

Advances in Feedback-Controlled Elastic Wave Manipulation: Mechanisms and Functionalities - Postprint

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Abstract

Elastic-wave metamaterials, owing to the carefully engineered design of their functional units, can realize wave phenomena unattainable in natural materials, and thus exhibit tremendous application potential in fields such as national defense equipment and precision instruments. In recent years, various passive metamaterials have been proposed, achieving novel functionalities including low-frequency wave suppression, super-resolution focusing, and negative refraction, but they suffer from limitations such as poor tunability and structural complexity. Actively controlled metamaterials based on multiphysical-field coupling are gradually attracting extensive attention, among which feedback-controlled active metamaterials demonstrate highly efficient tunability and even adaptive regulation capabilities. Therefore, research on this type of metamaterial is of great importance, as it can broaden the mechanisms and functionalities for elastic-wave manipulation.

This study, from the perspective of control mechanisms, introduces the role of incorporating feedback control in elastic-wave regulation from three aspects. First, the introduction of a feedback mechanism can effectively modify the effective parameters of the material and enhance the tunability of the band structure. Second, feedback control can construct effective gain and loss, endowing metamaterials with novel modes and giving rise to anomalous wave phenomena. In addition, feedback control makes it possible to change the amplitude, phase, and frequency of elastic waves via interference effects, thereby enabling richer wave functionalities. Finally, the study summarizes the limitations and application challenges of elastic-wave metamaterials based on feedback control and outlines potential future research directions. This work provides methodological strategies for the design of feedback-type active metamaterials and offers design concepts for enriching wave functionalities.

Full Text

Preamble

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Abstract

Elastic metamaterials, through careful design of their functional elements, can achieve wave manipulation effects unattainable in natural materials, demonstrating tremendous application potential in national defense equipment, precision instruments, and other fields. In recent years, various passive metamaterials have been proposed to realize novel functions such as low-frequency wave suppression, super-resolution focusing, and negative refraction. However, these suffer from poor tunability and structural complexity. Active metamaterials based on multi-physics coupling have gradually attracted widespread attention from researchers, among which feedback-controlled active metamaterials exhibit highly efficient tunability and even adaptive regulation capabilities. Therefore, studying this class of metamaterials is crucial for broadening the mechanisms and functionalities of elastic wave control.

Based on control mechanisms, this review introduces the role of feedback control in elastic wave manipulation from three aspects. First, the introduction of feedback mechanisms can effectively modify the effective parameters of materials and enhance the tunability of band structures. Second, feedback control constructs effective gain and loss, enabling metamaterials to exhibit novel modes and anomalous wave effects. Additionally, feedback control makes it possible to alter the amplitude, phase, and frequency of elastic waves through interference, further enabling rich wave functionalities. Finally, this review summarizes the limitations and application challenges of feedback-controlled elastic metamaterials and prospects future research directions. This study provides methodological strategies for designing feedback-based active metamaterials and offers design insights for enriching wave control functionalities.

Keywords: feedback control; elastic wave; non-reciprocity; interference; gain/loss; skin effect; vibration suppression

Introduction

Elastic waves are wave phenomena caused by particle vibrations propagating under mutual interaction forces when an elastic medium is disturbed or subjected to external forces. They widely exist in various mechanical systems. Many engineering applications, such as precision instruments or large industrial products, require high standards for vibration and noise control, necessitating the blocking or attenuation of elastic waves [1-2]. In energy harvesting applications, particularly in self-powered devices, there is a need to collect the mechanical energy of elastic waves and convert it into electrical energy [3-4]. The propagation properties of elastic waves encountering discontinuous boundaries during transmission can also be utilized for defect detection [5-6]. These engineering demands have made research on elastic wave control increasingly important.

The emergence of elastic metamaterials has endowed elastic wave control with greater freedom and better functional performance [7], enabling novel effects such as negative refraction [8], perfect absorption [9], cloaking [10-11], focusing [12], and waveguiding [13]. Based on whether parameters are tunable, elastic metamaterials can be divided into passive and active types. Passive metamaterials often require complex microstructural design to achieve novel functions, leading to limitations such as poor tunability, manufacturing difficulties, and effective parameters constrained by geometric and material properties. The introduction of external fields can increase control degrees of freedom, broaden application scenarios, and achieve superior performance at subwavelength scales.

