Instant determination of polar motion with tri-static common view lunar laser ranging (Post-print)

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Abstract

This paper proposes a method for determining instantaneous Earth polar motion (PM) values using lunar laser ranging (LLR) measurement datasets synchronously acquired by three European LLR stations operating in three-station common-view (TCV) mode. We developed an LLR TCV measurement model and derived linear equations for solving PM. Although no actual TCV events exist in the data, we conducted a two-stage study using actual LLR normal points (NPs) obtained by European stations during 2012–2022 to validate the method. The first stage simulated TCV events and PM solutions, evaluating the method's robustness by introducing Universal Time (UT1) errors and ranging errors at each station. The second stage employed '1+2' and '2+1' strategies with different data compositions, combining actual LLR NPs with simulated data to generate realistic TCV events and solution results. Results show that a 0.1 ms UT1 error results in PM errors of less than 18 mas, while a 50 mm uniform ranging error results in PM errors of less than 180 mas. In the enhanced stage, the maximum solution errors for the '1+2' and '2+1' strategies are 752 mas and 899 mas respectively, with 88.5% and 91.2% of solutions outperforming predicted values. This method relies on precise geodetic data and therefore does not intend to replace traditional methods. However, this study confirms the feasibility and robustness of instantaneous PM determination, although its accuracy still needs further improvement.

Full Text

Preamble

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Instant Determination of Polar Motion with Tri-Static Common View Lunar Laser Ranging

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Abstract

A method is presented for determining instant values of Earth' s polar motion (PM) using a set of lunar laser ranging (LLR) measurements acquired simultaneously by tri-static common view (TCV) at three LLR stations in Europe. We developed a model of the LLR TCV measurements, then formulated the linear equation for solving PM.

Although there was no actual TCV event in the data, we conducted a two-phase study to test our method using actual LLR normal points (NPs) acquired by the European stations during 2012-2022. In the first phase, we simulated TCV events and PM solutions. The robustness of our method was assessed by introducing Universal Time (UT1) errors and per-station range errors in this phase. In the second phase, we augmented the actual LLR NPs with simulated data to generate realistic TCV events and solutions, using the '1+2' and '2+1' strategies, which differed in terms of data composition. Results indicated that a UT1 error of 0.1 ms caused PM errors of <18 mas, while a uniform range error of 50 mm resulted in PM errors of <180 mas. In the augmentation phase, the maximum solution errors were 752 and 899 mas, and 88.5% and 91.2% of the solutions were better than the predictions for the '1+2' and '2+1' strategies, respectively. The presented approach relies on precise geodetic data, and therefore, it is not intended to replace the traditional method. However, this study demonstrated that instant determination of PM is feasible and robust, although the accuracy requires further enhancement.

Keywords: Equatorial coordinate system; LLR; PM; Satellite laser ranging; Space geodesy

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1. Introduction

Determining Earth' s orientation in inertial space, also known as the Earth Rotation Parameters (ERPs), is important in astrometric studies that relate terrestrial and celestial coordinates. The ERP data consists of three numbers: the PM components, and the UT1. Currently, ERPs are usually provided using the Very Long Baseline Interferometry (VLBI) technique. The international VLBI service can determine PM to an accuracy of 50 as and UT1 to an accuracy of 3 s, and delivery of the ERP product with temporal resolution of 1 day normally requires 8–10 days.

Determination of ERPs can also be conducted using the LLR technique, which involves measurement of the round-trip time between a ground-based laser ranging station (hereafter, station) and a lunar retro-reflector array (hereafter, reflector). The use of LLR data to determine ERPs was first explored soon after the data initially became available. Over the past decade, a number of LLR stations have produced precise data, with range precision enhanced to the centimeter level. Recent LLR analysis studies typically selected observation nights with 10–15 NPs, with the minimum requirement of 5 NPs per night. The most up-to-date studies have achieved an accuracy of several milliarcseconds for PM coordinates, with temporal resolution of 1 day.

The concept of organizing LLR observations in a common view manner, or LLRCV, was proposed by Leick. It was claimed that the LLRCV method could provide ERP components with measurement accuracy on the Earth' s surface of approximately 30 cm for LLR in the early 1980s, equivalent to 10 mas of PM or 0.6 ms of UT1. Another form of LLRCV, proposed by Müller et al., involves the placement of an optical transponder on the lunar surface to illuminate multiple laser ranging stations simultaneously. However, follow-up studies on the LLRCV concept have been limited.

