A Beam Pointing Error Compensation Method for Large Beam Waveguide Reflector Antennas Using an Adjustable Mirror (Postprint)

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Abstract

Deep space exploration imposes extremely stringent requirements on the beam pointing accuracy of large reflector antennas fed by beam waveguide systems. When the reflector is deformed by unavoidable external loads, compensating for beam pointing errors becomes critically important. This study proposes an innovative compensation method for such large beam waveguide reflector antennas by employing an adjustable ellipsoidal mirror within the beam waveguide system. First, the influence of the ellipsoidal mirror's positional deviation on the outgoing focal point of the beam waveguide system is analyzed. Next, an equivalent single-reflector model is established for the dual-reflector configuration of the beam waveguide reflector antenna, and the deviation of the single-reflector focal position induced by the outgoing focus deviation of the beam waveguide system is derived. An electromechanical coupling model (EMCM) is then formulated, incorporating both reflector deformation and the ellipsoidal mirror' s position error. Finally, assuming that the ellipsoidal mirror is adjustable, the beam pointing error compensation problem under reflector deformation is investigated based on the EMCM. A 35 m aperture beam waveguide reflector antenna is used as a case study. The results show that the proposed EMCM can accurately predict the main beam direction, thereby validating the effectiveness of the proposed beam pointing error compensation method for large antennas.

Full Text

Preamble

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Abstract

Deep space exploration demands extremely high beam pointing accuracy from large reflector antennas fed by beam waveguide systems. Beam pointing error compensation becomes critically important when the reflector deforms due to inevitable external loads. This study introduces an innovative compensation method for such large beam waveguide reflector antennas utilizing an adjustable ellipsoidal mirror within the beam waveguide system. Initially, the effect of the ellipsoidal mirror's position deviation on the beam waveguide system's outgoing focus is analyzed. Subsequently, an equivalent single reflector system for the dual reflector system of the beam waveguide reflector antenna is considered, and the derivation of the focus position deviation for the single reflector caused by the position deviation of the beam waveguide system' s outgoing focus is derived. Then, an electromechanical coupling model (EMCM) is formulated, incorporating both reflector deformation and the ellipsoidal mirror' s position error. Finally, with the assumption of adjustability of the ellipsoidal mirror, the study explores the beam pointing error compensation problem based on the EMCM in scenarios of reflector deformation. This paper presents a case study on a 35 m aperture beam waveguide reflector antenna. The results demonstrate that the proposed EMCM achieves high accuracy in calculating the main beam direction, thereby validating the effectiveness of the proposed beam pointing error compensation method for large antennas.

Key words: telescopes -methods: numerical -Astronomical Instrumentation, Methods and Techniques

1. Introduction

Large reflector antennas, renowned for their simplicity, high gain, and narrow beamwidth, have become a staple in deep space exploration and radio astronomy (Rahmat-Samii & Haupt 2015; Baars & Karcher 2017). However, owing to their substantial apertures, such large-scale reflector antennas are typically deployed in open-air environments and are consequently subject to significant influence from external loads, including gravity, thermal gradients, and wind (Von Hoerner & Wong 1975; Greve & Bremer 2010; Li et al. 2012; Zhang et al. 2017; Ukita et al. 2017; Wei et al. 2021). These factors can lead to reflector deformations, which in turn can significantly degrade beam pointing accuracy (Duan & Wang 2009).

For active main reflector antennas, each panel is equipped with four actuators and can be adjusted to the ideal reflector or best-fitting reflector positions, effectively compensating for beam pointing errors caused by reflector deformations (Wang et al. 2010, 2023; Lian et al. 2021a, 2023). However, the implementation of active main reflector antennas is costly, and as a result, many antennas lack this feature. To improve beam pointing error compensation to a certain extent for such antennas, an alternative approach involves the rigid body adjustment of the subreflector (Greve & Bremer 2005; Wang et al. 2014). Although this method may not achieve performance levels comparable to those of an active main reflector, it nonetheless provides a measurable degree of improvement. It is critical to recognize, however, that the subreflector—which is positioned near the focal point of the main reflector and is typically supported by four long legs—is itself susceptible to structural displacements and deformations under complex environmental conditions. These induced distortions can consequently degrade the effectiveness of beam pointing error compensation that has been applied to a deformed main reflector.

The beam waveguide system, with its high-power handling capabilities and the absence of rotary joints, is particularly well-suited for deployment in high-power microwave systems. This makes the large reflector antenna, when fed by a beam waveguide system, an ideal choice for deep space exploration applications (Han 2020; Wu et al. 2020). As depicted in Figure 1 [Figure 1: see original paper], the beam waveguide is an assembly of strategically positioned planar and curved mirrors that direct the radiation beam from a feed located in an underground machine room along a predetermined path to the antenna. This configuration not only ensures a favorable working environment but also facilitates maintenance, eliminates the need for a long feeder, reduces transmission loss, and minimizes cross polarization (Duan 2015).

