Effect of spin-orbit interaction on structure and two-proton radioactivity of \(^{18}\mathrm{Mg}\)

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Abstract

The influence of spin-orbit coupling strength (W) on the structure and two-proton ($2p$) radioactivity of ^{18}Mg is investigated using the spherical Skyrme-Hartree-Fock-Bogoliubov (SHFB) approach with the SLy4 interaction and a mean-field cluster potential model. Our calculations reveal that increasing W enhances the splitting of the single-proton $1d$ orbitals. Concurrently, the $2s_{1/2}$ proton state evolves from a weakly bound state into a resonance in the continuum. As W increases, the occupation probability of the $2s_{1/2}$ proton state decreases, and its density distribution near the nuclear surface becomes less diffuse. Furthermore, both the spectroscopic factor $S^{'}_{2p}$ and the decay energy Q_{2p} for $2p$ radioactivity gradually decrease with increasing W, resulting in a longer half-life. When Q_{2p} is held constant, the half-life is significantly enhanced by $S^{'}_{2p}$. At the mean time, it is found that the depth of the diproton cluster potential well increases with W, while the corresponding $S^{'}_{2p}$ becomes smaller, indicating that the diproton cluster is considerably looser than the α-cluster. Additionally, a clear linear correlation is observed between $\log_{10}S'_{2p}$ and Q_{2p}, as well as between $\log_{10}S'_{2p}$ and W. The logarithmic half-lives, both with and without inclusion of $S^{'}_{2p}$, exhibit good linear relationships with W and $Q_{2p}^{-1/2}$, respectively. Finally, using the experimental Q_{2p} value of ^{18}Mg (3.440(34) MeV), the optimal W is determined to be 1.152(8)W_0 with W_0=123 MeV fm^5.

Full Text

Effect of spin-orbit interaction on structure and two-proton radioactivity of ^{18}Mg

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The influence of spin-orbit coupling strength (W) on the structure and two-proton ($2p$) radioactivity of ^{18}Mg is investigated using the spherical Skyrme-Hartree-Fock-Bogoliubov (SHFB) approach with the SLy4 interaction and a

mean-field cluster potential model. Our calculations reveal that increasing W enhances the splitting of the single-proton $1d$ orbitals. Concurrently, the $2s_{1/2}$ proton state evolves from a weakly bound state into a resonance in the continuum. As W increases, the occupation probability of the $2s_{1/2}$ proton state decreases, and its density distribution near the nuclear surface becomes less diffuse. Furthermore, both the spectroscopic factor S'_{2p} and the decay energy Q_{2p} for $2p$ radioactivity gradually decrease with increasing W, resulting in a longer half-life. When Q_{2p} is held constant, the half-life is significantly enhanced by S'_{2p}. At the mean time, it is found that the depth of the diproton cluster potential well increases with W, while the corresponding S'_{2p} becomes smaller, indicating that the diproton cluster is considerably looser than the α-cluster. Additionally, a clear linear correlation is observed between $\log_{10} S'_{2p}$ and Q_{2p}, as well as between $\log_{10} S'_{2p}$ and W. The logarithmic half-lives, both with and without inclusion of S'_{2p}, exhibit good linear relationships with W and $Q_{2p}^{-1/2}$, respectively. Finally, using the experimental Q_{2p} value of ^{18}Mg (3.440(34) MeV), the optimal W is determined to be $1.152(8)W_0$ with $W_0 = 123$ MeV fm^5.

Keywords: $2p$ radioactivity; spin-orbit coupling strength; SHFB theory; mean-field cluster potential approach

I. INTRODUCTION

The $2p$ radioactivity is a rare decay mode observed in nuclei near the proton drip line, which was predicted by Zel'dovich and Goldansky in the 1960s [1–3]. Owing to the pairing correlation effect, the direct (true) $2p$ radioactivity occurs in even-Z nuclei. Because the one-proton emission is energetically forbidden while the $2p$ radioactivity is allowed [4], the true $2p$ radioactivity was first observed from the ground state of ^{45}Fe at the beginning of the 21st century [5, 6]. Later, more events of the $2p$ radioactivity from the ground states were discovered from ^{48}Ni [7], ^{54}Zn [8], ^{67}Kr [9] and ^{19}Mg [10]. Moreover, the $2p$ emission may occur from the excited states populated in β decay, which was usually named as the β-delayed $2p$ radioactivity [11–20]. Besides this, some $2p$ emitters, such as ^{14}O [21], ^{17,18}Ne [22–26], ^{22}Mg [27, 28] and ^{28,29}S [29, 30], were discovered from the excited states fed by nuclear reactions. In addition, the $2p$ emission from the 21^+ isomer state of ^{94}Ag was reported by the group of Mukha [31].

