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## Abstract

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## Full Text

# Level-anticrossing and new relationship in the B(E2) anomaly

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Recently, a new mechanism for explaining the B(E2) anomaly was given by F. Pan *et al.* (PRC, 110, 054324, 2024), which is realized in the parameter region from the SU(3) symmetry limit to the O(6) symmetry limit, and seems to be not related to the SU(3) symmetry. However, through SU(3) analysis, a new technique proposed recently, we found that it is not so. The new mechanism is related to the level-anticrossing phenomenon, which is related to the level-crossing phenomenon in the SU(3) symmetry limit (new relationship). By incorporating previous ideas, we have a new explanatory framework for the B(E2) anomaly, which is important for understanding some higher-order interactions in the interacting boson model. Through analysis, it is shown that level-anticrossing in this mechanism mainly results from the third-order interaction $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$. Finally, the B(E2) anomalies in $^{166}$W, $^{168,170}$Os, and $^{172}$Pt are also discussed within this new framework.

**Keywords:** B(E2) anomaly, SU3-IBM, level-anticrossing, new relationship, new explanatory framework

## I. INTRODUCTION

The interacting boson model (IBM) was proposed by Arima and Iachello [1, 2], which is a widely influential algebraic model for describing the collective behaviors of various nuclei. In the simplest model, the $s$ bosons ($L = 0$) and $d$ bosons ($L = 2$) are considered, which has the U(6) symmetry. Four dynamical symmetric limits exist: (1) the U(5) symmetry limit can describe the spherical vibration; (2) the SU(3) symmetry limit can present the prolate rotation; (3) the O(6) symmetry limit can describe the $\gamma$-soft rotation; and (4) the $\overline{\text{SU(3)}}$ symmetry limit can present the oblate rotation [3].

In this model, shape phase transitions from the spherical shape to various quadrupole deformations or among the deformed shapes can be studied [4–17]. These studies have received a lot of attention in the past few years with the discoveries of critical point dynamical symmetries, which are the approximate exact solutions of the Bohr Hamiltonian [18]. E(5) presents the critical point of the spherical to the $\gamma$-soft shape phase transition [19]. X(5) describes the critical point of the spherical to the prolate shape phase phase transition [20]. Y(5) depicts the critical point of the prolate to the rigid triaxial shape phase transition [21]. Z(5) presents the critical point of the prolate to the oblate shape phase transition [22]. T(5) describes the critical point of the spherical to the rigid triaxial shape phase transition [23]. In the IBM, these shape transitions can be also realized [4–17], and these critical point symmetries can be simulated with finite-$N$ effect.

A particularly interesting scenario is the shape phase transition from the prolate shape to the oblate shape [24], where the O(6) symmetry limit is also the first-order phase transitional point [3]. In this description of the IBM, the energy spectra of the prolate and oblate shapes are the same for the same boson number $N$ [17]. However this mirror symmetry is not found in realistic nuclei. In Ref. [25], the energy ratio $E_{4/2} = E_{4_1^+}/E_{2_1^+}$ of the $4_1^+$ and $2_1^+$ states of the realistic nuclei in the Hf-Hg region is 3.33 for the prolate shape, while it is 2.55 for the oblate shape. Clearly they are asymmetric.

Although previous IBM did give many good descriptions of the low-energy collective excitations of some nuclei, some recent new experiments have been found to conflict with the results of previous IBM and other nuclear structure theories. To solve these anomalies, an extended interacting boson model with SU(3) higher-order interactions (SU3-IBM) was proposed [26, 27]. The SU3-IBM combines the idea of the IBM [2] and the SU(3) correspondence of the rigid triaxial rotor [28–32]. The SU(3) symmetry dominates all the quadrupole deformations (prolate, oblate and various rigid triaxial). In the SU(3) symmetry limit, the SU(3) second-order Casimir operator $-\hat{C}_2[\text{SU(3)}]$ can describe the prolate shape, and the SU(3) third-order Casimir operator $\hat{C}_3[\text{SU(3)}]$ can present the oblate shape. Moreover, when the square of the second-order Casimir operator $\hat{C}_2^2[\mathrm{SU}(3)]$ is added into previous two ones, the rigid triaxial deformations with any SU(3) irrep $(\lambda, \mu)$ can be described. Other dynamical higher-order interactions $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ and $[(\hat{L} \times \hat{Q})^{(1)} \times (\hat{L} \times \hat{Q})^{(1)}]^{(0)}$ are also needed ($\hat{Q}$ is the SU(3) quadrupole operator).

The SU3-IBM can successfully explain the B(E2) anomaly [26, 33–43], which is the main topic discussed in this paper. The SU3-IBM can describe the Cd puzzle [27, 44–46], in which experimental researchers found that the vibrational phonon excitations of the spherical nucleus cannot be confirmed experimentally [47–52]. The SU3-IBM can also explain the prolate-oblate asymmetric shape phase transitions in the Hf-Hg region and the Xe nuclei [53–56], can describe the properties of $^{196}$Pt at a better level [57], can describe the E(5)-like spectra in $^{82}$Kr [58], and can describe the unique boson number odd-even phenomenon in $^{196\text{-}204}$Hg [59], which cannot be explained by previous theories [60, 61].

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Otsuka *et al.* proposed that strongly deformed nuclei previously regarded as prolate, such as $^{166}$Er, should instead be rigidly triaxial with $\gamma \approx 8^\circ$ [62–64]. Recently, they also found that $^{154}$Sm is a rigidly triaxial nucleus [64]. Two experiments have confirmed this small rigid triaxiality in $^{238}$U [65] and $^{154}$Sm [66]. This rigid triaxiality has also been explained by the SU3-IBM [67, 68], providing decisive support for the SU3-IBM.

Among these anomalous phenomena, the most prominent is the B(E2) anomaly [69–73]. In general, the E2 transition ratio $B_{4/2} = B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$ is usually larger than 1.0 for collective excitations, and the ratio $E_{4/2}$ is larger than 2.0. The B(E2) anomaly found in $^{168,170}$Os [69, 72], $^{166}$W [70], and $^{172}$Pt [71] changes this old view. (The Pt, Os, and W isotopes are the main paradigms for studying shape phase transitions in nuclei.) In these nuclei, the $B_{4/2}$ value can be smaller than 1.0, even decreasing to 0.33, while the $E_{4/2}$ value remains larger than 2.0. Recently, the B(E2) anomaly was also found in the odd-even nuclei $^{167,169}$Os [74, 75] and $^{119}$Te [76]. These B(E2) anomalies challenge our understanding of shape phase transitions in nuclei [4–17].

When investigating the SU(3) correspondence of the rigid triaxial rotor, Ref. [77] found the B(E2) anomaly theoretically for the first time, but did not believe that this anomaly really exists in realistic nuclei. Inspired by the experimental findings of the B(E2) anomaly in $^{168,170}$Os, $^{166}$W, and $^{172}$Pt, Ref. [26] presented the first theoretical description using the SU3-IBM. In this explanation, the SU(3) third-order interaction $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ plays a key role. In the SU(3) symmetry limit, the $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ interaction can make the ratio $B_{4/2}$ zero, which is a level-crossing phenomenon and a possible origin of the B(E2) anomaly.

Subsequently, Ref. [33] used the SU(3) correspondence of the rigid triaxial rotor to explain the B(E2) anomaly. Thus, two very different theories exist, but they are difficult to distinguish because of insufficient experimental data. It becomes more important to further explore these anomalous mechanisms. A central question in these studies is whether the emergence of the B(E2) anomaly is related to SU(3) symmetry.

In Ref. [34], up to fourth-order interactions, the B(E2) anomaly cannot be explained by O(6) symmetry. Recently, a deeper understanding of the B(E2) anomaly was obtained by F. Pan *et al.* [36], which investigates the possibility between the SU(3) symmetry limit and the O(6) symmetry limit. They found a new mechanism for the emergence of the B(E2) anomaly. In their paper, this new mechanism seems not to be related to SU(3) symmetry because, in the SU(3) symmetry limit, the $B_{4/2}$ value is larger than 1.0. It directly appears that, in this parameter region, the B(E2) anomaly can occur alone.

