Disentangling the global multiplicity and spectral shape fluctuations in radial flow

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Date: 2026-01-12T01:38:37+00:00

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Full Text

Disentangling Global Multiplicity and Spectral Shape Fluctuations in Radial Flow

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(Dated: January 11, 2026)

Abstract. Radial flow, a key collective phenomenon in heavy-ion collisions, manifests through event-by-event fluctuations in transverse momentum (p_T) spectra. The p_T-differential radial flow observable, $v_0(p_T)$, was originally designed to capture local spectral shape fluctuations but is inevitably influenced by global multiplicity fluctuations. Using the HIJING model, we demonstrate that different definitions of event activity for centrality classification and different spectral normalization schemes induce a constant vertical offset in $v_0(p_T)$ without altering its shape. This demonstrates that only the shape of $v_0(p_T)$, or equivalently its derivative $dv_0(p_T)/dp_T$, carries physical information about radial flow dynamics. Practical implications include the necessity to vertically align measurements from different experiments before comparison and the advantage of using the derivative observable to eliminate normalization ambiguities when constraining QGP properties.

Introduction

Radial and anisotropic flows are critical signatures of collective behavior in the quark-gluon plasma (QGP) formed in high-energy heavy-ion collisions [1]. While anisotropic flow (v_n, $n = 1, 2, ...$) has been extensively studied, radial flow (v_0)—the isotropic collective expansion driven by pressure gradients—remains comparatively underexplored. Event-by-event (EbE) fluctuations in the initial collision geometry and local energy densities produce corresponding fluctuations in radial flow, which manifest as variations in the transverse momentum (p_T) spectra, $N(p_T) = dN/dp_T$ [2, 3]. These radial flow fluctuations encode valuable information about fundamental QGP properties, including its equation of state and transport coefficients such as shear and bulk viscosities [4–6].

Historically, radial flow studies focused on integral quantities such as the average transverse momentum $[p_T]_R$ or particle multiplicity N_R within a momentum range R. These studies quantify radial flow fluctuations through multi-particle cumulants, $c_0\{k\}$ and $v_0\{k\}$ ($k = 1, 2, ...$), formulated analogously to anisotropic flow:

$$
\begin{aligned}
c_{0,x}\{1\} &= v_{0,x}\{1\} = \langle x \rangle, \\
c_{0,x}\{2\} &= v_{0,x}\{2\}^2 = \frac{\langle (\delta x)^2 \rangle}{\langle x \rangle^2}, \\
c_{0,x}\{3\} &= v_{0,x}\{3\}^3 = \frac{\langle (\delta x)^3 \rangle}{\langle x \rangle^3}, \\
c_{0,x}\{4\} &= v_{0,x}\{4\}^4 = \frac{\langle (\delta x)^4 \rangle - 3\langle (\delta x)^2 \rangle}{\langle x \rangle^4},
\end{aligned}
$$

where $x = N_R$ or $[p_T]_R$, and $\delta x = x - \langle x \rangle$. For brevity, we denote $v_0\{k\}$ when $x = N_R$ and $v_{0,p}\{k\}$ when $x = [p_T]_R$. Most early measurements focused on the mean and variance of $[p_T]_R$ (i.e., $v_{0,p}\{1\}$ and $v_{0,p}\{2\}$), though investigations of higher-order cumulants have recently emerged [6–11]. These integrated quantities exhibit strong sensitivity to both initial-state geometry and final-state QGP properties.

Recent measurements have advanced beyond integral observables by studying two-particle radial flow fluctuations differentially in p_T or pseudorapidity η [12, 13]. Through rigorous tests of factorization relations [12], such as

$c_0\{2\}(p_T^1, p_T^2) = v_0\{2\}(p_T^1)v_0\{2\}(p_T^2)$ and $c_0\{2\}(\eta_1, \eta_2) = v_0\{2\}(\eta_1)v_0\{2\}(\eta_2)$, these measurements confirmed the collective and long-range nature of radial flow. The measured $v_0(p_T)$ [12] displays characteristic behavior: it starts negative at low p_T, crosses zero near $p_T = p_T^0 \approx \langle [p_T] \rangle$ around 1 GeV, then peaks around 3–4 GeV before decreasing at higher p_T. Furthermore, the p_T-dependent shape of $v_0\{2\}(p_T)$ was identified as uniquely sensitive to bulk viscosity [12, 14], offering a new avenue for constraining QGP transport properties. This paper focuses exclusively on $v_0\{2\}(p_T)$, hereafter denoted simply as $v_0(p_T)$.

