, the formation of massive stars is steeply constrained even at fairly high metallicities.

The IGIMF theory generically predicts gas-rich dwarf galaxies with SFRs < $10^{-5} M_{\odot}$ yr^{-1} to have Hα-invisible star formation with a true SFR that can be higher because only stars with masses $m > 10 M_{\odot}$ can form (Figure 13). The existence of dwarf galaxies with Hα-dark SFR has already been noted by Pflamm-Altenburg et al. (2007). These authors also point out that dwarf galaxies have, due to their top-light gwIMFs, significantly larger true SFRs than the estimates via the standard Hα luminosity implies (see also Jeřábková et al. 2018; Haslbauer et al. 2024), with important repercussions for the cosmological matter cycle.

But as is also evident from Figure 13, if the linear low-mass IMF slope is the correct description then Hα-invisible star formation will also occur when [Z] > 0.6. This means we would detect massive disk galaxies with a large amount of cooling supersolar-metallicity gas with SFRs > $1 M_{\odot}$ yr^{-1} but with stars that only have $m < 1 M_{\odot}$ that form. Such galaxies, if they exist, would have an extremely slowed chemical enrichment because only AGB stars and stellar mergers (Wang et al. 2020) would be contributing new metals to the interstellar medium.

3.3. A Special Case of the IGIMF Theory at High Redshifts

In Figure 14 [Figure 14: see original paper] we show that the IMF proposed in van Dokkum & Conroy (2024) is a special case of the IGIMF. The formulation by van Dokkum & Conroy (2024, and references therein) constructs an average gwIMF (referred by them as the "Concordance IMF") for elliptical galaxies that are bottom-heavy. Above 1 $M_{\odot}$, they adopt a top-heavy gwIMF to reproduce the small mass-to-light ratios inferred for $z > 10$ galaxies observed with JWST. Such low ratios are required in the standard ΛCDM framework for young, low-mass galaxies to match the observed luminosities at these redshifts (but see Haslbauer et al. 2022 for a discussion).

In order to link the high-redshift data with the present-day elliptical galaxies and star-forming disk galaxies, they suggest that $\alpha_{1,2,3}$ are functions of the present-day velocity dispersion of a galaxy (Haslbauer et al. 2022, their Equation (6)). The van Dokkum & Conroy (2024) approach thus captures aspects that are contained in the IGIMF theory. To demonstrate this, their gwIMF formulation is rewritten in terms of IGIMF solutions that match their solution shown in van Dokkum & Conroy (2024), their Figure 3. We achieve this agreement through specific combinations between the metallicity Z and SFR, as shown in Figure 14. Namely, the "bottom-heavy" portion occurs at supersolar metallicities, while the "top-heavy" portion is due to high SFR.

The IGIMF theory provides physical motivations to the variability of the gwIMF, and can be calculated for any combination of these parameters, allowing also self-consistent chemical enrichment histories to be computed for ultra-faint dwarf galaxies to massive ellipticals starting from their formation times (Yan et al. 2019, 2020, 2021) together with photometry and spectra that match observed galaxies (Haslbauer et al. 2024; Zonoozi et al. 2025). The IGIMF theory provides a self-consistent framework for computing the stellar population evolution in any galaxy, including a formulation for how young, low-dispersion galaxies evolve into the massive systems observed today. Meanwhile, the "Concordance IMF" is tailored to elliptical galaxies, with extensions to other systems parameterized only by their present-day velocity dispersion, without a clear connection to the evolution of a galaxy.

Given the above approaches to the gwIMF, it is noteworthy that all cases we examined have in common being top-heavy at high redshift and bottom-heavy in deep potential wells. From the propositions, only the IGIMF explicitly links the physical properties of molecular cloud clumps to the galaxy-wide stellar population within a self-consistent computable theory.

4. Discussion

In this section we first briefly overview other approaches to a variable gwIMF. We then discuss three aspects of the IGIMF: its conceptual grounding, its empirical evidence, and the concept of optimal sampling.