However, most existing related studies have primarily focused on structural error analysis and beam pointing error compensation of traditional dual reflector antenna systems (Smith & Bastian 1997; Xu et al. 2009; Rocca et al. 2014; Lian et al. 2015, 2021b). Studies on electromechanical coupling models (EMCM) and compensation methods for beam waveguide reflector antennas are much less common compared to those for traditional single or dual reflector antennas (Wang et al. 2024).

For beam waveguide reflector antennas with non-active main reflectors, when the reflector deforms, the traditional compensation method involves adjusting the subreflector to the corresponding location of the best-fitting reflector while recalibrating the azimuth and pitch servo systems to correct for pointing errors induced by the subreflector’ s movement, which is somewhat complex (Lian et al. 2019; Wang et al. 2019). In this study, we propose an alternative approach by employing one of the mirrors within the beam waveguide system as an adjustable mirror. This adjustment is implemented to enhance the beam pointing accuracy of the deformed reflector. The advantages of employing an adjustable mirror are manifold compared to the conventional approach of adjusting the subreflector and servo systems. Located within the underground equipment room, the adjustable mirror operates within a more stable environment and offers significantly improved accessibility for maintenance purposes. Its smaller size also means that the associated adjustment mechanism is less costly. Moreover, the adjustment process is streamlined and eliminates the need for additional adjustments to the azimuth and pitch servo systems.

A critical aspect of achieving beam pointing error compensation with the adjustable mirror is the precise calculation of its adjustment amount. To address this, we must understand the relationship between the position error of the adjustable mirror and the beam pointing error, and then establish the EMCM when both the position error of the adjustable mirror and reflector deformation exist simultaneously. Based on the EMCM, the optimal adjustment amount for the adjustable mirror can be determined through an optimization process, thereby providing an effective solution for compensating beam pointing errors in the presence of reflector deformation.

This paper is structured as follows. Section 1 introduces the relevant research background. In Section 2, the ellipsoidal mirror is selected as the adjustable mirror through analysis. Section 3 presents a focus error analysis of the beam waveguide system. The EMCM is established in Section 4. The calculation of adjustment amounts is detailed in Section 5. The correctness and validity of the proposed method are subsequently demonstrated through several simulation studies in Section 6. Finally, a comprehensive summary is provided in Section 7.

2. Beam Waveguide Adjustable Mirror Selection

Figure 2 [Figure 2: see original paper] illustrates the geometry of a dual reflector antenna fed by a beam waveguide system, a prevalent setup in engineering applications. The feed is positioned at S_1, R_1 is a planar mirror, R_2 is an ellipsoidal mirror, S_1' is the mirror image of S_1, and S_1' and S_2 are the two focal points of the ellipsoidal mirror R_2. The radiation beam emanating from S_1 will converge at S_2 after sequential reflections from R_1 and R_2. R_3 and R_6 are additional planar mirrors, while R_4 and R_5 are parabolic mirrors sharing an identical focal length. The focus of R_4 is the mirror image of S_2 in planar mirror R_3, and the focus of R_5 is the mirror image of S_3 in planar mirror R_6. Thus, the radiation beam emanating from S_2 is refocused at point S_3. The paraboloid M_1 and the hyperboloid M_2 configuration symbolize the main reflector and the subreflector, respectively. S_3 is positioned at one of the focal points of the subreflector M_2. This positioning ensures that the beam, upon reflection from M_2 and M_1, is directed outwards in a parallel fashion.

2.1. Analysis of Position Error

The classical mirror configuration of the beam waveguide reflector antenna includes a plane mirror, a parabolic mirror, and an ellipsoidal mirror. These mirrors may have position and attitude errors during the assembly process or during antenna operation. To assess the feasibility of compensating for electrical performance degradation by adjusting beam waveguide reflector positions and attitudes, a series of GRASP simulations were performed to analyze how these adjustments influence the far-field pattern. The position errors of all the beam waveguide reflectors are analyzed in GRASP, and the positive x-axis, positive y-axis, and positive z-axis directions are discussed respectively. The range is \$±\$0.5 times the wavelength. The final far-field patterns are shown in Figures 3-5 [Figure 3: see original paper][Figure 4: see original paper][Figure 5: see original paper], and the gain loss, beam pointing offset, LFSLLI, and RFSLLI in the corresponding figures are shown in Tables 1–3 .

By analyzing the influence of the position error of all reflectors in the three coordinate directions on the far-field pattern, it can be seen that the position error of R_2 in the three coordinate directions will lead to beam pointing offset, and the position error of R_2 has the greatest influence on gain loss among all reflectors. The y-axis position errors of R_1, R_3, and R_6 have no effect on the far-field pattern. The x-axis and z-axis position errors will lead to E-plane beam pointing offset of the far-field pattern, which will lead to decreased far-field pattern gain. The plane mirror R_1 with the lowest distance from the ground has the least influence on beam pointing, followed by plane mirror R_3, and plane mirror R_6 has the greatest influence.

