Isogeometric-based geometrically nonlinear topology optimization of plate-shell structures postprint

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Abstract

For stiffening design of plate-shell structures, a geometrically nonlinear topology optimization method based on isogeometric analysis is proposed. The geometry of the plate-shell structure is represented by NURBS surfaces, on the basis of which a layered model (skin and stiffening design domain) is constructed using a through-thickness integration scheme for degenerated solid-shell elements. A three-field SIMP nodal density interpolation model is employed in the stiffening design domain to perform topology optimization and search for the optimal configuration. Geometric nonlinearity of the plate-shell structure is taken into account during the topology optimization process: a spline-based degenerated shell element formulation capable of capturing large rotations is established, and fully analytical sensitivities are derived. Compared with conventional solid-element models, the layered shell-element model significantly reduces the model size, and the use of isogeometric analysis enables seamless integration of topology optimization for plate-shell structures with CAD systems. Numerical examples demonstrate that the proposed method can identify clear stiffening regions for plate-shell structures while accounting for geometric nonlinear effects. Meanwhile, this study discusses the differences between small-deformation topology optimization and geometrically nonlinear topology optimization, and verifies the necessity of considering geometric nonlinearity in topology optimization of plate-shell structures.

Full Text

Isogeometric Geometrically Nonlinear Topology Optimization of Plate-Shell Structures

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Abstract

This study proposes an isogeometric analysis-based geometrically nonlinear topology optimization method for reinforcement design of plate-shell structures. The geometry of plate-shell structures is represented using NURBS surfaces, and a layered model (skin and reinforcement design domain) is constructed based on the through-thickness integration scheme of degenerated solid shell elements. In the reinforcement design domain, topology optimization is performed using a three-field nodal density interpolation model to find the optimal configuration, where the geometric nonlinearity of plate-shell structures is considered. A spline degenerated shell element formulation accounting for large rotations is established, and fully analytical sensitivities are derived. Compared with conventional solid element models, the hierarchical model based on shell elements significantly reduces the model scale, and isogeometric analysis enables seamless integration between topology optimization of plate-shell structures and CAD systems. Numerical examples demonstrate that the proposed method can obtain clear reinforcement regions for plate-shell structures under geometrically nonlinear effects. Additionally, this study discusses the differences between small-deformation topology optimization and geometrically nonlinear topology optimization, verifying the necessity of considering geometric nonlinearity in topology optimization of plate-shell structures.

Keywords: isogeometric analysis; geometric nonlinearity; topology optimization; plate-shell structures

1. Introduction

Plate-shell structures are widely used in aerospace and other engineering fields. Reinforcement regions have a significant impact on the performance of plate-shell structures. Adding stiffeners is a common reinforcement method for plate-shell structures, where scholars have conducted extensive research on the distribution of reinforcement regions. Some studies treat the number, position, size, and type of stiffeners as design variables combined with optimization algorithms. However, this method cannot fully exploit structural performance due to its dependence on initial configurations. To better utilize materials and design superior structural forms, scholars have employed various continuum topology optimization methods to explore optimal configurations.

In research on reinforcement design of plate-shell structures, some scholars use solid element models for both the skin and design domain, performing threedimensional continuum topology optimization with extrusion constraints. However, the use of solid element models greatly increases the number of mesh elements and reduces analysis efficiency. Consequently, some researchers have begun using shell element-based hierarchical models, adding or removing materials in specific layers. Lee et al. and Belblidia et al. used shell elements that can simulate multi-layer structures for structural topology design, providing a new approach for reinforcement design of plate-shell structures.

The Solid Isotropic Material with Penalization (SIMP) method, which uses element or nodal density to construct artificial materials with penalization factors to drive the material toward solid or void, is currently a common topology optimization approach. When using element density interpolation models for topology optimization, two numerical issues arise: the checkerboard phenomenon caused by discontinuous densities between elements, and mesh-dependent problems where the topological structure varies with mesh refinement. Sensitivity filtering or density filtering are effective methods to address these issues. Although the continuous density field approach suppresses the checkerboard phenomenon, island phenomena appear in topology structures using nodal density as design variables.

To suppress the island phenomenon, Kang et al. proposed a topology optimization method combining Q4/Q4M elements with adaptive mesh refinement, using internal averaging strategies to limit island formation. Tavakkoli et al. used NURBS basis function equations to distribute material continuously in the design domain, with control point densities as design variables, significantly reducing model size. However, low-density regions in linear elastic topology problems may not bear loads, leading to numerical analysis difficulties such as singular tangent stiffness matrices and convergence issues in geometrically nonlinear topology optimization.