The radial flow in each event is encoded in the p_T-dependent yield, $N(p_T) = N\, n(p_T)$, where $N = \int N(p_T) dp_T$ is the global multiplicity and $n(p_T)$ is the normalized fractional spectrum satisfying $\int n(p_T) dp_T = 1$.

Decomposition of Fluctuations

Events with stronger (weaker) radial flow exhibit flatter (steeper) $N(p_T)$ distributions (see [Figure 1: see original paper] top). The EbE fluctuations of $N(p_T)$ can be decomposed as:

$$\delta N(p_T) = \langle n(p_T) \rangle \delta N + \langle N \rangle \delta n(p_T) + \delta N \delta n(p_T) \approx \langle n(p_T) \rangle \delta N + \langle N \rangle \delta n(p_T),$$

where the δN term reflects centrality-dependent global multiplicity fluctuations, while the $\delta n(p_T)$ term captures genuine radial flow fluctuations. However, a fundamental ambiguity remains in separating δN from $\delta n(p_T)$ when studying spectral fluctuations. We demonstrate that $v_0(p_T)$ is meaningful only up to an arbitrary constant reflecting irreducible centrality fluctuations. Crucially, this offset does not affect integrated observables such as $[p_T]$ fluctuations, confirming that only the shape or derivative $dv_0(p_T)/dp_T$ carries physical information about radial flow. These findings have important practical implications: $v_0(p_T)$ measurements from different experiments or theoretical calculations must be vertically shifted to align their zero-crossing points before meaningful comparisons can be made.

[Figure 1: see original paper] shows a schematic illustration of how radial flow fluctuations create correlations between the EbE p_T-differential yield $n(p_T)$ and the EbE average transverse momentum $[p_T]$. The blue curve represents an event with larger-than-average radial flow, the red curve represents smaller-than-average radial flow, and the black curve represents the ensemble average. The zero-crossing point of $v_0(p_T)$, approximately at $\langle [p_T] \rangle$, depends on the normalization range chosen to obtain $n(p_T)$. The bottom panel shows fractional spectra obtained over a wider range F and its subrange R are normalized differently. The spectral fluctuation in range R can be decomposed into a global multiplicity fluctuation term and a reduced spectral shape fluctuation term (Eq. (7)).

Impact of Global Multiplicity and Spectral Shape Fluctuations on $v_0(p_T)$

Radial flow information is extracted from the correlation between $n(p_T)$ and $[p_T]$, which can be expressed in a factorizable form [17]:

$$\frac{\langle \delta n(p_T)\delta[p_T]\rangle}{\langle n(p_T)\rangle\langle[p_T]\rangle} = v_0(p_T)v_{0,p},$$

where $v_{0,p} = \sqrt{\langle(\delta[p_T])^2\rangle_A}/\langle[p_T]\rangle_A$ is measured within a reference p_T range $p_T^{\rm ref} \in A$, and $v_0\{1\}(p_T) = \langle n(p_T)\rangle$ according to Eq. (1). Recent ATLAS measurements indicate that $v_0(p_T)$ remains independent of the choice of $p_T^{\rm ref}$ [12], consistent with the collective nature of radial flow.

The connection between $v_0(p_T)$ and local spectral shape fluctuations can be explicitly formulated as:

$$v_0(p_T) = \rho(n(p_T), [pT])\frac{\sqrt{\langle(\delta n(p_T))^2\rangle}}{\langle n(p_T)\rangle},$$

where $\rho(x,y) = \langle \delta x \delta y\rangle/\sqrt{\langle(\delta x)^2\rangle\langle(\delta y)^2\rangle} \in [-1,1]$ is the Pearson correlation coefficient between x and y. Since $|v_0(p_T)| \leq \sqrt{\langle(\delta n(p_T))^2\rangle}/\langle n(p_T)\rangle$, $v_0(p_T)$ captures only the component of spectral fluctuation that is correlated with $[p_T]$.