To understand the mechanism of the $2p$ radioactivity, quite a few microscopic and phenomenological models have been developed [32–65]. Generally, the $2p$ radioactivity mechanism includes the following three types: (i) Strongly correlated emission. This is due to the attraction of the two protons, also called the diproton emission. So, the emission undergoes through the penetration of a preformed ^{2}He cluster. The emitted two protons have a small angular distribution and share the same energy [32–50]. (ii) Three-body simultaneous emission. The simultaneously emitted two protons distribute isotropically but usually have similar energies [47–65]. (iii) Two-body sequential emission. A parent nucleus first emits a proton and decays to an intermediate state, which itself then emits another proton to the final state [47–65].

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A few years ago, the four proton ($4p$) radioactivity was discovered firstly from the ground state of ^{18}Mg by the invariant-mass reconstruction of ^{14}O + $2p$ events. The ground-state decay energy Q_{4p} and width Γ_{4p} were measured as 4.865(34) MeV and 115(100) keV, respectively [66]. In fact, the $4p$ radioactivity of ^{18}Mg can be seen as two sequential $2p$ emissions through the intermediate nucleus ^{16}Ne. To understand the $2p$ decay mechanism of ^{18}Mg, its structure was investigated using a three-body Gamow coupled-channel method [67]. The results showed that the ground state of ^{18}Mg is significantly influenced by the continuum, resulting in a significant s-wave component. Recently, we explored the tensor force effect on the $2p$ radioactivity of ^{18}Mg in the framework of the spherical SHFB theory and mean-field cluster potential approach [68, 69]. It showed that a $2p$ halo-like structure appears in ^{18}Mg and its spectroscopic factor becomes larger with the increase of the $2p$ halo-like size caused by the tensor force. Furthermore, the decay mode prefers to the ^{2}He emission [68, 69]. Although the $2p$ emission of ^{18}Mg has been investigated by a series of models, its mechanism has not yet been fully understood [50, 67–70]. Therefore, investigating the $2p$ emission mechanism of ^{18}Mg remains a topic of significant interest.

The spin-orbit force is a key ingredient in the nucleon-nucleon interaction, which plays a crucial role in nuclear structure and radioactivity [71–74]. For the nuclei around the drip-line, the spin-orbit coupling strength W is not only a core parameter in nuclear models but also a key to understanding exotic nuclear structures, decay mechanisms and shell evolution [71–74]. However, its magnitude has not yet been precisely determined. Therefore, it is essential to investigate the influence of W on nuclear structure and decay processes. Very recently, we explored the effect of W on the α-decay of ^{210}Po within the spherical SHFB theory. It was found that the localization of α-cluster and α-decay energy are enhanced obviously by the rise of W [75]. This drives us to think whether the $2p$ radioactivity is also influenced substantially by W. Moreover, the mechanism how the spin-orbit interaction affects the $2p$ radioactivity has not been understood clearly. And the observables of $2p$ emission can be used to constrain the magnitude of W.

Driven by the above mentioned motivation, in this article we will explore how the structure and $2p$ emission of ^{18}Mg are influenced by the spin-orbit interaction within the spherical SHFB theory and mean-field cluster potential approach. This article is organized as follows. In Sec. II, the theoretical framework is presented. Sec. III shows the calculated results and discussions. And some conclusions are drawn in the last section.

II. THEORETICAL FRAMEWORK

A. SHFB theory

It is well known that the SHFB theory is a powerful tool to describe the structure of nuclei [76-79]. It can provide a unified and self-consistent description of both the mean-field and the pairing field in terms of the Bogoliubov quasi-particles. A series of properties of nuclei are described successfully with the SHFB approach.