The x-axis position errors of the parabolic mirrors R_4 and R_5 have little effect on the beam pointing of the far-field pattern, and the effect on gain is not much different from that of the plane mirror. The y-axis and z-axis position errors of the parabolic mirrors R_4 and R_5 affect the beam pointing of both the E and H planes. The positional adjustment of the ellipsoidal mirror along its respective coordinate axes can be utilized to achieve compensation for beam pointing offsets, while similarly, the positional adjustment of the parabolic mirror along its x- and z-axes can also be effectively employed to realize the correction of beam pointing deviations.

2.2. Attitude Error Analysis

The attitude errors of all mirrors of the beam waveguide are analyzed in GRASP. The x-axis direction and y-axis direction are discussed respectively, and the range is set to $±$0.3°. The final far-field patterns are shown in Figures 6 and 7 [Figure 6: see original paper][Figure 7: see original paper]. The corresponding gain loss, beam pointing offset, LFSLLI, and RFSLLI are shown in Tables 4 and 5 .

Through an analysis of the influence of attitude errors along the x- and y-axes for all reflectors on the far-field pattern of the beam waveguide system, it can be concluded that the attitude errors of the ellipsoidal mirror, parabolic mirror, and plane mirror will collectively result in beam pointing offset. The attitude error of R_2 has the greatest influence on beam pointing, and the position error of the ellipsoidal mirror has the greatest influence on gain loss among the attitude errors of all reflectors, followed by the parabolic mirrors R_4 and R_5, but the beam pointing offset caused by the same rotation direction of the two parabolic mirrors is opposite. The attitude error of the plane mirror has the least influence on beam pointing. Among them, the plane mirror R_1 with the lowest distance from the ground has the least influence on beam pointing, and the plane mirrors R_3 and R_6 have relatively large influence. Compared with position adjustment, the attitude adjustment of parabolic mirror, plane mirror, and ellipsoidal mirror can realize the adjustment of beam pointing offset.

Considering the influence of position and attitude errors on the far-field pattern, for beam pointing offset, the ellipsoidal mirror can achieve beam pointing compensation through attitude or position adjustment, and the parabolic mirror can also achieve beam pointing compensation. The plane mirror can realize beam pointing adjustment by position plus attitude adjustment or separate attitude adjustment, but the adjustment method of the plane mirror is the simplest. Considering the convenience of adjusting all mirrors, the ellipsoidal mirror is closer to the ground than the parabolic mirror, so the adjustable mirror is determined to be the ellipsoidal mirror.

3. Focus Error Analysis of Beam Waveguide System

Next, we employ geometrical optics to analyze the position deviations of S_2 and S_3 induced by slightly adjusting R_2. It should be noted that the adjustment amount of R_2 is very small in engineering because the reflector deformation is generally very small.

3.1. Position Deviation of S_2 Caused by the Movement of R_2

As depicted in Figure 8 Figure 8: see original paper, (b), and (c), a small displacement of the ellipsoidal mirror along the x-axis, y-axis, or z-axis results in a corresponding change to the central axis of the radiation beam. This minor disturbance will cause a significant shift in the location of the outgoing focus S_2.

Given the typically minuscule magnitude of the ellipsoidal mirror R_2' s movement, edge beam leakage is disregarded for simplification. As shown in Figure 8(a), the movement of the elliptical mirror R_2 along the x-axis will cause position errors of S_2 along both the x-axis and the z-axis. The position deviation of S_2 can be derived as:

$$\Delta x_{21} = \Delta x_1 \cdot \frac{r_1}{f} \cdot \cos\beta_1$$
$$\Delta z_{21} = \Delta x_1 \cdot \frac{r_1}{f} \cdot \sin\beta_1$$

where Δx_{21} and Δz_{21} are the displacements caused by the movement of the ellipsoidal mirror R_2 along the x-axis, with Δx_{21} representing the displacement of S_2 along the x-axis and Δz_{21} representing the displacement of S_2 along the z-axis, Δx_1 is the displacement of the ellipsoidal mirror R_2 along the x-axis, α is the angle between the two central axes of the incident radiation beam and the outgoing radiation beam for the ellipsoidal mirror R_2, β is the tilt angle of the ellipsoidal mirror, r_1 is the distance from the central reflection point O_3 to the focus S_2' when the ellipsoidal mirror R_2 undergoes a displacement along the x-axis, as shown in Figure 8(a), and can be expressed as $r_1 = f \cdot \cos\beta$, in which f is the focal length of the ellipsoidal mirror R_2 and r_1 is the distance from the feed mirror image point S_1' to the central reflection point O_3; r_1 can be expressed as $r_1 = \sqrt{(f^2 + d^2 - 2fd \cdot \cos\alpha)}$, in which d is the distance from the feed mirror image point S_1' to the central reflection point O_1 of the ellipsoidal mirror R_2, β_1 is the deflection angle of the central axis of the outgoing radiation beam when R_2 undergoes a displacement along the x-axis and it can be approximately expressed as $\beta_1 = \arccos[({r_1}^2 + {f_1}^2 - (\Delta x_1)^2)/(2r_1f_1)]$, f_1 is the distance between S_1' and S_2' when R_2 undergoes a displacement along the x-axis; f_1 can be derived by $f_1 = \sqrt{({r_1}^2 + f^2 - 2r_1f \cdot \cos\beta_1)}$.