While using the normalized spectrum $n(p_T)$ appears necessary to isolate spectral shape fluctuations from global multiplicity effects, this separation is non-unique. The key issue is that experimental measurements are always restricted to a limited p_T range R. The fractional spectrum defined within this range, $n_R(p_T) = N(p_T)/N_R$ where $N_R = \int_R N(p_T)dp_T$, differs from the ideal full-range spectrum by a rescaling factor: $n_R(p_T) = n(p_T)N/N_R$ (see [Figure 1: see original paper] bottom). Even if the total multiplicity N were fixed, the multiplicity within the restricted range, N_R, would still fluctuate statistically due to the stochastic nature of particle production. This residual fluctuation introduces an irreducible normalization ambiguity.

To quantify this ambiguity, consider an alternative definition of p_T-differential radial flow using the unnormalized spectrum $N(p_T)$ instead of $n(p_T)$:

$$\frac{\langle \delta N(p_T)\delta[p_T]\rangle}{\langle N(p_T)\rangle\langle[p_T]\rangle} = v_0'(p_T)v_{0,p}.$$

Substituting Eq. (2) and using the approximation $\langle N(p_T)\rangle = \langle Nn(p_T)\rangle = \langle N\rangle\langle n(p_T)\rangle(1+\tau)$, where the correlation term $\tau(p_T) = \langle \delta N \delta n(p_T)\rangle/(\langle N\rangle\langle n(p_T)\rangle)$ is negligible, we obtain:

$$\Delta v_0 \equiv v_0'(p_T) - v_0(p_T) = \rho(N,[p_T])\frac{\sqrt{\langle(\delta N)^2\rangle}}{\langle N\rangle}.$$

This result demonstrates that the two definitions differ by a constant. The sign and magnitude of this offset depend on how the global multiplicity N correlates with $[p_T]$, which is sensitive to the specific centrality definition employed, for example, whether centrality is based on forward calorimeters or midrapidity charged particles [12].

The offset Δv_0 can be further understood by recognizing that N_R fluctuates even when N is fixed (see [Figure 1: see original paper] bottom). Using $n_R(p_T) = N(p_T)/N_R$, the fluctuations can be decomposed as:

$$\delta N(p_T) \approx \delta N_R \langle n_R(p_T)\rangle + \langle N_R\rangle \delta n_R(p_T).$$

Assuming particles are independently sampled from the underlying momentum distribution, N_R follows a binomial distribution with sampling probability $\epsilon = \langle N_R\rangle/\langle N\rangle$. The variance of N_R at fixed N is given by:

$$\langle(\delta N_R)^2\rangle = N(1-\epsilon)\epsilon \approx \langle N_R\rangle(1-\epsilon).$$

Applying Eq. (6), the $v_0(p_T)$ calculated for R, denoted $v_{0,R}(p_T)$, differs from $v_0(p_T)$ defined in the full range by:

$$\Delta v_{0,R} = v_{0,R}(p_T) - v_0(p_T) \approx \rho(N,[p_T])\frac{\sqrt{\langle(\delta N)^2\rangle}}{\langle N\rangle} - \rho(N_R,[p_T])\frac{\sqrt{\langle(\delta N_R)^2\rangle}}{\langle N_R\rangle} \approx -\rho(N_R,[p_T])\frac{\sqrt{\langle(\delta N_R)^2\rangle}}{\langle N_R\rangle}(1-\epsilon),$$

where the subscript N denotes evaluation at fixed N. The approximation from Eq. (9) to Eq. (10) holds because changes in N induce proportional changes in N_R, such that the first term in Eq. (9) becomes negligible.

This vertical offset translates into a horizontal shift in the zero-crossing point of $v_0(p_T)$ from $p_T^0 \approx \langle[p_T]\rangle$ to $p_T^0 \approx \langle[p_T]\rangle_R$. Importantly, this shift does not affect the factorization property (Eq. (3)), which remains valid regardless of the p_T^{ref} range used to calculate $v_{0,p}$. As demonstrated in the Appendix, this offset also leaves integrated observables such as $[p_T]$ fluctuations unchanged due to sum rule constraints. Therefore, only the p_T-dependent variation of $v_0(p_T)$ contains meaningful information about collective radial flow fluctuations.

Model Setup

To investigate the interplay between global multiplicity and spectral shape fluctuations, we employ the HIJING model [18], which simulates heavy-ion collisions as a superposition of independent $p+p$ collisions. The absence of genuine collective radial flow in HIJING, combined with the presence of high-p_T particles from hard-scattered jets, provides an ideal framework for studying the non-flow baseline of $v_0(p_T)$ and isolating the effects of multiplicity fluctuations.