The Skyrme interaction V_{Skyrme} is written as [76-79]

$$\begin{aligned} V_{Skyrme} = & t_0(1 + x_0\mathbf{P}_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2) \\ & + \frac{1}{2}t_1(1 + x_1\mathbf{P}_\sigma)\left[\mathbf{k}'^2\delta(\mathbf{r}_1 - \mathbf{r}_2) + \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{k}^2\right] \\ & + t_2(1 + x_2\mathbf{P}_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{k} \\ & + \frac{1}{6}t_3(1 + x_3\mathbf{P}_\sigma)\rho^\gamma\delta(\mathbf{r}_1 - \mathbf{r}_2) \\ & + iW(\sigma_1 + \sigma_2) \cdot \left[\mathbf{k}' \times \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{k}\right], \end{aligned} \tag{1}$$

where t_i, x_i $(i = 1, 2, 3)$ and W are the parameters of the interaction, P_σ is the spin-exchange operator, and σ_j $(j = 1, 2)$ are the Pauli matrices. The operator $\mathbf{k} = (\nabla_1 - \nabla_2)/2i$ acts on the right and $\mathbf{k}' = -(\nabla_1 - \nabla_2)/2i$ on the left.

In the particle-particle (pairing) channel, a density-dependent δ interaction is usually employed, whose form is [77]

$$V_{pair}(\mathbf{r}_1, \mathbf{r}_2) = \left(t_0' + \frac{t_3'}{6}\rho^{\gamma'}\right)\delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{2}$$

where $\gamma' = 1$, $t_3' = 0$, $-37.5t_0'$, $-18.5t_0'$ corresponds to the volume, surface, and mixed pairing force, respectively. t_0' is the pairing strength parameter. Usually, its value is determined by the empirical pairing energy gap [80-82]. ρ is the isoscalar local density.

In the framework of the SHFB approach, the expressions of the effective mass, abnormal effective mass, particle-hole field, particle-particle field and Coulomb field can be found from Refs. [76-79]. As to the spin-orbit coupling fields,

whose forms are expressed as

$$B_q = -\frac{1}{8}(t_1x_1 + t_2x_2)\,J + \frac{1}{8}(t_1 - t_2)\,J_q + \frac{W}{2}\nabla(\rho + \rho_q), \tag{3}$$

$$\tilde{B}_q = \left[\frac{t_2'}{2}(1 + x_2') + W'\right]\tilde{J}_q. \tag{4}$$

In Eqs. (3-4), ρ and J are the particle and spin-current vector densities. $\tilde{J}$ is the corresponding abnormal spin-current vector density in the particle-particle channel. The subscript q stands for neutrons (protons).

The bulk and microscopic properties can be obtained by solving the following SHFB equation in the coordinate representation

$$\left[-\frac{d}{dr}\mathcal{M}\frac{d}{dr}+\mathcal{U}+\mathcal{M}\frac{l(l+1)}{r^2}+\frac{\mathcal{M}'}{r}+\mathcal{U}_{so}\right]\begin{pmatrix}u_1\\u_2\end{pmatrix}=E\begin{pmatrix}u_1\\u_2\end{pmatrix}, \tag{5}$$

where $\mathcal{M}'$ is the first order derivative with respect to radial coordinate r.

Furthermore, the $2p$ decay energy (Q_{2p}) is calculated by

$$Q_{2p}(Z,A)=-S_{2p}(Z,A)=E(Z,A)-E(Z-2,A-2), \tag{6}$$

where $E(Z,A)$ and $E(Z-2,A-2)$ are the binding energies of a parent nucleus and its daughter nucleus, respectively. S_{2p} is the $2p$ separation energy.

B. Mean-field cluster potential approach with Skyrme interaction

For the $2p$ radioactivity, the angular momentum carried by the ^{2}He-cluster is zero. So, the centrifugal potential vanishes. Based on the SHFB theory, the di-proton cluster potential can be decomposed as [83, 84]

$$V(r)=V_N(r)+V_C(r), \tag{7}$$

where $V_N(r)$ and $V_C(r)$ are the nuclear and Coulomb potentials, respectively, between the di-proton cluster and the core. $V_N(r)$ and $V_C(r)$ are expressed approximately by

$$V_N(r)=\lambda\left[U_p(r)+B_p(r)\right], \tag{8}$$

$$V_C(r)=2U_c(r), \tag{9}$$

where λ is a folding factor; $U_p(r)$, $B_p(r)$ and $U_c(r)$ refer to the single-proton potential without the spin-orbit potential, the proton spin-orbit potential and the single-proton Coulomb potential generated by the core, respectively. These expressions can be found from Refs. [76-79].