Figure 8(b) illustrates the central axis change of the radiation beam when slightly moving the ellipsoidal mirror R_2 along the y-axis. It should be noted that the mirror curve in Figure 8(b) represents the intersection line of mirror R_2 with the o–yz plane, whereas the corresponding mirror curve in Figure 8(a) or Figure 8(c) signifies the intersection line of mirror R_2 with the o–xz plane. This difference in plane intersection accounts for the distinct appearance of the curve in Figure 8(b) compared to those in Figure 8(a) and 8(c).

When the ellipsoidal mirror R_2 moves a short distance along the y-axis, the length of r_1 in the o–xz plane approximately remains unchanged. The movement of the elliptical mirror R_2 along the y-axis will cause position errors of S_2 along both the y-axis and the z-axis. The position deviation of S_2 can be derived as:

$$\Delta y_{22} = \Delta r_2 \cdot \sin \phi_y$$
$$\Delta z_{22} = \Delta r_2 \cdot \cos \phi_y$$

where Δy_{22} and Δz_{22} are the displacements caused by the movement of the ellipsoidal mirror R_2 along the y-axis, with Δy_{22} representing the displacement of S_2 along the y-axis and Δz_{22} representing the displacement of S_2 along the z-axis, Δr_2 represents the distance from the central reflection point to the outgoing focus S_2 of the ellipsoidal mirror R_2 before the movement, ϕ_y is the deflection angle of the central axis of the outgoing radiation beam when R_2 undergoes a displacement along the y-axis in the o–yz plane, and j_x is the rotation angle of the normal vector around the x-axis in the o–yz plane at the point determined by the intersection of the central axis of the incident radiation beam with the ellipsoidal mirror R_2 when R_2 undergoes a displacement along the y-axis, and it can be calculated by j_x = arccos($n_1 \cdot n_2/(|n_1||n_2|)$), where n_1 and n_2 are the projection vectors of the normal vectors of the intersection points of the incident radiation beam on the central axis of the ellipsoidal mirror R_2 before and after its movement, onto the o–yz plane.

As shown in Figure 8(c), the movement of the elliptical mirror R_2 along the z-axis will result in position errors of S_2 along the y-axis and z-axis. The position deviation of S_2 can be derived as:

$$\Delta x_{23} = \Delta z_1 \cdot \frac{r_1}{f} \cdot \sin \alpha$$
$$\Delta z_{23} = \Delta z_1 \cdot \frac{r_1}{f} \cdot \cos \alpha$$

where Δx_{23} and Δz_{23} are the displacements caused by the movement of the ellipsoidal mirror R_2 along the z-axis, with Δx_{23} representing the displacement of S_2 along the x-axis and Δz_{23} representing the displacement of S_2 along the z-axis, Δz_1 is the displacement of the central reflection point of the ellipsoidal mirror R_2 along the z-axis, r_z is the distance from the central reflection point O_2 to the focus S_2' when R_2 undergoes a displacement along the z-axis, as shown in Figure 8(c), and ϕ_z is the deflection angle of the central axis of the outgoing radiation beam when R_2 undergoes a displacement along the z-axis, in which r_z = $\sqrt{}(r_1^2 + (\Delta z_1)^2 - 2r_1\Delta z_1 \cdot \cos(\pi/2 - \beta))$, ϕ_z can be expressed as ϕ_z = arccos[(r_1^2 + r_z^2 - $(\Delta z_1)^2$)/($2r_1$r_z)].

Considering that the displacement of the elliptical mirror R_2 is generally small, the displacement of S_2 along the x-axis can be approximated as the sum of the x-axis displacements caused by the movements of the elliptical mirror R_2 along the x-axis and z-axis. The displacement of S_2 along the y-axis can be approximated as the sum of the y-axis displacements caused by the movement of the elliptical mirror R_2 along the y-axis. The displacement of S_2 along the z-axis can be approximated as the sum of the z-axis displacements caused by the movements of the elliptical mirror R_2 along the x-axis, y-axis, and z-axis. The final position error of S_2 can be expressed as follows:

$$\Delta x_2 = \Delta x_{21} + \Delta x_{23}$$
$$\Delta y_2 = \Delta y_{22}$$
$$\Delta z_2 = \Delta z_{21} + \Delta z_{22} + \Delta z_{23}$$

3.2. Position Deviation of S_3 Caused by Position Deviation of S_2

The displacement of the outgoing focus S_2 of the ellipsoidal mirror R_2 will cause the position deviation of S_3, the focus of the whole beam waveguide system. In an ideal beam waveguide scenario, free from structural imperfections, the feed' s radiation beam at S_1 would converge precisely at point S_3, and its electromagnetic (EM) performance would be equivalent to that of a conventional dual reflector antenna with a feed situated at S_3. Consequently, determining the positional deviation of S_3, which results from the displacement of S_2, is of paramount importance.

