

Bayesian Neural Network Evaluation Method for Neutron-Induced Fission Product Yields of ^{232}Th

Authors: Chun-Yuan Qiao, Ya-Xuan Wang, Chun-Wang Ma, Jun-Chen Pei, Yong-Jing Chen, Chun-Wang Ma

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Abstract

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Full Text

Preamble

Bayesian neural network evaluation method on the neutron-induced fission product yields of ^{232}Th

Chun-Yuan Qiao,¹ Ya-Xuan Wang,¹ Chun-Wang Ma,^{1,2,†} Jun-Chen Pei,³ and Yong-Jing Chen⁴

¹School of Physics, Centre for Theoretical Physics, Henan Normal University, Xinxiang 453007, China

²Institute of Nuclear Science and Technology, Henan Academy of Sciences, Zhengzhou 450046, China

³State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China

⁴China Nuclear Data Center, China Institute of Atomic Energy, Beijing 102413, China

Research on neutron-induced fission product yields of ^{232}Th is crucial for understanding the competition between symmetric and asymmetric fission in actinide nuclei. However, obtaining complete isotopic yield distributions over a wide range of neutron energies remains a challenge. In this study, a Bayesian neural network (BNN) model was developed to predict the independent (IND) and cumulative (CUM) fission yields of ^{232}Th under neutron irradiation at various incident energies. To address the limited availability of experimental data for the analysis of IND mass distributions, we substituted mass-number-based yields with the yields of specific isotopes. Furthermore, physical phenomena or quantities, such as the odd-even effect and isospin, were introduced as constraints to enhance the physical consistency of the predictions. The impact of these constraints was evaluated using mass-chain yield distributions and their dependence on energy. Incorporating physical constraints significantly improves the prediction accuracy, yielding more reliable and physically meaningful fission yield data for nuclear physics and reactor design applications.

Keywords: Bayesian neural network, ^{232}Th , Independent fission yield, Cumulative fission yield, Odd-even effect, Isospin

Introduction

Nuclear fission, a fundamental phenomenon in nuclear physics, plays a crucial role in a wide range of applications, including nuclear energy, national defense, and medical isotope production. It is also essential for studies on the synthesis of superheavy nuclei [?, ?], the analysis of reactor antineutrino energy spectra [?], and the investigation of nucleosynthesis via the r-process in neutron star mergers [?]. Accurate and comprehensive fission yield data are critical for understanding the dynamic evolution of the fission process. In 2011, the Chinese Academy of Sciences launched the Strategic Priority Science and Technology Project entitled “Future Advanced Nuclear Fission Energy–Thorium Molten Salt Reactor (TMSR) Nuclear Energy System,” employing ^{232}Th as the primary fuel [?]. The experimental TMSR reactor achieved its first criticality in October 2023. In this system, ^{232}Th serves as a key component, participating mainly in neutron-induced reactions via the Th-U fuel cycle [?]. In particular, high-energy neutrons may induce the direct fission of ^{232}Th . Consequently, for an accurate burn-up credit analysis, special attention must be paid to the fission product yields to reliably assess the reactivity feedback and characteristics of the spent fuel.

Currently, major international evaluated nuclear data libraries (such as JENDL

[?], ENDF [?], CENDL [?], and JEFF [?]) provide comprehensive data for neutron-induced fission yields only at specific energies, that is, 0.0253 eV (thermal neutron), 0.5 MeV, and 14 MeV, respectively. However, the measured neutron-induced fission data are often sparse, with incomplete isotopic coverage, significant uncertainties, and notable discrepancies [?]. Obtaining complete isotopic yield distributions across a continuous range of incident neutron energies remains a challenge. Phenomenological models of nuclear fission (such as the Brosa [?] and GEF [?] approaches) rely heavily on experimental data for parameter calibration and validation. When such data are limited or absent, the underlying assumptions and approximations of these models become weakly constrained, resulting in increased parameter uncertainty and diminished predictive reliability. In the case of ^{232}Th , the lack of systematic fission yield measurements at various neutron energies further hampers the direct application of such semi-empirical models, highlighting the urgent need for alternative approaches to obtain complete yield data.