For the folding factor λ, it can be determined by the following Bohr-Sommerfeld quantization condition

$$\int_0^{r_1} dr\sqrt{\frac{2\mu}{\hbar^2}\left|Q_{2p}-V_{2p}(r)\right|}=(2n+1)\frac{\pi}{2}=(G+1)\frac{\pi}{2}, \tag{10}$$

where r_1 (and r_2 later) is the classical inner (and outer) turning point obtained by $V_{2p}(r) = Q_{2p}$, μ is the reduced mass for the system of the cluster and daughter nucleus, and n is the node number of the radial wave function of the cluster motion within the potential. A quasibound state of the cluster orbiting the core is characterized by a global quantum number G, whose value is estimated by the Wildermuth rule

$$G = 2n = \sum_{i=1}^{2} g_i, \tag{11}$$

where g_i is the oscillator quantum number of a cluster nucleon orbiting the core. For ^{18}Mg, the g_i and G values are 2 and 4, respectively.

The width of the $2p$ radioactivity can be calculated by

$$\Gamma_{2p} = S'_{2p} F \frac{\hbar^2}{4\mu} \exp\left[-2\int_{r_1}^{r_2} dr\, k(r)\right], \tag{12}$$

where S'_{2p} is the spectroscopic factor.

For ^{18}Mg, we assume that the two emitted protons originate from the $2s_{1/2}$ orbitals. The S'_{2p} value is calculated by a BCS model [85]

$$S'_{2p} = V^2_{2s_{1/2}} U^2_{2s_{1/2}}, \tag{13}$$

$V^2_{2s_{1/2}}$ and $U^2_{2s_{1/2}}$ are the occupation probability and unoccupation probability of the $2s_{1/2}$ proton states for ^{18}Mg and ^{16}Ne, respectively.

In Eq. (12), F is the normalization factor determined by

$$F \int_0^{r_1} dr\, \frac{1}{k(r)} \cos^2\left[\int_0^{r} dr'\, k(r') - \frac{\pi}{4}\right] = 1, \tag{14}$$

with the wave number $k(r)$ calculated by

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2} \left|Q_{2p} - V_{2p}(r)\right|}. \tag{15}$$

Finally, the $2p$ radioactivity half-life is calculated by

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_{2p}}. \tag{16}$$

III. RESULTS AND DISCUSSIONS

To solve the SHFB equation, the HFBRAD code is employed [77]. In this work, the SLy4 Skyrme parameterization is used because the spin-current densities J and $\tilde{J}$ in Eqs. (3) and (4) are equal to zero. In this case, the spin-orbit field B_q in Eq. (3) is then just relevant to W and ρ. Therefore, using the SLy4 interaction, the effect of W on the structure and $2p$ radioactivity of ^{18}Mg can be seen even more clearly. Besides, our previous studies showed that the properties of $2p$ radioactivity of ^{18}Mg are different largely for the three types of pairings (volume, mixed and surface pairings) [68]. Since in this article we focus on the $2p$ radioactivity of ^{18}Mg caused by W, the pairing interaction is frozen. Eq. (2) shows that the volume and surface pairings are the two limiting cases. The former is not dependent on ρ. However, the latter depends strongly on ρ. Therefore, the mixed pairing that lies between the surface and volume pairings is more widely employed to investigate the structure and decay of unstable nuclei [69, 86-89]. So, in this work, the mixed pairing is used. In calculations, the spherical box and mesh sizes are selected as 20 fm and 0.1 fm, respectively. The quasiparticle energies are cut off at 60 MeV. The maximum angular momenta of the quasi-neutron and quasi-proton are set to be $\frac{39}{2}\hbar$ and $\frac{25}{2}\hbar$, respectively. All the calculations of this article are converged with these conditions.