First, as illustrated in Figure 9(a) [Figure 9: see original paper], when S_2 is minutely shifted to S_2' along the x-axis, it results in a corresponding approximate relocation of S_3 to S_3', with displacements occurring in both the x-axis and z-axis directions. The position deviation of S_3 along the x-axis can be derived as:

$$\Delta x_3 = \Delta x_2 \cdot \frac{O_5 O_6}{O_5 S_3} \cdot \sin \theta_x$$

where O_5O_6 is the distance between points O_5 and O_6 in Figure 9(a), O_5S_3 is the distance between the central point of the planar mirror R_6 and the outgoing focus S_3 of the beam waveguide system when there is no position deviation of the ellipsoidal mirror R_2, θ_x is the angle between the central axes of the two outgoing wave beams on the plane mirror R_6 before and after the movement of S_2 along the x-axis, ϕ_x is the angle between the central axes of the two outgoing wave beams on the parabolic mirror R_4 before and after the movement of S_2 along the x-axis, where n_3 and n_3' represent the normal vectors at the reflection points of the incident wave beam central axes on the parabolic mirror R_4, before and after the movement of S_2 in the o–xz plane; $\Delta\theta$_x is the angle between the normal vectors of n_5 and n_6 in the o–xz plane, where n_5 and n_6 represent the normal vectors of the incident beam' s central axis at the reflection points O_5 and O_6 on the parabolic mirror R_5, and $\Delta\theta$_x is negative if point O_6 is on the left side of point O_5, and positive otherwise.

Second, as depicted in Figure 9(b), when S_2 is moved slightly along the y-axis, it results in concurrent movements of S_3 in both the y-axis and z-axis directions. The position deviation of S_3 along the y-axis can be formulated as:

$$\Delta y_3 = \Delta y_2 \cdot \frac{O_5 O_6}{O_5 S_3} \cdot \sin\theta_y$$

where θ_y is the angle between the central axes of the two outgoing wave beams on the plane mirror R_6 before and after the movement of S_2 along the y-axis, ϕ_y is the angle between the central axes of the two outgoing wave beams on the parabolic mirror R_4 before and after the movement of S_2 along the y-axis, where n_7 and n_8 are the projection vectors of the normal vectors of the incident beam' s central axis at the reflection points on the parabolic mirror R_4 in the o-yz plane before and after the movement of S_2; $\Delta\theta$_y is the angle between the normal vectors of n_9 and n_{10} in the o-yz plane, where n_9 and n_{10} are the projection vectors of the normal vectors of the incident beam' s central axis at the reflection points on the parabolic mirror R_5 in the o-yz plane before and after the movement of S_2, and $\Delta\theta$_y is negative if point O_6 is in front of point O_5, and positive otherwise.

Third, as illustrated in Figure 9(c), the position deviation of S_2 along the z-axis will lead to a corresponding displacement of S_3 along the same axis. Additionally, the position deviations of S_2 along the x-axis and y-axis both contribute to a minor displacement of S_3 along the z-axis. Consequently, the overall position deviation of S_3 along the z-axis can be approximated as the aggregate of the three individual displacements along the z-axis, each resulting from the position deviations of S_2 in the x-, y-, and z-axes. This total deviation can be expressed as:

$$\Delta z_3 = \Delta z_2 \cdot \frac{O_5 O_6}{O_5 S_3} \cdot \cos\theta_z + \Delta x_2 \cdot \frac{O_5 O_6}{O_5 S_3} \cdot \sin\theta_{xz} + \Delta y_2 \cdot \frac{O_5 O_6}{O_5 S_3} \cdot \sin\theta_{yz}$$

where θz *is the angle between the central axes of the two outgoing wave beams on the plane mirror R_6 before and after the movement of S_2 along the z-axis, and* θ{xz} and θ_{yz} are the coupling angles from x- and y-axis displacements to the z-axis displacement, respectively.

When subjected to external loads, the reflector undergoes deformation, which introduces phase errors in the aperture plane. Assuming Δz represents the node deformation of the reflector in the z-axis direction, the phase error can be expressed as:

$$\Phi_d = \frac{2\pi}{\lambda} \cdot 2\Delta z \cdot \cos\xi$$

where the angle ξ is shown in Figure 10 [Figure 10: see original paper].

For a prime focus reflector antenna, the feed is ideally positioned at the focal point of the reflector. If a position deviation of the feed occurs, a new phase error will emerge in the aperture plane, and the phase error can be derived as:

$$\Phi_f = \frac{2\pi}{\lambda} \cdot (\Delta x_f \sin\theta\cos\phi + \Delta y_f \sin\theta\sin\phi + \Delta z_f \cos\theta)$$

where Δx_f, Δy_f, and Δz_f are the feed position deviations along the x-, y-, and z-axes, respectively.

4. Electromechanical Coupling Modeling

Section 2 presents the mathematical formulation for calculating the positional deviation of the outgoing focus in the beam waveguide system, which results from a positional deviation of the ellipsoidal mirror. Furthermore, this section includes a discussion on the consequential effects that such a deviation of the outgoing focus has on the overall electromagnetic performance (EMP) of the system.