With the development of artificial intelligence, machine learning (ML) techniques have found extensive applications in nuclear physics research [?], such as prediction and analysis of atomic nuclear mass [?], charge radius [?], α -decay and β -decay half-lives [?], ground-state spin [?], fission product yields [?], and reaction cross sections [?]. ML algorithms can uncover hidden patterns in complex datasets and extract the underlying physical correlations [?]. Among these approaches, the Bayesian neural network (BNN) has shown particularly strong potential for fission yield evaluation [?, ?]. The BNN rigorously quantifies uncertainties through the probabilistic treatment of model parameters [?], incorporates prior physical knowledge to enhance learning from limited datasets, and captures nonlinear features in yield distributions, such as the transition between asymmetric and symmetric fission modes [?]. Furthermore, its probabilistic framework ensures direct compatibility with modern nuclear data evaluation practices that require covariance data. The capability of BNN to integrate experimental uncertainties and missing data further enhances its robustness and predictive accuracy while mitigating overfitting [?]. This makes it a promising tool for reliable ^{232}Th fission yield predictions, where experimental data are sparse.

In this study, a Bayesian neural network (BNN) framework was employed to model and predict the fission product yields of ^{232}Th [?]. Guided by the findings of Ref. [?], the hyperbolic tangent (tanh) activation function was selected for its superior capability in capturing yield trends and reducing prediction errors. To balance the model complexity and accuracy, a two-hidden-layer architecture was adopted, which offers improved performance on complex fission datasets with a manageable number of trainable parameters. Therefore, a two-hidden-layer BNN architecture with tanh activation functions was adopted in this study. The network structure was further optimized by analyzing the error distributions to determine the appropriate number of neurons in each layer. In scenarios where experimental data are sparse, the inclusion of relevant physical constraints, such as the odd-even effect and isospin, has been shown to significantly enhance the

predictive accuracy [?]. The BNN predictions incorporating the odd-even effect and isospin showed significant improvement compared to the uncorrected case. Based on the established BNN framework, this study investigated the mass distributions of IND for ^{232}Th at incident neutron energies of 1 MeV and 13 MeV, and further explored the energy dependence of CUM for the selected nuclides.

The remainder of this paper is organized as follows. Section II introduces the theoretical framework of the BNN method. Section III provides detailed descriptions of the network construction and analysis of results, with Sect. III A focusing on IND and Sect. III B providing a comprehensive analysis of the CUM. Finally, Sect. IV summarizes the key findings and methodological innovations of this study.

II. BNN Model

A Bayesian neural network (BNN) is adopted in this study to predict fission product yields by combining Bayesian inference with the structure of traditional neural networks [?]. Unlike conventional neural networks, in which model parameters are fixed after training and produce a single deterministic output, BNN represents model parameters as probability distributions. This probabilistic formulation enables the BNN to provide both predictions and reliable estimates of the associated uncertainties.

The core of BNN lies in the posterior probability distribution of the model parameters, given by Bayes' theorem:

$$p(\theta|x, t) = \frac{p(x, t|\theta)p(\theta)}{\int p(x, t|\theta)p(\theta)d\theta}$$

where $p(x, t|\theta)$ is the likelihood function that describes the probability of the observed data given the model parameters θ , and $p(\theta)$ is the prior distribution that represents prior beliefs about these parameters. The integral in the denominator serves as a normalization constant to ensure that the posterior distribution is valid. The posterior and predictive distributions are obtained via Markov chain Monte Carlo (MCMC) sampling methods, including Metropolis, Hybrid Monte Carlo, and Slice sampling, which enables rigorous uncertainty quantification of the network parameters. As the training data accumulate, the influence of the prior diminishes in favor of the likelihood.

The likelihood function $p(x, t|\theta)$ is given by:

$$p(x, t|\theta) = \exp(-\chi^2/2)$$

The discrepancy between the target and predicted values is quantified by the χ^2 loss function:

$$\chi^2(\theta) = \sum_i \frac{(t_i - f(x_i, \theta))^2}{\Delta t_i^2}$$

where t_i is the target value, $f(x_i, \theta)$ is the network prediction for input x_i , and Δt_i is the associated experimental uncertainty or noise term [?]. Minimizing χ^2 during training corresponds to maximizing the likelihood. This choice of loss function is motivated by its statistical consistency with Gaussian measurement errors and its emphasis on accurately reproducing high-yield regions, such as the bimodal peaks in fission product distributions, which are of primary physical interest in this study.