We generate Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV with a hard scale of 2 GeV (parameter HIPR1(10)=2) to match LHC kinematics. Charged particles with $|\eta| < 2.5$ and $p_T < 10$ GeV are selected, consistent with typical experimental acceptances. Following the two-subevent method employed by ATLAS [12] and STAR [11], we divide particles into two pseudorapidity regions: $-2.5 < \eta_a < -\eta_{\mathrm{gap}}/2$ and $\eta_{\mathrm{gap}}/2 < \eta_b < 2.5$. We use two pseudorapidity gap sizes, $\eta_{\mathrm{gap}} = 0$

and 3, to evaluate the effects of short-range non-flow correlations and to assess possible longitudinal multiplicity decorrelation effects.

The normalization ambiguities are studied by varying three key ingredients in our analysis: (1) We calculate $[p_T]$ using three different $p_T^{\rm ref}$ ranges denoted by "A" : 0.5-2, 0.5-5, and 1-5 GeV. This variation allows us to study how non-flow correlations, which are more prominent at high p_T, affect the extracted $v_0(p_T)$. (2) We define the fractional spectrum $n(p_T)$ over three p_T ranges: 0–10 GeV (full range), 0.5-10 GeV, and 1-10 GeV (sub-ranges denoted by "R"). (3) We classify events using $N_{\rm ch}$ in various η ranges (listed in) or the number of participating nucleons $N_{\rm part}$ from the Glauber model. Both the second and third variations probe the impact of residual multiplicity fluctuations.

summarizes the systematic variations: p_T ranges for defining $n(p_T)$ or $N(p_T)$, reference ranges for calculating $[p_T]$, and event activity variables for centrality classification. For each subevent, we calculate the mean transverse momentum $[p_T]$, the unnormalized spectrum $N(p_T)$, and the fractional spectrum $n(p_T)$. Particle observables are evaluated in narrow bins of the event activity estimators to minimize detector and acceptance effects.

Following the standard two-subevent methodology [12], the $[p_T]$ fluctuations are measured as $\langle(\delta[p_T])^2\rangle = \langle\delta[p_T]_a\delta[p_T]_b\rangle$, and the covariance between spectral and $[p_T]$ fluctuations is $\langle\delta n_a(p_T)\delta[p_T]_b + \delta n_b(p_T)\delta[p_T]_a\rangle$. The offset term in Eqs. (6) and (9) is estimated using:

$$\Delta v_0 = \frac{\langle\delta N_a\delta[p_T]_b\rangle}{\langle N_a\rangle\langle[p_T]_b\rangle}\frac{\langle\delta[p_T]_a\delta[p_T]_b\rangle}{\langle[p_T]_a\rangle\langle[p_T]_b\rangle}.$$

The independent source picture underlying HIJING provides useful scaling relations that serve as consistency checks. If particles used to define event activity are statistically independent of those used in the correlation analysis, the two-subevent quantities in Eq. (12) should exhibit simple power-law scaling with the number of sources N_s (approximately proportional to $N_{\rm part}$ in the HIJING framework):

$$\begin{aligned}
\langle\delta N_a\delta N_b\rangle &\propto N_s,\\
\langle\delta[p_T]_a\delta[p_T]_b\rangle &\propto 1/N_s,\\
\frac{\langle\delta N_a\delta[p_T]_b\rangle}{\langle N_a\rangle\langle[p_T]_b\rangle} &\propto 1/N_s,\\
\Delta v_0 &\propto 1/N_s,\\
\rho(N_a,[p_T]_b) \equiv \frac{\langle\delta N_a\delta[p_T]_b\rangle}{\sqrt{\langle(\delta N_a)^2\rangle\langle(\delta[p_T]_b)^2\rangle}} &\sim \text{const},\\
N_a\Delta v_0 &\sim \text{const}.
\end{aligned}$$

Here, "const" denotes a value determined by particle production within individual sources (i.e., within each $p+p$ collision). These scaling relations provide important consistency checks for our analysis and help distinguish genuine correlations from statistical artifacts.

