

## Bayesian evaluation of photofission product yields of Th isotopic chains

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### Abstract

The fragment yields in photon-induced fission reaction of thorium (Th) isotopes are important in modern nuclear energy applications, as well as in the evolution of the nuclear structure in its isotopic chains. Bayesian neural networks (BNN) models have been constructed to describe fragment yields in the photonuclear fission reactions of thorium isotopes ranging from  $^{216}\text{Th}$  to  $^{232}\text{Th}$ , including those of  $^{232}\text{Th}$  at various incident photon energies. The predicted results of the optimized BNN models show good agreement with the measured data in the reactions. The double-layer BNN models successfully illustrate the systematic transition from asymmetric to symmetric fission in thorium isotopes, including associated odd-even effects, energy dependence, and the leftward shift in mass yield distributions. The developed BNN models provide new tools for predicting fragment yields in thorium photonuclear fission reactions.

### Full Text

### Preamble

#### Bayesian Evaluation of Photofission Product Yields of Thorium Isotopic Chains

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The fragment yields in photon-induced fission reactions of thorium (Th) isotopes are important for modern nuclear energy applications as well as for understanding the evolution of nuclear structure along isotopic chains. Bayesian neural network (BNN) models have been constructed to describe fragment yields in photonuclear fission reactions of thorium isotopes ranging from  $^{216}\text{Th}$  to  $^{232}\text{Th}$ , including those of  $^{232}\text{Th}$  at various incident photon energies. The optimized BNN models show good agreement with measured data. The double-layer BNN models successfully capture the systematic transition from asymmetric to symmetric fission in thorium isotopes, including associated odd-even effects, energy dependence, and the leftward shift in mass yield distributions. These BNN models provide new tools for predicting fragment yields in thorium photonuclear fission reactions.

**Keywords:** Photonuclear fission reaction, Thorium isotopes,  $^{232}\text{Th}$ , Bayesian Neural Network, Total element yield, Mass chain yield, Odd-even staggering phenomenon

## Introduction

Photon-induced nuclear fission (PNF) of a heavy nucleus is a process in which high-energy photons interact with a nucleus, causing it to split into two or more lighter hot fragments, followed by decay processes that release neutrons, gamma rays, and energy [1, 2]. The mechanism of PNF differs from other particle-induced nuclear fissions because only electromagnetic interaction occurs between the photon and the nucleus [3-5]. This interaction can transfer clear angular momentum and directly affect the distribution of fission products, providing a clean probe for investigating nuclear structure and reaction mechanisms. The PNF process has significant scientific and applied importance in fields such as nuclear physics research, energy development, nuclear medicine, and nuclear astrophysics [6, 7]. PNF reactions also enable unique studies of the interplay between macroscopic and microscopic degrees of freedom in nuclei. At low excitation energies, the reaction mechanism is strongly influenced by nuclear structure and shell effects. The pairing effect plays an important role in the mass and charge distributions of fission fragments [8-10]. However, experimental data can only indirectly explain the influence of nuclear structure on the fission process, making additional PNF reaction data crucial for improving our understanding [11, 12].

Machine learning technology provides algorithms for analyzing large datasets and uncovering patterns in complex high-dimensional data. By constructing sophisticated computational models, it can effectively capture nonlinear relationships inherent in the data [13, 15, 36]. The Bayesian Neural Network (BNN)

method applies Bayesian statistical techniques to neural network models. Its core principle involves assigning prior probability distributions to model parameters, then updating these priors through Bayes' theorem using observed data to obtain posterior parameter distributions. Machine learning has demonstrated high-quality performance in predicting and analyzing nuclear masses [16-19], nuclear physics phenomena [20, 21], photoneutron reactions [22], and residual nuclide cross sections in nuclear spallation [23-25] and projectile fragmentation [26-30] reactions. New theories based on machine learning have also been developed to describe fission mechanisms in heavy nuclides [31-33], including extrapolating nuclear masses, neutron-induced fission fragment yields, various nuclear structures, and reaction observables [34, 35].

Existing fission data are often incomplete and subject to large uncertainties. No study has yet applied Bayesian neural networks or similar frameworks to systematically predict both charge and mass yields of photofission fragments across the entire thorium isotopic chain. Due to the difficulties in describing fragment production in PNF reactions, BNN technology is systematically applied for the first time to construct models that evaluate and predict fragment yields in photonuclear fission reactions of thorium isotopes. This work addresses the important applications in modern nuclear reactors, nuclear medicine, and nuclear structure studies, while also evaluating incomplete charge and mass distributions of fragments. The excitation functions of fragments of interest are also investigated.

The article is organized as follows. Section II describes the method for building BNN models. Section III presents the BNN evaluation of total element yields and mass chain yields of PNF fragments. A summary is given in Section IV.