This new result motivates us to investigate it further. In a recent paper, the concept of "SU(3) analysis" was proposed [42], which is a useful technique for discussing the relationship between the B(E2) anomaly and SU(3) symmetry. In the SU(3) analysis, only the corresponding SU(3) symmetry limit is considered if it really exists (the non-SU(3) symmetry parts in the original Hamiltonian are removed), and the low-lying levels and E2 transition rates are studied when the parameter in front of the $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ interaction varies. Three new results have been found: (1) the third-order interaction $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ is vital for the B(E2) anomaly; (2) the level-crossing phenomenon is important for the B(E2) anomaly; and (3) not only the E2 transition $B(E2; 4_1^+ \rightarrow 2_1^+)$, but also $B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 6_1^+ \rightarrow 4_1^+)$ can be anomalous (the B(E2) value is 0). The third discovery expands the scope of the B(E2) anomaly, which is related not only to the $0_1^+$, $2_1^+$, and $4_1^+$ states but also to all levels in the ground band. The $B(E2; 2_1^+ \rightarrow 0_1^+)$ anomaly was found in $^{166}$Os [43, 73].

We found that the new mechanism in Ref. [36] is also related to SU(3) symmetry. In the SU(3) analysis, when the parameter of the $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ interaction decreases further, level crossing between the $4_1^+$ state and another $4^+$ state can occur, and the $B_{4/2}$ value can be zero. However, in that paper, the new mechanism was still unknown because the problem was discussed only within the SU(3) symmetry limit.

In this paper, we further investigate the new mechanism in Ref. [36]. This is very important. In Ref. [42], the new mechanism is found to be related to SU(3) symmetry, so it greatly improves the understanding of the B(E2) anomaly from a much broader perspective. We find that level anticrossing plays an important role here; it occurs in the parameter region from the SU(3) symmetry limit to the O(6) symmetry limit and is related to the level-crossing phenomenon in the SU(3) symmetry limit (new relationship).

Based on this new relationship, we find that the two anomalous mechanisms in Refs. [26] and [36] can merge together. When the parameter varies from the SU(3) symmetry limit to the O(6) symmetry limit, a deep $B_{4/2}$ dip can appear. SU(3) symmetry and O(6) symmetry can have a deeper relationship. This new mechanism is related to the third interaction $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ and the prolate-oblate asymmetric shape phase transition from the SU(3) symmetry limit to the O(6) symmetry limit. These works will lead to a new explanatory framework for the B(E2) anomaly. Finally, the B(E2) anomalies in $^{166}$W, $^{168,170}$Os, and $^{172}$Pt are discussed.

## II. LEVEL-ANTICROSSING PHENOMENA IN THE IBM AND THE SU3-IBM

In the IBM, the angular momentum $L$ is a conserved quantity. The evolutions of states with different angular momenta are not related to one another, and level crossing can arise. Along the evolution from the U(5) symmetry limit to the O(6) symmetry limit, O(5) symmetry always exists, so states with the same angular momentum can have different O(5) quantum numbers $\tau$. The evolutions of these states with different $\tau$ are also not related to one another, and level crossing can occur. If we discuss only the evolutions between states with the same quantum numbers, level crossing does not occur, but another effect can appear, namely level repulsion, or level anticrossing. Level crossing and level anticrossing are two fundamental and important phenomena in the IBM, and even in quantum mechanics.

Level anticrossing is a universal phenomenon, which was discussed in Refs. [78, 79] to understand the quantum phase tran-

**Fig. 1.** (a) The evolutionary behaviors of the $0^+$ states as a function of $\eta$ in $\hat{H}_1$ for $N = 7$, from the U(5) symmetry limit to the O(6) symmetry limit; (b) The evolutionary behaviors of the partial $0^+$ states as a function of $\eta$ in $\hat{H}_2$ for $N = 15$, from the U(5) symmetry limit to the SU(3) symmetry limit. From the inset, only level-anticrossing can occur.

···sitions in the IBM. When the states have the same quantum numbers, these states cannot be further distinguished; thus they may be related to each other. In some cases, two levels can become very close in energy, but because the repulsion between them also increases, they cannot cross over. This phenomenon is called level-anticrossing. In [78, 79], the quantum phase transition of the ground state was mainly considered, and its relationship with level-anticrossing was discussed.

For a finite quantum system, the evolutions of low-energy excitations can be more complex and can also show interesting phenomena. To understand these, we should consider not only the ground state but also the excited states. In nuclear structure, an excited state may have a new shape different from that of the ground state, or may have a different rotational mode with different angular momentum.

For the evolution of $0^+$ states from the U(5) symmetry limit to the O(6) symmetry limit, the $0_2^+$ and $0_3^+$ states can cross over with each other; see Fig. 1(a). The Hamiltonian describing the evolution is as follows:

$$\hat{H}_1 = c\left[(1-\eta)\hat{n}_d - \frac{\eta}{N}\hat{Q}_0 \cdot \hat{Q}_0\right], \tag{1}$$

where $\hat{n}_d = d^\dagger \cdot \tilde{d}$ is the $d$-boson number operator and $\hat{Q}_0 = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)}$ is the O(6) quadrupole operator. $\eta$ and $c$ are two fitting parameters. $N$ is the total boson number of the quantum system, which is half of the valence nucleon number of a nucleus. In the IBM, the Hamiltonian is composed of an $s$ boson with angular momentum $L = 0$ and a $d$ boson with angular momentum $L = 2$, and therefore it has U(6) symmetry.

O(5) symmetry is a common subgroup of U(5) symmetry and O(6) symmetry, so it is the symmetry group of Hamiltonian (1). Each state evolving with the parameter $\eta$ has a good O(5) quantum number $\tau$, which is the number of $d$-boson pairs that are not coupled to total angular momentum $L = 0$ [2]. On the U(5) symmetry side ($\eta = 0$), the $0_2^+$ state has O(5) quantum number $\tau = 2$, and the $0_3^+$ state has $\tau = 3$. However, on the O(6) symmetry side ($\eta = 1$), they are exactly the opposite. Thus level-crossing arises. In [17], this level-crossing was studied, and it can be regarded as a signature of the evolution from the U(5) symmetry limit to the O(6) symmetry limit.

A general Hamiltonian that includes $\hat{H}_1$ is the $Q$-consistent formalism, as in [4-17]:

$$\hat{H}_2 = c\left[(1-\eta)\hat{n}_d - \frac{\eta}{N}\hat{Q}_\chi \cdot \hat{Q}_\chi\right], \tag{2}$$

where $\hat{Q}_\chi = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi[d^\dagger \times \tilde{d}]^{(2)}$ is the general quadrupole operator $\left(-\frac{\sqrt{7}}{2} \leq \chi \leq \frac{\sqrt{7}}{2}\right)$. This Hamiltonian can describe various quadrupole shapes. If $\eta = 0$, it can describe the spherical shape (the U(5) symmetry limit). If $\eta = 1$ and $\chi = -\frac{\sqrt{7}}{2}$ $\left(\hat{Q}_{-\frac{\sqrt{7}}{2}} = \hat{Q}\right)$, it can describe the prolate shape (the SU(3) symmetry limit). If $\eta = 1$ and $\chi = 0$, it can describe the $\gamma$-soft rotation (the O(6) symmetry limit). If $\eta = 1$ and $\chi = \frac{\sqrt{7}}{2}$, it can describe the oblate shape (the $\overline{\text{SU(3)}}$ symmetry limit).