Major geodynamic events, such as earthquakes, can cause rapid changes in ERPs. While conventional LLR analysis extracts a daily solution using single-night observations, the common view method provides an instant solution at higher temporal resolution, which is defined by the accumulation time of LLR NPs, i.e., typically 15 min or less. This represents a substantial improvement in monitoring capability for transient geodynamic phenomena by LLR.

In this paper, we explore the instant determination of Earth' s PM using LLR data in the form of TCV measurements. Linear equations can be formulated upon three simultaneous range residuals from three cooperating stations to solve for PM in near real time.

Three European stations were selected from the International Laser Ranging Service network to form a station group that we named GMW. The stations are located in Matera in Italy (MATM/7941), Grasse in France (GRSM/7845), and Wettzell in Germany (WETL/8834). The baselines between the stations are 753 km (GW), 877 km (GM), and 990 km (MW), see Fig. 1 [Figure 1: see original paper].

Our approach is grounded in contemporary geodetic infrastructures, which include the International Terrestrial Reference Frame (ITRF), International Earth Rotation Service (IERS), high-precision planetary ephemerides, fundamental astronomical software, and the IERS Conventions 2010, which provide key algorithms. The reference ERP data were extracted from the IERS C04 20 series, which was accessed via the IERS official website.

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2. Method and Data

This paper explores instant determination of Earth' s PM using LLR data in the form of TCV measurements. We first established the method for determining Earth' s PM from the LLR model, the range derivatives to ERPs, the PM prediction model, and the least squares solution. Then, we conducted a two-phase study to evaluate the method. The first or simulation phase involved generating LLR TCV simulation data to test the robustness of the PM determination method by introducing UT1 and range errors. In the second or augmentation phase, we combined simulated data with actual LLR NPs to create realistic TCV events and solve for PM. These results were compared with reference values to generate solution error data. Statistical analysis was conducted to assess the accuracy of the solutions.

2.1 LLR Model and Data

The lunar ranging measurement can be formulated as:

$$\tau = \frac{s_{12}}{c} + \frac{s_{23}}{c} + \tau_{GR} + \tau_{tropo} + \Delta\tau_{emp} \quad (1)$$

where τ represents the time of flight measured at the ranging station. The distances s_{12} and s_{23} represent the light path from the station to the reflector and from the reflector to the station, respectively, accounted for in barycentric coordinates. The relativistic delay τ_{GR} represents the gravitational delay effect on the flying pulse. The tropospheric delay τ_{tropo} represents atmospheric refraction delay during the propagation of light. Tags 12 and 23 represent the outbound flight and the inbound flight, respectively. An empirical term $\Delta\tau_{emp}$ was introduced and fitted to compensate for the remaining periodic effect.

Time-of-flight measurements are performed by ground stations in the clock rate of International Atomic Time (TAI), while planetary motions are calculated using barycentric dynamical time (TDB). The difference in the clock rate between TAI and TDB is considered negligible in this paper.

Light paths s_{12} and s_{23} are in barycentric inertial vacuum space, where s_{12} represents the path from the station at transmit time t_1 to the reflector at hit time t_2, and s_{23} represents the path from the reflector at t_2 to the station at receive time t_3. Thus, we have the following decompositions:

$$s_{12} = \|s_{12}\| = \|r_{ES}(t_1) - r_{BE}(t_1) + r_{BM}(t_2) + r_{MA}(t_2)\| \quad (2)$$

$$s_{23} = \|s_{23}\| = \|r_{MA}(t_2) - r_{BM}(t_2) + r_{BE}(t_3) + r_{ES}(t_3)\| \quad (3)$$

where r_{ES}, r_{BE}, r_{BM}, and r_{MA} are vectors in barycentric inertial space with B/E/M/S/A meaning barycenter, Earth center, moon center, Earth observing station, and lunar reflector array, respectively. The barycentric vectors are computed with position vectors and lunar libration angles provided by DE430 planetary ephemeris, using TDB. The geocentric vector is computed with station coordinates from ITRF2020 and IERS Earth Orientation Parameter data.