[Figure 1: see original paper] (Color online) A schematic diagram of a neural network with double hidden layers of 3-3 neurons ( $H_1=3, H_2=3$ ) and two input variables ( $I=2$ ).

## II. BNN Models

BNNs can automatically avoid overfitting by incorporating prior distributions, quantifying uncertainties in predictions, and assessing correlations among model parameters [21, 36]. To construct a BNN model for reproducing and predicting fragment cross sections, we begin with the prior distribution of model parameters given sample data  $D = (x_1, t_1) \cdots (x_N, t_N)$ , where  $x_n$  and  $t_n$  ( $n = 1, 2, \dots, N$ ) are the input and output datasets, respectively, and  $N$  is the total sample size. According to Bayes' theorem, the posterior probability distribution of model parameters is obtained as:

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

where the model parameter  $\theta$  is described probabilistically,  $p(D|\theta)$  is the likeli-

hood function,  $p(\theta)$  is the prior distribution introduced over all possible values of  $\theta$ , and  $p(D)$  is the normalization constant representing the marginal density of observed samples, defined as:

$$p(D) = \int f(D|\theta)f(\theta)d\theta$$

For regression tasks where the goal is to predict the target variable given an input vector, the likelihood function is typically defined as a Gaussian distribution:

$$p(D|\theta) = \exp\left(-\frac{\chi^2}{2}\right)$$

The objective function  $\chi^2$  is defined as:

$$\chi^2 = \sum_n \frac{(t_n - f(x; \theta))^2}{\Delta t_n^2}$$

where  $\Delta t_n$  represents the noise error.

The BNN model typically employs a multilayer perceptron (MLP) network, also known as a “backpropagation” or “feedforward” network. These networks consist of input variables  $x_i$ , one or several hidden layers with different numbers of neurons, and output variables  $f_k(x; \theta)$ . The functional equation for a typical MLP network with a single hidden layer is:

$$f(x; \theta) = a + \sum_{j=1}^H b_j \tanh\left(c_j + \sum_{i=1}^I d_{ji}x_i\right)$$

where  $H$  is the number of hidden neurons,  $I$  is the number of input variables,  $x = \{x_i\}$  is the input dataset, and  $\theta = \{a, b_j, c_j, d_{ji}\}$  defines the MLP network parameters corresponding to biases and weights of the output layer, and biases and weights of the hidden layer, respectively. The total number of parameters is  $1 + (2 + I) \times H$ . In the BNN model, each output variable  $f(x)$  is obtained by taking the weighted sum of hidden unit values and adding the output layer bias, while each hidden unit value is calculated by taking the weighted sum of input values and applying a nonlinear activation function.

Two independent BNN models were constructed to predict yields of two different PNF fragment distributions: one for charge yields (BNN-CY) for photonuclear fission of thorium isotopes ( $^{216-232}\text{Th}$ ), and another for mass yields (BNN-MY) of PNF fragments for  $^{232}\text{Th}$  at different  $\gamma$  incident energies. All calculations adopted  $10^5$  sampling iterations of BNN, with uncertainty quantification presented as 80% confidence intervals (CI). The optimal number of neurons in each layer was determined by systematically varying the number of neurons per layer

and comparing the corresponding standard deviations  $\sigma_N$  to find the best configuration, where  $\sigma_N$  is determined by:

$$\chi_N^2 = \sum_i \frac{[t_i - f(x_i)]^2}{N}$$

### III. Results and Discussion

#### A. The Fission Mechanism of Actinide Elements

Before discussing the BNN model predictions, it is important to review the basic phenomena of fragment yield distributions discovered experimentally to better understand the evaluation of fragment yields in  $\gamma$ -induced thorium fission reactions.

The phenomenological Brosa model [37, 38], based on fission product mass distributions, postulates the existence of two asymmetric fission modes. The origin of these distinct asymmetric modes is attributable to the influence of fission barriers. The Brosa model further describes the broadening of fission observables through stochastic neck rupture, with the total prompt neutron multiplicity providing evidence for a new compact symmetric fission mode in the light thorium isotopic chain [47]. Based on BNN predictions, a detailed discussion of the photofission mechanism of Th isotopes will be presented in Sections III B and III C.

#### B. Total Element Yield of Fission Products

The BNN-CY model was constructed to predict PNF fragment charge yields by learning experimental charge yield data from the EXFOR database, which includes 1,437 data points from PNF products of 14 nuclides ( $^{217-230}\text{Th}$ ). All measured data are taken from Refs. [41, 43]. The BNN-CY input dataset consists of  $\mathbf{x}_i = \{Z_{\text{fi}}, Z_i, N_i, E_i\}$ , referring to the charge number of fission fragments, the charge and neutron numbers of the fissioning nucleus, and the excitation energy ( $E_i$ ) of the compound nucleus, respectively. The output dataset  $t_i$  is the charge yield of fission fragments.

