DOI: 10.11776/j.issn.1000-4939.2025.06.018

ZHANG Xiantong, CHEN Jianbing, LIU Cong, TIAN Zhuangyan

2025-12-18

ChinaRxiv

Abstract

In steel-concrete composite structures, the plane section assumption is no longer applicable due to the presence of interface slip and web shear deformation. To scientifically investigate the effects of shear deformation and interface slip on the deflection and slip of composite beams, this study derives the elastic bending differential equations for double inverted T-shaped steel-concrete composite beams considering shear deformation and interface slip by incorporating the strain relationship and mechanical equilibrium of an infinitesimal beam element based on Goodman' s assumption and the Timoshenko beam dual generalized displacement assumption. Additionally, theoretical formulas for calculating the elastic shear stiffness of web-embedded connectors are developed using equivalent spring models and equivalent truss-spring models. By applying the known deformation and constraint conditions of composite beams, analytical solutions for deflection and slip are obtained for simply supported composite beams under mid-span concentrated loads, which are then validated against experimental results from four double inverted T-shaped steel-concrete composite beams with different parameters. The results demonstrate that the theoretically calculated deflection and slip values agree well with the measured values, thereby confirming the validity of the theoretical formulas for the elastic shear stiffness of web-embedded connectors. For double inverted T-shaped composite beams with a depth-to-span ratio of 1/10, the deflection caused by bending accounts for approximately 56% of the total deflection, the deflection induced by interface slip contributes about 36%, and the deflection resulting from shear deformation represents roughly 8%. This research comprehensively considers the influence of both shear deformation and interface slip on the deflection and slip of composite beams, showing significant improvement over models that neglect these effects.

Full Text

DOI: 10.11776/j.issn.1000-4939.2025.06.018

Theoretical Model for Double Inverted T-Shaped Composite Beams Considering Shear Deformation and Interface Slip

ZHANG Xiantong, CHEN Jianbing, LIU Cong, TIAN Zhuangyan

(School of Civil Engineering, Suzhou University of Science and Technology, 215011 Suzhou)

Abstract: In steel-concrete composite structures, due to the existence of certain interface slip and slab shear deformation, the plane-section assumption is no longer applicable. To scientifically study the effects of shear deformation and interface slip on the deflection and interface slip of composite beams, by adopting the Goodman assumption and the Timoshenko beam double generalized displacement assumption, introducing the strain relations of composite beams and the mechanical equilibrium of an element micro-segment, the elastic bending differential equation for double inverted T-shaped steel-concrete composite beams considering shear deformation and interface slip is derived. Based on the equivalent spring model and the equivalent truss spring model, theoretical formulas for the elastic shear stiffness of embedded connectors in the slab are derived. Using the known deformation and constraint conditions of composite beams, analytical solutions for the deflection and slip of simply supported composite beams under a midspan concentrated load are obtained, and the results are verified using test results from four double inverted T-shaped steel-concrete composite beams with different parameters. The results show that the deflection and slip values obtained from theoretical calculations agree well with the measured values, while also verifying the correctness of the theoretical calculation formula for the elastic shear stiffness of embedded connectors in the slab. For the deflection deformation of a double inverted T-shaped composite beam with a height-to-span ratio of 1/10, the deflection value caused by bending accounts for about 56% of the total deflection, the deflection value caused by interface slip accounts for about 36% of the total deflection, and the deflection value caused by shear deformation accounts for about 8% of the total deflection. This study comprehensively considers the effects of shear deformation and interface slip on the deflection and slip of composite beams, and the model has clear improvements compared with models that do not consider shear deformation and interface slip.

Keywords: double inverted T-shaped steel-concrete composite beam; shear deformation; elastic shear stiffness; slip; deflection

Chinese Library Classification No.: O313; TU398.6 **Document Identification Code:** A **Article No.:** 1000-4939(2025)06-1387-10

Theoretical model of double inverted T-shaped composite beam considering shear deformation and interface slip

ZHANG Xiantong, CHEN Jianbing, LIU Cong, TIAN Zhuangyan

(School of Civil Engineering, Suzhou University of Science and Technology, 215011 Suzhou, China)

Abstract: In steel-concrete composite structures, due to the existence of certain interface slip and web shear deformation, the assumption of flat section is no longer applicable. In order to scientifically study the effects of shear deformation and interface slip on the deflection and interface slip of composite beams, this paper adopts Goodman' s assumption and Timoshenko beam' s double generalized displacement assumption, introduces the strain relationship of composite beams and element microsegment mechanical equilibrium, and derives the elastic bending differential equation of double inverted T-shaped steel-concrete composite beams considering shear deformation and interface slip. Then based on the equivalent spring model

Date received: 2023-09-26
Fund projects: National Natural Science Foundation of China (No. 52208187); Jiangsu Province Graduate Research and Practice Innovation Program (No. KYCX21_3039)
Author information: ZHANG Xiantong, professor. E-mail: szctt2009@163.com
Citation format: ZHANG Xiantong, CHEN Jianbing, LIU Cong, et al. Theoretical model for double inverted T-shaped composite beams considering shear deformation and interface slip [J]. Chinese Journal of Applied Mechanics, 2025, 42(6): 1387-1396.
ZHANG Xiantong, CHEN Jianbing, LIU Cong, et al. Theoretical model of double inverted T-shaped composite beam considering shear deformation and interface slip [J]. Chinese Journal of Applied Mechanics, 2025, 42(6): 1387-1396.

and the equivalent rod spring model, a theoretical calculation formula for the elastic shear stiffness of the embedded web connection is derived. By using the known deformation and constraint conditions of the composite beam, we obtain the analytical solution of deflection and slip of the simply supported composite beam under concentrated load in the span and verify it through the experimental results of four double inverted T-shaped steel-concrete composite beams with different parameters. The results show that the deflection and slip values obtained from theoretical calculations are in good agreement with the measured values, and the correctness of the theoretical calculation formula for the elastic shear stiffness of the embedded web connection is verified. In the deflection deformation of double inverted T-shaped composite beams, the deflection value caused by bending accounts for about 56% of the total deflection, the deflection value caused by interface slip accounts for about 36% of the total deflection, and the deflection value caused by shear deformation accounts for about 8% of the total deflection. This article comprehensively considers the effects of shear deformation and interface slip on the deflection and slip of composite beams, and makes a significant improvement compared to the model structure that does not consider shear deformation and interface slip.