Minor adjustments were made to the terrestrial and lunar coordinates to minimize the range residuals.

We selected 9,747 LLR NP data records, which were retrieved from the EUROLAS Data Center. The data were acquired by European LLR stations GRSM, MATM, and WETL, with the observation time spanning from May 2, 2012 to December 7, 2022.

The data were recorded in the Consolidated Laser Ranging Data (CRD) format. For each NP data record, the CRD format includes the following details: the observing station, the observed reflector, the mean epoch of the data, the NP duration (or NP bin size), the number of photoelectrons accumulated within the NP bin, and the root mean square (RMS) value. The NP RMS value is equivalent to the 1-σ standard deviation of the range measurements in the NP.

We discarded 0.56% (55 of 9,747) outlier NPs for which the range residuals were >0.4 m. In the remaining 99.44% (9,692 of 9,747) of the NP data, the overall range measurement residual against our LLR model was 35.9 mm in RMS. The RMS values were 36, 48, and 22 mm for stations GRSM, MATM, and WETL, respectively.

The number of NPs used was 8,191, after applying the lunar declination limit, as mentioned in Section 3.1.

2.2 ERPs

The ERPs represent the rotation axis of Earth in the terrestrial reference frame and the rotation angle in the celestial reference frame. The three ERP components, x_p, y_p, and ERA (Earth' s rotation angle), are related to frame rotations about the X, Y, and Z axes, respectively. ERA is quantified within the celestial reference frame and represents the angle between Earth' s prime meridian and the equinox. It is conventionally represented by UT1.

The formula for conversion from the Earth rotation angle (modulo 2π) is:

$$ERA = 15 \times (1.00273781191135448) \times UT1 \quad (4)$$

with units of seconds and arcseconds for UT1 and the Earth rotation angle, respectively.

The rotation axis is represented by celestial intermediate pole coordinates, or PM, denoted as (x_p, y_p). The celestial intermediate pole is the point of intersection between Earth' s axis of rotation and its ellipsoid, as measured within the ITRF, and PM denotes the angular deviation of the celestial intermediate pole from the north pole of the ITRF. The PM predictions used by modeling and determination are adopted from the IERS Conventions 2010 mean pole model. After epoch 2010.0, the model is formulated as follows:

$$x_p(t) = 23.513 + 7.6141(t - t_0) \quad (5)$$

$$y_p(t) = 358.891 - 0.6287(t - t_0) \quad (6)$$

where t and t_0 are Besselian epochs of current time and 2000.0, respectively. The results are presented in units of milliarcseconds.

In this paper, the reference Earth Orientation Parameter data, including the PM components and the UT1-UTC series, were derived from the IERS C04 20 series using linear interpolation. The data files were retrieved from the IERS website.

During 2012-2022, the PM prediction errors relative to the reference data were both within acceptable limits.

2.3 Determination of PM by LLR

To investigate the variation of lunar range measurements with respect to variations in ERP components, or the partial derivatives, we consider the following geometrical relation:

$$\Delta\tau = H(\alpha, \delta) \cdot \Delta r_{GCRS} \quad (6)$$

where $H(\alpha, \delta) = (-\cos\alpha\cos\delta; -\sin\alpha\cos\delta; -\sin\delta)$ is the unit vector along the line of sight from the reflector to the station, transposed; α and δ are the celestial right ascension and declination of the reflector, respectively, as seen from the station; and the variation vector Δr_{GCRS} is the variation of the station' s geocentric inertial vector, caused by ERP variation.

According to the IERS Conventions 2010, the formula for transformation from an international terrestrial reference system (ITRS) to a geocentric celestial reference system (GCRS) is as follows:

$$r_{GCRS} = Q(t)R_3(-s')[R_3(-ERA)R_2(x_p)R_1(y_p)]r_{ITRS} \quad (7)$$

where the bracketed term is the transformation matrix arising from the motion of the celestial pole in the celestial reference system, i.e., precession and nutation, while R_1, R_2, and R_3 are matrices for the coordinate frame rotation about axes X, Y, and Z, respectively.