The fission charge yields of the compound nucleus  $^{217}\text{Th}$  predicted by the BNN-CY model are illustrated in Fig. 2 [Figure 2: see original paper]. For clarity, prediction results from neural networks without odd-even parameters display only central values without confidence intervals. The predictive results from single-layer neural networks excluding odd-even parameters demonstrate two notable anomalies: negative values emerge near fission fragment charge  $Z_f = 35$ , while an abnormal upward tail becomes apparent for  $Z_f > 60$ . Moreover, double-layer neural networks without odd-even parameters generate aberrant negative predictions near  $Z_f = 54$ .

The production of fission fragments with even proton numbers is usually enhanced because fully paired proton configurations tend to survive scission with

high probability. To describe the odd-even effect in the reaction system, we implemented methods described in Refs. [16, 32], adding an additional input  $\delta = \pm 0.1$  to the BNN-CY learning dataset to represent even and odd fragments, respectively. The input parameter set becomes  $x_i = \{Z_{fi}, Z_i, A_i, E_i, \delta_i\}$ . As shown in Fig. 2, BNN-CY evaluations without the odd-even parameter  $\delta$  fail to capture the odd-even effect in the charge yield distribution of PNF fragments, while inclusion of  $\delta$  allows BNN-CY to accurately reproduce this effect.

Based on  ${}^2_N$  values from Eq. (6), we screened the number of neurons in hidden layers and ultimately adopted a double-layer network with 30 neurons for this study. Generally, a double-layer network significantly enhances learning performance due to its larger number of connection parameters compared to a single-layer network, even when both have the same total number of neurons. To verify this and better describe the odd-even effect, we conducted a detailed comparison between single-layer neural networks with odd-even parameters and double-layer networks with odd-even parameters. Specifically, we compared a single-layer network with 30 neurons and a double-layer network with 15-15 neurons, performing  $10^5$  BNN-CY sampling iterations. For both configurations, the standard deviations  ${}^2_N$  obtained from Eq. (6) were  $1.7 \times 10^{-2}$  and  $1.3 \times 10^{-2}$ , respectively, indicating similar uncertainties. As shown in Figs. 3 and 4, the single-layer network can effectively reproduce the odd-even effect in fission fragment charge distributions, similar to the double-layer network. However, the confidence interval of the double-layer network is slightly larger than that of the single-layer network.

Figure 3 presents a comparison between BNN-CY evaluation results and experimental charge yield data for PNF fragments from targets  ${}^{217-229}\text{Th}$  at an average excitation energy of 11 MeV, as reported by Schmidt et al. [41]. The BNN-CY results show good agreement with experimental data. Notably, neural networks incorporating odd-even parameters successfully reproduce the characteristic odd-even staggering in the charge distribution of the reaction system. Furthermore, these calculations accurately capture the transition from asymmetric to symmetric fission in the PNF mechanism across the thorium isotopic chain. The number of peaks in the fission fragment charge distribution evolves with increasing neutron number of target nuclei: single-peak distributions for neutron-deficient thorium targets transition through triple-peak patterns in intermediate-mass thorium nuclei, eventually becoming double-peak distributions for heavier thorium isotope targets. This systematic evolution demonstrates the strong dependence of fission mechanisms on neutron number along the Th isotopic chain. Experimental data on charge yields of  ${}^{221-230}\text{Th}$  fission fragments at an average excitation energy of around 14 MeV by Chatillon et al. [43], shown in Fig. 4 [Figure 4: see original paper], exhibit similar phenomena.

The results predicted by the double-layer network with the smallest  ${}^2_N$  value were selected for further evaluation and prediction, focusing on charge yield values associated with the three prominent peaks observed in the fission fragment

charge distribution across the thorium isotopic chain. Fragments with  $Z_f = 36$  (Kr), 45 (Rh), and 54 (Xe) were particularly analyzed because of their critical role in quantifying competition between symmetric and asymmetric fission modes. These yields serve as direct indicators of the evolving fission mechanism. Furthermore, the fission product  $^{135}\text{Xe}$  has a substantial neutron absorption cross section, acting as a reactor “poison” that significantly reduces operational power, making evaluation of its production highly important.

Figure 5 [Figure 5: see original paper] presents the evolution of fission fragment yields for  $Z_f = 45$  (Rh) as a function of target mass number. The observed systematic decrease in Rh fragment yields with increasing target neutron number clearly demonstrates the diminishing contribution of symmetric fission channels. This trend culminates in near-zero fragment charge yields for  $^{230-232}\text{Th}$ , establishing asymmetric fission as the dominant mechanism in heavier thorium isotopes, while symmetric fission prevails in lighter Th isotopes. Notably, higher charge yields were observed at 11 MeV excitation energy in heavier Th isotopes compared to the 13–15 MeV range.

