A Preliminary Study on Stellar Energy for High School Students

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Abstract

Stellar energy generation constitutes a crucial aspect of stellar astrophysics. The description of stellar energy sources typically necessitates substantial advanced mathematics. In this work, however, the authors ingeniously reformulate the portions requiring advanced mathematical treatment into descriptions employing elementary high school physics and mathematics. In our discussion of stellar energy, a key function requiring description is the power density (, T, X, Y, Z). To prove that the energy source underlying must necessarily be nuclear burning, we shall examine alternative possibilities.

Full Text

A Preliminary Study on Stellar Energy for High School Students

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Stellar energy represents a crucial component of stellar physics. Traditional descriptions of stellar energy typically require extensive advanced mathematics; however, this paper skillfully adapts these concepts for presentation using simple high school-level physics and mathematics. In our investigation of stellar energy, an important function we must describe is the power density $\varepsilon(\rho, T, X, Y, Z)$. To demonstrate that the energy source behind ε must be nuclear burning, we will examine alternative possibilities.

Keywords: stellar energy; nuclear energy; function; weak interaction

Introduction

Under certain conditions, convection rather than radiation becomes the dominant energy transport mechanism in some regions of a star. Convection occurs when a volume element of matter is displaced from its equilibrium position and continues moving in the direction of displacement rather than returning to its original location. For instance, if an upward displacement brings the element to a region of lower density, and its density becomes lower than that of its surroundings, the element will continue to rise due to buoyancy. Once convection initiates, it can transport heat with remarkable efficiency, establishing itself as the primary transport mechanism.

Consider the hypothesis that solar energy originates from gravitational contraction—that is, the energy the Sun has radiated thus far represents the potential energy released as it contracted from infinity to its present radius. According to the virial theorem, the thermal energy generated by such contraction equals negative one-half of the gravitational potential energy.

The internal timescale is 10^{10} years. This remarkably long timescale arises primarily because the reactions proceed via weak interactions (as evidenced by neutrino emission). The positron, deuteron, and neutrino each carry 0.425 MeV of energy. Following the reaction, the positron rapidly annihilates with an electron, producing two gamma-ray photons of 0.511 MeV each. Neutrinos, interacting only weakly with matter, escape from the Sun, carrying away their average energy of 0.26 MeV. The remaining kinetic energy and photons quickly thermalize through frequent matter-matter and matter-photon collisions.

The Proton-Proton Chain Reactions

Within seconds, deuterium merges with another proton to form helium-3: $p + d \rightarrow {}^3\mathrm{He}$. The total energy released is 5.49 MeV. Finally, on a timescale of 300,000 years, we have ${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p$, releasing kinetic energy of 12.86 MeV. Each time this three-step chain occurs twice, four protons are converted into one helium nucleus, two neutrinos, photons, and kinetic energy. Therefore, each helium nucleus releases 26.73 MeV. Subtracting 2×0.511 MeV from the annihilation of two pre-existing electrons, this represents the rest mass difference between four free protons and a He nucleus: $[m(4p) - m({}^4\mathrm{He})]c^2 = 25.71$ MeV $=$ 0.7% $m(4p)c^2$. Thus, the rest-mass energy conversion efficiency of the p-p chain is 0.7%.

The time to utilize 10% of the available energy is 3.3×10^{17} s. In other words, in terms of energy budget, hydrogen fusion can easily produce a luminosity equivalent to the entire lifetime of the solar system. Next, we must examine whether conditions in the solar interior are suitable for these reactions to actually occur.

Consider two nuclei with atomic numbers (i.e., protons per nucleus) Z_A and Z_B. The strong interaction produces short-range attractive forces between nuclei at distances less than $r_0 \approx 1.4 \times 10^{-13}$ cm. As the separation increases, the strong interaction vanishes, and Coulomb repulsion between nuclei becomes dominant. The Coulomb barrier is $E_{\mathrm{coul}} = \frac{Z_A Z_B e^2}{r}$. At r_0, $E_{\mathrm{coul}}(r_0) \approx Z_A Z_B$ MeV, representing the combination of nuclear potential (strong force) and electrostatic potential (Coulomb force). In the reference frame of one nucleus, another nucleus with kinetic energy E can classically approach only to a certain distance. It will be repelled at that point. At typical stellar interior temperatures of 10^7 K, the kinetic energy of nuclei is $1.5kT \approx 1$ keV. Thus, the characteristic kinetic energy is on the order of 10^{-3} of the energy required to overcome the Coulomb barrier. Typical nuclei can only approach to a distance $r_1 \approx 10^{-10}$ cm, which is 1000 times larger than the distance at which the strong nuclear binding force operates. Perhaps those nuclei located in the Maxwell-Boltzmann distribution could overcome the Coulomb barrier. The fraction of nuclei possessing such energy is $e^{-E/kT} \approx e^{-1000} \approx 10^{-434}$. The number of protons in the Sun is $N \approx 10^{57}$. Consequently, not a single nucleus in the Sun (or indeed, in all observable stars in the universe) possesses classical kinetic energy sufficient to overcome the Coulomb barrier and undergo nuclear fusion with another nucleus. Fortunately, quantum tunneling through the barrier ultimately enables nuclear reactions to occur.