We adapted the equation to its current form using commutativity between the terrestrial intermediate origin locator $R_3(-s')$, and $Q(t)$ and the Earth rotation $R_3(-ERA)$. The variations in astronomical angles α and δ due to ERP changes are negligible. If we apply partial derivative rules on Equation (7), and let the ERP components be predicted values (y_p^0, x_p^0, ERA^0), then we have the following partial derivatives:

$$\frac{\partial r_{GCRS}}{\partial y_p} = MHQR_3(-ERA^0)R_2(x_p^0)R_1'(y_p^0)r_{ITRS} \quad (8)$$

$$\frac{\partial r_{GCRS}}{\partial x_p} = MHQR_3(-ERA^0)R_2'(x_p^0)R_1(y_p^0)r_{ITRS} \quad (9)$$

$$\frac{\partial r_{GCRS}}{\partial ERA} = -MHQR_3'(-ERA^0)R_2(x_p^0)R_1(y_p^0)r_{ITRS} \quad (10)$$

where $MHQ = H(\alpha, \delta)Q(t)R_3(-s')$ and the matrices R_k' $(k = 1, 2, 3)$ denote the derivatives of each rotation matrix with respect to the rotation angle.

We consider that the variation in ERPs represented by the vector $p = (\Delta y_p, \Delta x_p, \Delta ERA)^T$ is the exclusive source of the variation in the laser ranging measurements, $\Delta\tau$, i.e., $\Delta\tau = \frac{\partial\tau}{\partial p} \cdot p$, where $\Delta\tau$ is the change in the laser ranging measurement, and the partial derivatives represent the sensitivity of the ranging measurement to changes in each ERP component. To determine the PM components, we assume UT1 is known; hence, $\Delta ERA = 0$. At the epoch of the common view measurement, with the group of stations A, B, and C, we can form a linear equation $Mp = q$, where the following holds:

$$M = \begin{bmatrix} \frac{\partial \tau_A}{\partial y_p} & \frac{\partial \tau_A}{\partial x_p} \\ \frac{\partial \tau_B}{\partial y_p} & \frac{\partial \tau_B}{\partial x_p} \\ \frac{\partial \tau_C}{\partial y_p} & \frac{\partial \tau_C}{\partial x_p} \end{bmatrix}, \quad p = \begin{bmatrix} \Delta y_p \\ \Delta x_p \end{bmatrix}, \quad q = \begin{bmatrix} \Delta \tau_A \\ \Delta \tau_B \\ \Delta \tau_C \end{bmatrix} \qquad (11)$$

M is called the design matrix, while q is the range residual vector, i.e., the difference between the measurements acquired and the measurements predicted by the theoretical model. It is an overdetermined problem, with three equations to solve for two unknowns. We apply the least squares technique to the problem, and the normal equation is formed as $M^T M p = M^T q$, giving the least squares estimate $p = (M^T M)^{-1} M^T q$.

If any perturbation Δq is added to the measurement residual, then the solution would be correspondingly perturbed: $\Delta p = (M^T M)^{-1} M^T \Delta q$.

The PM predictions $(\bar{y}_p, \bar{x}_p)$ were calculated with a linear model as described in subsection 2.2. After solving for $p = (\Delta y_p, \Delta x_p)^T$, we adjust the prediction to form estimates $(\hat{y}_p, \hat{x}_p) = (\bar{y}_p + \Delta y_p, \bar{x}_p + \Delta x_p)$. The estimated ERP components were then compared with the reference data to form the solution error.

2.4 TCV Simulation

Regarding the simulation phase, we simulated TCV events for the GMW station group, according to the existing NP data at the European stations. First, we identified the available NPs at the European stations in the LLR data, taking each NP epoch as the TCV epoch. Second, ranging measurement data were generated at the NP epochs from the GMW stations to the observed reflector. Finally, the same number of PM solutions were generated from the simulated TCV events by solving the underlying equation $Mp = q$.

To assess the robustness of the determination method, we introduced perturbations to the range residual vector q. The perturbed solution was compared with the original solution to determine the effects of the perturbation. Table 1 lists the settings of all the simulation cases.

We started with the control case denoted as S0. Regarding the perturbation on UT1, an error of \$±\$100 s was introduced to the UT1 prediction to observe its effect, referred to as SU1 and SU2 in Table 1. Regarding the perturbation on the ranging measurement, we have six cases ranging from SG1 to SW2, for which uniform range errors of \$±\$50 mm were introduced to each of the three stations.