Results

Impact of Non-Flow Correlations

[Figure 2: see original paper] presents $v_0(p_T)$ calculated using three different p_T^{ref} selections in central collisions. The results reveal significant differences at high p_T (>3 GeV), as expected from the dominance of non-flow correlations from jet production. These differences persist into the low-p_T region (see inset), where they affect the zero-crossing point depending on the choice of p_T^{ref}. Importantly, the overall magnitude of $v_0(p_T)$ in HIJING is much smaller than that observed in experimental data [12], confirming that collective radial flow—absent in HIJING—dominates the experimental measurements.

Residual Multiplicity Fluctuations and Offset

The central result of this paper is illustrated in [Figure 3: see original paper], which compares $v_0(p_T)$-like observables obtained using three different approaches: (a) using the unnormalized spectrum $N(p_T)$ via Eq. (5), (b) using the normalized spectrum $n(p_T)$ via Eq. (3), and (c) using $N(p_T)$ but applying the offset correction Δv_0 calculated from Eq. (12).

[Figure 3: see original paper]a displays $v_0'(p_T)$ obtained from the unnormalized spectrum $N(p_T)$ for different event activity definitions. While all curves show a similar increasing trend with p_T, they exhibit different vertical offsets. These event-activity-dependent offsets can drastically alter the zero-crossing point p_T^0: for event classes based on N_{part} or N_{ch} measured in forward rapidity, $v_0'(p_T)$ never crosses zero, indicating the strong influence of global multiplicity fluctuations.

In stark contrast, [Figure 3: see original paper]b shows that results based on the normalized spectrum $n(p_T)$ cluster together with a common zero-crossing point near the average transverse momentum of the inclusive spectrum ($ $1 GeV). This demonstrates that $v_0(p_T)$ obtained from fractional spectra is largely insensitive to the event activity definition, precisely as expected if it successfully isolates spectral shape fluctuations from global multiplicity effects.

The key validation appears in [Figure 3: see original paper]c: when the offset correction Δv_0 via Eq. (6) is applied to the $N(p_T)$-based results, they collapse onto the $n(p_T)$-based results. This confirms that the difference between the two approaches is indeed a p_T-independent constant offset, exactly as predicted by our formalism.

Further quantitative validation of the offset prediction is provided in [Figure 4: see original paper]. [Figure 4: see original paper]a compares the measured difference $v_0'(p_T) - v_0(p_T)$ (symbols) with the predicted offset Δv_0 (dotted lines) as a function of p_T for various event activity selections. The measured difference is remarkably flat across the entire p_T range and is accurately reproduced by the simple analytical expression in Eq. (6). Notably, the offset increases with the pseudorapidity separation between particles used for event activity classification and those used in the correlation analysis. This trend is consistent with the behavior of longitudinal multiplicity decorrelation, $\langle \delta N(\eta_1)\delta N(\eta_2)\rangle / [\langle N(\eta_1)\rangle\langle N(\eta_2)\rangle]$, measured previously by ATLAS [19], and can be attributed to inherent forward-backward fluctuations in the initial state of heavy-ion collisions [20-22].

[Figure 4: see original paper]b extends this validation across centrality by plotting $N_{\text{part}}(v_0'(p_T) - v_0(p_T))$ at $p_T = 1$ GeV compared with $N_{\text{part}}\Delta v_0$. The excellent agreement between the two validates Eq. (6) across the full centrality range. Moreover, the approximately constant behavior of $N_{\text{part}}\Delta v_0$ across centrality demonstrates excellent consistency with the independent source scaling expectation in Eq. (13).

Verification of Independent Source Scaling

To further validate the HIJING model' s independent source picture and test the consistency of our framework, we examine the centrality dependence of the three key components entering Δv_0 in Eq. (12): the covariance $\langle \delta N_a \delta[p_T]_b\rangle$, the correlation coefficient $\rho(N_a, [p_T]_b)$, and the scaled offset $N_a\Delta v_0$. [Figure 5: see original paper] displays these quantities for $\eta_{\text{gap}} = 0$ (left column) and $\eta_{\text{gap}} = 3$ (right column) across various event activity definitions.

According to the independent source scaling in Eq. (13), these quantities should remain approximately constant across centrality when particles defining event activity are statistically independent of those used in the correlation analysis. [Figure 5: see original paper] confirms this expectation for most configurations: the three components are either constant or vary slowly across centrality.