4.1. Other Approaches to the gwIMF

The IGIMF approach is distinct from other gwIMF approaches that do not build the galaxy-wide stellar population from the fact that star formation occurs primarily within embedded clusters (Dinnbier et al. 2022; Cook et al. 2023, and references therein). However, that star formation occurs primarily within embedded clusters is non-trivial. In fact, many young stars are observed outside of embedded clusters (e.g., Gieles et al. 2012), leading some to believe that stars may form elsewhere within the molecular clouds. However, such young stars were likely not born at their current location. Several authors (Oh et al. 2015; Dinnbier et al. 2022) have shown that the rapid early evolution of the embedded cluster disperses young stars widely throughout the star-forming region. This process occurs as quickly as one Myr since their birth (Oh & Kroupa 2016).

The formulation by Jermyn et al. (2018), Steinhardt et al. (2022a, 2022b) assumes the form of the gwIMF to be invariantly canonical but the masses at which the power-law indices $\alpha_{1,2,3}$ shift to different stellar masses depend on the temperature of the gas in the galaxy which depends also on the redshift. As discussed in Kroupa et al. (2026) this approach has the drawback that the temperature is not readily accessible once the stars have formed such that reconstructing the sIMFs from a gwIMF becomes difficult. This approach thus does not link the gwIMF to the physics of star-forming sites in different locations in a galaxy. The IGIMF in contrast constructs a gwIMF through the integration at a given time of all the stellar masses born in embedded clusters across the mass distribution of the very same embedded clusters, and it contains an implicit temperature dependency through the distribution of molecular cloud clump densities that enter the SFR of the galaxy.

4.2. Stars are Born in Embedded Clusters within Molecular Clouds

The occurrence of star formation within embedded clusters has been confirmed by Galactic surveys (Carpenter et al. 1995, 2000; Lada & Lada 2003; Megeath et al. 2016; Porras et al. 2003; Winston et al. 2020), by Galactic binary star properties (Kroupa 1995a, 1995b; Marks & Kroupa 2011; Dabringhausen et al. 2022; Kroupa 2025), and extragalactic observations (Dinnbier et al. 2022; Cook et al. 2023, and references therein). However, that star formation occurs primarily within embedded clusters is non-trivial. In fact, many young stars are observed outside of embedded clusters (e.g., Gieles et al. 2012), leading some to believe that stars may form elsewhere within the molecular clouds. However, such young stars were likely not born at their current location. Several authors (Oh et al. 2015; Dinnbier et al. 2022) have shown that the rapid early evolution of the embedded cluster disperses young stars widely throughout the star-forming region. This process occurs as quickly as one Myr since their birth (Oh & Kroupa 2016).

Figure 15 [Figure 15: see original paper] contains simulation results, represented by the purple lines and shaded regions, as well as observations from the LEGUS survey on dwarf galaxies, represented by the red and black bins (Cook et al. 2023). Black bins represent data for resolved stellar populations (where individual stars are detected), while red bins correspond to results for unresolved populations (where the SFR is inferred from integrated light). All young resolved clusters fall within the limits predicted by dynamical evolution models, while unresolved populations preferentially reside above the conservative simulation results. The axis limits were chosen to match those in Figure 12 of Cook et al. (2023). Both observations and simulations consider young ($\lesssim$ 10 Myr) embedded clusters. The purple solid line is the IGIMF theory rationale: all stars are born in embedded clusters. Dynamical N-body evolution reduces the fraction of stars remaining within the embedded clusters (purple shaded area) to values consistent with observational uncertainties for similarly young clusters, namely, between 40% and 90% of all stars are dynamically lost from the cluster within the short timeframe of 10 Myr (Dinnbier et al. 2022). Aside from typical molecular clouds, embedded clusters can also form in filamentary distributions along probable shock fronts, provided that the required density and temperature conditions are satisfied. Possible examples are the 100 to 300 pc-long relic filaments discovered by Jerabkova et al. (2019), Beccari et al. (2020).