These perturbations were applied independently, although the linear nature of the model allows them to accumulate.

2.5 TCV Augmentation

In the augmentation phase, we combined the existing NP data with the simulated data. The aim was to assess the accuracy of the solutions with common view events occurring in real-world observation epochs. In this phase, two augmentation strategies were employed in what were termed the '2+1' stage and the '1+2' stage; their settings are listed in Table 1.

In the '1+2' stage, we identified the available NPs at the European stations in the LLR data, which were regarded as monostatic events. Taking each NP epoch as the TCV epoch, we simulated the missing data with range residuals of $±$36 mm, $±$48 mm, and $±$22 mm for the stations GRSM, MATM, and WETL, respectively. The '1+2' stage yielded 12 cases, from AG1 to AW4, which are detailed in Table 1.

In the '2+1' stage, we first identified the bi-static common view events at the European stations in the LLR data. The search for bi-static common view events was based on the criterion that the temporal separation between two NPs at different stations should be <1 h. We then calculated the mid-point of the pair of NP epochs to be the TCV event epoch. Finally, we simulated the range residual at the missing station in the same way as in the '1+2' stage. The '2+1' stage yielded four cases, as listed in Table 1.

The outputs of both stages were evaluated using statistical methods to estimate how the TCV solution would perform under real-world conditions. If a TCV event occurs in reality, the solution should behave similarly to that of our results.

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3. Results

3.1 Preliminary Analysis

Prior to solution of the determination problem, two important attributes can be assessed: the sensitivity and the stiffness. The sensitivity is represented by the partial derivatives, and the stiffness is represented by the condition number.

The sensitivity of a station' s ranging measurement to variations in the ERPs can be quantified by the mean absolute values of the partial derivatives, which are measured in units of meters per arcsecond (m arcsec^{-1}). In our dataset, the mean absolute value of $\frac{\partial\tau}{\partial y_p}$ varies from 14 to 17 m arcsec^{-1} among the stations, larger than that of $\frac{\partial\tau}{\partial x_p}$, which varies from 8 to 10 m arcsec^{-1}. The WETL station exhibited the largest mean absolute values for both partial derivatives. The difference can be attributed to the geographic location of the stations, specifically their distance from the X, Y, and Z axes of the ITRF. The lower sensitivity of the range measurement to x_p compared with those to y_p is due to the stations being located closer to the X axis than to the Y axis.

Another key attribute is the stiffness of the least squares estimate problem

$M^T M p = M^T q$, which can be quantified by the condition number c_{Normal} of the normal matrix $M^T M$. Here, $c_{Normal} = \|M^T M\| \cdot \|(M^T M)^{-1}\|$, where the matrix norm is the 2-norm. The condition number c_{Normal} is inversely related to the lunar declination δ_{moon}. When δ_{moon} approaches zero, the condition number soars to infinity. As an example, the relation between c_{Normal} and δ_{moon} is shown in Fig. 2 [Figure 2: see original paper], where the condition number is plotted on a logarithmic scale.

After applying the criterion $|\delta_{moon}| > 5°$, 8,191 TCV events remained out of the original 9,692. The condition numbers of the remaining TCV events could be confined to an upper limit of 3.2×10^4. Among the remaining 8,191 events, the GRSM station accounted for 93% (7,646 events), the MATM station accounted for 4% (331 events), and the WETL station accounted for 3% (214 events).

The effects were quantified by using the mean absolute error (MAE), RMS error (RMSE), and absolute maximum (MaxAbs) metrics for the PM components x_p and y_p, and the pole position error $\sqrt{\Delta x_p^2 + \Delta y_p^2}$. With the introduction of the UT1 error of $±$100 s, the resulting solution errors were only several milliarcseconds, as indicated by both the RMSE and the MAE. The maximum angular separation between the perturbed pole and the reference pole reached 18 mas, with an average of 6 mas. For comparison, range perturbations of $±$50 mm resulted in solution errors that were 4-10 times greater. Detailed statistics are provided in Table 2 .

3.2 Solutions Overview

A total of 106,845 TCV solutions were obtained for simulation and augmentation cases, as listed in Table 1. In each solution, a set of PM components $(\hat{y}_p, \hat{x}_p)$ for a TCV event was evaluated.