One configuration, however, requires special attention. When the event activity strongly overlaps with the analysis particles (e.g., N_{ch} in $|\eta| < 2.5$, blue symbols), we observe negative values that deviate significantly from the general trend. This behavior arises from a statistical autocorrelation artifact. The total multiplicity N_{ch} comprises contributions from three regions: subevents a, b, and the middle region c, such that $N_{\text{ch}} = N_a + N_b + N_c$. Requiring a fixed N_{ch} induces anticorrelations between N_a, N_b, and N_c, described by a multinomial distribution. When $\eta_{\text{gap}} = 0$, the middle region vanishes ($N_c = 0$), and the anticorrelation between N_a and N_b or $[p_T]_b$ is strongest. As η_{gap} increases, more particles populate the middle region, weakening the anticorrelation between N_a and N_b (see bottom row of [Figure 5: see original paper]). Crucially, this autocorrelation artifact affects only $v_0'(p_T)$ calculated from the unnormalized spectrum $N(p_T)$, while $v_0(p_T)$ calculated from the normalized spectrum $n(p_T)$ remains unaffected by construction.

When particles defining event activity are separated from the analysis particles (no overlap), the correlations are close to zero or slightly positive. The correlation strength increases for N_{ch} measured in more forward η regions and is strongest when using N_{part} from the Glauber model. When N_{part} is employed for event selection, the correlation shows a significant enhancement toward peripheral collisions. These positive correlations reflect genuine longitudinal dynamics encoded in the HIJING model rather than statistical artifacts.

Consequence of Varying p_T Acceptance

Finally, we address a practical experimental issue: different experiments employ different p_T acceptance ranges. For example, ALICE typically measures $p_T > 0.1$ GeV, CMS uses $p_T > 0.3$ GeV, and ATLAS employs $p_T > 0.5$ GeV in Pb+Pb collisions. The fractional spectra defined in these different ranges have distinct normalizations, leading to systematic differences in the extracted $v_0(p_T)$. According to Eq. (11), even when N is fixed, the multiplicity N_R within a restricted range still fluctuates due to binomial sampling. This residual fluctuation induces a downward shift in $v_0(p_T)$, moving the zero-crossing point from $p_T^0 \approx \langle [p_T] \rangle$ to $p_T^0 \approx \langle [p_T] \rangle_R$.

[Figure 6: see original paper]a confirms this expectation: measurements of $v_{0,R}(p_T)$ based on $n_R(p_T)$ show clear dependence on the p_T range used for normalization, with zero-crossing points shifting systematically with the range choice. Such a constant difference is indeed observed between ATLAS and ALICE measurements [23]. However, [Figure 6: see original paper]b demonstrates that applying the offset correction from Eq. (9) collapses these curves onto a common baseline, validating our prediction.

These results demonstrate that the location of the zero-crossing point of $v_0(p_T)$ is purely a kinematic effect determined by the average spectrum $\langle n(p_T) \rangle$ in the normalization range, and does not reflect genuine dynamical radial flow fluctuations. This has important implications for comparing $v_0(p_T)$ measurements across different experiments: direct comparison of absolute values is meaningless unless measurements are corrected to a common normalization range or, equivalently, vertically shifted to align their zero-crossing points.

Discussion and Summary

Event-by-event fluctuations in the final particle spectrum $N(p_T)$ serve as a sensitive probe of collective radial flow in the quark-gluon plasma. Experimentally, these fluctuations are accessed through the observable $v_0(p_T)$ (or $v_0'(p_T)$), which characterizes the correlation between the normalized spectrum $n(p_T)$ (or the unnormalized spectrum $N(p_T)$) and the event-by-event spectral shape, encoded in the average transverse momentum $[p_T]$. The spectral fluctuations can be decomposed into contributions from global multiplicity fluctuations and local spectral shape fluctuations:

$$\delta N(p_T) = \langle n(p_T) \rangle \delta N + \langle N \rangle \delta n(p_T).$$

We demonstrate that the separation between δN and $\delta n(p_T)$ is fundamentally ambiguous. Using HIJING simulations, we systematically varied the event activity definition, the spectral normalization range, and the reference range for $[p_T]$ calculations. We found that $v_0'(p_T)$ based on the unnormalized spectrum $N(p_T)$ is sensitive to the event activity variable and affected by autocorrelation effects when centrality and analysis particles overlap. In contrast, $v_0(p_T)$ based on the normalized spectrum $n(p_T)$ is only sensitive to the normalization range. However, all formulations are related by a constant offset Δv_0 that reflects the correlation between δN and $[p_T]$. Since the $[p_T]$ fluctuations are insensitive to these constant offsets, only the p_T-dependent variation of $v_0(p_T)$ contains meaningful physical information about collective radial flow.