Within the mixed pairing force, the single-proton energy spectra near the Fermi energies of ^{18}Mg are calculated when W is taken in the range of 0-$1.4W_0$ MeV fm^5 with an interval of $0.2W_0$ MeV fm^5, which are plotted in Fig. 1. Note that W_0 is the spin-orbit coupling strength of the SLy4 interaction and equals to 123 MeV fm^5. Although the t_0' value is generally determined by the experimental pairing gap [80-82], the proton pairing gap of ^{18}Mg is unavailable experimentally because ^{18}Mg is located beyond the proton drip-line. So, we have to use the empirical proton pairing gap to determine its t_0' value. This gap is obtained by the following way: Firstly, its t_0' value is determined by fitting the experimental neutron pairing gap (1.31 MeV) of ^{120}Sn when $W = 1.0W_0$ [68, 69, 86-88]. In this case, the determined t_0' value can be seen as the pairing strength for all nuclei. Next, within the SHFB theory, by inputting the determined t_0' value we can extract the proton pairing gap of ^{18}Mg, whose value is 1.921 MeV. The t_0' values for different W values determined by the proton pairing gap of ^{18}Mg (1.921 MeV) are shown in column 2 of Table I.

In Fig. 1, the single-proton energies refer to those in the canonical basis. From Fig. 1, one can see that the splitting of the $1d$ orbital becomes increasingly pronounced with the increase of W. Moreover, the $2s_{1/2}$ energy levels are below the Fermi energies for $W = 0W_0$, $0.2W_0$, $0.4W_0$ and $0.6W_0$, which indicates that the $2s_{1/2}$ states are weakly bound. However, with the enhancement of the $1d$ orbital splitting the $2s_{1/2}$ energy level exceeds the $1d_{5/2}$ orbital

FIG. 1: (Color online) The single-proton energy spectra of ^{18}Mg versus different W values within the SLy4 interaction. The Fermi energies are depicted by the red dashed lines.

TABLE I: The calculated t_0', $V^2_{2s_{1/2}}$, S'_{2p}, Q_{2p}, $T_{1/2}$ and $T'_{1/2}$ values of ^{18}Mg with different W values. $U^2_{2s_{1/2}}$ is the unoccupation probability of the $2s_{1/2}$ proton state for ^{16}Ne. W_0 is the spin-orbit coupling strength of the SLy4 interaction, whose value is 123 MeV fm^5. The experimental Q_{2p} value of ^{18}Mg is $3.440^{+0.034}_{-0.034}$ MeV [66].

W (MeV fm^5)	t_0' (MeV fm^{-3})	$V^2_{2s_{1/2}}$	$U^2_{2s_{1/2}}$	S'_{2p}	Q_{2p} (MeV)	$T_{1/2}$ (s)	$T'_{1/2}$ (s)
$0W_0$	-295.07	0.827	0.654	0.541	5.132	–	–
$0.2W_0$	-294.92	0.807	0.679	0.548	5.076	–	–
$0.4W_0$	-293.75	0.743	0.738	0.548	4.958	–	–
$0.6W_0$	-291.23	0.633	0.809	0.512	4.805	4.320×10^{-23}	8.431×10^{-23}
$0.8W_0$	-288.60	0.482	0.873	0.421	4.533	1.050×10^{-22}	2.494×10^{-22}
$1.0W_0$	-287.80	0.313	0.923	0.289	3.989	2.336×10^{-22}	8.083×10^{-22}
$1.2W_0$	-289.87	0.174	0.954	0.166	3.157	7.967×10^{-22}	4.799×10^{-21}
$1.4W_0$	-291.96	0.090	0.967	0.087	2.095	1.524×10^{-20}	1.752×10^{-19}
$1.152^{+0.008}_{-0.008}W_0$	$-289.20^{+0.10}_{-0.10}$	$0.203^{+0.005}_{-0.003}$	$0.946^{+0.001}_{-0.002}$	$0.192^{+0.005}_{-0.004}$	$3.440^{+0.034}_{-0.034}$	$4.678^{+0.358}_{-0.193} \times 10^{-22}$	$2.438^{+0.250}_{-0.155} \times 10^{-21}$