4.1. EMCM for Prime Focus Reflector Antenna

Figure 10 [Figure 10: see original paper] illustrates the geometry of a prime focus reflector antenna with a diameter D and focal length F. D = 2a, where a is the radius. σ is the projected circular region of the curved surface on the aperture plane with polar coordinates (r, θ). The unit vector $\hat{r}$ represents the direction of observation with coordinates (θ, ϕ) corresponding to the observation point. The vector $\mathbf{r}_f$ denotes a point on the reflector surface. Considering the relationship between the aperture field and the far-field to be a Fourier transform pair (Rahmat-Samii 1983), the far-field pattern can be derived as follows:

$$\mathbf{E}(\theta, \phi) = \int_\sigma Q(r, \theta_f) e^{j\Phi(r,\theta_f)} e^{jk\hat{r}\cdot\mathbf{r}_f} dS$$

where $Q(r, \theta_f)$ is the aperture amplitude distribution, $\Phi(r, \theta_f)$ is the aperture phase distribution, $k = 2\pi/\lambda$, λ is the wavelength, and dS is the differential surface element. The unit vector $\hat{r}$ is the unit vector of the observation direction.

Thus, when both the reflector' s deformation and the feed' s position deviation exist, the modified far-field pattern can be represented as:

$$\mathbf{E}(\theta, \phi) = \int_\sigma Q(r, \theta_f) e^{j(\Phi(r,\theta_f)+\Phi_d+\Phi_f)} e^{jk\hat{r}\cdot\mathbf{r}_f} dS$$

where Φ_d and Φ_f are the phase errors caused by reflector deformation and feed position deviation, respectively.

4.2. EMCM for Beam Waveguide Reflector Antenna

In Section 2, the position deviation of focus S_3 caused by the position deviation of the ellipsoidal mirror R_2 for the beam waveguide reflector antenna in Figure

2 is derived. Given that the antenna in Figure 2 functions as a dual reflector system, the position deviation of focus S_3 cannot be directly substituted into Equation (17) for calculating the EMP. Figure 11 [Figure 11: see original paper] shows the geometry of the dual reflector system in Figure 2. Point S_3 is the outgoing focus of the beam waveguide system, point S_4 is the focus of the main reflector M_1, and S_3 and S_4 are also the two foci of the subreflector M_2, which is a hyperboloid. The effect of the position deviation of S_3 on the phase error in the aperture plane for the dual reflector system can be equivalent to the effect of the position deviation of S_4 for the prime focus system.

The position deviation of S_4 can be derived as:

$$\Delta\mathbf{r}_4 = \mathbf{M} \cdot \Delta\mathbf{r}_3$$

where $\Delta\mathbf{r}_3 = [\Delta x_3, \Delta y_3, \Delta z_3]^T$ and $\Delta\mathbf{r}_4 = [\Delta x_4, \Delta y_4, \Delta z_4]^T$ are the position deviations of S_3 and S_4, respectively. $\mathbf{M}$ is the equivalent coefficient matrix, where:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

and $m = L_1/L_2$ is the reflector antenna amplification factor, with L_1 and L_2 being the distances from focus S_3 and focus S_4 to the vertex of the subreflector, respectively.

Then, we replace the vector $\mathbf{d}$ in Equation (16) by $\Delta\mathbf{r}_4$ and substitute Equations (15) and (16) into Equation (17) to derive the EMCM when both reflector deformation and position deviation of the ellipsoidal mirror are considered. The EMCM can be described as:

$$\mathbf{E}(\theta,\phi) = \int_\sigma Q(r,\theta_f) e^{j(\Phi(r,\theta_f)+\Phi_d+\Phi_{\Delta r_4})} e^{jk\hat{r}\cdot\mathbf{r}_f} dS$$

where $\Phi_{\Delta r_4}$ is the phase error caused by the position deviation of S_4, which is a function of $\Delta\mathbf{r}_1$ (the position deviation of the ellipsoidal mirror). According to Equation (20), the power pattern of the beam waveguide reflector antenna can be calculated and the corresponding gain loss and pointing error can be easily derived.

5. Optimal Adjustment Amount Calculation

Equation (20) provides the computational formulation for the beam waveguide antenna under concurrent conditions of reflector surface deformation and positional deviation of the ellipsoidal mirror. The deformation of the main reflector induces a deviation in the beam direction, which can be effectively compensated for by implementing positional adjustments to the ellipsoidal mirror. Consequently, by engineering the ellipsoidal mirror, which is situated in the underground machine room, as an adjustable component capable of precise translational movement along its x-, y-, and z-axes, it becomes possible to perform manual realignment to counteract the detrimental effects of these errors. This corrective adjustment serves to reduce the associated antenna gain loss and thereby enhance the overall EMP of the system.