Key words: double inverted T-shaped steel-concrete composite beam; shear deformation; elastic shear stiffness; deflection; slip

Traditional T-shaped steel-concrete composite beams are prone to overturning and damage of the steel web, and generally use stud shear connectors for steel-concrete connection; the welding workload is large, and the welding fatigue effect is significant. Based on this, this study investigates a new type of composite beam structure—web-embedded double inverted T-shaped steel-concrete composite beam (abbreviated as WDSCB). WDSCB eliminates the welding process for shear connectors. The embedded connectors are slotted beneath the steel-beam web and embedded into the concrete flange; together with the concrete dowels and the transverse reinforcement inside the dowels, they form connectors that jointly resist shear. This ensures construction quality, enables the steel-beam web and the T-shaped concrete beam to be cast as an integral unit and participate together in interfacial shear resistance, improves the load-carrying capacity of the composite beam, and at the same time ensures the stability of the steel-beam web.

However, even in composite beams with complete shear connection, a certain bending-induced interfacial slip still exists. This slip reduces the overall stiffness of the composite beam, increases the deflection, and, when the span-to-depth ratio of the composite beam is small, shear deformation cannot be neglected[^1]. Therefore, it is necessary to comprehensively consider the coupled effects of interface slip and shear deformation in calculations. Regarding the influence of shear deformation and interface slip on the deformation of composite beams, scholars have conducted extensive research. Zhou Lingyu et al.[^2] studied interface slip and shear deformation in I-shaped composite beams, and established a deflection-governing differential equation by using the minimum potential energy principle and the variational method. Yan Qingqing et al.[^3] used the force equilibrium principle and deformation compatibility conditions to establish a dynamic model for I-shaped composite beams, and discussed the degree to which interface slip and shear deformation affect the vibration characteristics and dynamic response of composite continuous beams. SCHNABL et al.[^4] proposed a computational model for analyzing double-layer composite beams with different material and geometric properties; this model considered the influence of interface slip and shear deformation on the displacement of each layer. Zhou Shijun et al.[^5], based on the energy variational method, derived the governing differential equation for a simply supported double-box composite box girder considering interface slip and web shear deformation.

Existing studies on interface slip and shear deformation in composite beams are mainly based on the principle of minimum potential energy, with integration or matrix solution methods; the solution process is relatively complicated and has certain limitations in engineering applications. Moreover, few studies have examined the correlation between interface slip and shear deformation in double inverted T-shaped steel-concrete composite beams. Therefore, on the basis of the Goodman assumption and the Timoshenko beam generalized-displacement assumption, this study establishes a deformation model for double inverted T-shaped steel-concrete composite beams with web-embedded shear connectors by means of the steel-concrete strain relationship and the force-equilibrium relationship of infinitesimal beam segments, and obtains an analytical solution for a simply supported composite beam under a midspan concentrated load[^6]. By comparing experimental values with theoretical values, the correctness of the derived formula is verified; by comparing the theoretical values in this study with existing theoretical values, the accuracy of the formula is verified.

Fig. 1 Coordinate system for the calculation of steel-composite beams

Figure 1: Fig. 1 Coordinate system for the calculation of steel-composite beams

Fig. 2 Schematic diagram of the combined beam strain

Figure 2: Fig. 2 Schematic diagram of the combined beam strain

1 Establishment of the theoretical model for the composite beam

1.1 Basic assumptions

1) The steel-concrete composite beam is in the elastic working stage, and both the steel and concrete are ideal elastic bodies.
2) The curvatures of the steel and concrete are the same during deformation, and each satisfies the plane-section assumption.
3) The horizontal shear force on the steel-concrete interface is proportional to the relative slip.

1.2 Establishment of the governing differential equation under vertical loading

Figure 1 shows the web-embedded double inverted T-shaped steel-concrete composite beam used.

For the beam schematic, to facilitate calculation, the $\{o, x, y, z\}$ coordinate system is introduced, with the x-axis coinciding with the neutral axis of the composite beam.

Figure 1 Coordinate system for the calculation of steel-concrete composite beams

Fig. 1 Coordinate system for the calculation of steel-composite beams

According to the Goodman assumption, when the interface between the concrete and the steel beam is regarded as a continuous elastic interlayer, the strain composition of the composite beam is as shown in Fig. 2[7].

Figure 2 Schematic diagram of the strain of the composite beam

Fig. 2 Schematic diagram of the combined beam strain

The bending slips generated in the concrete and the steel beam are

$$\begin{cases} u_c(x) = -z\theta(x) \\ u_s(x) = -z\theta(x) \end{cases} \tag{1}$$

where θ is the rotation angle of the section.

Let the relative slip between the steel beam and the concrete be $s(x)$; then the strain generated by interface slip is $s'(x)$. Under the action of slip, the composite beam should maintain equal axial-force magnitudes and opposite directions[1], and therefore

$$\begin{cases} E\varepsilon_s'(y,z)A_s + E_c\varepsilon_c'(y,z)A_{co} = 0 \\ \varepsilon_s'(y,z) - \varepsilon_c'(y,z) = s'(x) \end{cases} \tag{2}$$

where $\varepsilon_c'(y,z)$ is the slip strain of the concrete slab; $\varepsilon_s'(y,z)$ is the slip strain of the steel beam; A_{co} is the sectional area of the concrete; A_s is the sectional area of the steel beam; E is the elastic modulus of the steel beam; and E_c is the elastic modulus of the concrete.