and is located in the resonant states of the continuum energy region when $W \geq 0.8W_0$. Although the $2s_{1/2}$ orbit is independent of the spin-orbit potential, it is influenced by the particle-hole field and pairing (particle-particle) field. The particle-hole field is contributed by the spin-orbit interaction and the pairing field is associated with the spin-orbit interaction since when the spin-orbit strength W changes the pairing strength t_0' changes as well in order to reproduce the proton pairing gap ^{18}Mg (see Table 1). That is the reason why the $2s_{1/2}$ level gradually gets higher when W increases. No matter whether the $2s_{1/2}$ state is weakly bound or not, a certain number of protons are occupied on it. For the $2s_{1/2}$ state, its centrifugal barrier vanishes. So, the valence protons can extend to a large r region and then a $2p$ halo or $2p$ halo-like structure appear. This situation is similar to the formation of the neutron halo [90, 91]. Therefore, the contribution of the $2s_{1/2}$ state to the full density in the large r region is dominant. This can be seen clearly from Fig. 2, which shows the density distribution of each orbit near the Fermi energy as a function of r for $W = 0.6W_0$, $1.0W_0$ and $1.4W_0$. Our recent studies with the T31 and SLy5 interactions suggested that the deformation of ^{18}Mg is very weak [68, 69]. Furthermore, relevant studies indicate that the $2p$ halo-like structure rather than a deformation is responsible for the $2p$ correlation and the mechanism of $2p$ emission [92–96]. So, the microscopic single-proton energy spectra are enough to examine the $2p$ halo or $2p$ halo-like structure of ^{18}Mg.

FIG. 2: (Color online) The density distributions of each orbit near the Fermi energy as a function of r for the spin-orbit coupling strength $0.6W_0$, $1.0W_0$ and $1.4W_0$ MeV fm^5 within the SLy4 interaction.

With the enhancement of the $1d$ orbital splitting, less protons occupy the $2s_{1/2}$ orbital. As shown in column 3 of Table 1, the occupation probability of the $2s_{1/2}$ state ($V^2_{2s_{1/2}}$) decreases with increasing W. As a result, the tail of the density distribution of the $2s_{1/2}$ orbit is less extended, which can be seen clearly from Fig. 2. Hence, the $2p$ halo or $2p$ halo-like sizes of ^{18}Mg are depressed by the increase of W. Similarly, the occupation probability of the $2s_{1/2}$ state of its daughter nucleus ^{16}Ne falls with the rise of W. It means that the unoccupation probability $U^2_{2s_{1/2}}$ of ^{16}Ne increases with the rise of W, which can be seen from column 4 of Table I. Using Eq. (13), it is easy to obtain the corresponding S'_{2p} values, which are listed in column 5 of Table I. As can be seen from column 5 of Table I, on the whole, the S'_{2p} value is getting smaller and smaller with the increase of W. Nevertheless, when W is weak the S'_{2p} value changes very little with W. Note that the diproton cluster preforms more easily under the condition that the density on the nuclear surface is low. The weakening sizes of the $2p$ halo or $2p$ halo-like structure in Fig. 2 suggest that the density distribution on the nuclear surface is less extended with the enhancement of W. As a result, the corresponding S'_{2p} value becomes smaller and smaller. Therefore, the S'_{2p} value may be correlated strongly to the $2p$ halo or $2p$ halo-like size. To study this kind of correlation, in our recent work, we had to use a virtual charge radius to represent the $2p$ halo-like size of ^{18}Mg because it is difficult to distinguish a halo from a gaslike structure for an unbound nucleus [68, 69]. We found that there exists a pronounced linear correlation between the S'_{2p} and the $2p$ halo-like size [68, 69]. So, we will not discuss the correlation between the S'_{2p} and the $2p$ halo or $2p$ halo-like size in this article.