Figure 6 [Figure 6: see original paper] presents the evolution of fission fragment yields for  $Z_f = 36$  (Kr) and 54 (Xe) as a function of target mass number. The BNN-CY results successfully reproduce the symmetry characteristics of charge yield distributions. Experimental data reveal a perfectly symmetric charge yield distribution where Kr and Xe fragment yields are identical, a feature accurately reproduced by BNN-CY predictions. Moreover, charge yields representing asymmetric fission mechanisms show systematic increase with target neutron number, with more pronounced growth observed at 14 MeV excitation energy. These results provide clear evidence for the transition from symmetric fission in neutron-deficient thorium isotopes to asymmetric fission in their heavier counterparts.

### C. Mass Chain Yield Distribution

The BNN-CY predictions in Section III B indicate that the fission mechanism of  $^{232}\text{Th}$  will be dominated by asymmetric fission. To explore the energy dependence of mass yield distributions, the BNN-MY model was constructed by learning experimental mass yield data from the EXFOR database, which includes 771 data points within an energy range of 8–80 MeV for PNF products of  $^{232}\text{Th}$  [48–55]. The mass yields of  $^{232}\text{Th}$  fission fragments were measured by Naik et al. [50–55] using the activation method. To avoid systematic inconsistencies between different experimental datasets that could artificially widen model confidence intervals, we selected data exclusively from the same group (Naik et al.) for training, while using data from other groups for validation.

The charge distribution exhibits a pronounced odd-even effect, whereas this effect appears obscured in the mass distribution. Consequently,  $\delta$  parameters were not needed as input corrections for mass yield data. The network input variables are  $x_i = \{A_{fi}, Z_i, N_i, E_i\}$ , denoting the mass number of fission fragments, the charge and neutron numbers of the fissioning nucleus,

and incident  $\gamma$  energy ( $E_i$ ), respectively. The output data  $t_i$  is the mass yield of fission fragments.

We compared a single-layer network with 38 neurons and a double-layer network with 18–20 neurons, performing  $10^5$  BNN-MY sampling iterations. The standard deviations obtained were  $6.4 \times 10^{-3}$  and  $3.8 \times 10^{-3}$  for single-layer and double-layer networks, respectively. As shown in Fig. 7 [Figure 7: see original paper], the CI of the double-layer network is significantly narrower than that of the single-layer network. The double-layer network successfully reproduces peak structures, whereas single-layer network results appear smoother. Based on these considerations, we selected the double-layer network as the more suitable model for further data evaluation.

Figure 7 presents BNN evaluation for regions with sparse experimental data. In the mass region  $A_f < 90$ , experimental measurements are particularly scarce, demonstrating that the trained BNN model can predict physical behaviors in regions where experimental data are currently limited or inaccessible. The results show that fission yield exhibits monotonic increase with energy in the valley region (fragment mass number  $A_f = 100-130$ ), while twin peaks corresponding to asymmetric fission modes diminish in prominence.

In Fig. 8 [Figure 8: see original paper],  $A_f = 91$  and  $140$  were selected for BNN prediction because these fragments are located in the light and heavy peak regions of the asymmetric fission mode of  $^{232}\text{Th}$ , respectively, and their yields are relatively high. Most importantly, these fragments have sufficient experimental data for comparison with BNN predictions. Generally, fragment yields for  $A_f = 91$  and  $140$  representing asymmetric fission increase gradually within the incident  $\gamma$  energy range of 5–40 MeV. However, when  $\gamma$  energy exceeds 40 MeV, yields decrease significantly rather than continuing to increase.

In 2010, Demekhina et al. [39] compared PNF fragment mass yields of  $^{232}\text{Th}$  at  $\gamma$  energies of 50 MeV and 3500 MeV, observing that symmetric fission dominated as energy increased, with its contribution increasing by almost an order of magnitude. Moreover, additional neutron emission from the fission nucleus under higher excitation energy decreases the mass numbers of fission fragments, leading to a leftward shift in the mass-yield distribution. Figure 9 [Figure 9: see original paper] displays mass yield distributions evaluated by BNN-MY for  $^{232}\text{Th}$  PNF fragments at incident  $\gamma$  energies of 8 MeV, 25 MeV, and 80 MeV. The yield of symmetric fission ( $A_f = 100-130$ ) increases with energy, while contributions from the light fragment peak ( $A_f = 80-100$ ) and heavy fragment peak ( $A_f = 130-150$ ) decrease progressively. This suggests that with increasing nuclear excitation energy, particle evaporation (predominantly neutron emission) from the excited nucleus opens a new decay channel leading to formation of neutron-deficient fission fragments (primarily through symmetric fission) [56, 57]. However, no obvious leftward shift of the light fragment peak toward lower mass numbers was observed, indicating that the PNF mechanism of  $^{232}\text{Th}$  changes with incident  $\gamma$  energy. In the low-energy region, asymmetric fission dominates, while symmetric fission becomes dominant in the high-energy

region.