Although this Hamiltonian is very simple, it is very valuable for understanding nuclear collectivity. When $\chi = -\frac{\sqrt{7}}{2}$, this Hamiltonian can be used to discuss the shape phase transition from the spherical shape (the U(5) symmetry limit) to the prolate shape (the SU(3) symmetry limit), which was studied in [80], where the case with $N = 10$ was shown. Thus the $0^+$ states evolving with the parameter $\eta$ have no other good quantum numbers, and only level-anticrossing can occur. Clearly, in [80], the level-anticrossing of the $0_4^+$ and $0_5^+$ states is not very clear. However, in [79], the case with $N = 50$ was studied, and the $0_4^+$ and $0_5^+$ states can become very close in energy at a specific $\eta$ value, and level-anticrossing can be observed. In [79], the level-anticrossing is very similar to level-crossing, but it is not. The same result can also arise when $N = 15$; see Fig. 1(b), especially the inset (no level-crossing occurs). Thus the first mechanism for the emergence of level-anticrossing is to increase the boson number $N$.

**Fig. 2.** (a) Evolutionary behavior of the $0^+$ states as a function of $\eta$ in $\hat{H}_3$ for $N = 7$, from the U(5) symmetry limit to the SU(3) symmetry limit; (b) evolutionary behavior of the $0^+$ states as a function of $\eta$ in $\hat{H}_4$ for $N = 6$, from the U(5) symmetry limit to the SU(3) symmetry limit.

When $\chi = 0$, this is just $\hat{H}_1$. This Hamiltonian can describe the shape phase transition from the spherical shape to $\gamma$-soft rotation. The $0_2^+$ and $0_3^+$ states can cross over with each other (Fig. 1(a)). In [17], it was found that, if $\chi$ deviates somewhat from 0, the level crossing between the $0_2^+$ and $0_3^+$ states becomes level anticrossing because of the breaking of the O(5) symmetry and the complete distinguishability of the two states. This provides the second mechanism for the emergence of level anticrossing. If two states can be partially distinguished, level anticrossing can occur.

In any case, two energy levels repel each other because of the correlations between them; see [78, 79]. In nuclei, level crossing and level anticrossing are not easy to distinguish because of the discrete change in the nucleon number $N$. The SU3-IBM can present much more complicated evolutionary behavior. In [27], the new spherical-like $\gamma$-soft spectrum was first proposed, which was confirmed recently [45]. The Hamiltonian is

$$\hat{H}_3 = c\left[(1-\eta)\hat{n}_d + \eta\left(-\frac{\hat{C}_2[\mathrm{SU(3)}]}{2N} + \kappa\frac{\hat{C}_3[\mathrm{SU(3)}]}{2N^2}\right)\right], \tag{3}$$

where $\kappa$ is the parameter of the SU(3) third-order Casimir operator. When $\eta = 1$, the Hamiltonian can describe the shape phase transition from the prolate shape to the oblate shape, which differs from the SU(3)-$\overline{\mathrm{SU(3)}}$ description. $-\hat{C}_2[\mathrm{SU(3)}]$ describes the prolate shape, and $\hat{C}_3[\mathrm{SU(3)}]$ describes the oblate shape. Because both have the SU(3) symmetry, the shape transition can be discussed analytically, even for finite boson number $N$ [54]. There exists a first-order phase-transition point at $\kappa_0 = \frac{3N}{2N+3}$. The $0^+$ states with SU(3) irreducible representation (irrep) $(\lambda, \mu)$ satisfying $\lambda + 2\mu = 2N$ all cross over at this point, so it is also called the SU(3) degeneracy point. This is a simple example of level crossing. It should also be noted that the energy levels evolve linearly because the two quantities $-\hat{C}_2[\mathrm{SU(3)}]$ and $\hat{C}_3[\mathrm{SU(3)}]$ commute with each other.

If $\kappa_0 = \frac{3N}{2N+3}$, the evolutionary behavior from the U(5) symmetry limit to the SU(3) degeneracy point was discussed in [27], in which the shape phase transition from the spherical shape to the new spherical-like $\gamma$-soft phase can be found

[46]. Fig. 2(a) shows the evolutionary behavior of the low-lying $0^+$ states for $N = 7$. Obviously, among the $0_3^+$, $0_4^+$, $0_5^+$, and $0_6^+$ states, level anticrossing can occur at various positions. At $\eta = 0.5$, the energy of the $0_3^+$ state is nearly twice that of the $0_2^+$ state, which results from level anticrossing. The lowest five $0^+$ states have been confirmed in $^{106}$Pd [45]. Thus level anticrossing plays a vital role in the SU3-IBM. Here the emergence of level anticrossing results from the addition of the SU(3) third-order interaction and the specific SU(3) degeneracy point.

If the SU(3) fourth-order interaction is added, some new results can be obtained, as discussed in [58]. This Hamiltonian is as follows:

$$\hat{H}_4 = c\left[(1-\eta)\hat{n}_d + \eta\left(-\frac{\hat{C}_2[\mathrm{SU(3)}]}{2N} + \zeta\frac{\hat{C}_2^2[\mathrm{SU(3)}]}{2N^3}\right)\right], \tag{4}$$

where $\zeta$ is the parameter of the fourth-order interaction. When $\eta = 1$, this Hamiltonian can be used to describe the shape phase transition from the prolate shape to any rigid triaxial deformation. This is an important finding [58]. If rigid triaxial rotation really exists, the fourth-order interaction must be needed. Recently, Otsuka *et al.* believed that the largely deformed nuclei, such as $^{166}$Er and $^{154}$Sm, are in fact rigid triaxial shapes with about $\gamma = 8^\circ$ and $\gamma = 3.5^\circ$ [62-64], and we have indeed proved them within the SU3-IBM [67, 68]. In particular, this proposition was confirmed by two experiments [65, 66].

When $\zeta = 0.2232$, the SU(3) irrep of the ground state is $(4, 4)$, which presents a rigid triaxial shape with $\gamma = 30^\circ$. Fig. 2(b) shows the evolutionary behavior of the low-lying $0^+$ states from the spherical shape to the rigid triaxial deformation. Prominent level anticrossing of the $0_2^+$ and $0_3^+$ states arises, which seems to be similar to level crossing of the $0_2^+$

and $0_3^+$ states from the U(5) symmetry limit to the O(6) symmetry limit in Fig. 1(a). Thus, in the SU3-IBM, adding fourth-order interactions within the SU(3) symmetry limit can also induce the emergence of level anticrossing.

A summary is needed here. Levels with the same angular-momentum quantum number may undergo level crossing if they possess other distinct good quantum numbers. Otherwise, only level anticrossing can occur. Three mechanisms for the emergence of level anticrossing have been found in the IBM and the SU3-IBM: (1) increasing the boson number $N$; (2) breaking the O(5) symmetry; and (3) adding SU(3) higher-order interactions within the SU(3) symmetry limit. When quantum states cannot be completely distinguished by good quantum numbers, these states become coupled, which is a common phenomenon in quantum mechanics. In the IBM or SU3-IBM, if the system is not in one of the four dynamical-symmetry limits, such quantum states are correlated and produce level anticrossing. Thus the B(E2) anomaly and other accompanying anomalous phenomena can occur.

In this paper, the evolutionary behavior from the SU(3) symmetry limit to the O(6) symmetry limit is discussed mainly; hence level anticrossing can occur. This is the reason for the B(E2) anomaly in the new mechanism [36], and it results from the addition of SU(3) higher-order interactions and the quasi-SU(3) symmetry of the O(6) higher-order interactions. We also find that this level anticrossing is related to level crossing in the SU(3) symmetry limit (a new relationship).

## III. NEW MECHANISM OF THE B(E2) ANOMALY

If the energy ratio $E_{4/2} \geq 2.0$, it is a signature of collective excitation in nuclei. In general, the E2 transition ratio $B_{4/2}$ is larger than 1.0. In the B(E2) anomaly, this value can be smaller than 1.0, and may even decrease to 0.33. If the $0_1^+$, $2_1^+$, and $4_1^+$ states belong to the same rotational band, this indeed seems very strange. However, in the rigid triaxial rotor, this phenomenon can in fact occur, as was first discovered in [77]. In that finding, when the angular momentum $L$ of the state in the yrast band increases, the E2 transition rate to the $L-2$ state can decrease, but it changes from slowly to rapidly. The value of $B_{4/2}$ appears to be larger than 0.5 and cannot decrease to 0.33. This needs to be discussed in the future. (It should be noted that only a specific rigid triaxiality can exhibit the B(E2) anomaly.)