4.3. Extragalactic Evidence that Star Formation Occurs in Embedded Clusters

There is compelling evidence that stars form in localized discrete regions, i.e., in the regions of molecular clouds that undergo gravitational collapse (Kroupa et al. 2026, their Section 3.1). Such regions must, by necessity for collapse (to form stars) be very compact, since they need to achieve a sufficient density for the collapse to ensue and to form stars. During the collapse, always more than one star forms, since these regions contain much more mass than that which ends up in a single low-mass star (star formation being inefficient, less than some 30% of the gas ending up in stars, Battisti & Heyer 2014). These regions thus must form embedded clusters that may contain just a few multiple systems to many millions of stars.

Dynamical population synthesis simulations support observational evidence, and demonstrate that the binary star properties observed in star-forming regions naturally lead to the observed Galactic-field binary population (Kroupa 1995a, 1995b). Kroupa (2005) advanced the hypothesis that embedded star clusters can be treated as the fundamental building blocks of galaxies. Embedded clusters are dynamically very active and so they expel stars quite rapidly (in less than a Myr, Oh & Kroupa 2012; Dinnbier et al. 2022) due to the high birth multiplicity fraction. In fact, a binary can contain much binding energy that can be transformed into kinetic energy when an encounter with another star or binary occurs in the embedded cluster, e.g., Kroupa (2025). The embedded clusters expel most of their mass as gas through the feedback from the forming stars therewith they expand, e.g., Dinnbier & Kroupa (2020). Embedded clusters do typically interact while forming, since molecular clouds have long free-fall times such that the embedded clusters form and expel residual gas well before merging (Kroupa 1998a, 1998b), while their merging becomes relevant only in massive star cluster complexes (Mahani et al. 2021).

Any gwIMF must be consistent with galaxy-scale properties as well as the observed sIMFs in molecular clouds. The IGIMF does just that: by considering the resolved and dynamically well-modeled star clusters and ultra-compact dwarf galaxies, it gauges the dependency of the sIMF from the physical attributes of embedded clusters (Kroupa et al. 2026, their Section 4.3). What emerges is a theory that accounts for many galaxy properties.

4.4. The sIMF Must be Variable

Metal-rich gas cools significantly faster than low-metallicity gas due to a larger number of allowed electron transitions (Ploeckinger & Schaye 2020). From a theoretical standpoint, inefficient cooling at low metallicity reduces gas fragmentation, resulting in fewer low-mass stars compared to higher-metallicity environments. Consequently, the sIMF at low-metallicities is expected to be deficient in low-mass stars relative to the canonical form (for the definition of the canonical parameters see Section 2.1). Additional physical quantities that are expected to affect the shape of the sIMF are cosmic-ray heating of the clump (Papadopoulos 2010), magnetic fields and rotational shear which would inhibit the formation of low-mass stars near a SMBH (for reviews see Kroupa et al. 2013; Hennebelle & Grudić 2024; Kroupa et al. 2026) and especially the coalescence of massive cores in the dense innermost regions of massive clumps, before said cores can collapse to a protostar (Dib et al. 2007).

Concerning observational evidence, in starburst conditions, embedded clusters may range in mass up to a few hundred thousand to many hundreds of millions of stars (Banerjee & Kroupa 2018), with radii of <1 pc (Marks & Kroupa 2012). On the opposite end, the smallest embedded clusters generate a total of 5 $M_{\odot}$ in newly formed low-mass stars (Kirk & Myers 2011; Joncour et al. 2018). Using binary stars as the tracers of the maximum possible density an open cluster could have had at birth, Marks & Kroupa (2012) found the birth half-mass radii of the embedded clusters to follow the relation $r_{\mathrm{h}} \approx 0.1(M_{\mathrm{ecl}}/M_{\odot})^{0.13}$ pc (Marks & Kroupa 2012, Equation (7)).