To determine the optimal adjustment amount, an optimization method is adopted to minimize the gain loss of the ellipsoidal mirror. The optimization model can be described as:

$$\begin{aligned} &\text{Find} && \Delta\mathbf{r}_1 = [\Delta x_1, \Delta y_1, \Delta z_1]^T \\ &\text{Minimize} && \Delta G = G_0 - G(\Delta\mathbf{r}_1) \\ &\text{Subject to} && \Delta\mathbf{r}_1^{\text{lower}} \leq \Delta\mathbf{r}_1 \leq \Delta\mathbf{r}_1^{\text{upper}} \end{aligned}$$

where $G(\Delta\mathbf{r}_1)$ is the main beam gain of the deformed reflector antenna after adjusting the ellipsoidal mirror; G_0 is the main beam gain of the ideal reflector antenna; the gain loss ΔG is taken as the optimization objective to be minimized; and $\Delta\mathbf{r}_1^{\text{lower}}$ and $\Delta\mathbf{r}_1^{\text{upper}}$ are the lower and upper bounds of the optimization variables. It should be noted that the main beam gain in Equation (21) can be calculated by the derived EMCM in Equation (20) during each iteration when performing the optimization.

6. Simulation Studies

6.1. Simulation Object

A Cassegrain reflector antenna configuration, which is fed by a beam waveguide system, has been adopted as the test platform to validate the correctness and demonstrate the practical effectiveness of the proposed compensation method. The main reflector of the Cassegrain antenna is a paraboloid with an aperture D of 35 m and a focal length f of 11.2 m. The subreflector is a hyperboloid with a diameter of 3.5 m, a distance L_2 of 1.0222 m, and an amplification factor m of 9.9567, as shown in Figure 11. The beam waveguide system is shown in Figure 12 [Figure 12: see original paper] and the corresponding values of the structural parameters are presented in Table 6 . The operating frequency in the following simulations is 8.45 GHz.

6.2. Correctness Validation of EMCM

The correctness and effectiveness of the presented method will be demonstrated by results generated by GRASP Software, a professional tool for reflector antenna analysis. Before performing the correctness validation of the derived EMCM, we should first ensure that the excitations are consistent when calculating the radiation patterns by GRASP and EMCM. However, a special feed pattern can be directly applied in GRASP for the beam waveguide system, but the aperture amplitude distribution defined by Equation (14) is adopted in the EMCM. Given this inherent uncertainty, there is no definitive assurance that the field excitations utilized in both computational methods are identical. Consequently, an approximate methodology is deliberately employed to generate the aperture amplitude distribution for the EMCM. This approach is implemented specifically to establish an equitable basis for comparison, thereby enabling a more rigorous and accurate assessment of the proposed method' s validity and performance.

First, for the beam waveguide system shown in Figure 12, the feed is typically situated at S_1. In fact, the EMP when the feed is directly positioned at S_3 is approximately the same as that when the feed is situated at S_1, which means that the beam waveguide reflector antenna can be equivalent to a dual reflector antenna. Then, the dual reflector antenna can be further approximately equivalent to a single reflector antenna with the same feed (Zhu & Ye 1980), as shown in Figure 13 [Figure 13: see original paper]. The equivalent single reflector antenna has the same aperture size, but the focal length is magnified m times, where the parameter m is consistent with that defined in Equation (19). Consequently, the aperture amplitude distribution for the system can be approximately derived through a comprehensive analysis that takes into account both the specific characteristics of the feed horn implemented within the beam waveguide system and the corresponding EM properties of the equivalent single reflector configuration.

The derived aperture amplitude distribution can be taken as Q in Equation (14) and directly applied in the EMCM. In this case, the radiation pattern calculated using EMCM will be approximately the same as that simulated by GRASP for the beam waveguide reflector antenna directly fed by the feed mentioned above when there are no structural errors.

Figure 14 [Figure 14: see original paper] illustrates a comparative analysis of the power patterns calculated by the derived EMCM and GRASP for an ideal beam waveguide reflector antenna. The detailed EM parameters are listed in Table 7 . The results clearly show that the power patterns derived by both methods are almost identical, which demonstrates the correctness of the approximation calculation method of the aperture amplitude distribution. Figure 14 also indicates the presence of some minor discrepancies in the sidelobe levels, which are most likely attributable to the cumulative effect of the series of approximations introduced in the modeling methodology. However, given that the primary focus of this paper is on the compensation of beam pointing errors rather than on ultra-precise sidelobe prediction, these observed differences in sidelobe levels exert a negligible impact on the central findings and conclusions of this study.

Then, the position deviation of the ellipsoidal mirror will be considered. Figure 15 [Figure 15: see original paper] presents the power patterns in the horizontal plane and the vertical plane calculated by the EMCM and GRASP when the ellipsoidal mirror is moved along the x-axis by 0.2λ, and the corresponding power patterns when the ellipsoidal mirror is moved along the y-axis and z-axis by 0.2λ are presented in Figures 16 and 17 [Figure 16: see original paper][Figure 17: see original paper], respectively. The results clearly show that the power patterns calculated by the EMCM and GRASP do not perfectly coincide when displacement of the ellipsoidal mirror exists. Nonetheless, it is observed that the main beams calculated by both EMCM and GRASP exhibit a near-perfect overlap, as evidenced by the local magnification of the main beam area, and the errors mainly exist in the sidelobe region. The specific pointing errors and gain losses for the main beams in Figures 15, 16, and 17 are detailed in Tables 8 and 9 , respectively.