In Bayesian neural networks, a feed-forward neural network is used:

$$f(x, \theta) = a + \sum_j b_j \tanh \left(\sum_i d_{ji} x_i + c_j \right)$$

where $x = (x_1, x_2, \dots, x_l)$ is the input vector consisting of l characteristics, typically including the proton number (Z), neutron number (N), and mass number (A) of the fissioning nuclei and fission fragments, and the excitation energy (E) of the compound nucleus. $\theta = \{a, b_j, c_j, d_{ji}\}$ represents the values of the model parameters, where d_{ji} is the connection weight between the input layer and the hidden layer, indicating the influence of the input characteristics on the hidden neurons. c_j is the bias of the hidden layer, which determines the baseline level when there is no contribution from the input layer to the activation function. b_j is the connection weight between the hidden layer and the output layer, representing the contribution weight of each hidden neuron to the final output. a is the bias of the output layer. The nonlinear tanh activation function enables an accurate approximation of complex continuous functions. Figure 1 shows the schematic structure of the single-hidden-layer Bayesian neural network used for the fission yield evaluation.

Given a new input x_n , the network prediction is the expected value over the posterior:

$$\langle f(x_n, \theta) \rangle = \int f(x_n, \theta) p(\theta | x, t) d\theta$$

Each prediction corresponding to a specific set of parameters is weighted by its posterior probability, and the integration yields the final predictive output of the BNN, which incorporates both the central estimate and uncertainty.

[Figure 1: see original paper]

III. Results and Discussion

Independent Fission Yield

Based on the fission yield dataset selected in this study, BNN models with both single and double hidden layers were constructed and evaluated. Comparative analysis revealed that the two-hidden-layer architecture provides better generalization performance and results in lower prediction errors than the single-hidden-layer configuration. To further optimize the architecture and prevent overfitting from excessive complexity or underfitting from insufficient model capacity, the number of neurons per layer was systematically examined. For both simplicity and consistency, the same number of neurons was assigned to each of the two hidden layers of the network. This symmetric configuration not only ensures a balanced parameter distribution across the network but also reduces the number of hyperparameters to be tuned during model optimization. The neuron count varied from 16 to 40 in increments of four. Each configuration was trained under identical conditions using a fixed dataset and a hyperparameter set. During training, the total error in the training dataset was monitored across iterations to evaluate the convergence behavior and prediction stability. The resulting error trends are shown in Fig. 2, which provides a basis for selecting the optimal network configuration used in subsequent fission yield predictions.

Figure 2 illustrates the variation in the mean square error (MSE) during training for networks with different neuron counts per hidden layer. Under a fixed network architecture, the error gradually decreased with an increase in training iterations and stabilized as the number of iterations approached 10^5 , indicating that convergence had been reached. Within the Bayesian neural network framework, we mitigate the risk of converging to local minima by employing a substantially large number of iterations, allowing the sampling process to thoroughly explore the parameter space. At a given number of training iterations, increasing the number of neurons per hidden layer leads to a noticeable reduction in the prediction error. However, this trend began to plateau when the neuron count reached 28, beyond which the additional reduction in error became marginal. This observation suggests that increasing the model complexity beyond this point does not lead to significant performance gains and may potentially introduce the risk of overfitting. To balance the predictive accuracy and model generalization, a configuration with two hidden layers and 28 neurons per layer was selected as the optimal network architecture.

In IND modeling, the mass distribution of fission product yields is of central importance. However, experimental data directly reporting the yield as a function of the mass number are limited. To address this issue and maximize the use of the available data, the training input was designed to include the neutron number of the fissioning nuclei, proton number and mass number of the fission fragments, excitation energy of the compound nucleus, and corresponding IND of individual nuclides. The dataset used in IND includes 4,128 evaluated data points extracted from the JENDL database [?] and 112 experimental data points

obtained from the EXFOR database [?]. The mass-yield data from EXFOR were employed for validation. To further enhance the predictive performance of the BNN, additional physical constraints were added. Specifically, the odd-even effect is incorporated by assigning a value of +0.2 for even-proton-number fragments and -0.2 for odd-proton-number fragments, following the methodology described in Ref. [?].

In addition, isospin symmetry is accounted for by including the third component of isospin, defined as $(Z - N)/2$, as an input feature of the model. Once the BNN predicts the yields of individual nuclides, the mass yield distribution is reconstructed by summing the predicted yields of nuclides with the same mass numbers. This enables the generation of independent fission product mass distributions induced by incident neutrons on thorium isotopes, even when direct experimental data on mass yields are not available.