These findings have important practical implications. When comparing $v_0(p_T)$ across experiments or between data and theory, curves must be vertically shifted to align their zero-crossing points, or corrected to a common normalization range using Eq. (9). Direct comparison of absolute values without such corrections could be misleading, especially considering different experimental acceptances at the LHC (ALICE: $p_T > 0.1$ GeV, CMS: $p_T > 0.3$ GeV, ATLAS: $p_T > 0.5$ GeV).

To eliminate the normalization ambiguity entirely, we propose the derivative observable:

$$v_0^s(p_T) = \frac{dv_0(p_T)}{dp_T}.$$

According to Eq. (3), $v_0^s(p_T)$ directly quantifies the correlation between the local spectral slope $s(p_T) = dn(p_T)/dp_T$ and $[p_T]$:

$$\frac{\langle \delta s(p_T) \delta [p_T] \rangle}{\langle s(p_T) \rangle \langle [p_T] \rangle} = v_0^s(p_T) v_{0,p}.$$

Since constant offsets vanish upon differentiation, $v_0^s(p_T)$ is free from normalization ambiguities, providing an unambiguous alternative for characterizing radial flow fluctuations. Recent experimental results [12, 13] imply that $v_0^s(p_T)$ is positive at low p_T, crosses zero at $p_T \sim$ 3-4 GeV, and approaches zero at higher p_T.

This work is supported by DOE Research Grant Number DE-SC0024602.

Appendix

In this Appendix, we derive sum rules relating the differential $v_0(p_T)$ to integrated $[p_T]$ fluctuations, and show how $v_0(p_T)$ measured in different p_T ranges are related. We consider the full p_T range "F", a restricted experimental range "R" (e.g., 0.5-10 GeV in ATLAS), and the reference p_T range "A" used to calculate $[p_T]$ (see [Figure 7: see original paper]). Quantities calculated in F carry no subscript, while those in other ranges are labeled by subscript R or A. We use $N(p_T)$ and $n(p_T)$ to denote the particle yield and fractional yield at p_T, and

N to denote the total multiplicity. The notation $\int_A$ represents integration of p_T over range A. This discussion borrows heavily from Refs. [14, 17].

A. Observables in Range A Expressed Using Variables in the Full Range

The average transverse momentum in an event and its ensemble average in A is given by:

$$[p_T]_A = \frac{N_A}{\int_A p_T N(p_T)} = \frac{\int_A p_T N(p_T)}{N_A}.$$

The ensemble average is:

$$\langle [p_T] \rangle_A \equiv \langle [p_T]_A \rangle = \frac{\langle \int_A p_T N(p_T) \rangle}{\langle N_A \rangle} = \frac{\int_A p_T \langle N(p_T) \rangle}{\langle N_A \rangle}.$$

Plugging this into the fluctuation expression, we obtain:

$$\delta [p_T]_A = \frac{\int_A p_T \delta N(p_T)}{\langle N_A \rangle} - \frac{\int_A p_T \langle N(p_T) \rangle}{\langle N_A \rangle^2} \delta N_A.$$

The integral involving $\langle n_A(p_T) \delta N_A \rangle$ is a higher-order term corresponding to the correlated fluctuation of spectral shape and integrated multiplicity. The other term represents the per-particle average transverse momentum. If we calculate $\langle p_T \rangle$ on a per-track basis instead of per-event basis, the expression would be exact.

From these, we obtain:

$$\delta [p_T]_A = \frac{\int_A p_T \delta N(p_T)}{\langle N_A \rangle} - \langle p_T \rangle_A \frac{\delta N_A}{\langle N_A \rangle} = \frac{\int_A (p_T - \langle p_T \rangle_A) \delta N(p_T)}{\langle N_A \rangle}.$$

The appearance of $\langle p_T \rangle_A$ is critical because it absorbs the contribution from δN_A. Note that since A is a subrange, N_A still fluctuates even if the total multiplicity N is fixed.