One of the most direct reasons for the emergence of the B(E2) anomaly is that, in the SU(3) symmetry limit, these three states, $0_1^+$, $2_1^+$, and $4_1^+$, do not actually belong to the same rotational band. Ref. [26] first provided such an explanation. In the SU(3) symmetry limit, the SU(3) third-order interaction $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ can induce level crossing between the $4_1^+$ state and another $4^+$ state. It should be noted that this level crossing is not easily realized by other higher-order interactions. Ref. [34] showed that, in the O(6) symmetry limit, a similar result cannot be obtained. This imposes a strong limitation on the explanation of the B(E2) anomaly when interactions up to fourth order are considered.

**Fig. 3.** (a) Evolutionary behavior of selected low-lying levels as a function of $\eta$ in $\hat{H}_5$ for $N = 9$ and $\chi = -\frac{\sqrt{7}}{2}$ (SU(3) analysis); (b) evolutionary behavior of $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line), $B(E2; 4_1^+ \to 2_1^+)$ (red dashed line), and $B(E2; 6_1^+ \to 4_1^+)$ (green dash-dotted line) as functions of $\eta$ in $\hat{H}_5$ for $N = 9$ and $\chi = -\frac{\sqrt{7}}{2}$ (SU(3) analysis). Other parameters are deduced from [36].

Recently, the B(E2) anomaly has been studied further. To determine whether these new mechanisms are related to the SU(3) symmetry, and to better distinguish among these different mechanisms, the concept of "SU(3) analysis" was proposed in a previous paper [42]. If a Hamiltonian has the SU(3) symmetry limit (with the non-SU(3)-symmetry parts removed), we should first discuss whether the B(E2) anomaly exists in the SU(3) symmetry limit. When the parameter of the SU(3) third-order interaction $[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ varies from 0, the evolutionary behavior of the low-lying levels and the E2 transition rates between these levels are studied. If level crossing can occur between the $4_1^+$ state and another $4^+$ state, then $B_{4/2} = 0$, and the B(E2) anomaly exists.

In a recent paper by F. Pan *et al.*, a new mechanism was found for explaining the B(E2) anomaly [36]. In their study,

the O(6) symmetry limit is also included. The B(E2) anomaly was found in the parameter region from the SU(3) symmetry limit to the O(6) symmetry limit. This seems very different from previous explanations. They used the following Hamiltonian:

$$\hat{H}_5 = \varepsilon_d \hat{n}_d - \kappa \hat{Q}_\chi \cdot \hat{Q}_\chi + \eta [\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)} + f \hat{L}^2. \tag{5}$$

Compared with $\hat{H}_2$, the interaction $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ is added. $\hat{L} = \sqrt{10}[d^\dagger \times \tilde{d}]^{(1)}$ is the angular-momentum operator. $\varepsilon_d$, $\kappa$, $\eta$, and $f$ are four fitting parameters. In $\hat{H}_2$, the B(E2) anomaly cannot appear, so the third-order interaction is important for the emergence of the B(E2) anomaly. In the SU(3) symmetry limit, this interaction can generate level crossing and the B(E2) anomaly, but in the O(6) symmetry limit this cannot occur [34]. These results imply that SU(3) symmetry is vital for the B(E2) anomaly. In [36], however, this does not seem to be the case.

In the previous paper, the case in [36] was also discussed using the SU(3) analysis with the example of $^{168}$Os [42]. Fig. 3(a) shows the results for $^{170}$Os with the parameters in [36], when $\chi = -\frac{\sqrt{7}}{2}$ and $\eta$ varies from 0 to $-29.74$ keV ($\varepsilon_d = 0.0$ for the SU(3) analysis, $\kappa = 30.0$ keV, and $f = 15.5$ keV); the middle point is the case of the SU(3) analysis in [36]. Clearly, the $4_1^+$ state and another $4^+$ state intersect at $\eta = -27.15$ keV, and the $6_1^+$ state and another $6^+$ state intersect at $\eta = -17.9$ keV (see the two black circles in Fig. 3(a)).

To understand the B(E2) anomaly, the $B(E2)$ values are necessary. The $E2$ operator is defined as

$$\hat{T}(E2) = q\hat{Q}_\chi, \tag{6}$$

where $q$ is the boson effective charge. In Fig. 3(b), the $E2$ transition rates $B(E2; 2_1^+ \rightarrow 0_1^+)$, $B(E2; 4_1^+ \rightarrow 2_1^+)$, and $B(E2; 6_1^+ \rightarrow 4_1^+)$ are shown. When $\eta = 0$, the $0_1^+$, $2_1^+$, $4_1^+$, and $6_1^+$ states belong to the same SU(3) irrep $(18, 0)$ and have the same shape; thus the B(E2) values between them are large. After the level-crossing point, another quadrupole-deformed shape can occur, and the B(E2) value is zero according to the SU(3) electric-quadrupole selection rule. Clearly, the B(E2) anomaly really exists. The value of $B(E2; 4_1^+ \rightarrow 2_1^+)$ is zero when $\eta \leq -27.15$ keV, and the value of $B(E2; 6_1^+ \rightarrow 4_1^+)$ is zero when $\eta \leq -17.96$ keV. The level crossings in Fig. 3(a) induce the sudden change of the B(E2) values to zero in Fig. 3(b). At the middle point, which is the SU(3) symmetry limit of the new mechanism in [36], the B(E2) anomaly cannot occur. Thus the new mechanism is related to SU(3) symmetry, but in a *hidden* way. The level-crossing points lie to the right of the middle point, but they exist. This is very interesting. This implies that the previous understanding with $B(E2; 4_1^+ \to 2_1^+) = 0$ and $B(E2; 2_1^+ \to 0_1^+) \neq 0$ is insufficient.

## IV. LEVEL-ANTICROSSING IN B(E2) ANOMALY AND NEW RELATIONSHIP

Now we discuss the emergence of the B(E2) anomaly in [36]. Fig. 4(a) shows the evolutionary behavior of the low-lying levels when the parameter $\chi$ changes from $-\frac{\sqrt{7}}{2}$ (the SU(3) symmetry limit) to 0 (the O(6) symmetry limit). The parameters are deduced from [36] ($\kappa = 30.0$ keV, $\eta = -14.87$ keV, and $f = 15.5$ keV). In [36], $\varepsilon_d = 60.0$ keV, but here $\varepsilon_d = 0$ keV in order to discuss the evolutions from the SU(3) symmetry limit to the O(6) symmetry limit. Thus the middle point ($\eta = -14.87$ keV) in Fig. 3(a) is the SU(3) symmetry limit of Fig. 4(a). Prominent level anticrossing can be seen for the $4_1^+, 4_2^+$ states, the $6_1^+, 6_2^+$ states, and even the $2_1^+, 2_2^+$ states (see the three black circles). The parameter region between the two dashed lines corresponds to $B_{4/2} \leq 0.38$. The value 0.38 is the $B_{4/2}$ value in $^{170}$Os. The $B(E2; 6_1^+ \to 4_1^+)$ anomaly also exists. Thus the new mechanism in [36] is related to level anticrossing.

**Fig. 4.** (a) Evolutionary behavior of the partial low-lying levels as a function of $\chi$ in $\hat{H}_5$ for $N = 9$, from the SU(3) symmetry limit to the O(6) symmetry limit when $\eta = -14.87$ keV. (b) Evolutionary behavior of $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line), $B(E2; 4_1^+ \to 2_1^+)$ (red dashed line), and $B(E2; 6_1^+ \to 4_1^+)$ (green dash-dotted line) as functions of $\chi$ in $\hat{H}_5$ when $\eta = -14.87$ keV. Other parameters are deduced from [36].