In Fig. 3, BNN0 denotes the baseline model without additional physical constraints; BNN1-1 incorporates the odd-even effect; BNN1-2 includes isospin; and BNN2 incorporates both constraints simultaneously by introducing the two corresponding input columns (odd-even and isospin) into the network. In Fig. 3(a), corresponding to thermal neutron-induced fission of ^{227}Th , BNN1-1 reduces the unphysical negative values present in BNN0 but introduces an abnormal upturn near mass number 170. Similarly, BNN1-2 suppresses the negative values and upturn but fails to reproduce the fine structure in the light fragment region. In contrast, BNN2 not only eliminates these unphysical features but also retains the peak structure originally predicted by BNN0, indicating that the simultaneous inclusion of both constraints results in a more precise representation of the fission mechanism and improves the model predictive accuracy. In Fig. 3(b), for thermal neutron-induced fission of ^{229}Th , BNN1-1 and BNN1-2 exhibit similar behaviors as in Fig. 3(a), whereas BNN2 successfully reproduces the peak structure of the heavy fragment for the first time, which aligns well with the evaluated data. This suggests that the formation of heavy fission fragments is significantly influenced by the combined effects of the odd-even pattern and isospin. These results underscore the necessity of jointly incorporating both constraints to advance the theoretical modeling of the production of heavy fission fragments. In Fig. 3(c), which shows neutron-induced fission of ^{232}Th at 0.5 MeV, BNN0 exhibits an abnormal upturn around mass number 70 and significantly underestimates the yields in the 138-144 mass range relative to the evaluated data. BNN1-1 corrects the heavy fragment underestimation, whereas BNN1-2 mitigates the abnormal upturn and reduces negative yields. However, only BNN2 resolves all these deficiencies simultaneously, demonstrating the best overall performance in reproducing the fission fragment mass yield distribution.

In Fig. 3(d), corresponding to 14 MeV neutron-induced fission of ^{232}Th , BNN2 provides the most accurate description of the symmetric fission region and matches the evaluated data well in the light fragment region (mass 80-100), outperforming the other models in both respects. Across all four reaction systems, compared to BNN0, BNN1-1 results in a slightly narrower confidence

band, whereas BNN1-2 shows a modest increase in uncertainty. In contrast, BNN2 yields a substantial reduction in the confidence band width, indicating significantly improved predictive reliability. This reduction in uncertainty further suggests that the combination of both physical constraints enhances model generalizability. In summary, while the odd-even effect and isospin each contribute to improving the BNN model performance, they have individual limitations. Their simultaneous incorporation into BNN2 leads to the most substantial improvement in both yield prediction accuracy and uncertainty reduction, closely aligning with the evaluated data and demonstrating the robustness of the combined-constraint model. Despite the incorporation of additional physical constraints, YBNN still exhibited unphysical negative values. Reference [?] demonstrated that applying a penalty on negative values improves both the representation of yield peak structures and the overall predictive accuracy. This penalty mechanism will be considered in future network design.

[Figure 2: see original paper]

Figure 4 shows the predicted fission fragment mass distributions for ^{232}Th induced by 1 MeV and 13 MeV neutrons, with both energy points excluded from the training data. YBNN2 are compared with YBNN0 and the YGEF [?]. In Fig. 4(a), BNN2 eliminates the unphysical negative yields observed in BNN0 around mass numbers $A = 150 - 160$. Moreover, in the mass range $A = 90 - 100$, YBNN2 shows an improved agreement with YGEF. In Fig. 4(b), YBNN2 also suppresses the negative values in the mass range $A = 100 - 120$ and produces results significantly closer to YGEF in the $A = 120 - 140$ region. In both panels, the confidence intervals of YBNN2 are noticeably narrower than those of YBNN0, indicating reduced predictive uncertainty and improved generalization performance. These results suggest that the simultaneous inclusion of the odd-even effect and isospin enhances the physical realism of the model and underscores the importance of incorporating both effects in fission yield modeling.

[Figure 3: see original paper]

[Figure 4: see original paper]

Cumulative Fission Yield

The training dataset includes both the proton and neutron numbers of the fissioning nuclei and fission fragments, excitation energy of the compound nucleus, and cumulative fission yields. It comprises 4,128 evaluated data points from the JENDL database [?] and 1,258 experimental data points from the EXFOR database [?]. Based on the collected cumulative fission yield data, various BNN architectures, including single-hidden-layer and two-hidden-layer networks with varying numbers of neurons per layer, were systematically evaluated. Optimal performance was achieved using a two-hidden-layer architecture with 20 neurons per layer, which was adopted as the baseline model. Four configurations were tested: (1) without additional physical constraints (BNN0), (2) with only the odd-even effect (BNN1-1), (3) with only the isospin effect (BNN1-2), and (4)

with both physical constraints simultaneously (BNN2). The implementation of these physical constraints follows the same approach as that in IND [?]. Comparative analysis against the evaluated data shows that the BNN2 model, which incorporates both effects, achieves the best predictive accuracy and generalization capability among the three models. Consequently, only the BNN2 model was retained for the subsequent analysis.