Active metamaterials based on multi-physics coupling have attracted increasing attention. Some studies utilize multi-field coupling materials such as piezoelectric materials [14-15] and magnetostrictive materials [17] to build tunable or even reconfigurable metamaterials. A crucial aspect of active metamaterial design is how to utilize multi-field coupling for regulation, and feedback control strategies that process output information through control laws before re-inputting are particularly noteworthy [18-19]. Feedback-controlled elastic wave research primarily employs three mechanisms: changing effective dynamic parameters, constructing gain and loss in media, and utilizing wave interference for regulation. This review summarizes relevant work according to these three mechanisms.

1. Application of Feedback Control in Spring-Mass Models

1.1 Research on Band Structures and Effective Dynamic Parameters

Band theory, originating from solid-state physics, describes electron motion and states in crystals to explain fundamental physical properties such as electrical and thermal conductivity. Kushwaha et al. [20] and Sigalas et al. [21] analogized

from photonic crystals [22] to introduce the band concept into periodic structures for elastic wave propagation characterization. The frequency range where no band curves exist in the band structure is called a band gap, indicating that elastic waves cannot propagate effectively in the structure at those frequencies. Common band gaps are generally divided into Bragg band gaps [20] and locally resonant band gaps [23]. The Bragg mechanism is closely related to structural periodicity, with the corresponding wavelength typically on the same order as the phononic crystal lattice constant. The locally resonant mechanism breaks this limitation, enabling miniaturization of elastic metamaterials [23].

The calculation of effective parameters is crucial for studying locally resonant phononic crystals. Born et al. [26] detailed the vibration characteristics of periodic spring-mass models. Wen et al. [27-29] applied the lumped-mass method to phononic crystal band gap calculations, simplifying continuous media such as beams and plates into discrete models that reduce computational difficulty while clearly revealing the principles of band gap formation. Many studies [18, 30-38] have introduced feedback control strategies into spring-mass models, calculating their equivalent dynamic density or stiffness and band structures, thereby demonstrating the feasibility of this strategy in metamaterials.

Baxa et al. [30] performed similar studies in two-dimensional spring-mass structures, achieving broadband elastic wave attenuation. Bera et al. [31] introduced feedback control within a unit cell, multiplying the measured displacement and velocity at mass 1 by feedback coefficient matrix \mathbf{C} and applying the result as interaction forces between masses 1 and 2 [FIGURE:2(a)]. They calculated the corresponding effective dynamic parameters under different structural parameters and further studied the damping enhancement effect of this local feedback [32], using machine learning to quantify stochastic parameters and prove robustness in the control process [33]. Gao et al. [34] explored practical applications using time-delayed control, combined with genetic algorithms to optimize time-delayed control for broadband wave suppression. Gao et al. [35] also proposed nonlocal feedback control, applying feedback between unit cells by multiplying the displacement of adjacent mass blocks by feedback coefficient matrix \mathbf{G} and applying it as external forces on mass blocks [FIGURE:2(d)]. These studies provide a foundation for feedback control applications in continuous medium metamaterials.

1.2 Application in Beam and Plate Phononic Crystals

In beam and plate phononic crystals, feedback control is generally implemented by attaching piezoelectric patches as sensors and actuators. Some studies attach pairs of sensing and actuating piezoelectric patches on the upper and lower surfaces of plates [FIGURE:3(a)], with blue indicating sensing and red indicating actuation. These studies discuss in detail the band structure changes under displacement feedback, acceleration feedback, and their combinations, using machine learning to optimize feedback control coefficients for wider band gaps [FIGURE:3(b)]. For beam structures, similar research has been published

[47-55], though attachment methods may vary.

To increase tunability and broaden band gaps, some studies [39-45] attach periodic piezoelectric patches on structures with shunting circuits to change effective parameters, representing an early form of feedback control where sensing and actuation are performed by the same piezoelectric patch. Li et al. [46] proposed a two-dimensional phononic crystal plate as shown in FIGURE:3(a), discussing different control combinations and their effects on band structures. Other studies embed piezoelectric materials in structures or use them as structural components to design phononic crystals [FIGURE:3(c)]. He et al. [56] added out-of-plane resonators with feedback on plates, using a controller to achieve programmable feedback coefficients, establishing relationships between feedback coefficients and resonator stiffness, and fitting curves between effective stiffness and band gap frequencies to achieve broadband flexural wave transmission suppression within the fitting range.