In the simulation part, 8,191 simulated TCV events were solved one time in the unperturbed control case S0, and 8 more times in the perturbed cases from SU1 to SW2.

In the augmentation part, two stages were investigated. In the '1+2' stage, 8,191 TCV events were augmented in four combinations. Augmentation solutions were categorized by station and residual signs into 12 cases, from AG1 to AW4. In the '2+1' approach, there were 181 TCV events, each solved twice.

3.3 Solutions and Assessment of TCV Simulations

In the simulation phase, we simulated TCV events at 8,191 epochs selected from the actual LLR data. Then, we compared the solved PM to the reference values to generate the solution error data. In the control case S0, the solution errors were trivially zero. The positively perturbed cases (SU1, SG1, SM1, and SW1) are depicted in Fig. 3 [Figure 3: see original paper]. The negatively perturbed cases are symmetrical with the positive cases about the origin. It can be seen from the plots that each perturbation type displays a distinct directional pattern in the error distribution.

The focus in the simulation phase was to examine the effects of perturbations in UT1 and range on the solution. The results imply that the UT1 prediction error of $±$100 s could, on average, introduce an error of several milliarcseconds into the PM solutions. Furthermore, the range error of $±$50 mm could result in RMSEs of around 20 mas in the x_p solution and nearly 50 mas in the y_p solution. The range error at the WETL station contributed less to the pole position error because of its higher latitude and therefore the greater distance from the X and Y axes of the ITRF, which is related to the fact that the partial derivatives were larger at the WETL station. The perturbation results are useful for assessing solution errors under a certain amount of ranging error.

3.4 Solutions and Assessment of TCV Augmentations

The augmentation phase focuses on evaluating solution errors under real-world settings. Table 3 displays all the augmentation results by case ID, the settings of which are listed in Table 1.

In the '1+2' stage of the augmentation phase, we simulated two range residuals for each selected range residual data. This stage yielded 12 cases from AG1 to AW4. Among the cases, the maximum pole position error was 752 mas. When comparing the pole position data to the linear pole prediction, 88.5% of the events (29,000 out of 32,764) yielded more accurate results than the prediction. The x_p component errors were markedly larger than those of the y_p component owing to the geographical locations of the GRSM and MATM stations, i.e., the distance to the X-axis of the ITRF is less than that to the Y-axis. The WETL cases, labeled AW1–AW4, yielded the lowest errors, while the MATM cases ranked in the middle and the GRSM cases exhibited the highest errors. This is partly attributable to the low range residuals of the WETL station, and also to the limited size of the WETL dataset.

In the '2+1' stage of the augmentation phase, we identified 181 bi-static common view events from the actual NP data without applying the lunar declination filter. In the selected bi-static events, the range residuals varied from -0.097 m to +0.141 m and the RMS was 0.034 m. Of all the bi-static events, 69.6% (126 of 181) were between GRSM and MATM and 30.4% (55 of 181) were between GRSM and WETL. This stage yielded four cases from AGM1 to AGW2, as listed in Table 1. The statistical values of the x_p component are obviously larger than those of the y_p component owing to the same geographical reasons. Comparison of the linear PM predictions with the two perturbed cases combined reveals that 91.2% of the solutions provided values that were more accurate than the predictions. Among all cases, the maximum pole position error was 899 mas, which was attributable primarily to the high value of the maximum x_p error. The AGW1&2 cases yielded lower errors owing to the low range of the residual values of the WETL station.

In summarizing the two stages, it is evident that the MAE statistics of x_p, y_p, and the pole position, as well as the percentage of the solutions that outperformed the predictions, were comparable. This indicates that the two augmentation strategies did not introduce substantial bias in simulating reality.

Fig. 4 [Figure 4: see original paper] illustrates the solution errors for the two augmentation strategies, where Fig. 4A displays 99.35% (32,552 of 32,764) of the solutions in the '1+2' augmentation, and Fig. 4B displays 95.6% (346 of 362) of the solutions in the '2+1' augmentation. Note the difference in the scale of the axes.