Although small deformation assumptions can meet some engineering requirements, structural deformations are often large in practical engineering, particularly when structures experience significant deformation. Small deformation assumptions cannot obtain the true structural response, leading to unreasonable optimization results. Considering geometric nonlinearity effects in topology optimization is essential. Moreover, the influence of geometric nonlinearity increases with load magnitude.

Traditional finite element analysis requires mesh discretization of geometry to obtain simulation models. Due to the high sensitivity of complex shell structures to geometric features, this leads to increased mesh numbers and computational costs. The isogeometric analysis (IGA) technique proposed by Hughes et al. effectively solves problems of high geometric errors and costs in standard mesh construction. NURBS, as a widely used method, was quickly introduced into topology optimization. Hassani and Tavakkoli used NURBS basis functions to distribute material continuously in the design domain, with control point coordinates constructing density fields that serve as design variables for topology problems. Gao et al. proposed an efficient topology optimization method based on isogeometric analysis using enhanced density distribution functions.

This study represents plate-shell structure geometry using NURBS surfaces, constructs a hierarchical model based on shell elements (skin and reinforcement design domain), and performs topology optimization in the reinforcement design domain using a nodal density interpolation model to find the optimal configuration. The geometric nonlinearity of plate-shell structures is considered, and the necessity of considering geometric nonlinearity in topology optimization is discussed.

2. Isogeometric Analysis Considering Geometric Nonlinearity

2.1 Isogeometric Analysis Based on NURBS NURBS basis functions are more flexible when describing geometric shapes. The binary vector piecewise rational basis function expression for NURBS is built upon spline basis functions. In the parameter space, to construct an ordered set of basis functions, a knot vector is defined as $\zeta = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, where ξ_i are one-dimensional coordinates, n is the number of basis functions, and p is the polynomial degree. The spline basis functions are defined recursively according to the Cox-de Boor formula:

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

A p-degree NURBS curve is defined as:

$$\mathbf{C}(\xi) = \frac{\sum_{i=1}^{n} N_{i,p}(\xi) w_i \mathbf{P}_i}{\sum_{i=1}^{n} N_{i,p}(\xi) w_i}$$

where $\mathbf{P}_i$ are control points and w_i are weights. Unless otherwise specified, the formula can be rewritten using rational basis functions:

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{j=1}^{n} N_{j,p}(\xi) w_j}$$

Based on the isoparametric concept, NURBS functions are used as element basis functions in structural analysis, with the displacement field expressed as:

$$\mathbf{u}(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi) \mathbf{u}_i$$

This achieves seamless integration between geometric and analysis models.

2.2 Isogeometric Degenerated Shell Element Considering Large Rotations This study introduces rotational degrees of freedom into the displacement field of degenerated shell elements to overcome the small-angle limitation during deformation. The coordinates and displacements of the degenerated shell element are interpolated using NURBS basis functions, ensuring accurate geometric information. In degenerated shell elements, the direction vectors at Gauss integration points and control points are the most important geometric information. Since control points are not located on the surface, Greville abscissae are calculated to obtain direction vectors at corresponding points, from which accurate direction vectors at Gauss integration points can be directly obtained.

For a degenerated shell element based on a 2-degree NURBS surface , the global coordinates of any point in the shell element can be expressed as an interpolation of control point coordinates. The position vector is:

$$\mathbf{x}(\xi,\eta,\zeta)=\sum_{i=1}^{n}N_i(\xi,\eta)\left(\mathbf{x}_i+\frac{\zeta h_i}{2}\mathbf{v}_i\right)$$

where $\mathbf{v}_i$ is the unit normal vector at the i-th control point, h_i is the thickness, and $\zeta\in[-1,1]$.

According to the fundamental assumptions of shell theory, the normal line remains straight after deformation of the middle surface, ignoring its length change. The displacement of any point on the middle surface is expressed through the translational degrees of freedom $\mathbf{u}_i$ and rotational degrees of freedom θ_i of control points in the global coordinate system:

$$\mathbf{u}(\xi,\eta,\zeta)=\sum_{i=1}^{n}N_i(\xi,\eta)\left(\mathbf{u}_i+\frac{\zeta h_i}{2}\mathbf{R}_i\theta_i\right)$$

where $\mathbf{R}_i$ is the rotation matrix at the i-th control point.

To overcome the small rotation limitation, appropriate nonlinear functions are used to define nodal rotations in the displacement field. The displacement field vector is:

$$\mathbf{u}_i=\mathbf{u}_i+\mathbf{F}(\theta_i)$$

where $\mathbf{F}(\theta_i)$ is a nonlinear function of rotations.