Sirota et al. [64-65] studied topological states of phononic crystals under feedback control. Zhou et al. [66-67] applied feedback control in acoustic metamaterials to achieve non-reciprocal propagation. Akl et al. [68-70] constructed tunable effective density and modulus, studying stability analysis of active acoustic metamaterials.

2. Gain and Loss

Metamaterials have long been considered closed systems based on energy conservation, with real observables corresponding to real eigen-energies. Real physical systems, however, are open systems that interact with the environment and possess complex eigenvalues. Bender et al. [71] discovered that purely real spectra can still exist in parity-time symmetric complex eigenvalue systems, which can be constructed through modulation of gain-loss elements [72-73]. The transition from real to complex parameter planes increases wave control degrees of freedom, enabling novel characteristics such as coherent perfect absorption [78-79], exceptional points [80-81], and asymmetric scattering [82].

In elastic wave fields, the required gain and loss can be theoretically introduced through the imaginary part of material moduli. However, the lack of gain materials has led to the development of pure-loss modulation schemes for complex eigenvalue systems. In active elastic metamaterials, gain and loss can be achieved through precisely controlled multi-physics interactions: using active devices to inject energy for effective gain, and employing lossy materials like viscoelastic materials or porous media for loss [74-77]. Piezoelectric systems, leveraging electromechanical coupling, are widely used methods for constructing complex eigenvalue systems.

2.1 Exceptional Points

Exceptional points are singularities in complex eigenvalue systems where two eigenvalues and their corresponding eigenvectors simultaneously coalesce into identical hybrid modes. They exhibit novel properties such as unidirectional invisibility [88], single-mode lasing [89], and enhanced sensing sensitivity [90]. In waveguide structures, when disturbances occur in the system, second-order exceptional points degenerate, causing originally degenerate frequencies to split again, with frequency splitting proportional to the square root of the perturbation. This response relationship is superior to general linear responses when perturbations are extremely small [84].

Rosa et al. [87] theoretically demonstrated that feedback-controlled external forces can achieve effective gain and loss and induce exceptional points. Essentially, gain and loss are manifested as conjugate imaginary parts in the diagonal terms of the system Hamiltonian. For elastic wave systems, this can be implemented by introducing imaginary parts in the stiffness term of the motion equation. Specifically, applying external forces with velocity information as feedback can achieve effective damping effects, as the dimensions of force and velocity product match those of damping.

Cai et al. [91] designed a column-structured beam as shown in FIGURE:4(a), achieving exceptional points through symmetric loading of feedback-controlled external forces. The external force was applied on the upper beam surface, while velocity information was collected at the corresponding lower surface position, multiplied by feedback coefficient γ , and input as external force. They calculated frequency response spectra under perturbation, demonstrating enhanced sensitivity relative to linear response [FIGURE:4(d)]. Jin et al. [92] designed a piezoelectric equivalent external force scheme to construct exceptional points in two-dimensional elastic media, using surface piezoelectric patches to generate external forces fed by velocity information detected at corresponding positions [FIGURE:4(b)]. As feedback coefficient γ changed, the real parts of eigenfrequencies in the spectrum gradually approached and coalesced [FIGURE:4(c)]. By introducing three-dimensional micro-cracks and calculating response spectra, frequency splitting showed a square-root trend with crack size variation [FIGURE:4(f)], demonstrating high-sensitivity crack detection.

2.2 Skin Effect and Non-reciprocal Propagation

The non-Hermitian skin effect in complex eigenvalue systems manifests as large-scale mode localization at boundaries, with extreme sensitivity to boundary conditions. The eigen-spectrum calculated using open boundary conditions differs significantly from that using periodic boundary conditions, profoundly altering bulk topology and bulk-boundary correspondence [73, 94-96]. In such systems, waves excited at any position always propagate in one direction while propagation in the opposite direction is strongly suppressed, showing good application prospects in vibration suppression [97-98] and energy harvesting [3].