Figure 10 [Figure 10: see original paper] evaluates the predictive capability of BNN-MY using a test set that excludes PNF fragment mass yield data for  $^{232}\text{Th}$  at  $\gamma$  energies of 7.64 MeV and 17.5 MeV. Comparison between predicted results and experimental measurements [58, 59] shows that BNN-MY can effectively predict mass yields of PNF fragments for unstudied reactions, and the trained model successfully reproduces results reported by other research groups.

## IV. Summary

Nuclear fission is an extremely complex non-equilibrium quantum many-body dynamical process, and gaining deeper insights into fission remains a well-recognized challenge in nuclear physics. Strong motivation for studying nuclear fission persists, driven by expanding nuclear applications in energy production and rare isotope generation, as well as fundamental physics domains including superheavy element synthesis and constraints on the r-process. This work presents the relationship between yield and target nucleus mass number, and the yield-energy correlation of fission fragments of interest, enabling an energy-dependent two-dimensional distribution of fission yields. The results reasonably reflect the evolution of fission modes with increasing energy.

In fission nuclei of lighter-mass regions, PNF fragment charge yields tend to be symmetric. In heavier-mass regions, charge yields begin to exhibit asymmetric components that gradually dominate, most notably in Th and Pa isotopes. The proton number of PNF fragments directly carries information about scission. Experimentally, production of PNF fragments with even proton numbers is typically enhanced, representing one of the most prominent features of fission fragment yields. To describe the odd-even effect, we added an input  $\delta = \pm 0.1$  in the BNN-CY learning dataset to represent even and odd fragments. This decreased uncertainty and showed a regular transition from symmetric fission in lighter Th isotopes to asymmetric fission in heavier Th isotopes.

The fission mechanism evolves with excitation energy. The mass distribution of PNF fragments predicted by the BNN-MY model indicates that asymmetric fission prevails at low excitation energies, whereas symmetric fission becomes dominant at higher energies. For  $^{232}\text{Th}$ , asymmetric fission dominates at incident  $\gamma$  energies below 40 MeV, while symmetric fission contribution gradually increases above 40 MeV. Furthermore, no leftward shift in fragment mass distribution was observed within the  $\gamma$  energy range of several tens of MeV.

The BNN serves as a powerful and reliable tool for predicting photofission fragment yields. It has successfully revealed the evolution of fission mechanisms and associated odd-even effects, providing critical theoretical support for nuclear databases and reactor design. Currently, numerous facilities have begun PNF reaction research, such as the High Intensity  $\gamma$ -ray Source (HI $\gamma$ S) [60-62], GSI [44, 63, 64], and the Extreme Light Infrastructure Nuclear Physics (ELI-NP) [65-67]. Building upon this theoretical work, future photofission experi-

ments could be conducted using laser Compton scattering (LCS)  $\gamma$  rays at the Shanghai Laser Electron Gamma Source (SLEGS) at the Shanghai Synchrotron Radiation Facility (SSRF), which provides monoenergetic  $\gamma$  beams.