From Eq. (2), one obtains

$$\begin{cases} \varepsilon_c'(y,z) = \left[-nA_s/(A_{co}+nA_s)\right]s'(x) \\ \varepsilon_s'(y,z) = \left[A_{co}/(A_{co}+nA_s)\right]s'(x) \end{cases} \tag{3}$$

where $n = E/E_c$.

For ease of calculation, the concrete section is hereinafter converted into an equivalent steel section.

Therefore, the total strains of the concrete slab and the steel beam are respectively

$$\begin{cases} \varepsilon_c(y,z) = z\theta'(x) + f_c s'(x) \\ \varepsilon_s(y,z) = z\theta'(x) + f_s s'(x) \end{cases} \tag{4}$$

where $f_c = -A_s/(A_c+A_s)$, $f_s = A_c/(A_c+A_s)$; and A_c is the converted sectional area of the concrete.

According to the linear elastic assumption, the normal stresses are

$$\begin{cases} \sigma_c = E\left[z\theta'(x) + f_c s'(x)\right] \\ \sigma_s = E\left[z\theta'(x) + f_s s'(x)\right] \end{cases} \tag{5}$$

Thus, the axial forces borne by the upper and lower layers of the composite beam are respectively

$$\begin{cases} N_c = \int_{A_c} \sigma_c \, \mathrm{d}A = EA_c \left[-d_c \theta'(x) + f_c s'(x)\right] \\ N_s = \int_{A_s} \sigma_s \, \mathrm{d}A = EA_s \left[d_s \theta'(x) + f_s s'(x)\right] \end{cases} \tag{6}$$

where d_c and d_s are respectively the distances from the neutral axes of the concrete and the steel beam to the neutral axis of the composite-beam section.

The bending moment caused by the axial force is opposite in direction to the external bending moment; therefore, the bending moment is

$$\begin{cases} -M_c = \int_{A_c} z\sigma_c \, \mathrm{d}A = EI_c \theta' + d_c EA_0 s'(x) \\ -M_s = \int_{A_s} z\sigma_s \, \mathrm{d}A = EI_s \theta' + d_s EA_0 s'(x) \end{cases} \tag{7}$$

where $A_0 = \dfrac{A_c A_s}{A_c + A_s}$; and I_c and I_s are the moments of inertia of the converted concrete section and the steel section, respectively.

The total bending moment of the composite beam is

$$-M = M_c + M_s = EA_0 d_0 s'(x) + EI_0 \theta' \tag{8}$$

where $I_0 = I_c + I_s$, and $d_0 = d_c + d_s$.

The relative slip strain on the lap joint is caused by the incompatibility between the strain ε_c of the adjacent point in the concrete member and the strain ε_s of the steel member[8], namely

$$\varepsilon_{\mathrm{sp}} = \frac{\mathrm{d}s}{\mathrm{d}x} = \varepsilon_c(y,z) - \varepsilon_s(y,z) \tag{9}$$

where $\varepsilon_{\mathrm{sp}}$ is the slip strain.

From Assumption 3), the longitudinal shear force $q(x)$ per unit length is, in the elastic working stage, proportional to the relative slip $s(x)$ at the lap joint. Its physical equation is

$$q(x) = Ks(x) \tag{10}$$

$$q(x) = N'(x) \tag{11}$$

where K is the longitudinal shear stiffness of the lap joint.

From Eqs. (9) and (10), one obtains

$$\varepsilon_{\mathrm{sp}} = \frac{\mathrm{d}s}{\mathrm{d}x} = \frac{1}{K}\frac{\mathrm{d}q}{\mathrm{d}x} = \frac{1}{K}N''(x) \tag{12}$$

Substituting Eq. (12) into Eq. (8) gives

$$-M = M_c + M_s = \frac{1}{K}EA_0d_0N''(x) + EI_0\theta' \tag{13}$$

Solving Eq. (6) for θ' and substituting it into Eq. (13), one obtains

$$-M = \frac{EA_0d_0A_cd_c - EI_0A_0}{KA_cd_c}N''(x) + \frac{I_0}{d_cA_c}N(x) \tag{14}$$

$$N''(x) - K\lambda N(x) = K\Lambda \tag{15}$$

where

$$\delta = \frac{EA_0(I_0 - d_0A_cd_c)}{A_cd_c}; \qquad \lambda = \frac{I_0}{d_cA_c\delta}; \qquad \Lambda = M(x)\frac{1}{\delta}.$$

Let $v^2 = K\lambda$. The above equation can be transformed into a second-order constant-coefficient nonhomogeneous differential equation, namely

$$N''(x) - v^2N(x) = K\Lambda \tag{16}$$

1.3 Solution for the longitudinal shear stiffness of the composite seam

For embedded shear connectors in the web plate, the local component of a single connector jointly bears the displacement effect between the external force and the various components; the steel bars and concrete deform compatibly. The shear stiffness of the steel bars and concrete can be equivalent to springs connected in series, which are then connected in parallel with the embedded connector [9]. After the shear member and the concrete tenon are subjected to shear, their deformations are not synchronized. To accommodate their respective deformation characteristics, the shear stiffnesses of the concrete and the reinforcing bars inside the concrete tenon are equivalent to spring models, considering only the compressive contribution of the springs; the shear member is equivalent to a rod-system spring model, while the bending and shear contributions of the force-transmitting rod are considered [10]. These are denoted by k_b and k_r, respectively, as shown in Fig. 3. Thus,

$$\frac{1}{k_s} = \frac{1}{k_b} + \frac{1}{k_r} \tag{17}$$

Fig. 3 Equivalent spring model of embedded composite connector

Figure 3: Fig. 3 Equivalent spring model of embedded composite connector

Fig. 4 Equivalent spring model of concrete tenon

Figure 4: Fig. 4 Equivalent spring model of concrete tenon

where k_s is the shear stiffness of the shear member; k_b is the bending-spring stiffness of the shear member; and k_r is the shear-spring stiffness of the shear member.