Interestingly, Hsu et al. (2012) found that even moderately massive embedded clusters which generate 50–100 $M_{\odot}$ in new stars do not generate massive stars. These observations indicate that there is immense variability in the sIMF. Consequently, the gwIMF will tightly depend on the mass distribution of the embedded clusters. Molecular clouds that form low-mass to moderately massive embedded clusters may have a systematically different sIMF than molecular clouds generating more massive embedded clusters. Weidner & Kroupa (2006) deduced that the most massive star in an embedded cluster, m_{max}, appears to be strongly related to the stellar mass of the embedded cluster itself, M_{ecl}. This relation is further investigated and compared to other scenarios in Yan et al. (2023).

On top of being dependent on M_{ecl}, the sIMF was shown to further depend on the molecular cloud clump metallicity, [Z], as well as its density, ρ_{cl}. The gas density ρ_{cl} eventually generates a total stellar mass, M_{ecl}. A first hint of this variability was advanced in Kroupa (2002). This metallicity dependence was robustly confirmed for the first time in the solar neighborhood by Li et al. (2023).

Detailed modeling of globular star clusters, ultra-compact dwarf galaxies and nearby massive starburst clusters, when compared to observational data indicates that the sIMF systematically varies with the density and metallicity of the star-forming gas (Dabringhausen et al. 2009, 2012; Marks et al. 2012; Wirth et al. 2022 and references therein). The variation is such that at large ρ_{cl} the sIMF becomes top-heavy. For metallicities as low as [Z] -1, the sIMF of moderately massive embedded clusters also becomes top-heavy, and additionally it becomes bottom-light, i.e., the number of massive stars is enhanced while the number of low-mass stars is suppressed. At larger Z, the opposite is true: the sIMF becomes bottom-heavy and for sufficiently low-mass embedded clusters, top-light, i.e., the number of massive stars is suppressed while the number of low-mass stars is enhanced.

4.5. The sIMF is Inconsistent with Stochastic Sampling

If fragmentation of a gas clump depends on the metallicity, then the shape of the sIMF should remain invariant at fixed Z in every single stellar population of mass $M_{\rm ecl}$. Since low-mass clumps can only form a small number of stars, the mass of the most-massive star ($m_{\rm max}$) is expected to depend on the total stellar mass ($M_{\rm ecl}$) of the embedded cluster in which it forms. Such a relation ($m_{\rm max}$-$M_{\rm ecl}$) has indeed been found at near-solar metallicity (Weidner & Kroupa 2006; Weidner et al. 2010, 2013b; Yan et al. 2023). Given the observational uncertainties, the distribution of $m_{\rm max}$ values at a given $M_{\rm ecl}$ informs us of the possible stochastic, probabilistic nature of the sIMF.

The sIMF has been shown to vary both with metallicity as well as with clump mass (Section 4.4), and it is inconsistent with a stochastic origin: by investigating the dispersion of $m_{\rm max}$ as a function of $M_{\rm ecl}$, Yan et al. (2023) found that this dispersion is too small to be consistent with the sIMF being a stochastic probability distribution function.

Similarly to the above, Xu et al. (2024), Jiao et al. (2025) identified a relation between the most massive core and the mass of the clump in which it forms. Regarding the core mass distribution within a clump, optimal sampling (see Section 4.6) is favored over stochastic sampling.

Additionally for solar to moderately subsolar metallicities, the sIMF of massive stars is characterized by a power-law index, α_3, for stars exceeding one solar mass (1 $M_\odot$). The various estimates of α_3 are found to be too close to the Salpeter slope (Equation (1), see also Kroupa 2002, their Figure 6), and therefore $\alpha_3 \approx \alpha_{\rm S55}$. Stochastic sampling is unable to generate the small observed standard deviation (Kroupa 2002; Yan et al. 2023). Kroupa et al. (2013) proposed that the sIMF might be an ODF. Drawing stars from an ODF leads to a distribution without Poisson scatter and can be interpreted physically to mean that the formation of an embedded cluster in a molecular cloud core is highly self-regulated.

4.6. Optimal Sampling

Kroupa et al. (2013) introduced the process of optimal sampling in the context of

Note: Figure translations are in progress. See original paper for figures.

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