The skin effect can be achieved through nonlocal feedback control. Rosa et al. [99] explored non-reciprocal wave attenuation and amplification in one-dimensional periodic spring-mass models using nonlocal feedback interactions [FIGURE:5(a)]. By applying additional forces on the mass at the n th unit cell proportional to the spring extension rate at distance a (feeding stiffness-related information from position $n-a$ into the n th unit cell), the stiffness matrix becomes real but asymmetric, yielding complex eigenvalues. The positive and negative imaginary parts in the complex dispersion spectrum correspond to effective loss and gain, respectively [FIGURE:5(b)]. This feedback mechanism allows reconfigurable local states as the proportionality factor serves as a tunable parameter.

Cai et al. [91] implemented skin effects in finite beams using a feedback external force scheme similar to Section 2.1, applying feedback-controlled forces only on columns at one side to break spatial symmetry [FIGURE:5(e)-5(f)]. Zhong et al. [100] achieved skin effects in two-dimensional structures using similar feedback control mechanisms. Reconfigurable complex eigenvalue systems can be flexibly controlled using micro-electro-mechanical systems [101]. In odd-elastic metamaterials based on piezoelectric sensing and control, establishing non-reciprocal coupling between bending deformation and shear forces enables active odd-elastic moduli, non-reciprocity, and vibration mode localization [96, 102].

3. Regulating Elastic Waves Using Interference

Programmable phononic crystals demonstrate extraordinary wave control possibilities. This programmable approach typically uses only one piezoelectric patch as a sensor and another as an actuator, with low periodicity requirements. The gain between input and output signals can be designed for different frequencies, significantly broadening the operating frequency range.

Chen et al. [106] proposed a piezoelectric metamaterial as shown in FIGURE:6(a), implementing specific transfer functions between sensing and actuation through microcontrollers to independently control asymmetric polarizations of metamaterial layers. This characteristic enables the metamaterial to have frequency-dependent asymmetric polarizabilities, achieving independent control of reflected and transmitted waves and non-reciprocal flexural wave suppression [FIGURE:6(b)]. Similar studies [107-109] have been used to generate unique acoustic transmission characteristics.

For wave phase modulation, Chen et al. [110] designed a metasurface for elastic wave phase modulation, where a single unit cell can achieve arbitrary phase modulation. As shown in FIGURE:7(a)-7(c), they designed a broadband flexural wave absorbing piezoelectric metamaterial, with feedback control principles illustrated in FIGURE:7(d). Time-domain experiments well validated the broadband absorption effect. Using multiple unit cells with nonlocal feedback control

[FIGURE:7(e)], elastic wave cloaking effects were achieved [111].

In addition to amplitude and phase control, Wu et al. [112] utilized the meta-material beam shown in FIGURE:8(a) to modulate elastic wave frequency, converting incident waves to arbitrary frequencies [FIGURE:8(b)]. The harmonics are decoupled, which is significant for avoiding structural resonance frequencies. The main challenge lies in control stability and control law programming, though this approach shows good prospects in frequency conversion applications.

4. Summary and Outlook

Feedback control-based elastic wave regulation is widely applied in active meta-materials. Different feedback interaction designs and tunable parameters provide rich possibilities for wave control. This review primarily elaborated on three mechanisms: modulation of effective dynamic parameters, construction of effective gain-loss systems, and utilization of interference effects.

Challenges and Future Directions:

1. **Stability Issues:** System output may grow infinitely or become uncontrollable under certain parameters. This requires rational selection and design of system parameters and control laws based on theoretical analysis of both the host structure and control system to achieve more stable and effective feedback control.
2. **Robustness:** Feedback control systems require robustness against parameter variations and external disturbances. Real-time acquisition of sensing data and computational control output are critical. Integration with rapidly developing artificial intelligence, machine learning, and optimization algorithms can achieve higher robustness and real-time performance.
3. **Adaptability:** Industrial development creates diverse engineering tasks and environments. Feedback control systems urgently need enhanced adaptability to meet varied requirements. This necessitates designing adaptive control algorithms and intelligent decision-making mechanisms, enabling elastic wave control under feedback to automatically adapt to tasks and environments for more functionalities.

Future research should focus on developing more sophisticated control strategies, exploring novel multi-physics coupling mechanisms, and integrating advanced computational methods to overcome current limitations and expand practical applications in vibration control, energy harvesting, and structural health monitoring.

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Figure 3

Figure 1: Figure 3

Figures

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