In Fig. 4, $\eta = -14.87$ keV. Now we change $\eta$ to study level anticrossing in detail. Fig. 5(a) shows the evolu-

**Fig. 5.** (a) Evolutionary behavior of selected low-lying levels as a function of $\chi$ in $\hat{H}_5$ for $N = 9$, from the SU(3) symmetry limit to the O(6) symmetry limit, with $\eta = -22.305$ keV; (b) evolutionary behavior of $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line), $B(E2; 4_1^+ \to 2_1^+)$ (red dashed line), and $B(E2; 6_1^+ \to 4_1^+)$ (green dash-dotted line) as functions of $\chi$ in $\hat{H}_5$ with $\eta = -22.305$ keV. The other parameters are taken from Ref. [36].

**Fig. 6.** (a) Evolutionary behavior of selected low-lying levels as a function of $\chi$ in $\hat{H}_5$ for $N = 9$, from the SU(3) symmetry limit to the O(6) symmetry limit, with $\eta = -29.74$ keV; (b) evolutionary behavior of $B(E2; 2_1^+ \to 0_1^+)$ (solid blue line), $B(E2; 4_1^+ \to 2_1^+)$ (red dashed line), and $B(E2; 6_1^+ \to 4_1^+)$ (green dash-dotted line) as functions of $\chi$ in $\hat{H}_5$ with $\eta = -29.74$ keV. The other parameters are taken from Ref. [36].

evolutionary behavior of the low-lying levels when the parameter $\chi$ changes from $-\frac{\sqrt{7}}{2}$ to 0 and $\eta = -22.305$ keV. The other parameters are the same as those in Fig. 4. In Fig. 3(a), as $\eta$ changes from $-14.87$ keV to $-22.305$ keV, the $6_1^+$ state crosses over with another $6^+$ state. In Fig. 5(a), the level anticrossing of the two $6^+$ states disappears. The level anticrossing of the $4_1^+$ and $4_2^+$ states becomes more pronounced and moves toward the SU(3) symmetry side (see the upper black circle). The parameter region between the two dashed lines for $B_{4/2} \leq 0.38$ becomes larger. However, over most of this anomalous region, the value of $B(E2; 6_1^+ \to 4_1^+)$ is nearly the same as that of $B(E2; 4_1^+ \to 2_1^+)$, which is an important signature of this new mechanism. However, at the position of the left dashed line, the value of $B(E2; 6_1^+ \to 4_1^+)$ is much smaller than that of $B(E2; 4_1^+ \to 2_1^+)$.

In Fig. 6, $\eta$ is changed to $-29.74$ keV. In Fig. 3(a), as $\eta$ changes from $-22.305$ keV to $-29.74$ keV, the $4_1^+$ state intersects with another $4^+$ state. Fig. 6(a) shows the evolutionary behavior of the low-lying levels when the parameter $\chi$ changes from $-\frac{\sqrt{7}}{2}$ to 0. The other parameters are the same as those in Fig. 4. Clearly, the level anticrossing of the $4_1^+$ and $4_2^+$ states also disappears, and only the level anticrossing between the $2_1^+$ and $2_2^+$ states remains (see the black circle). The left side of the dashed line corresponds to $B_{4/2} \leq 0.38$.

To better illustrate the evolutionary behavior of the $B_{4/2}$ values in Figs. 4-6, the corresponding evolutionary behaviors are shown in Fig. 7(a) and compared with one another. In Fig. 4(b) (the solid black line in Fig. 7(a)), at the

**Fig. 7.** (a) Evolution of the $B_{4/2}$ values as a function of $\chi$ in $\hat{H}_5$ for $N = 9$ when $\eta = -14.87$ keV (black solid line), $-22.305$ keV (red dashed line), and $-29.74$ keV (blue dash-dotted line); (b) evolution of the $E_{4/2}$ values as a function of $\chi$ in $\hat{H}_5$ for $N = 9$ when $\eta = -14.87$ keV (black solid line), $-22.305$ keV (red dashed line), and $-29.74$ keV (blue dash-dotted line). Other parameters are taken from [36].

At both the SU(3) symmetry limit and the O(6) symmetry limit, the $B_{4/2}$ values are larger than 1.0. Around $\chi = -0.35$, the B(E2) anomaly does arise. Thus, an anomalous $B_{4/2}$ dip exists. In Fig. 5(b) (the red dashed line in Fig. 7(a)), the B(E2) anomaly dip also appears more prominent and extends over a broader parameter region. In Fig. 6(b) (the blue dash-dotted line in Fig. 7(a)), the B(E2) anomaly occurs from the SU(3) symmetry limit and persists over an even broader parameter region. Fig. 7(b) shows the evolution of the $E_{4/2}$ values in Figs. 4-6, and we find that the $E_{4/2}$ values at the level-anticrossing points can be smaller than 2.0. However, this problem can be resolved when the parameter $f$ of the $\hat{L}^2$ interactions is further adjusted in Hamiltonian (5).

These discussions reveal a new relationship: level crossing in the SU(3) symmetry limit is related to level anticrossing in the parameter region from the SU(3) symmetry limit to the O(6) symmetry limit. For $\hat{H}_5$, when the third-order interaction $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ is introduced, level crossing from $-\hat{Q} \cdot \hat{Q}$ to $-\hat{Q} \cdot \hat{Q} + \frac{\eta}{\kappa}[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ in the SU(3) symmetry limit, or level anticrossing from $-\hat{Q} \cdot \hat{Q} + \frac{\eta}{\kappa}[\hat{L} \times \hat{Q} \times \hat{L}]^{(0)}$ to $-\hat{Q}_0 \cdot \hat{Q}_0 + \frac{\eta}{\kappa}[\hat{L} \times \hat{Q}_0 \times \hat{L}]^{(0)}$, must occur, and only one result can appear (see the number of black circles in Figs. 3-6). This is the most important finding of this paper. Thus, this new mechanism in [36] is indeed related to the SU(3) symmetry.

This finding is very interesting. In the previous IBM, from $-\hat{Q}\cdot\hat{Q}$ (the SU(3) symmetry limit) to $-\hat{Q}_0\cdot\hat{Q}_0$ (the O(6) symmetry limit), there is no level-crossing or level-anticrossing phenomenon. In [34], in the O(6) symmetry limit, it was found that when the negative $[\hat{L}\times\hat{Q}_0\times\hat{L}]^{(0)}$ interaction is added to $-\hat{Q}_0\cdot\hat{Q}_0$, it presents an oblate shape. Thus, the sequence from $-\hat{Q}\cdot\hat{Q}$ to $-\hat{Q}\cdot\hat{Q}+\frac{\eta}{\kappa}[\hat{L}\times\hat{Q}\times\hat{L}]^{(0)}$, and then to $-\hat{Q}_0\cdot\hat{Q}_0+\frac{\eta}{\kappa}[\hat{L}\times\hat{Q}_0\times\hat{L}]^{(0)}$, presents a prolate-oblate asymmetric shape phase transition.

## V. A NEW FRAMEWORK FOR THE B(E2) ANOMALY

Now we establish a new explanatory framework for the B(E2) anomaly with interactions up to third order. Fourth-order interactions will be considered in the future because their discussion is much more complicated. Thus, the Hamiltonian is as follows:

$$\hat{H}_6=\varepsilon_d\hat{n}_d-\kappa\hat{Q}_\chi\cdot\hat{Q}_\chi+\zeta[\hat{Q}_\chi\times\hat{Q}_\chi\times\hat{Q}_\chi]^{(0)}+\eta[\hat{L}\times\hat{Q}_\chi\times\hat{L}]^{(0)}+f\hat{L}^2. \quad (7)$$

where $\zeta$ is a new fitting parameter. If $\chi=-\frac{\sqrt{7}}{2}$, this Hamiltonian has been discussed for describing the B(E2) anomaly in [26]. Here the $\chi$ value can vary from $-\frac{\sqrt{7}}{2}$ to 0. This Hamiltonian combines the ideas in [26] and [36].

In [26], due to level crossing of the $4_1^+$ state and one other $4^+$ state in the SU(3) symmetry limit, the $B_{4/2}$ value can be 0. When the $d$-boson number operator $\hat{n}_d$ is added, the experimental $B_{4/2}$ value can be obtained. From the analysis of [36], we know that if the parameter $\chi$ in $\hat{Q}_\chi$ changes from the SU(3) symmetry limit to the O(6) symmetry limit, the anomalous experimental value $B_{4/2}$ may also be obtained. Now we extend the discussion in [26] with the new mechanism.