图 3 嵌入式组合连接件等效弹簧模型

Fig. 3 Equivalent spring model of embedded composite connector

In the elastic stage, the concrete, connector, and steel plate satisfy the linear superposition relationship; therefore, the shear stiffnesses of the shear member, the concrete tenon, and the reinforcing bars penetrating through the tenon must be obtained separately. The elastic shear stiffness k of the connector can be obtained from Eq. (18).

$$k = \left(\frac{1}{k_s} + \frac{1}{k_c + k_i}\right)^{-1} \tag{18}$$

where k_c is the stiffness of the concrete tenon; and k_i is the shear stiffness of the reinforcing bars penetrating through the tenon.

When the connector bears a relatively large load, the concrete around the shear key is locally crushed, and local damage develops in the direction of loading, producing splitting failure [11]. Therefore, the local shear failure of the concrete tenon should be considered, and the effective length of the concrete tenon is taken as $l_{c.2} - l_{c.1}$.

The concrete tenon is formed by casting concrete and is integral with the concrete at other locations. Therefore, when the concrete tenon is subjected to shear, the contribution of the surrounding concrete should be considered. Referring to the average minimum crack spacing when longitudinal splitting failure occurs in concrete, a width correction value of $0.15l_{c.1}$ is added [9], where $l_{c.1}$ is the bottom width of the concrete tenon. Thus, the equivalent cross-sectional width is $0.15l_{c.1} + t$, as shown in Fig. 4.

图 4 混凝土榫等效弹簧模型

Fig. 4 Equivalent spring model of concrete tenon

The shear stiffness of the concrete tenon is

Fig. 5 Equivalent rod spring model of shear members

Figure 5: Fig. 5 Equivalent rod spring model of shear members

$$k_c = \frac{E_c h_c (0.15 l_{c.1} + t)}{l_{c.2} - l_{c.1}} \tag{19}$$

The shear member is idealized as a cantilever structure. The cantilever length is the distance from the centroid of the concrete to the location of the maximum contact stress point of the shear member root. From finite element simulations [12], the maximum contact stress point is known to be an approximate value; it is taken as the centroidal position of the shear member, as shown in Fig. 5.

图 5　剪力件等效杆系弹簧模型

Fig. 5　Equivalent rod spring model of shear members

According to the force characteristics of the shear member, it is known that the force at the bottom is relatively large and shear failure is prone to occur. To simplify the calculation, the effective width of the shear member is taken as the bottom length l_s.

The bending displacement Δ_b of the rod-system spring of the shear member is

$$\Delta_b = \frac{Fl^3}{3EI_s} = \frac{4Fh_s^3}{Etb_1^3} \tag{20}$$

Thus, the bending-spring stiffness k_b of the shear member is

$$k_b = \frac{Etb_1^3}{4h_s^3} \tag{21}$$

where t is the plate thickness of the shear member.

The shear displacement Δ_r of the rod-system spring of the shear member is

$$\Delta_r = \frac{6Fh_s}{5G_s b_1 t} \tag{22}$$

then the shear spring stiffness k_r of the shear member is

$$k_\mathrm{r} = \frac{5G_\mathrm{s} b_1 t}{6h_\mathrm{s}} \tag{23}$$

where G_s is the shear modulus of the shear member, and $G_\mathrm{s} = \dfrac{E}{2(1+\mu)}$; μ is the Poisson' s ratio of the shear member.

Fig. 6 Model diagram of equivalent Winkler foundation beam with steel bars

Figure 6: Fig. 6 Model diagram of equivalent Winkler foundation beam with steel bars

The force borne by the reinforcing bars in the connector is related to the shear member and the passive reaction force provided by the concrete. When the reinforcing bars are subjected to an external force, they tend to deform. This deformation tendency produces compression in the surrounding concrete; the compressed concrete is also restrained by the nearby concrete, so the strength and ductility of the concrete in the compressed zone are greatly improved, and its effect on the reinforcing bars is equivalent to that of an elastic foundation[^13]. Therefore, in the elastic state, the reinforcing bars are equivalent to a Winkler foundation beam, and the reaction concrete around the reinforcing bars is equivalent to an elastic foundation, as shown in Fig. 6.

图 6 钢筋等效文克尔地基梁模型
Fig. 6 Model diagram of equivalent Winkler foundation beam with steel bars

According to the basic assumptions of the Winkler foundation beam, the compressive stress at any point on the foundation surface is proportional to the deflection at that point, namely

$$P = k_{\mathrm{i}} v_{\mathrm{i}} \tag{24}$$

where k_{i} is the foundation coefficient, related to the concrete of the elastic foundation, taken as $k_{\mathrm{i}} = 0.01\ \mathrm{kN/mm^3}$; v_{i} is the vertical displacement at the point corresponding to the point of force application. Except at the point where the shear member acts, the load $P(x) = 0$.

Using the solution method for a Winkler elastic foundation model, and combining the foundation equilibrium equation, the beam deflection differential equation, and the deformation compatibility equation, the equilibrium equation of the beam can be obtained as

$$E_{\mathrm{i}} I_{\mathrm{i}} \frac{\mathrm{d}^4 v_{\mathrm{i}}}{\mathrm{d}x^4} + D_{\mathrm{i}} k_{\mathrm{i}} v_{\mathrm{i}} = -P(x) \tag{25}$$

where E_{i} is the elastic modulus of the reinforcing bar; I_{i} is the sectional moment of inertia of the reinforcing bar; D_{i} is the diameter of the reinforcing bar.