For geometrically nonlinear problems, Green's strain must consider second-order terms:

$$\varepsilon = \varepsilon_L + \varepsilon_N$$

where ε_L and ε_N correspond to linear and nonlinear terms of Green' s strain, respectively.

The virtual work equation for the shell element is:

$$\int_V \sigma : \delta\varepsilon \, dV = \int_S \mathbf{t} \cdot \delta\mathbf{u} \, dS + \int_V \mathbf{b} \cdot \delta\mathbf{u} \, dV$$

Assuming linear elastic material, the stress-strain relationship in the local coordinate system is:

$$\sigma' = \mathbf{D}\varepsilon'$$

where $\mathbf{D}$ is the elasticity matrix, and strains in the thickness direction $\varepsilon_{z'}$ are ignored.

The relationship between Green' s strain in local and global coordinate systems is:

$$\varepsilon = \mathbf{T}^T \varepsilon' \mathbf{T}$$

The equilibrium equation can be written as:

$$\mathbf{R}(\mathbf{U}) = \mathbf{F}_{ext} - \mathbf{F}_{int}(\mathbf{U}) = \mathbf{0}$$

which is solved using the Newton-Raphson method:

$$\mathbf{K}_T \Delta\mathbf{U} = \mathbf{R}$$

where $\mathbf{K}_T$ is the tangent stiffness matrix.

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3. Topology Optimization Formulation

3.1 Topology Problem Description In this study, the initial model for topology optimization assumes perfect bonding between adjacent layers. The reinforcement design domain and skin share a common interface in the thickness direction. The structure is modeled using N NURBS patches with M Gauss points for integration .

The design variable is the nodal density ρ_i at the i-th control point. After smoothing, the physical density at the i-th control point is denoted as $\tilde{\rho}_i$. The artificial material constructed using elastic modulus E only affects the reinforcement design domain.

The mathematical formulation of the topology optimization problem is:

$$\begin{aligned} \min_{\rho} \quad & C(\rho) = \mathbf{F}^T\mathbf{U} \\ \text{subject to} \quad & \mathbf{R}(\mathbf{U}, \rho) = \mathbf{0} \\ & \frac{\int_\Omega \tilde{\rho}\, d\Omega}{V_0} \le v^* \\ & 0 \le \rho_i \le 1, \quad i = 1, 2, ..., N \end{aligned}$$

where C is structural compliance, v^* is the volume fraction upper bound, and V_0 is the total design domain volume.

3.2 Smooth Nodal Density and Projection Technique Using nodal density as design variables ensures continuity of element density, effectively limiting checkerboard phenomena. However, island phenomena appear in topology structures using nodal density. To improve overall smoothness of nodal density, a Shepard function is employed [43].

The physical density field in the design domain is calculated as:

$$\tilde{\rho}_m = \frac{\sum_{n=1}^{N} N_n(\xi_m, \eta_m)\rho_n}{\sum_{n=1}^{N} N_n(\xi_m, \eta_m)}$$

where N_n is the n-th control point basis function value at the m-th Gauss point.

The smoothed density for the i-th control point is calculated by:

$$\bar{\rho}_i = \frac{\sum_{j\in\Omega_j} w_{ij}\rho_j}{\sum_{j\in\Omega_j} w_{ij}}, \quad w_{ij} = \max(0, d_m - d_{ij})$$

where d_{ij} is the Euclidean distance between the j-th and i-th control points, and d_m is the smoothing radius [Figure 3: see original paper].

To obtain crisp structural features and reduce intermediate densities, a Heaviside projection technique is applied [30,31]. The projected density is:

$$\hat{\rho}_i = \frac{\tanh(\beta\eta) + \tanh(\beta(\bar{\rho}_i - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$

where β is the projection parameter that increases during optimization, and η is the threshold (typically 0.5).

The elastic modulus at the m-th Gauss point is interpolated as:

$$E_m = E_{min} + \hat{\rho}_m^p (E_0 - E_{min})$$

where E_0 is the solid material modulus, E_{min} is the void modulus (set as $10^{-9}E_0$ to prevent singular stiffness), and p is the penalization exponent.