The parameters in [26] are used ($\kappa=60.18$ keV, $\zeta=-9.9653$ keV, $\eta=-17.7193$ keV, and $f=-4.515$ keV) and let $\varepsilon_d=0$ for discussing the evolution from the SU(3) symmetry limit to the O(6) symmetry limit. In the SU(3) symmetry limit, the B(E2) anomaly exists for $B(E2;4_1^+\to2_1^+)=0$. The SU(3) analysis for this case can be found in [42]. Fig. 8(a) shows the evolution of the low-lying levels when the parameter $\chi$ changes from $-\frac{\sqrt{7}}{2}$ to 0. Clearly, a shape phase transition arises. On the SU(3) symmetry side, the ground state is indeed prolate, but its excitations are not similar to prolate spectra. On the O(6) symmetry side, the sign of the O(6) third-order interactions is negative, so it is an oblate shape [34]. level anticrossing

**Fig. 8.** (a) Evolutionary behavior of selected low-lying levels as a function of $\chi$ in $\hat{H}_6$ for $N=9$ with $\eta=-17.7193$ keV. (b) Evolutionary behavior

of $B(E2;2_1^+\to 0_1^+)$ (blue solid line), $B(E2;4_1^+\to 2_1^+)$ (red dashed line), and $B(E2;6_1^+\to 4_1^+)$ (green dash-dotted line) as functions of $\chi$ in $\hat{H}_6$ for $N=9$ with $\eta=-17.7193$ keV. The other parameters are taken from [26].

**Fig. 9.** (a) Evolutionary behavior of selected low-lying levels as a function of $\chi$ in $\hat{H}_6$ for $N=9$ with $\eta=-12.5293$ keV. (b) Evolutionary behavior of $B(E2;2_1^+\to 0_1^+)$ (blue solid line), $B(E2;4_1^+\to 2_1^+)$ (red dashed line), and $B(E2;6_1^+\to 4_1^+)$ (green dash-dotted line) as functions of $\chi$ in $\hat{H}_6$ for $N=9$ with $\eta=-12.5293$ keV. The other parameters are taken from [26].

of the $2_1^+$ and $2_2^+$ states can be readily identified (see the black circle). No similar phenomenon occurs for the $4_1^+$, $4_2^+$ states or the $6_1^+$, $6_2^+$ states, or else level crossing of these levels has already occurred in the SU(3) symmetry limit when the parameter of the third-order interaction $[\hat{L}\times\hat{Q}_\chi\times\hat{L}]^{(0)}$ decreases [42]. In Fig. 8(b), the $B(E2)$ anomaly exists for $\chi<-1.0$. The region to the left of the dashed line corresponds to $B_{4/2}\le 0.38$. This is similar to Fig. 6(b). In Sec. VIII, this case will be used to fit $^{170}$Os. Over most of the region $-\frac{\sqrt{7}}{2}<\chi<-1.0$, the value of $B(E2;6_1^+\to 4_1^+)$ is larger than that of $B(E2;4_1^+\to 2_1^+)$.

If the parameter of $[\hat{L}\times\hat{Q}_\chi\times\hat{L}]^{(0)}$ is increased, then, in the SU(3) analysis, level crossing of the $4_1^+$, $4_2^+$ states will not occur. We choose $\eta=-12.5293$ keV, with the other parameters the same as those in Fig. 8(a). In the SU(3) symmetry limit, the $B(E2)$ anomaly with $B_{4/2}=0$ does not exist, and the value of $B_{4/2}$ is larger than 1.0. Figure 9(a) shows the evolutionary behavior of the low-lying levels as the parameter $\chi$ changes from $-\frac{\sqrt{7}}{2}$ to 0. Clearly, level anticrossing of the $4_1^+$, $4_2^+$ states can be identified (see the upper black circle). In Fig. 9(b), the $B(E2)$ anomaly can be seen in the parameter region extending from the SU(3) symmetry limit to the O(6) symmetry limit. The parameter region between the two dashed lines corresponds to $B_{4/2}\le 0.38$. This case is similar to Fig. 5(b), and will also be used to discuss the $B(E2)$ anomaly in $^{170}$Os and $^{166}$W.

If the parameter of $[\hat{L}\times\hat{Q}_\chi\times\hat{L}]^{(0)}$ is increased further, then, from the SU(3) symmetry limit to the O(6) symmetry limit, level anticrossing of both the $4_1^+$, $4_2^+$ states and the $6_1^+$, $6_2^+$ states can be observed, which is similar to the result in Fig. 4(b) and

**Fig. 10.** (a) Evolution of the $B_{4/2}$ values as a function of $\chi$ in $\hat{H}_6$ for $N=9$, with $\eta=-17.7193$ keV (black solid line) and $-12.5293$ keV (blue dashed line). (b) Evolution of the $E_{4/2}$ values as a function of $\chi$ in $\hat{H}_6$ for $N=9$, with $\eta=-17.7193$ keV (black solid line) and $-12.5293$ keV (blue dashed line). Other parameters are taken from [26].

**Fig. 11.** The quadrupole-deformation part of the phase diagram in the extended model proposed by Fortunato *et al.* [9]. The region above the blue dashed line represents the oblate shape, while the region below it represents the prolate shape. The green dash-dotted line represents the evolution in $\hat{H}_7$, the red solid line represents that in $\hat{H}_8$, and the black dotted line represents that in

$\hat{H}_9$.

not shown here.

Fig. 10(a) shows the evolution of the $B_{4/2}$ values in Figs. 8 and 9. The B(E2) anomaly can indeed occur. These anomalous shapes are similar to those in Fig. 6(b) and Fig. 5(b), but they shift toward the SU(3) symmetry side and become narrower. Fig. 10(b) shows the evolution of the $E_{4/2}$ values in Figs. 8 and 9. In the new Hamiltonian (6), the $E_{4/2}$ values at the level-anticrossing points appear more natural than those in Fig. 7(b), and can be further adjusted by changing the parameter $f$.

Thus, the new mechanism in [36] can be incorporated into [26]. If level crossing arises in the SU(3) symmetry limit, level anticrossing from the SU(3) symmetry limit to the O(6) symmetry limit cannot occur. The converse is also true. The newly discovered relationship is universal and may provide a universal mechanism for the emergence of the B(E2) anomaly. The key finding is that, compared with $\hat{H}_5$, $\hat{H}_6$ makes the prolate-oblate asymmetric phase transition more prominent.

It is also necessary to distinguish the rigid-triaxial explanation [33, 40, 77] from the level-crossing—or level-anticrossing—explanation in the new framework proposed in this paper. In the rigid-triaxial explanation, only certain specific triaxial deformations can generate the B(E2) anomaly, whereas in the new framework, any quadrupole deformation can induce the B(E2) anomaly. Thus, the level-crossing—or level-anticrossing—explanation is more universal. In [42], it was found that, in the SU(3) analysis, not only can the $B(E2; 4_1^+ \rightarrow 2_1^+)$ anomaly be found, but the $B(E2; 2_1^+ \rightarrow 0_1^+)$ anomaly also exists. For $^{166}$Os, the $B(E2; 2_1^+ \rightarrow 0_1^+)$ value is very small—only 7(4) W.u.—which is much smaller than that in $^{168}$Os, 74(13) W.u. In another paper, we discuss this problem within the new framework [43].

## VI. SOME DISCUSSIONS ON $[\hat{Q}_\chi \times \hat{Q}_\chi \times \hat{Q}_\chi]^{(0)}$

In $\hat{H}_6$, one may wonder whether this third-order interaction, $[\hat{Q}_\chi \times \hat{Q}_\chi \times \hat{Q}_\chi]^{(0)}$, affects the B(E2) anomaly. This needs to be discussed here, and some interesting results are found.