Let

$$\lambda_{\mathrm{i}} = \sqrt[4]{\frac{D_{\mathrm{i}} k_{\mathrm{i}}}{4 E_{\mathrm{i}} I_{\mathrm{i}}}},$$

then Eq. (25) can be expressed as

$$\frac{\mathrm{d}^4 v_\mathrm{i}}{\mathrm{d}x^4} + 4\lambda_\mathrm{i}^4 v_\mathrm{i} = 0 \tag{26}$$

The end of the reinforcing bar is almost not subjected to force[^14], so it may be considered that the beam is not subjected to force at positions far from the action point of the embedded connector. Thus, the general solution of Eq. (26) is

$$v_\mathrm{i} = \mathrm{e}^{-\lambda_\mathrm{i} x} \left[C_1 \sin(\lambda_\mathrm{i} x) + C_2 \cos(\lambda_\mathrm{i} x)\right] \tag{27}$$

From the deformation compatibility condition, when $x = 0$, $\frac{\mathrm{d}v_\mathrm{i}}{\mathrm{d}x} = 0$, hence

$$C_1 = C_2.$$

From the approximate differential equation of the deflection curve,

$$v_\mathrm{i}'' = -\frac{M_\mathrm{i}(x)}{E_\mathrm{i} I_\mathrm{i}},$$

the shear force at any point can be obtained, namely

$$V_\mathrm{i} = -4E_\mathrm{i} I_\mathrm{i} C_2 \lambda_\mathrm{i}^3 \mathrm{e}^{-\lambda_\mathrm{i} x} \cos(\lambda_\mathrm{i} x) \tag{28}$$

From internal-force equilibrium, when $x = 0$,

$$V_\mathrm{i} = \frac{F_0}{2}.$$

Solving Eqs. (27) and (28) simultaneously gives

$$C_1 = C_2 = \frac{F_0}{8E_\mathrm{i} I_\mathrm{i} \lambda_\mathrm{i}^3} \tag{29}$$

Therefore, the deflection at $x = 0$ is

$$v_\mathrm{i} = \frac{F_0}{8E_\mathrm{i} I_\mathrm{i} \lambda_\mathrm{i}^3}.$$

The contribution of the reinforcing-bar part to the overall shear stiffness is

$$k_\mathrm{i} = 8\xi E_\mathrm{i} I_\mathrm{i} \lambda_\mathrm{i}^3 \tag{30}$$

Fig. 7 Simply supported beam subjected to concentrated load

Figure 7: Fig. 7 Simply supported beam subjected to concentrated load

where ξ is the correction coefficient for the contribution of the reinforcing bar to the actual stiffness of the connector, taken as $\xi = 4.0$[^15].

Substituting Eqs. (19), (21), (23), and (30) into Eqs. (17) and (18), the expression for the elastic shear stiffness of the embedded connector is obtained as

$$k = \left(\frac{4h_{\mathrm{s}}^3}{Etb_1^3} + \frac{6h_{\mathrm{s}}}{5G_{\mathrm{s}}b_1 t} + \frac{1}{\dfrac{E_{\mathrm{c}}h_{\mathrm{c}}(0.15l_{\mathrm{c.1}} + t)}{l_{\mathrm{c.2}} - l_{\mathrm{c.1}}} + 8\xi E_{\mathrm{i}} I_{\mathrm{i}} \lambda_{\mathrm{i}}^3} \right)^{-1} \tag{31}$$

The longitudinal shear stiffness K of the overlapping seam is obtained as

$$K = \frac{n_{\mathrm{s}}k}{u} \tag{32}$$

where n_{s} is the number of connector rows; u is the longitudinal spacing of the connectors.

1.4 Solution for deformation of a simply supported composite beam subjected to a midspan concentrated load

For a simply supported beam subjected to a concentrated load P (Fig. 7), set the coordinate origin at the left support, and the beam span is l.

图 7 简支梁受集中荷载作用

Fig. 7 Simply supported beam subjected to concentrated load

Due to the symmetry of the boundary, only the segment $0 \le x \le l/2$ needs to be considered; thus

$$M(x) = \frac{p}{2}x \tag{33}$$

Substituting Eq. (33) into Eq. (16) gives

$$N''(x) - v^2 N(x) = \frac{Kp}{2\delta}x \tag{34}$$

Solving Eq. (34) yields

$$N(x) = C_1 \sinh vx + C_2 \cosh vx - \frac{px}{2\delta\lambda} \tag{35}$$

From the symmetry of the load, it is known that, at the midspan boundary condition,

$$q\left(\frac{l}{2}\right) = N'\left(\frac{l}{2}\right) = 0.$$

Then, from the support boundary condition $N(0) = 0$, the axial force in the left half of the beam is obtained as

$$N(x) = \frac{p}{2\delta\lambda v \cosh(vl/2)} \sinh vx - \frac{px}{2\delta\lambda} \tag{36}$$

The slip distribution is

$$s(x) = \frac{p}{2K\delta\lambda \cosh(vl/2)} \cosh vx - \frac{p}{2K\delta\lambda} \tag{37}$$

Integrating both sides of Eq. (8) simultaneously gives

$$\frac{px^2}{4} = EA_0 d_0[s(x) - s(0)] + EI_0[\theta(x) - \theta(0)] \tag{38}$$

Substituting $x = l/2$ into Eq. (38) gives

$$\frac{pl^2}{16} = EA_0 d_0 s(0) + EI_0 \theta(0) \tag{39}$$

In Timoshenko beam theory, the relationship between internal force and generalized strain is

$$Q = C\gamma$$

where C is the stiffness coefficient, independent of beam deformation and related only to the cross-sectional shape and the materials used. Generally $C = GA_s/K_s$, where K_s is the shear-stress distribution nonuniformity coefficient, generally obtained by integration as

$$K_s = \frac{F}{J^2} \int \frac{S^2(z)}{b^2(z)} \,\mathrm{d}F$$

[16]; γ is the shear angle.