Energy Generation and the p-p Chain

For a rough estimate, we adopt the solar core mass density as the central density $\rho = 150$ g cm^{-3}. In the solar central region, some hydrogen has already been converted to helium through nuclear reactions. We assume a typical hydrogen abundance of $X = 0.5$, which we can use for both X_A and X_B. The first step of the p-p chain, the $p + p \rightarrow d + e^+ + \nu_e$ reaction, is the slowest of the three steps in the chain and therefore constitutes the bottleneck that determines the overall p-p process rate. The constant S_0 for this reaction is theoretically calculated to be $\approx 4 \times 10^{-46}$ cm^2 keV, characteristic of weak interactions.

For Q, we take the total thermal energy released per completion of the p-p chain, because once the first step occurs, the subsequent two reactions—with timescales of 1 second and 300,000 years, respectively—are essentially instantaneous on the 10^{10} year timescale. We see that each chain completion produces 26.73 MeV of energy and two neutrinos. Subtracting the 0.52 MeV carried away on average by the two neutrinos, the thermal energy released per p-p chain completion is $Q = 26.2$ MeV.

As mentioned previously, $E_G = 500$ keV for two protons, with a typical core temperature of $kT = 1$ keV. The atomic mass numbers are of course $A_A = A_B = 1$, and the reduced mass is $\mu = m_p/2$. Finally, since we are considering reactions between identical particles (i.e., proton-proton reactions), we must divide the collision rate by 2 to avoid double-counting. From these values, the resulting power is $\varepsilon = 10$ erg s^{-1}g^{-1}.

Isothermal behavior also governs the long-term evolution of stars. Eventually, when the primary nuclear fuel is exhausted, the energy density ε will decrease. The star then contracts, E_{th} increases, and T rises until a new nuclear reaction (involving nuclei with higher atomic numbers) can become effective. A key prediction of the picture we have outlined is that the Sun' s energy originates from the p-p chain, and therefore a constant neutrino flux should escape from the Sun. Unlike photons, the weak interaction of neutrinos with matter ensures they can escape from the solar core almost unimpeded.

This enormous particle flux passes through our bodies and the entire Earth almost unimpeded, making it extremely difficult to detect. Experiments measuring solar neutrino flux, beginning in the 1960s, have consistently indicated a deficit in the electron neutrino flux from the Sun. The total neutrino flux from the Sun likely matches predictions from solar models very closely; some of the original electron neutrinos transform into other neutrino types during their journey from the Sun to Earth. For completeness, I note that besides the specific p-p chain (the dominant nuclear reaction sequence in the Sun), other nuclear reactions occur and produce detectable neutrinos on Earth. In stars more massive than the Sun, hydrogen conversion to helium also proceeds through a different reaction sequence called the CNO cycle.

In the CNO cycle, trace amounts of carbon, nitrogen, and oxygen in the gas act as catalysts in the burning process that converts hydrogen to helium, without synthesizing any additional C, N, or O. The strong temperature dependence of the reactions further compensates for this. Although core temperature varies little with stellar mass, slightly higher core temperatures in more massive stars are sufficient to make the CNO cycle the dominant hydrogen-burning mechanism in main-sequence stars with masses of 1.2 $M_\odot$ and above.

We have derived many observable properties of main-sequence stars. Solving these equations means finding the functions $P(r)$, $T(r)$, and $\rho(r)$ that are generally believed to describe stellar structure. These equations have no analytical solutions unless one makes unrealistic assumptions. Nevertheless, numerical solutions can be obtained directly, and given the complexity of the functions P, κ, and ε when all relevant processes are included, numerical solutions are also the most reasonable approach. In numerical solutions, differentials in the equations are replaced by differences. Finally, an example of a possible computational scheme is to track the radial structure of the star layer by layer, either from the center outward or from the surface inward.

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