Figure 5 compares the evaluated cumulative fission yields (YJENDL) with the predictions of the BNN without additional physical constraints (YBNN0) and the physically constrained model (YBNN2) for four representative reactions. In the thermal neutron-induced fission of thorium isotopes (Fig. 5(a) and (b)), YBNN0 exhibits a systematic overestimation compared to YJENDL, particularly in low-yield (blue) regions, and fails to reproduce the characteristic yield peaks. In contrast, BNN2 provides a much improved prediction, especially for light fragments, where YBNN2 closely matches YJENDL. In particular, two pronounced peaks (red spots) corresponding to ^{137}Ba and ^{140}Ce appear in both YBNN2 and YJENDL, demonstrating the model's ability to capture the key heavy fragment yields. In Fig. 5(c), corresponding to a different actinide target, BNN0 again overpredicts the yields in the low-value regions. This trend of overestimation was significantly mitigated in BNN2, which was in better agreement with the evaluated distribution. Figure 5(d), corresponding to fission at 14 MeV incident neutron energy, shows that BNN2 substantially improves the yield predictions in the symmetric fission region, aligning more closely with YJENDL than BNN0. In general, across the four reactions, YBNN2 demonstrated significantly better agreement with YJENDL than YBNN0, particularly in reproducing the heavy fragment yield peaks. These results demonstrate that incorporating the odd-even effect and isospin enhances the physical reliability and the predictive precision of the model. The mean squared error was reduced from 0.565 to 0.364, further confirming the effectiveness of the added physical constraints.

[Figure 5: see original paper]

Figure 6 presents the cumulative fission yields for heavy fragments with mass numbers $A = 134 - 144$ from four neutron-induced Th fission reactions at different neutron energies. As fission products reach stability primarily through β -decay, these chain yields reflect the population of the final stable or long-lived nuclides along the decay chains. The evaluated data (YJENDL) show a clear odd-even staggering, with yields at odd mass numbers being notably higher than those of neighboring even- A isotopes. In addition, a pronounced enhancement was observed at $A = 140$, corresponding to the production of ^{140}Ce , which is a β -stable nuclide with a magic neutron number ($N = 82$) and one of the yield peaks identified in Fig. 5. BNN predictions without physical constraints (YBNN0) show a smoother trend, failing to capture the odd-even structure and underestimating the peak at $A = 140$. In contrast, the constrained model (BNN2), which incorporates both the odd-even effect and isospin, reproduces the evaluated pattern well (including the odd-even staggering and sharp rise

at $A = 140$). These results demonstrate that integrating physical features into the BNN framework significantly improves its ability to capture decay chain effects and local nuclear structure signatures. Thus, the BNN2 model offers a more accurate and physically interpretable prediction of the cumulative yields of fission products.

As shown in Fig. 6, the BNN2 model more accurately captures the mass-dependent features of the fission product yields, particularly the odd-even staggering and the notable deviation around $A = 140$. To quantitatively assess the improvement from the incorporation of physical constraints, Figure 7 compares the ratios of the model predictions to the JENDL evaluations of the four neutron-induced fission reactions in thorium isotopes. The results show that the yield ratios predicted by BNN2 (YBNN2/YJENDL) are consistently closer to unity than those predicted by the original BNN0 model (YBNN0/YJENDL), indicating a significant enhancement in the prediction accuracy. This improvement was particularly evident for several mass numbers. For instance, at $A = 137$, YBNN2 consistently aligns more closely with YJENDL in all four cases, corresponding to the enhanced production of ^{137}Ba observed in Fig. 5. Around $A = 140$, YBNN2 are nearly identical to YJENDL in Fig. 7(a) and (d), and also exhibit notable improvements over YBNN0 in Fig. 7(b) and (c), respectively. However, at $A = 136$, both YBNN0 and YBNN2 show notable deviations from YJENDL, suggesting that additional physical effects may influence the yields in this region and warrant further investigation. In summary, the overall trend of YBNN2/YJENDL being consistently closer to unity than YBNN0/YJENDL clearly demonstrates the benefit of incorporating the odd-even effect and isospin into the BNN framework. These physical enhancements significantly improved the model's ability to reproduce the evaluated cumulative fission yields for thorium-induced reactions, emphasizing their indispensable role in refining fission yield predictions.