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- [1] A. Zilges, D. L. Balabanski, J. Isaak et al., Photonuclear reactions-From basic research to applications, *Prog. Part. Nucl. Phys.* 122, 103903 (2022).
- [2] H. L. Wei, M. D. Zhou, P. Jiao et al., SPAGINS: semiempirical parameterization for fragments in gamma-induced nuclear spallation, *Nucl. Sci. Tech.* 34, 190 (2023).
- [3] M. Bender, Rémi Bernard, G. Bertsch et al., Future of nuclear fission theory, *J. Phys. G: Nucl. Part. Phys.* 47, 113002 (2020).
- [4] D. L. Balabanski, P. Constantin, 80 years of experimental photo-fission research, *Eur. Phys. J. A* 60, 39 (2024).
- [5] M. D. Zhou, Z. R. Hao, Q. K. Sun et al., Measurement of  $^{59}\text{Co}(\gamma, n)^{58}\text{Co}$  using a new flat-efficiency neutron detector at the Shanghai Laser Electron Gamma Source, *Phys. Rev. C* 111, 054612 (2025).
- [6] H. X. Zhang, T. R. Yeh, and H. Lancman, Intermediate Structure in the Photofission Cross Section of  $^{232}\text{Th}$ , *Phys. Rev. Lett.* 53, 1 (1984).
- [7] H. X. Zhang, T. R. Yeh, and H. Lancman, Photofission cross section of  $^{232}\text{Th}$ , *Phys. Rev. C* 34, 4 (1986).
- [8] J. A. Sheikh, W. Nazarewicz, and J. C. Pei, Systematic study of fission barriers of excited superheavy nuclei, *Phys. Rev. C* 80, 011302(R) (2009).
- [9] J. A. Silano and H. J. Karwowski, Near-barrier photofission in  $^{232}\text{Th}$  and  $^{238}\text{U}$ , *Phys. Rev. C* 98, 054609 (2018).
- [10] M. L. Yoneama, E. Jacobs, J. D. T. Arruda-Neto et al., Study of the  $^{232}\text{Th}$  fission barrier by electron-induced fission, *Nucl. Phys. A* 604, 3 (1996).
- [11] G. L. Wang, H. Y. Lan, X. M. Shi et al., A general framework for describing photofission observables of actinides at an average excitation energy below 30 MeV, *Chin. Phys. C*, 46 084102 (2022).
- [12] X. M. Shi, G. L. Wang, K. J. Luo et al., Geant4 development for actinides photofission simulation, *Nucl. Instrum. Meth. A* 1062, 169222 (2024).
- [13] H. F. Zhang, L. H. Wang, J. P. Yin et al., Performance of the Levenberg-Marquardt neural network approach in nuclear mass prediction, *J. Phys. G: Nucl. Part. Phys.* 44, 045110 (2017).
- [14] A. Boehnlein, M. Diefenthaler, N. Sato et al., Colloquium: Machine learning in nuclear, *Phys. Rev. Mod. Phys.* 94, 031003 (2022).
- [15] P. Vicente-Valdez, L. Bernstein, M. Fratoni, Nuclear data evaluation augmented by machine learning, *Ann. Nucl. Energy* 163, 108596 (2021).
- [16] Z. M. Niu and H. Z. Liang, Nuclear mass predictions based on Bayesian neural network approach with pairing and shell effects, *Phys. Lett. B* 778, 48

(2018).

- [17] Z. M. Niu, J. Y. Fang, Y. F. Niu, Comparative study of radial basis function and Bayesian neural network approaches in nuclear mass predictions, *Phys. Lett. B* 100, 054311 (2019).
- [18] Z. M. Niu, H. Z. Liang, B. H. Sun et al., Predictions of nuclear  $\beta$ -decay half-lives with machine learning and their impact on  $\gamma$ -process nucleosynthesis, *Phys. Rev. C* 99, 064307 (2019).
- [19] Z. M. Niu, H. Z. Liang, Nuclear mass predictions with machine learning reaching the accuracy required by  $\gamma$  process studies, *Phys. Rev. C* 106, L021303 (2022).
- [20] Y. G. Ma, L. G. Pang, R. Wang et al., Phase transition study meets machine learning, *Chin. Phys. Lett.* 40, 122101 (2023).
- [21] W. B. He, Q. F. Li, Y. G. Ma et al., Machine learning in nuclear physics at low and intermediate energies, *Sci. China-Phys. Mech. Astro.* 66, 282001 (2023).
- [22] Q. K. Sun, Y. Zhang, Z. R. Hao et al., Enhancing reliability in photonuclear cross-section fitting with Bayesian neural networks, *Nucl. Sci. Tech.* 36, 52 (2025).
- [23] C. W. Ma, D. Peng, H. L. Wei et al., Isotopic cross-sections in proton induced spallation reactions based on the Bayesian neural network method, *Chin. Phys. C* 44, 014104 (2020).
- [24] C. W. Ma, D. Peng, H. L. Wei et al., A Bayesian-neural-network prediction for fragment production in proton induced spallation reaction, *Chin. Phys. C* 44, 124107 (2020).
- [25] D. Peng, H. L. Wei, X. X. Chen et al., Bayesian evaluation of residual production cross sections in proton-induced nuclear spallation reactions, *J. Phys. G: Nucl. Part. Phys.* 49 085102 (2022).
- [26] C. W. Ma, H. L. Wei, X. Q. Liu et al., Nuclear fragments in projectile fragmentation reactions, *Prog. Part. Nucl. Phys.* 121, 103911 (2021).
- [27] X. B. Wei, H. L. Wei, C. W. Ma et al., Predictions from several models for the cross sections of light neutron-rich isotopes by Qg systematics in  $^{40}\text{Ar}$  projectile-fragmentation reactions, *Phys. Rev. C* 111, 034607 (2025).
- [28] C. W. Ma, X. B. Wei, X. X. Chen et al., Precise machine learning models for fragment production in projectile fragmentation reactions using Bayesian neural networks, *Chin. Phys. C* 46, 7 (2022).
- [29] X. B. Wei, H. L. Wei, Y. T. Wang et al., Multiple-models predictions for drip line nuclides in projectile fragmentation of  $^{40,48}\text{Ca}$ ,  $^{58,64}\text{Ni}$ , and  $^{78,86}\text{Kr}$  at 140 MeV/u, *Nucl. Sci. Tech.* 33, 155 (2022).
- [30] C. W. Ma, X. X. Chen, X. B. Wei et al., Systematic behavior of fragments in Bayesian neural network models for projectile fragmentation reactions, *Phys. Rev. C* 108, 044606 (2023).
- [31] J. C. Pei, Recent progress in microscopic nuclear fission dynamics, *Chinese Science Bulletin (in Chinese)*, 68, 9 (2023).
- [32] C. Y. Qiao, J. C. Pei, Z. A. Wang et al., Bayesian evaluation of charge yields of fission fragments of  $^{239}\text{U}$ , *Phys. Rev. C* 103, 034621 (2021).
- [33] C. Y. Qiao, J. C. Pei, Z. A. Wang et al., Study of energy dependence of