Using $N(p_T) = n(p_T) N$, we can rewrite:

$$\delta [p_T]_A = \frac{\int_A (p_T - \langle p_T \rangle_A) \delta N(p_T)}{\langle N_A \rangle} = \frac{\int_A (p_T - \langle p_T \rangle_A)[\delta n(p_T) N + \langle n(p_T) \rangle \delta N]}{\langle N_A \rangle}.$$

Squaring this equation, averaging over events, and then using $\langle (\delta [p_T])^2 \rangle / \langle p_T \rangle^2 = v_0^2$, we obtain:

$$\frac{\langle (\delta [p_T]_A)^2 \rangle}{\langle p_T \rangle_A^2} = \frac{\left(\int_A (p_T - \langle p_T \rangle_A) v_0(p_T) \langle n(p_T) \rangle \right)^2}{\langle p_T \rangle_A^2}.$$

From this, and using the relation $\langle p_T\rangle_A = \int_A p_T\langle n(p_T)\rangle + \int_A p_T\langle \delta N\delta n(p_T)\rangle/\langle N\rangle$ (but ignoring the $\langle \delta N\delta n(p_T)\rangle$ term), we obtain the sum rule:

$$v_{0,A} \equiv \frac{\sqrt{\langle(\delta[p_T])^2\rangle}}{\langle[p_T]\rangle_A} = f\times\frac{\int_A (p_T-\langle p_T\rangle_A)v_0(p_T)\langle n(p_T)\rangle}{\int_A p_T\langle n(p_T)\rangle},$$

where $f^2 = \langle (N/N_A)^2\rangle = \langle N\rangle^2/\langle N_A\rangle^2$. Assuming the fluctuation of N_A comprises a component fully correlated with N and a component at fixed N, then only the second component contributes, and we have $f \approx 1 + \langle(\delta N_A)^2\rangle/(2\langle N_A\rangle^2)|_N$. The deviation of f from unity should be very small.

From this we also reach the relation that connects the v_0 calculated between two p_T ranges from Ref. [14]:

$$v_{0,A} = C_A v_0, \quad \text{where} \quad C_A = f\frac{\langle N\rangle}{\langle N_A\rangle}\frac{\langle p_T\rangle_A}{\int_A p_T\langle n(p_T)\rangle}.$$

Note that due to condition of Eq. (21), Eqs. (25) and (26) remain valid under constant shift $v_0(p_T)\to v_0(p_T)+c$.

B. Relating the $v_0(p_T)$ Measured in Two p_T Ranges

In principle, the normalization of the fractional spectra can be defined in any p_T range. However, varying the choice of p_T range effectively changes the nature of global multiplicity fluctuations defined in that range, as discussed in the main text. This results in a vertical shift that adjusts the zero-crossing point of $v_0(p_T)$ in each range to its own $\langle p_T\rangle$ value. In particular, the $v_0(p_T)$ calculated in ranges F and R are related by a constant:

$$v_{0,R}(p_T) = v_0(p_T) - s.$$

This change of zero-crossing point ensures the basic sum rules are satisfied. Using $\int_R v_0(p_T) = 0$ and $\int_R v_{0,R}(p_T) = 0$, and $\langle n_R(p_T)\rangle = \langle N\rangle/\langle N_R\rangle\langle n(p_T)\rangle$, Eq. (25) is satisfied in both F and R:

$$v_{0,A} \equiv \frac{\int_A (p_T-\langle p_T\rangle_A)v_{0,R}(p_T)\langle n_R(p_T)\rangle}{\int_A p_T\langle n_R(p_T)\rangle} = \frac{\int_A (p_T-\langle p_T\rangle_A)(v_0(p_T)-s)\langle n(p_T)\rangle}{\int_A p_T\langle n(p_T)\rangle} = \frac{\int_A (p_T-\langle p_T\rangle_A)v_0(p_T)\langle n(p_T}{\int_A p_T\langle n(p_T)\rangle}$$

The insensitivity of $[p_T]$ fluctuations to the offset in $v_0(p_T)$ strengthens our conclusion that only the p_T-dependent variation of $v_0(p_T)$ is related to collective radial flow, because its baseline value is subject to a shift dictated by global multiplicity fluctuations.

Finally, we note that using $n(p_T)$ is more convenient than using $N(p_T)$, since for $N(p_T)$ the expression must account for additional multiplicity fluctuations:

$$\frac{\delta N(p_T)}{\langle N(p_T)\rangle} = v_0(p_T)\frac{\delta[p_T]}{\langle[p_T]\rangle},$$

which complicates the expression of Eqs. (25) and (26) in terms of $v_0'(p_T)$.

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