There are two primary reasons for the large errors in the sidelobe region. First, when the ellipsoidal mirror is adjusted, there is not only a translational displacement of the beam waveguide system' s outgoing focus but also a small rotational displacement, which has been overlooked. Second, the outgoing radiation beam of the beam waveguide system does not achieve precise focus at a single point, leading to defocusing, a factor that has also been neglected. Regarding the rotational displacement mentioned above, it impacts the aperture amplitude distribution but does not affect the aperture phase distribution, resulting in accurate main beam prediction but inaccurate sidelobes. The defocusing (not lateral shift of focus) mentioned above also does not change the pointing of the main beam, but its effect on the sidelobe levels is relatively large.

Thus, even though the above two influential factors are not considered in the EMCM, the calculations of the gain and the pointing of the main beam are still accurate, which is consistent with the simulation result. Because the emphasis of this paper is to realize EMP compensation of the main beam, the derived EMCM with good accuracy for main beam calculation can be applied in the compensation problem.

6.3. Beam Pointing Error Compensation for Deformed Beam Waveguide Reflector Antenna

Figure 18 [Figure 18: see original paper] presents the gravitational deformation profiles of the antenna' s main reflector at various elevation angles. These structural deformations are known to directly cause both antenna gain degradation and beam pointing errors. Because the derived EMCM is effective for the main beam, we can adjust the ellipsoidal mirror to improve the gain and pointing performance of the main beam. The adjustment amounts can be easily derived by solving the optimization model in Equation (21) based on the proposed EMCM in Equation (20).

For the three cases depicted in Figure 18(a), (b), and (c), the corresponding adjustment amounts of the ellipsoidal mirror along the x-axis, y-axis, and z-axis are listed in Table 10 , and the power patterns before and after adjustment are displayed in Figures 19, 20, and 21 [Figure 19: see original paper][Figure 20: see original paper][Figure 21: see original paper]. The corresponding gain losses and pointing errors of Figures 19, 20, and 21 are listed in detail in Tables 11 and 12 , respectively.

The reflector deformations due to gravity at different elevation angles exhibit both up-down asymmetry and left-right symmetry, as illustrated in Figure 18. The distribution form of deformation will result in a large gain loss for the power pattern in the horizontal plane and a large pointing error for the power pattern in the vertical plane, as evident from the power patterns before adjustment depicted in Figures 19, 20, and 21. To improve the gain and pointing performance, the ellipsoidal mirror is adjusted according to the derived adjustment amounts presented in Table 10. When compared with the radiation power patterns obtained prior to implementation of any adjustments, the results clearly indicate that the gain losses associated with the main beams in the horizontal plane have been significantly reduced following the compensation process. Similarly, pointing errors of the main beams observed in the vertical plane have also been markedly diminished after the adjustment procedure was applied. The comprehensive numerical data presented for the horizontal plane in Table 11 and for the vertical plane in Table 12 collectively demonstrate substantial improvements in both the gain and pointing accuracy of the main beam. These experimental results thereby provide a robust and conclusive demonstration of the correctness and practical validity of the derived EMCM as well as the overall effectiveness of the proposed compensation methodology.

7. Conclusions

This paper introduces an innovative beam pointing error compensation strategy for large-scale beam waveguide reflector antennas, achieved through precise adjustment of an ellipsoidal mirror within the beam waveguide system located in the underground machine room. The strategy derives the relationship between the ellipsoidal mirror' s adjustment magnitude and the displacement of the beam waveguide' s outgoing focus, enabling the development of an EMCM. This model effectively analyzes the impact of the ellipsoidal mirror' s position deviation on the EMP of the beam waveguide reflector antenna. Utilizing the EMCM, an optimization model is formulated and solved to determine the ellipsoidal mirror' s adjustment, with the objective of minimizing main beam gain loss. The simulation results obtained from a 35 m aperture beam waveguide reflector antenna comprehensively validate the computational accuracy and operational efficacy of the proposed compensation method. In comparison with traditional approaches, such as mechanical adjustment of the subreflector position, the proposed methodology demonstrates significant practical advantages, including reduced implementation cost, enhanced system reliability, improved accessibility for maintenance operations, and greater simplicity in executing engineering adjustments.

Two key points deserve emphasis. First, the primary objective of this method is to optimize the main beam' s pointing accuracy and gain performance, where the EMCM demonstrates high computational precision. However, larger errors are observed in the sidelobe region, which is consistent with expectations. In the design of large beam waveguide reflector antennas, main beam performance is critical for high-precision applications such as deep space exploration, whereas sidelobe optimization typically requires more complex structural adjustments, such as deformable mirrors or multipoint adjustment mechanisms. The proposed method, by adjusting the rigid body displacement of a single ellipsoidal mirror, provides a cost-effective and easily implementable engineering solution, and the limitation in sidelobe performance does not detract from its primary objective. Second, expecting a single rigid body adjustment to simultaneously optimize both main beam and sidelobe performance is impractical in engineering practice. Looking forward, integrating deformable mirror technology could be explored to further enhance sidelobe performance while maintaining the low-cost and adjustable nature of this approach.

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Note: Figure translations are in progress. See original paper for figures.