Using Eq. (6), we can obtain different Q_{2p} values when different W values are selected, whose values are shown in column 6 of Table I, where it is shown that the Q_{2p} values are gradually decreasing with the growth of W. Previous studies have suggested a dependence of S'_{2p} on Q_{2p} [4, 50, 52, 53, 97]. Therefore, we plot the logarithm S'_{2p} versus Q_{2p} as well as that versus W in Figs. 3(a) and 3(b), respectively. From Fig. 3(a), a good linear correlation between $\log_{10} S'_{2p}$ and Q_{2p} with the Pearson coefficient $R = 0.9975$ is found. Similarly, a less pronounced linear correlation between $\log_{10} S'_{2p}$ and W with $R = -0.8899$ is seen from Fig. 3(b).

To see the W effect on the diproton cluster potentials V_{2p}, we calculate the V_{2p} with different W values using the mean-field cluster potential approach. The V_{2p} values with $W = 0.6W_0$, $1.0W_0$ and $1.4W_0$ are shown in Fig. 4(a). To examine the potential in the nuclear surface region influenced by W, the V_{2p} curves in the surface region with $W = 0.6W_0$, $1.0W_0$ and $1.4W_0$ are shown in Fig. 4(b). From Figs. 4(a) and 4(b), one can see that the potential well becomes deeper and deeper with the increase of W. However, the potential barriers in the surface region grow gradually with the rising W. Studies on α-decay showed that the preformation probability of α-cluster is enhanced with the increasing potential well depth [75, 98]. Nevertheless, for the S'_{2p} of the diproton cluster of ^{18}Mg, it gets

FIG. 3: (Color online) (a) The logarithm spectroscopic factors $\log_{10} S'_{2p}$ versus the corresponding Q_{2p} values. (b) Same as 3(a), but for the W values.

smaller and smaller when the potential well becomes deeper and deeper. It implies that the diproton cluster is different largely from the α-cluster and should be much looser than the α-cluster.

Next, we calculate the $2p$ radioactivity half-lives without and with S'_{2p} ($T_{1/2}$ and $T'_{1/2}$) using the mean-field cluster potential approach by inputting the Q_{2p} values with different W values. The $T_{1/2}$ and $T'_{1/2}$ values with different W values are listed in column 7 and the last column of Table I. For the cases of $W < 0.6W_0$, the Q_{2p} value exceeds the height of the diproton potential barrier, thus the half-life cannot be calculated within the tunneling model and is not listed. Besides, the logarithm half-lives without and with S'_{2p} ($\log_{10} T_{1/2}$ and $\log_{10} T'_{1/2}$) versus W and those versus $Q_{2p}^{-1/2}$ are shown in Figs. 4(c) and 4(d), respectively. From Figs. 4(c) and 4(d), we can see that $\log_{10} T_{1/2}$ and $\log_{10} T'_{1/2}$ show good linear relationships with W and $Q_{2p}^{-1/2}$, respectively. As the picture of the tunneling effect, the decreasing Q_{2p} and S'_{2p} values may result in longer half-lives, which can be seen from Table I, Fig. 4(c) and Fig. 4(d). Moreover, from Table I, Fig. 4(c) and Fig. 4(d) we can see that the logarithm half-lives are enhanced by S'_{2p}. For the case of $0.8W_0$, the half-life has increased by a factor of 2.375. When $W = 1.4W_0$, the half-life has been extended by 11.496 times.

The Q_{2p} value of ^{18}Mg was measured as 3.440(34) MeV [66]. So, using the experimental Q_{2p} value the W value can be determined. We found that when $W = 1.152(8)W_0$, the calculated Q_{2p} value exactly equals to the corresponding experimental one. Besides, with $W = 1.152(8)W_0$, the corresponding t'_0, $V^2_{2s_{1/2}}$, $U^2_{2s_{1/2}}$, S'_{2p}, $T_{1/2}$ and $T'_{1/2}$ values with errors are calculated. These values are listed in the last line of Table I, from which it is easy to obtain that $T'_{1/2}$ is 5.212 times as large as $T_{1/2}$.