3.3 Sensitivity Formulation The design variables are nodal densities ρ_i. The objective is to minimize structural compliance $C = \mathbf{F}^T\mathbf{U}$. Using the adjoint method, the sensitivity of the objective function is:

$$\frac{dC}{d\rho_i} = \frac{\partial C}{\partial \rho_i} + \lambda^T \frac{\partial \mathbf{R}}{\partial \rho_i}$$

where the Lagrange multiplier λ is obtained from:

$$\mathbf{K}_T \lambda = -\frac{\partial C}{\partial \mathbf{U}} = -\mathbf{F}$$

The sensitivity can be rewritten as:

$$\frac{dC}{d\rho_i} = \lambda^T \frac{\partial \mathbf{R}}{\partial \rho_i}$$

Using the chain rule:

$$\frac{\partial \mathbf{R}}{\partial \rho_i} = \sum_{m=1}^{M} \frac{\partial \mathbf{R}}{\partial \hat{\rho}_m} \frac{\partial \hat{\rho}_m}{\partial \bar{\rho}_i} \frac{\partial \bar{\rho}_i}{\partial \rho_i}$$

The derivatives of the projection and smoothing operations are:

$$\frac{\partial \hat{\rho}_i}{\partial \bar{\rho}_i} = \frac{\beta \operatorname{sech}^2(\beta(\bar{\rho}_i - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))}$$

$$\frac{\partial \bar{\rho}_i}{\partial \rho_j} = \frac{w_{ij}}{\sum_{k \in \Omega_i} w_{ik}}$$

4. Numerical Examples

4.1 Cantilever Beam A cantilever beam with length $L = 2.5$ m, height $h = 0.25$ m, and thickness $t = 0.1$ m is considered [Figure 4: see original paper]. The elastic modulus is 3 GPa, Poisson' s ratio is 0.4, and a concentrated force of 144 kN is applied at the free end. The structure is discretized using a 60×15 NURBS patch with 144 control points.

The topology optimization results under geometric nonlinearity are shown in [Figure 5: see original paper]. The structural deformation is illustrated in [Figure 6: see original paper]. The iteration curve is shown in [Figure 7: see original paper], where fluctuations in compliance are caused by changes in the projection parameter β. After reconstruction, a clear topology is obtained with almost no intermediate densities [Figure 8: see original paper].

4.2 Plate Structure Design A plate structure with dimensions $3 \times 3 \times 0.1$ m is optimized [Figure 9: see original paper]. The reinforcement design domain is Ω_r. Four different boundary conditions (a, b, c, d) are considered [Figure 10: see original paper]. The elastic modulus is 3 GPa and Poisson' s ratio is 0.4. The structure is discretized using a 60×60 NURBS patch.

Topology optimization results under different loads are compared with those based on small deformation assumptions . As the load increases, the difference between geometrically nonlinear and small-deformation results becomes more significant. The compliance values show increasing divergence with load magnitude . Iteration curves for both approaches are shown in , where jumps in intermediate volume fraction are caused by β changes.

The geometrically nonlinear topology results for boundary conditions b, c, and d are shown in [Figure 11: see original paper]. The presence of skin provides support for low-density shell elements.

4.3 Shell Structure Design A shell structure with four fixed corners under concentrated loads is optimized [Figure 12: see original paper]. The initial structure is modeled using quadratic NURBS surfaces. Control point information is listed in .

Topological design results under various loads are shown in [Figure 13: see original paper]. For comparison, results based on small deformation assumptions are shown in [Figure 14: see original paper]. Since small-deformation results are independent of load magnitude, only the 500 kN case is displayed.

The load-displacement curves for structures optimized under different assumptions are compared in [Figure 15: see original paper]. At 850 kN load, the displacement is 0.21 mm for the small-deformation optimized structure, 0.20 mm for the geometrically nonlinear structure at 500 kN, and 0.19 mm for the geometrically nonlinear structure at 850 kN. The geometrically nonlinear optimized structure not only meets design loads but also shows increased load-bearing capacity compared to the small-deformation optimized structure. The small-deformation assumption appears potentially dangerous as it cannot determine if the structure remains within safe load-bearing capacity.

5. Conclusions

This study proposes an isogeometric geometrically nonlinear topology optimization method for plate-shell structures. A hierarchical model based on degenerated solid shell elements is constructed with skin and reinforcement design domains. Topology optimization is performed using a nodal density interpolation model to find optimal reinforcement configurations, considering geometric nonlinearity. A large rotation degenerated shell element formulation is established with fully analytical sensitivity derivation.

The method is applied to topology optimization of plate-shell structures, yielding clear reinforcement patterns. The differences between small-deformation and geometrically nonlinear topology optimization are discussed. Small-deformation topology results are load-independent, while geometrically nonlinear topology results diverge increasingly from small-deformation results as loads increase. Geometrically nonlinear topology optimization not only satisfies design loads but also provides enhanced load-bearing capacity compared to small-deformation topology optimization, verifying the necessity of considering geometric nonlinearity.

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