In 1999, Isacker discussed the O(6) third-order interaction $[\hat{Q}_0 \times \hat{Q}_0 \times \hat{Q}_0]^{(0)}$ [81]. This work, together with discussions of the SU(3) correspondence of the rigid triaxial rotor [31], inspired the emergence of the SU3-IBM. The O(6) third-order interaction has O(6) symmetry, but the O(5) symmetry in the O(6) symmetry-reduction chain is broken. Isacker showed that this interaction can describe the prolate shape and its rotation (see Fig. 11), and can replace the SU(3) second-order interaction $-\hat{Q} \cdot \hat{Q}$. Here we further study the relationship

**Fig. 12.** (a) Evolution of selected low-lying levels as a function of $\eta$ in $\hat{H}_7$ along the green dash-dotted line in Fig. 11; (b) evolution of $B(E2; 2_1^+ \rightarrow 0_1^+)$

(blue solid line), $B(E2;4_1^+\to 2_1^+)$ (red dashed line), and $B(E2;6_1^+\to 4_1^+)$ (green dash-dotted line) as functions of $\eta$ in $\hat{H}_7$ along the green dash-dotted line in Fig. 11.

**Fig. 13.** (a) Evolution of selected low-lying levels as a function of $\eta$ in $\hat{H}_8$ along the red solid line in Fig. 11; (b) evolution of $B(E2;2_1^+\to 0_1^+)$ (blue solid line), $B(E2;4_1^+\to 2_1^+)$ (red dashed line), and $B(E2;6_1^+\to 4_1^+)$ (green dash-dotted line) as functions of $\eta$ in $\hat{H}_8$ along the red solid line in Fig. 11.

between the SU(3) interaction and the O(6) third-order interaction, which can be described by the Hamiltonian

$$\hat{H}_7 = c\left[-(1-\eta)\hat{Q}\cdot\hat{Q} + \eta\left[\hat{Q}_0\times\hat{Q}_0\times\hat{Q}_0\right]^{(0)}\right]. \quad (8)$$

Fig. 12(a) shows the evolution of the low-lying levels as the parameter $\eta$ changes from 0 to 1 for $N = 9$ (see the green dash-dotted line in Fig. 11). Clearly, the levels evolve in an approximately linear manner. This result appears very interesting and somewhat puzzling. On the SU(3) symmetry side, the levels have good quantum numbers $(\lambda,\mu)$, whereas on the O(6) symmetry side only the O(6) quantum number $\sigma$ is used. Fig. 12(b) shows the evolution of the E2 transition rates $B(E2;2_1^+\to 0_1^+)$, $B(E2;4_1^+\to 2_1^+)$, and $B(E2;6_1^+\to 4_1^+)$; the results are similar for all $\eta$. Thus, here the prolate rotational spectra described by the parameters along the green dash-dotted line in Fig. 11 are very similar, and the emergence of the $B(E2)$ anomaly should be found on the upper side of the green line.

In the O(6) symmetry limit, $\left[\hat{Q}_0\times\hat{Q}_0\times\hat{Q}_0\right]^{(0)}$ represents the prolate shape, whereas $-\left[\hat{Q}_0\times\hat{Q}_0\times\hat{Q}_0\right]^{(0)}$ represents the oblate shape [53]. In [34], it was found that the spectra of the two shapes are also the same. Now we have such a Hamiltonian

$$\hat{H}_8 = c\left[-(1-\eta)\hat{Q}\cdot\hat{Q} - \eta\left[\hat{Q}_0\times\hat{Q}_0\times\hat{Q}_0\right]^{(0)}\right], \quad (9)$$

which represents the prolate-oblate asymmetric shape phase transition. In Fig. 11, it is shown that any evolution path through the blue dashed line will generate the prolate-oblate shape phase transition, and $\hat{H}_8$ is a typical example. Fig. 13(a) shows the evolution of the low-lying lev-

**Fig. 14.** (a) Evolution of selected low-lying levels as a function of $\eta$ in $\hat{H}_9$ along the black dotted line in Fig. 11; (b) Evolution of $B(E2;2_1^+\to 0_1^+)$ (solid blue line), $B(E2;4_1^+\to 2_1^+)$ (red dashed line), and $B(E2;6_1^+\to 4_1^+)$ (green dash-dotted line) as functions of $\eta$ in $\hat{H}_9$ along the black dotted line in Fig. 11.

els when the parameter $\eta$ changes from 0 to 1 (see the red solid line in Fig. 11). A prominent quantum phase transition can be observed, and, importantly, level anticrossing between the $2_1^+$ and $2_2^+$ states, or between the $4_1^+$ and $4_2^+$ states, or between the $6_1^+$ and $6_2^+$ states, can be found. In Fig. 13(b), the $B(E2)$ anomaly appears in a narrow parameter region. However, this anomaly cannot explain the experimental data. When evolving along the dashed blue line, this kind of level anticrossing can be found on the SU(3) symmetry side, but it occurs only within a narrow parameter region and is not useful.

We also consider a prolate-oblate shape phase transition arising on the O(6) symmetry side of the dashed blue line. The evolutionary path of the black dotted line in Fig. 11 is studied.

**Fig. 15.** Evolution of selected levels of the $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ interaction as a function of $\chi$.

Thus we have the Hamiltonian

$$\hat{H}_9 = c\left[-\frac{1}{N}\hat{Q}_\chi \cdot \hat{Q}_\chi - \frac{\kappa}{N^2}[\hat{Q}_\chi \times \hat{Q}_\chi \times \hat{Q}_\chi]^{(0)}\right], \tag{10}$$

From the $-\hat{Q}\cdot\hat{Q}$ to the $-[\hat{Q}\times\hat{Q}\times\hat{Q}]^{(0)}$, the prolate-oblate shape phase transition can occur, and there is an SU(3) degenerate point at $\kappa_0 = \frac{\sqrt{35}N}{3(2N+3)}$ [27, 54]. In Fig. 14, $\kappa = \kappa_0/2$ is used. It is shown that level anticrossing and the $B(E2)$ anomaly cannot be found.

Thus, the most important interaction is $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ in $\hat{H}_6$ for the emergence of the $B(E2)$ anomaly. However, the third-order interaction $[\hat{Q}_\chi \times \hat{Q}_\chi \times \hat{Q}_\chi]^{(0)}$ is still necessary. In other papers related to the oblate shape [55, 59], we can see that this interaction plays an important role, especially in the SU(3) symmetry limit. The $-[\hat{Q}_\chi \times \hat{Q}_\chi \times \hat{Q}_\chi]^{(0)}$ interaction induces the prolate-oblate asymmetric shape phase transition [55] and the new spherical-like $\gamma$-soft spectra [27]. The $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ interaction mainly induces the $B(E2)$ anomaly. When fourth-order interactions are considered, it will be more complex and will be discussed in the future (rigid triaxial shapes can be found).

**Table 1.** Fitting parameters of results 1 and 2 for $^{170}$Os, $^{166}$W, $^{168}$Os, and $^{172}$Pt. The unit is keV except for $\chi$.

| Result 1 | $\chi$ | $\varepsilon_d$ | $\kappa$ | $\zeta$ | $\eta$ | $f$ |
|---|---|---|---|---|---|---|
| $^{170}$Os | −1.200 | 138.0 | 67.240 | −11.130 | −19.800 | −5.040 |
| $^{166}$W | −1.065 | 0.0 | 88.040 | −14.570 | −18.330 | −6.610 |
| $^{168}$Os | −0.960 | 0.0 | 97.130 | −16.084 | −28.599 | −7.287 |
| $^{172}$Pt | −0.931 | 0.0 | 146.170 | −24.204 | −43.037 | −10.966 |

| Result 2 | $\chi$ | $\varepsilon_d$ | $\kappa$ | $\zeta$ | $\eta$ | $f$ |
|---|---|---|---|---|---|---|
| $^{170}$Os | $-1.172$ | 0.0 | 93.560 | $-15.490$ | $-19.470$ | $-7.020$ |
| $^{166}$W | $-1.171$ | 0.0 | 82.206 | $-13.613$ | $-17.115$ | $-6.167$ |
| $^{168}$Os | $-1.229$ | 0.0 | 83.470 | $-13.822$ | $-24.577$ | $-6.262$ |
| $^{172}$Pt | $-1.232$ | 0.0 | 11.960 | $-18.540$ | $-32.965$ | $-8.400$ |

**Fig. 16.** The experimental data for the yrast bands in $^{170}$Os and $^{166}$W, together with fitting results 1 and 2.