[Figure 6: see original paper]

[Figure 7: see original paper]

Figure 8 presents the cumulative fission yields of four representative fission products— ^{95}Zr , ^{99}Mo , ^{132}Te , and ^{131}I —as a function of the incident neutron energy from 0 to 14 MeV. These nuclides were selected because of their significance in nuclear applications, environmental safety, and human health. Specifically, ^{95}Zr is a precursor of ^{95}Nb , which is a widely used redox indicator in reactor monitoring. ^{99}Mo , with a high decay branching ratio (88%) to $^{99\text{m}}\text{Tc}$, plays a critical role in nuclear medicine. ^{132}Te promotes corrosion, shortening the service life of structural materials, whereas ^{131}I is a radiotoxic nuclide with substantial biological impact. For ^{95}Zr [Fig. 8(a)], the yield initially increases, then gradually decreases at higher neutron energies (4-14 MeV). The BNN0 model consistently underestimates the experimental data, whereas BNN2 offers more accurate predictions across the energy range, with only a slight overestimation at 14 MeV. In the case of ^{99}Mo [Fig. 8(b)], the yield remains nearly constant from 2 to 12 MeV, with a small drop at 14 MeV. The BNN2 predictions ac-

curately captured this behavior and corrected the underestimation observed in BNN0 at thermal energy (0.0253 eV). For ^{132}Te [Fig. 8(c)], the yield shows a pronounced increase at low energies, followed by a gradual decline beyond 8 MeV. The BNN2 model matches the experimental trends more closely than BNN0, particularly in the 0-4 MeV region. As shown in Fig. 8(d), the yield of ^{131}I rises with increasing neutron energy. The predictions of BNN2 align well with the experimental and evaluated data above 5 MeV and at thermal energy; however, both BNN predictions slightly overestimate the experimental results between 2 and 4 MeV. Overall, all experimental points fell within the 95% confidence intervals of the BNN predictions. Notably, the confidence bands associated with BNN2 are narrower than those of BNN0, demonstrating that incorporating odd-even effects and isospin corrections not only improves the predictive accuracy but also reduces the model uncertainty.

[Figure 8: see original paper]

IV. Summary

In this study, a Bayesian neural network (BNN) was employed to evaluate both independent (IND) and cumulative fission product yields (CUM), with the goal of improving the predictive accuracy beyond the limitations posed by sparse and inconsistent experimental data available only at a few neutron energies (thermal, 0.5 MeV, and 14 MeV). In the evaluation of IND, the input features were optimized by incorporating the proton and mass numbers of the fission fragments, along with their corresponding yields. This enabled the use of a broader set of experimental data for model training. A two-hidden-layer architecture with 28 neurons per layer was adopted based on the error minimization analysis. The incorporation of the odd-even effect and isospin significantly enhanced the predictive performance of the model. When both physical constraints were applied simultaneously, the model effectively suppressed unphysical negative yields, mitigated abnormal upturns, and reproduced the fine structures in the heavy fragment region. The uncertainty of the prediction, as reflected in the 95% confidence intervals, was also substantially reduced.

For CUM evaluation, a two-hidden-layer architecture with 20 neurons per layer was selected after comprehensive testing of various configurations, including single-hidden-layer and two-hidden-layer networks with different neuron counts. Comparative studies were performed, including single-nuclide yield comparisons, analysis of odd-even staggering in mass yields (particularly in the $A = 134 - 144$ region), and yield ratio Y_{BNN}/Y_{JENDL} analysis. These results demonstrate that the simultaneous incorporation of odd-even effects and isospin constraints significantly improved the reliability of the yield predictions. The model more accurately captured the features of stable nuclide production and successfully reproduced the anomalous structure near $A = 140$. Overall, our study demonstrates that combining physical constraints, particularly the odd-even effect and isospin, significantly enhances the performance and robustness of BNN-based fission yield evaluations. These findings suggest that such physical features should

be systematically incorporated into future data-driven models and theoretical approaches to improve the accuracy of fission yield predictions.

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