- neutron induced fission yield  $^{235}\text{U}$  with Bayesian machine learning (in Chinese), Atomic Energy Science and Technology 56, 5 (2022).
- [34] Y. Qiang, J. C. Pei, and K. Godbey, Quantum entanglement in nuclear fission, Phys. Lett. B 861, 139248 (2025).
- [35] Z. A. Wang, J. C. Pei, Y. Liu et al., Bayesian evaluation of incomplete fission yields, Phys. Rev. Lett. 123, 122501 (2019).
- [36] A. Boehnlein, M. Diefenthaler, N. Sato et al., Colloquium: Machine learning in nuclear physics, Rev. Mod. Phys. 94, (2022).
- [37] U. Brosa, Multimodal fission and neutron evaporation, Phys. Rev. C 38, 1944 (1988).
- [38] U. Brosa, S. Grossmann, A. Müller et al., Nuclear scission, Phys. Rep. 4, 167-262 (1990).
- [39] N. A. Demekhina and G. S. Karapetyan, Symmetric and asymmetric modes of  $^{232}\text{Th}$  photofission at intermediate energies, Physics of Atomic Nuclei, 73, 1 (2010).
- [40] P. Moller, Odd-multipole shape distortions and the fission barriers of elements in the region  $84 \leq z \leq 120$ , Nucl. Phys. A 192, 3 (1972).
- [41] K. H. Schmidt, S. Steinhäuser, C. Böckstiegel et al., Relativistic radioactive beams: A new access to nuclear-fission studies, Nucl. Phys. A 665, 3 (2000).
- [42] K. H. Schmidt, B. Jurado, C. Amouroux et al., General Description of Fission Observables: GEF Model Code, Nucl. Data Sheets, 131, 107 (2016).
- [43] A. Chatillon, J. Taïeb, H. Alvarez-Pol et al., Experimental study of nuclear fission along the thorium isotopic chain: From asymmetric to symmetric fission, Phys. Rev. C 99, 054628 (2019).
- [44] J. F. Martin, J. Taieb, A. Chatillon et al., Studies on fission with ALADIN, Eur. Phys. J. A 51, 174 (2015).
- [45] J. L. Rodríguez-Sánchez, J. Benlliure, J. Taieb et al., Complete characterization of the fission fragments produced in reactions induced by  $^{208}\text{Pb}$  projectiles on proton at 500 MeV, Phys. Rev. C 91, 064616 (2015).
- [46] E. Pellereau, J. Taieb, A. Chatillon et al., Accurate isotopic fission yields of electromagnetically induced fission of  $^{238}\text{U}$  measured in inverse kinematics at relativistic energies, Phys. Rev. C 95, 054603 (2017).
- [47] A. Chatillon, J. Taïeb, H. Alvarez-Pol et al., Evidence for a new compact symmetric fission mode in light Thorium isotopes, Phys. Rev. Lett. 124, 202502 (2020).
- [48] J. C. Hogan, A. E. Richardson, J. L. Meason et al., Photofission of  $^{232}\text{Th}$  with 9, 15, and 38 MeV peak bremsstrahlung, Phys. Rev. C 16, 6 (1977).
- [49] A. Chattopadhyay, K. A. Dost, I. Krajbich, Mass-yield distributions in photofission of  $^{232}\text{Th}$  and  $^{238}\text{U}$ , J. Inorganic Nucl. Chem. 35, 8 (1977).
- [50] H. Naik, V. T. Nimje, D. Raj et al., Mass distribution in the bremsstrahlung-induced fission of  $^{232}\text{Th}$ ,  $^{238}\text{U}$  and  $^{240}\text{Pu}$ , Nucl. Phys. A 853, 1 (2011).
- [51] H. Naik, A. Goswami, G. N. Kim et al., Mass-yield distributions of fission products from photofission of  $^{232}\text{Th}$  induced by 45- and 80-MeV bremsstrahlung, Phys. Rev. C 86, 054607 (2012).
- [52] H. Naik, T. N. Nathaniel, A. Goswami et al., Mass distribution in the 50-, 60-, and 70-MeV bremsstrahlung-induced fission of  $^{232}\text{Th}$ , Phys. Rev. C 85,

024623 (2012).