The total shear force carried by the composite beam is

$$Q = Q_c + Q_s = \left(\frac{GA_c}{K_s^c} + \frac{GA_s}{K_s^s}\right)\left(\frac{\mathrm{d}\omega}{\mathrm{d}x} - \theta\right) \tag{40}$$

where $\mathrm{d}\omega/\mathrm{d}x$ is the rotation of the centroidal axis; K_s^c is the shear-stress distribution nonuniformity coefficient for the concrete slab; and K_s^s is the shear-stress distribution nonuniformity coefficient for the double inverted T-shaped steel beam.

Integrating Eq. (40) gives

$$M(x) + M(0) = \left(\frac{GA_c}{K_s^c} + \frac{GA_s}{K_s^s}\right)\left[\omega(x) - \omega(0) - \int_0^x \theta(\xi)\mathrm{d}\xi\right] \tag{41}$$

From Eqs. (38) and (39), one obtains

$$\theta(x) = -\frac{px^2}{4EI_0} - \frac{A_0 d_0}{I_0}[s(x) - s(0)] + \theta(0) \tag{42}$$

$$\theta(0) = \frac{pl^2}{16EI_0} - \frac{A_0 d_0}{I_0}s(0) \tag{43}$$

Substituting Eqs. (42) and (43) into Eq. (41) gives

$$\omega(x) = \frac{px}{48EI_0}(3l^2 - 4x^2) + \frac{A_0 d_0}{I_0}\left[\frac{px}{2K\delta\lambda} - \frac{P\sinh vx}{2K\delta\lambda v\cosh(vl/2)}\right] + \frac{1}{\left(\frac{GA_c}{K_s^c} + \frac{GA_s}{K_s^s}\right)}\frac{px}{2} \tag{44}$$

Equation (44) is the deflection expression for the left half of the composite beam under a concentrated load. In the equation, the first term is the deflection caused by bending, the second term is the deflection caused by the interface slip effect, and the third term is the deflection caused by shear deformation.

2 Experimental Verification

2.1 Test overview

Four test specimens of concrete-slab-embedded double inverted T-shaped steel-concrete composite beams were designed and numbered according to different structural parameters. Among them, WDTSCB-1 was the reference specimen, and the parameter variables were the width of the concrete flange slab, the thicknesses of the web and lower flange plate of the double inverted T-shaped steel beam, and the spacing between shear connectors, corresponding respectively to

WDTSCB-2 to WDTSCB-4. The members were designed mainly according to the *Code for Design of Composite Structures* (JGJ 138—2016), and the member dimensions are shown in Fig. 8. The beam length was $L = 3\,200$ mm, the calculated span was $L_0 = 3\,000$ mm, and the beam height was $H = 300$ mm, so the height-to-span ratio of the specimen was 1/10. The thickness of the concrete flange slab was $h_c = 80$ mm; the slab width B had two values, 640 mm and 760 mm. The steel plate thickness t had two values, 4 mm and 6 mm. Five transverse diaphragms were arranged at equal intervals along the beam length, with diaphragm spacing $d_h = 800$ mm. The lower flange width of the inverted T-shaped steel beam was $b_s = 64$ mm, and the lower flange width of the double inverted T-shaped member was $b = 130$ mm. The height of the steel plate embedded in the concrete was $d_e = 35$ mm, of which the height of the embedded shear key was $d_t = 30$ mm. The upper length of the shear connector was $b_u = 88$ mm, and the lower length was $b_l = 100$ mm. The connector spacing d had two values, 200 mm and 300 mm. The distance from the slotted shear connector to the member end was $b_e = 50$ mm. The nominal diameters of the threaded reinforcing bars in the slab had two values, 8 mm and 12 mm; the transverse reinforcing-bar spacing was 100 mm, the longitudinal reinforcing-bar spacing was 100 mm, and vertical single-leg stirrups were arranged.

The length was 280 mm, the spacing was 100 mm, and the concrete cover to the reinforcement was 5 mm. The detailed dimensional parameters of each specimen are shown in Table 1.

Table 1 WDSCB specimen design dimensions

Tab. 1 Detailed dimensions of WDSCB

Specimen No.	B/mm	t/mm	b_e/mm	b_u/mm	b_l/mm	d/mm
WDSCB-1	640	4	50	100	88	200
WDSCB-2	760	4	50	100	88	200
WDSCB-3	640	6	50	100	88	200
WDSCB-4	640	4	31	150	138	300

Fig. 8 Dimensions of embedded double inverted T-shaped steel concrete composite beam with web plate

The main steel components of the specimens were made of Q345B bridge steel. The steel beam web plates and flange plates had two plate thicknesses, 4 mm and 6 mm. The reinforcing bars were all HRB400, with two diameters, 8 mm and 12 mm. In accordance with the provisions of *Metallic Materials—Tensile*

Testing (GB/T 228.1—2010), tensile tests were conducted on the components; the test results are shown in Table 2.

Table 2 Mechanical properties of steel

Tab. 2 Table of mechanical properties of steel

Material type	Diameter (plate thickness) /mm	Yield strength σ_s /MPa	Tensile strength σ_b /MPa	Elastic modulus E /GPa
Steel plate	4	421.0	530.9	211.6
	6	477.0	567.7	219.6
Reinforcing bar	8	527.6	649.8	227.2
	12	434.9	610.4	191.6

According to the *Standard for Test Methods of Concrete Physical and Mechanical Properties* (GB 50081—2019), the measured cubic compressive strength of concrete was $f_{\mathrm{cu},k} = 40.7$ MPa. According to the *Code for Design of Concrete Structures* (GB 50010—2010), the axial compressive strength of concrete was obtained as $f_{\mathrm{c},k} = 27.2$ MPa, and the elastic modulus of concrete was $E_c = 3.28 \times 10^4$ MPa.