**Fig. 17.** $B_{J/2}$ as a function of the parameter $J$ in $^{170}$Os and $^{166}$W. The experimental data are shown as black circles. The theoretical results are fitting results 1 and 2 from this work, as well as those of [26] (Wang), [36] (Pan), and [37] (Zhang).

## VII. MORE EXPLANATIONS ON LEVEL-ANTICROSSING

In Section II, the mechanisms of level anticrossing in the IBM or SU3-IBM are discussed. In Fig. 12, we show that the quantum numbers $\lambda, \mu$ in $-\hat{Q} \cdot \hat{Q}$ are closely related to the quantum number $\tau$ in $[\hat{Q}_0 \times \hat{Q}_0 \times \hat{Q}_0]^{(0)}$, but are not exactly equivalent. Between the SU(3) symmetry and the O(6) symmetry, no significant relationship was found, except in Ref. [81]. For the $[\hat{Q}_0 \times \hat{Q}_0 \times \hat{Q}_0]^{(0)}$ interaction, more symmetry structures may be found. By intuitively analyzing the spectra of the $[\hat{Q}_0 \times \hat{Q}_0 \times \hat{Q}_0]^{(0)}$ interaction, quasi-SU(3) symmetry can be confirmed. Thus, in Fig. 13, the prolate shape has SU(3) symmetry, while the oblate shape has quasi-SU(3) symmetry; level anticrossing results from the slight breaking of SU(3) symmetry, similar to the case in [17].

In Figs. 8 and 9, similar situations occur. The introduction of the $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ interaction can produce effects similar to those of quasi-SU(3) symmetry. Fig. 15 shows the evolution of some levels under the $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ interaction as a function of $\chi$, which also presents a nearly linear behavior. For the prolate-oblate shape phase transition in Figs. 8 and 9, level anticrossing must occur. A detailed discussion of the quasi-SU(3) symmetry of the $[\hat{Q}_\chi \times \hat{Q}_\chi \times \hat{Q}_\chi]^{(0)}$ and $[\hat{L} \times \hat{Q}_\chi \times \hat{L}]^{(0)}$ interactions is needed in future work, because this is important for the general mapping of the rigid triaxial rotor in the IBM [36, 82]. Thus, the level anticrossing in this paper results from the addition of SU(3) higher-order interactions and the quasi-SU(3) symmetry of the O(6) higher-order interactions.

**Fig. 18.** The experimental data of the yrast band in $^{168}$Os and $^{172}$Pt and the fitting results 1 and 2.

**Fig. 19.** $B_{J/2}$ as a function of the parameter $J$ in $^{168}$Os and $^{172}$Pt. The experimental data are presented by black circles. The theories are the fitting results 1 and 2 in this paper, and 33, 36 and 37.

## VIII. $B(E2)$ ANOMALIES IN $^{170}$Os, $^{166}$W, $^{168}$Os AND $^{172}$Pt

Now we fit the $^{170}$Os, $^{166}$W, $^{168}$Os and $^{172}$Pt with the Hamiltonian $\hat{H}_6$. The fitting parameters are deduced from section V. The energy of the $2_1^+$ state is the same as the experimental value. The parameters $f$ and $\varepsilon_d$ are adjusted to satisfy the $B_{4/2}$ values and all the parameters are multiplied by the same factor. The boson number $N$ is half of the number of the valence nucleons. For $^{170}$Os, there are 6 valence proton holes and 12 valence neutrons, resulting in boson number 9. Similarly, $^{166}$W has boson number 9, while $^{168}$Os and $^{172}$Pt both have boson number 8.

**Table 1 shows the fitting parameters of the result 1 and 2 for $^{170}$Os, $^{166}$W, $^{168}$Os and $^{172}$Pt. For $^{170}$Os, the parameters of the result 1 are deduced from Fig. 8, and the ones of the result 2 are deduced from Fig. 9. For $^{166}$W, the parameters of the result 1 and 2 are both deduced from Fig. 9. For $^{168}$Os and $^{172}$Pt, we choose new parameters. Figs. 16 and 18 present the fitting results. These results are similar to the ones in [26, 36]. When the two SU(3) fourth-order interactions $\hat{C}_2^2[\mathrm{SU}(3)]$, $[(\hat{L}\times\hat{Q})^{(1)}\times(\hat{L}\times\hat{Q})^{(1)}]^{(0)}$ are considered, the levels in the ground band can fit more precisely [44, 45].**

Defining $B_{J/2} = B(E2; J_1^+ \to (J-2)_1^+)/B(E2; 2_1^+ \to 0_1^+)$. Figs. 17 and 19 present the $B_{J/2}$ evolutions with the parameter $J$. The experiment results of $B_{4/2}$ in $^{170}$Os, $^{166}$W, $^{168}$Os and $^{172}$Pt are 0.38(11), 0.33(5), 0.34(18) and 0.55(14) (see the black circles), so these theoretical results fit well for $B_{4/2}$. The key difference is $B_{6/2}$. In [26, 37], the $B_{6/2}$ is smaller than the $B_{4/2}$. In result 2, the $B_{6/2}$ is much smaller than the $B_{4/2}$, which can be also found in [36] but not mentioned. In Fig. 5, the $B_{6/2}$ can reduce to a very small value 0.015 when $B_{4/2}$ is 0.38. Result 1 and [36] show an opposite trend. In [70], the $B_{6/2}$ value in $^{166}$W is 0.12(4) (see the black circle). Theoretically, only the result 2 fits this experi-

mental value. In [33], the two SU(3) fourth-order interactions are also considered, and the $B_{6/2}$ can be reduced to 0.12; see Fig. 19. Thus, by comparison with the experimental result for $B_{6/2}$, some theoretical mechanisms can be excluded. We also look forward to more experimental data on the $B_{6/2}$, $B_{8/2}$, and $B_{10/2}$ values in these four nuclei.

## IX. CONCLUSION

Using the SU(3) analysis technique proposed in a previous paper, we observed that the new mechanism found in [36] is also related to the SU(3) symmetry. In this paper, we illustrate the reason for this. We find that the third-order interaction $\left[\hat{L}\times\hat{Q}_\chi\times\hat{L}\right]^{(0)}$ is vital for the emergence of the B(E2) anomaly, which is found to have quasi-SU(3) symmetry. A specific new relationship is found. If level crossing occurs in the SU(3) symmetry limit when the parameter of $\left[\hat{L} \times \hat{Q} \times \hat{L}\right]^{(0)}$ decreases, level anticrossing from the SU(3) symmetry limit to the O(6) symmetry limit cannot appear. The opposite result is also true. The two ideas in [26] and [36] can merge into a new explanatory framework, and the B(E2) anomalies in $^{170}$Os, $^{166}$W, $^{168}$Os, and $^{172}$Pt are discussed. In [42], an important finding was that not only does the $B(E2; 4_1^+ \to 2_1^+)$ anomaly exist, but the $B(E2; 2_1^+ \to 0_1^+)$ anomaly also exists. In another paper, the $B(E2; 2_1^+ \to 0_1^+)$ anomaly in $^{166}$Os is also discussed within the new explanatory framework [43], which is vital for understanding the real source of the B(E2) anomaly. In future work, the two SU(3) fourth-order interactions are also considered in relation to the level-anticrossing phenomena.

**This new relationship is highly intriguing and complicates our efforts to identify the underlying mechanism responsible for the B(E2) anomaly. This requires further development of new experimental techniques [83–87] and analysis of additional anomalous experimental results [52, 60, 61].**

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*Note: Figure translations are in progress. See original paper for figures.*