[53] H. Naik, B. S. Shivashankar, H. G. Raj Prakash et al., Measurements of fission yield in 8 MeV bremsstrahlung induced fission of  $^{232}\text{Th}$  and  $^{238}\text{U}$ , *J. Radioanal Nucl. Chem.* 299, 127-137 (2014).

[54] H. Naik, G. N. Kim, R. Schwengner et al., Fission product yield distribution in the 12, 14, and 16 MeV bremsstrahlung-induced fission of  $^{232}\text{Th}$ , *Eur. Phys. J. A* 51, 150 (2015).

[55] H. Naik, G. N. Kim and K. Kim, Mass-yield distributions of fission products in bremsstrahlung-induced fission of  $^{232}\text{Th}$ , *Phys. Rev. C* 97, 014614 (2018).

[56] W. Günther, K. Huber, U. Kneissl et al., Symmetric and asymmetric yields in the photofission of  $^{232}\text{Th}$ ,  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$ , *Zeitschrift für Physik A Atoms and Nuclei*, 295, 4 (1980).

[57] J. Randrup and P. Moller, Energy dependence of fission-fragment mass distributions from strongly damped shape evolution, *Phys. Rev. C* 88, 064606 (2013).

[58] K. X. Jing, Z. Li, C. G. Liu et al., Fission yields in 7.64 MeV gamma induced fission of  $^{232}\text{Th}$ , *Chin. J. Nucl. Phys.* 10, 244 (1988).

[59] O. O. Parlag, V. T. Maslyuk, E. V. Oleynikov et al., Asymmetric modes of photofission fragments mass distribution of Th-232, *Naukovyi Visnyk Uzhgorods' kogo Univ., Ser.Fiz.*, 49, 54 (2021).

[60] Krishichayan, M. Bhike, A. P. Tonchev et al., Fission product yield measurements using monoenergetic photon beams, *EPJ Web of Conferences*, 146, 04018 (2017).

[61] Krishichayan, S. W. Finch, C. R. Howell et al., Monoenergetic photon-induced fission cross-section ratio measurements for  $^{235}\text{U}$ ,  $^{238}\text{U}$ , and  $^{239}\text{Pu}$  from 9.0 to 17.0 MeV, *Phys. Rev. C* 98, 014608 (2018).

[62] Krishichayan, M. Bhike, C. R. Howell et al., Fission product yield measurements using monoenergetic photon beams, *Phys. Rev. C* 100, 014608 (2019).

[63] J. F. Martin, J. Taïeb, G. Boutoux et al., Fission-fragment yields and prompt-neutron multiplicity for Coulomb-induced fission of  $^{234,235}\text{U}$  and  $^{237,238}\text{Np}$ , *Phys. Rev. C* 104, 044602 (2021).

[64] J. F. Martin, J. Taïeb, G. Boutoux et al., Fission-fragment yields and prompt-neutron multiplicity for Coulomb-induced fission of  $^{234,235}\text{U}$  and  $^{237,238}\text{Np}$ , *Phys. Rev. C* 104, 044602 (2021).

[65] D. Choudhury, D. L. Balabanski, A. Krasznahorkay et al., High-resolution photofission studies with the gamma beam system at ELI-NP, *AIP Conf. Proc.* 1852, 070003 (2017).

[66] P. Constantin, D. L. Balabanski, P. V. Cuong et al., Simulation of photofission experiments at the ELI-NP facility, *Nucl. Instrum. Meth. A* 372, 78-85 (2016).

[67] S. Gales, D. L. Balabanski, F. Negoita et al., New frontiers in nuclear physics with high power lasers and brilliant monochromatic gamma beams, *Phys. Scr.* 91, 093004 (2016).

[68] H. W. Wang, G. T. Fan, H. H. Xu et al., Construction and trial operation of Shanghai laser electron gamma source (in Chinese), *Nuc. Phys. Rev.* 41, 1

(2024).

[69] R. Z. Tai, Z. T. Zhao, Overview of SSRF phase-II beamlines, Nucl. Sci. Tech. 35, 137 (2024).

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