The test adopted a symmetric loading method at one-third points. The test loading diagram is shown in Fig. 9, and the test loading device is shown in Fig. 10.

In Fig. 9: hydraulic jack; force sensor; distribution beam; sliding hinged support; support structure; fixed hinged support; reaction frame.

Fig. 9 Test loading diagram

Fig. 10 Specimen loading device

2.2 Solution of parameters for the theoretical calculation model

To verify the correctness of the theoretical formulas in this study, the theoretical calculated values were compared with the test values. The parameters of each test beam are shown in Table 3.

Table 3 Basic parameters of test beam

Tab. 3 Basic parameters of test beam

Specimen No.	A_0/mm^2	d_0/mm	I_0/mm^4	K_s^c	K_s^s
WDSCB-1	1 829.4	142.5	9.692×10^7	3.13	1.4
WDSCB-2	1 873.9	146.4	1.02×10^8	2.28	1.4
WDSCB-3	2 472.6	142.9	1.119×10^8	3.05	1.4
WDSCB-4	1 830.1	142.5	9.692×10^7	3.13	1.4

The calculated values of the longitudinal shear stiffness of the composite joints of each specimen are shown in Table 4.

Table 4 Calculated values of longitudinal shear stiffness of composite joints of test beams

Tab. 4 Calculation value of longitudinal shear stiffness for composite joints of test beams

Specimen No.	$k/(\mathrm{kN}\cdot\mathrm{mm}^{-1})$	$K/(\mathrm{kN}\cdot\mathrm{mm}^{-1})$
WDSCB-1	760.2	7.6
WDSCB-2	760.2	7.6
WDSCB-3	969.8	9.7
WDSCB-4	942.3	6.5

2.3 Deflection verification

Substituting the above basic parameters into Eq. (44) gives the deflection values of the WDSCB under concentrated loading at midspan, as shown in Fig. 11. Here, P_p

is the peak load. It can be seen from the figure that, when the load is in the elastic stage ($0.6P_\mathrm{p}$) [9], the deflection values calculated by the theory agree well with the measured values, and the calculated values are slightly larger than the measured values. When the load is greater than $0.6P_\mathrm{p}$, as the load increases, the deviation between the test values and the theoretical calculated values increases. This may be because the formula is established on the basis of the linear-elastic assumption; after the member enters the elastoplastic stage, the derived theoretical calculation formula is no longer applicable.

Table 5 lists the variation of the deflection at the mid-span section of the specimens in the elastic stage. It can be seen from the table that, during the linear-elastic deflection development of the WDSCB members, the deflection value caused by bending accounts for about 56% of the total deflection, the deflection

Fig. 11 Comparison of measured deflection and calculated deflection of WDSCB

Figure 8: Fig. 11 Comparison of measured deflection and calculated deflection of WDSCB

value caused by the interfacial slip effect accounts for about 36% of the total deflection, and the deflection value caused by shear deformation accounts for about 8% of the total deflection.

Fig. 11 Comparison of measured deflection and calculated deflection of WDSCB

Table 5 Deflection values of WDSCB mid-span section

Specimen No.	Load / kN	Bending deflection ω_1 / mm	Slip deflection ω_2 / mm	Shear deflection ω_3 / mm	Total deflection ω / mm	(ω_1/ω) / %	(ω_2/ω) / %
WDSCB-1	0.2 P_p	2.41	1.46	0.20	4.06	59	36
	0.4 P_p	4.82	2.91	0.39	8.13	59	36
	0.6 P_p	7.24	4.37	0.59	12.19	59	36
WDSCB-2	0.2 P_p	2.27	1.44	0.14	3.84	59	37
	0.4 P_p	4.54	2.87	0.28	7.69	59	37
	0.6 P_p	6.81	4.31	0.42	11.53	59	37
WDSCB-3	0.2 P_p	2.47	1.56	0.20	4.23	58	37
	0.4 P_p	4.95	3.11	0.40	8.46	58	37
	0.6 P_p	7.42	4.67	0.60	12.69	58	37
WDSCB-4	0.2 P_p	2.27	1.60	0.18	4.06	56	40
	0.4 P_p	4.54	3.20	0.37	8.11	56	40
	0.6 P_p	6.81	4.81	0.55	12.17	56	40

2.4 Slip Verification

Figure 12 gives a comparison between the measured and theoretical values of the slip of WDSCB from the slip point at the mid-span section to the maximum slip point under peak load. The data agree well. In the shear-span segment, the measured values are larger than the theoretical values; in the pure-bending segment, the measured values are smaller than the theoretical values. It is inferred that this is because WDSCB is symmetrically loaded: there is no shear force in the pure-bending segment, and the interface slip is caused by the slip in

Fig. 12

Figure 9: Fig. 12

the shear-span segment. When calculating the interface stiffness of the lap seam, considering the shear resistance of the whole beam is relatively conservative, and the calculation of interface slip is relatively safe.

Figure 12 Comparison between measured and calculated slip values of WDSCB under peak load

Fig. 12 Comparison between measured and calculated values of sliding of WDSCB under peak load

3 Comparison and Analysis of Existing Methods for Calculating Composite-Beam Deflection

The existing methods for calculating the deflection of steel–concrete composite beams are as follows.

1) Stiffness superposition method for composite beams

7

$$\omega(x) = \frac{Fx}{48\left(E_c I_c + E_s I_s\right)}\left(3l^2 - 4x^2\right) \tag{45}$$

2) Transformed-section method for composite beams

17

$$\omega(x) = \frac{Fx}{48E_s\left(I_s + I_s'/\alpha_E + A_s d_s^2 + A_c d_c^2/\alpha_E\right)} \cdot \left(3l^2 - 4x^2\right) \tag{46}$$

Taking the reference specimen (WDSCB-1) model as an example, according to Eq. (44), Eq. (45), and Eq. (46), the deflection was calculated, and the load-deflection curves were plotted (Fig. 13) for comparative analysis.