It is worth mentioning that in this article we assume that the two emitted protons are from the $2s_{1/2}$ orbital. However, from Fig. 1, one can see that the two correlated protons might be emitted from the $1d_{5/2}$ orbital when $W < 0.8W_0$. Using Eq. (13), the corresponding S'_{2p} values with different W values can be extracted. For example, when $W = 0.6W_0$, the extracted S'_{2p} value is 0.298. Furthermore, if the two correlated protons are emitted from the $1d_{5/2}$ orbital, the centrifugal potential should be taken into account. As a result, the half-life $T'_{1/2}$ value with that S'_{2p} value is 2.852×10^{-20} s, which is much longer than the corresponding half-life (8.431×10^{-23} s) that the two correlated protons are emitted from the $2s_{1/2}$ orbital. This indicates that the protons in the $1d_{5/2}$ orbital are significantly harder to be emitted compared with those in the $2s_{1/2}$ orbital. So, our assumption that the two emitted protons are mainly from the $2s_{1/2}$ orbital is reasonable, which is consistent with the conclusions of Refs. [66, 67].

FIG. 4: (Color online) (a) Diproton potentials V_{2p} for $W = 0.6W_0$, $1.0W_0$, and $1.4W_0$. (b) Same as 4(a), but for the potentials in the nuclear surface region. (c) Logarithmic half-lives without and with S'_{2p} versus the corresponding W values. The red dashed line represents the logarithmic half-lives without S'_{2p} $\left(\log_{10} T_{1/2}\right)$. The red solid line represents those with S'_{2p} $\left(\log_{10} T'_{1/2}\right)$. (d) Same as 4(c), but for the corresponding $Q_{2p}^{-1/2}$ values.

IV. CONCLUSIONS

In this article, the impact of the spin-orbit coupling strength W on the structure and $2p$ radioactivity of ^{18}Mg has been investigated within the spherical SHFB theory and the mean-field cluster potential approach using the SLy4 interaction. Specifically, the single-proton spectra, density distributions, spectroscopic factors, diproton cluster potentials, and half-lives of $2p$ radioactivity have been calculated when the W value is taken in the range of 0–$1.4W_0$. The following conclusions can be drawn:

(i) The splitting of the $1d$ proton orbital becomes increasingly evident with increasing W. Moreover, the $2s_{1/2}$ proton state is weakly bound for the cases of $W = 0W_0$, $0.2W_0$, $0.4W_0$, and $0.6W_0$. However, the $2s_{1/2}$ state becomes a resonant state in the continuum energy region when $W \geq 0.8W_0$.

(ii) With the enhancement of the $1d$ orbital splitting, the occupation probabilities of the $2s_{1/2}$ state decrease gradually. As a result, the tail of the density distribution of the $2s_{1/2}$ orbital is less extended. Correspondingly, the spectroscopic factor S'_{2p} becomes progressively smaller.

(iii) The diproton cluster potential well becomes deeper with increasing W. Moreover, the deeper the diproton potential well, the smaller the S'_{2p}. This implies that the diproton cluster is much looser than the α-cluster.

(iv) The declining S'_{2p} and Q_{2p} values caused by increasing W lead to longer half-lives of $2p$ radioactivity. Moreover, the half-life is enhanced by S'_{2p} when Q_{2p} is kept constant.

(v) A pronounced linear correlation between $\log_{10} S'_{2p}$ and Q_{2p}, and that between $\log_{10} S'_{2p}$ and W, are found. Besides, $\log_{10} T_{1/2}$ and $\log_{10} T'_{1/2}$ show good linear relationships with W and $Q_{2p}^{-1/2}$, respectively.

(vi) Using the experimental Q_{2p} value of ^{18}Mg, the optimal W value is constrained to be $1.152(8)W_0$.

Finally, it is necessary to point out that Ref. [66] showed that the Q_{2p} value of ^{16}Ne is measured as 1.42 MeV. Thus, its experimental Q_{2p} value provides a basis for testing the SLy4 interaction. Within the SLy4 interaction, when $W = 1.0W_0$, our calculation suggests that the Q_{2p} value of ^{16}Ne is 1.19 MeV, which is close to the experimental data from Ref. [66] and the experimental Q_{2p} value 1.401(20) MeV from Ref. [99]. Hence, it is reliable to continue discussing the structure and $2p$ radioactivity of ^{16}Ne caused by W. In addition, both cluster radioactivity [83, 100–104] and one-proton radioactivity [105–108] are important decay modes for unstable nuclei. Therefore, it is interesting to extend our approach to study the effect of W on these radioactivities, which is work in progress.

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Note: Figure translations are in progress. See original paper for figures.