It can be seen from Fig. 13 that the deflection calculation formula derived in this study has higher accuracy and better agreement. The values calculated by the stiffness superposition method and the transformed-section method differ greatly from the measured values. Therefore, when checking the deflection of composite-beam members, the effects of interface slip and shear deformation should be considered.

Figure 13 Comparison of load–mid-span section deflection for various calculation methods

Fig. 13 Comparison of load mid-span section deflection for different calculation methods

Fig. 13

Figure 10: Fig. 13

4 Conclusions

This study considered the effects of shear deformation and interface slip on the deflection and slip of composite beams, derived formulas for calculating the deflection and slip of embedded-deck double inverted T-shaped steel-concrete composite beams, and, based on the equivalent spring model and the equivalent rod-system spring model, derived the 弹性 [[unclear: continuation of sentence beyond visible page]]

theoretical formulas for flexural rigidity. The main conclusions are as follows.

1) Based on the Goodman assumption and the Timoshenko beam generalized displacement assumption, computational expressions for the deflection and slip of embedded double inverted T-shaped steel-concrete composite beams under concentrated load at midspan were obtained. The expressions are clear and easy to apply in engineering practice.

2) The calculated values from the derived theoretical formulas for the deflection and slip of embedded steel-concrete composite beams were compared with measured values. For the elastic loading stages of composite beams under different parameters, the theoretical values agreed well with the measured values, verifying the correctness of the theoretical calculation formulas.

3) In the deflection deformation of an embedded double inverted T-shaped steel-concrete composite beam with a span-to-depth ratio of 1/10, the deflection caused by bending accounts for approximately 56% of the total deflection, the deflection caused by the interface slip effect accounts for approximately 36% of the total deflection, and the deflection caused by shear deformation accounts for approximately 8% of the total deflection.

4) The derived theoretical calculation formulas were used to calculate deflection together with other existing theoretical formulas, and the results were compared with measured results. The comparison shows that the theoretical formulas derived in this study can effectively reflect the influence of interface slip and shear deformation effects on the deflection of composite beams, and can serve as a basis for structural design.

References:

[1] HUANG Qiao, LI Wenxian, WANG Bing. Analytical method of steel-concrete composite beam based on interface slip and shear deformation[J]. Journal of Nanjing University of Aeronautics & Astronautics, 2018, 50(1): 131-137(in Chinese).

[2] ZHOU Lingyu, YU Zhiwu, JIANG Lizhong. Analysis of composite beams of steel and concrete with slip and shear deformation[J]. Engineering mechanics, 2005, 22(2): 104-109(in Chinese).

[3] QI Jingjing, JIANG Lizhong, ZHANG Chuanzeng, et al. Effects of interface slip, vertical uplift and shear deformation on dynamic behavior of steel-concrete composite continuous beams[J]. Journal of Central South University(science and technology), 2010, 41(6): 2334-2343(in Chinese).

[4] SCHNABL S, SAJE M, TURK G, et al. Analytical solution of two-layer beam taking into account interlayer slip and shear deformation[J]. Journal of structural engineering, 2007, 133(6): 886-894.

[5] ZHOU Shijun, JIANG Yao, WU Yundan, et al. Shear lag effect of twin-cell composite beams considering slip and shear deformation[J]. Journal of civil, architectural & environment engineering, 2017, 39(3): 20-27(in Chinese).

[6] ZHANG Junfeng, WEN Junbo, LI Jie, et al. Derivation of element stiffness matrix of Timoshenko beam element[J]. Chinese journal of applied mechanics, 2020, 37(6): 2625-2633(in Chinese).

[7] ZHANG Shibo. Study on mechanical behavior of steel–concrete composite beam bridges considering slip effects[D]. Guangzhou: South China University of Technology, 2012.

[8] ZHU Pingru. Principles of Steel–Concrete Composite Beam Design[M]. 2nd ed. Beijing: China Architecture & Building Press, 2006.

[9] ZHANG Xiantong. Experimental study on flexural performance of embedded double inverted T-shaped steel–concrete composite beams[D]. Suzhou: Suzhou University of Science and Technology, 2023.

[10] CLASSEN M, HEGGER J. Shear-slip behaviour and ductility of composite dowel connectors with pry-out failure[J]. Engineering structures, 2017, 150: 428-437.

[11] LI Xianhui, LI Guoqiang. Push-out experimental study on shear connectors in embedded composite beams with notched web[J]. Journal of building structures, 2009, 30(4): 78-84(in Chinese).

[12] CLAβEN M, GALLWOSZUS J. Concrete fatigue in composite dowels[J]. Structural concrete, 2016, 17(1): 63-73.

[13] DU Yunwei, WANG Ronghui, ZHEN Xiaoxia, et al. Calculation of elastic stiffness of shear connector in steel concrete bridge tower joint section[J]. Journal of South China University of Technology(natural science edition), 2023, 51(2): 76-87(in Chinese).

[14] DING Faxing, WANG En, LÜ Fei, et al. Composite action of steel-concrete composite beams under lateral shear force[J]. Engineering mechanics, 2021, 38(7): 86-98(in Chinese).

[15] ZHU Wanyu. Study on mechanical mechanism and design method of steel plate-concrete composite structural bridge towers[D]. Beijing: Tsinghua University, 2020.

[16] SHI Binghua. Formula for calculating the non-uniform coefficient of shear stress distribution in commonly used sections[J]. Journal of building structures, 1984, 5(2): 66-70(in Chinese).

[17] Ministry of Construction of the People' s Republic of China; General Administration of Quality Supervision, Inspection and Quarantine of the People' s Republic of China. Code for Design of Steel Structures: GB 50017—2003[S]. Beijing: China Planning Press, 2003.

(Editor: Zhang Lu)

Note: Figure translations are in progress. See original paper for figures.