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4. Discussion and Conclusions

This paper presents a method for determining Earth' s PM using tri-static range measurements in real time. We formulated the linear equations for this purpose and applied the least squares method to solve them. The solutions were initiated using linear PM predictions. Preliminary analysis showed that the partial derivatives were associated with the geographical locations of the stations, and that the condition numbers were strongly correlated with the lunar declination. The solutions were compared with the IERS Earth Orientation Parameter 20 C04 time series to generate solution error data. Our method was examined through a two-phase study, including a simulation phase and an augmentation phase.

During the simulation phase, we simulated TCV events to the actual range data epochs. Predefined errors were introduced in the UT1 and range to observe the consequent variations in the solution. The results showed that the UT1 prediction error of $±$100 s led to minor variations of several milliarcseconds in the solutions, and that the uniform range errors of $±$50 mm could lead to errors of 18–23 mas in the x_p solution and 24–45 mas in the y_p solution (RMSE). Perturbation patterns were revealed in the solution variations.

During the augmentation phase, two strategies were used to simulate realistic TCV scenarios. In the first strategy named '1+2' , we simulated two additional measurements for each NP entry. Realistic range errors were introduced to yield solution errors of 26 and 53 mas in x_p and y_p (RMSE), respectively. In the second strategy named '2+1' , we identified the actual bi-static common view events and augmented them into tri-static events using simulated data, to yield solution errors of 34 and 99 mas in x_p and y_p (RMSE), respectively. Both strategies revealed that the solution error of the x_p component was markedly larger than that of the y_p component. The largest pole position error was 899 mas. The realistic scenarios within the '1+2' and '2+1' augmentation strategies yielded solutions that were more accurate than the linear pole prediction for 88.5% and 91.2% of the cases, respectively. Cases involving WETL data had notably smaller error statistics owing to the low range of the residual values of the WETL station. The results presented in this paper are compared with those of other reported methods in Table 4 . Our method could yield errors that are 20–60 times larger than LLR long-term solutions, or 300–2,000 times larger than VLBI solutions. However, the temporal resolution of our method is the LLR NP bin size, i.e., approximately 15 min, which is much shorter than that of the other methods considered.

Overall, these results indicate that tri-static determination of PM coordinates is possible. Our method requires one set of data from three stations tracking the moon simultaneously, instead of a long period of data accumulation. This makes it a potentially viable tool for detecting transient changes in Earth' s rotation. It could also prove useful for validating ERPs to accuracy of several milliarcseconds.

However, our method cannot replace traditional long-term determination methods, partly because of its dependence on precise geodetic data and partly because it lacks the capability to estimate additional parameters.

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AI Disclosure Statement

Deepseek was employed for language and grammar checks within the article. The authors carefully reviewed, edited, and revised the Deepseek-generated texts to their own preferences, assuming ultimate responsibility for the content of the publication.

Author Contributions

Zhipeng Liang was responsible for the methodology, software development, validation processes, and the preparation of the original draft. Chengzhi Liu and

Xue Dong undertook the supervision of the project and contributed to its conceptualization. Zhipeng Liang and Yanning Zheng were involved in data curation, visualization creation, and investigative tasks. Chengzhi Liu, Xue Dong, and Yanning Zheng participated in the reviewing and editing of the manuscript. All authors have read and approved the final manuscript.

Declaration of Interests

Chengzhi Liu is an editorial board member for Astronomical Techniques and Instruments and he was not involved in the editorial review or the decision to publish this article. The authors declare no competing interests.

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Supplementary Material

For the sake of completeness, additional data tables and figures that are not included in the main text of the manuscript are provided here.

S.1 Preprocessing of LLR Data

In the case of traditional LLR analysis studies, it is usual to estimate numerous physical and empirical model parameters in addition to the ERP components. Our approach, however, cannot solve for extra parameters while solving for instant ERP components. For the long-term physical effects, we performed a global preprocessing on the complete 11-year dataset of LLR residuals. In the preprocessing of the LLR residuals, we fitted for the following parameters:

1. ITRF coordinate adjustments of the GRSM, MATM, and WETL stations: 9 parameters;
2. Lunar coordinate adjustments of the Apollo-11/14/15 and Luna-17/21 reflectors, X component only: 5 parameters;

3. Periodical terms, including triannual, biannual, annual, synodic, semi-synodic, lunar-day, and solar-day terms: 14 parameters;
4. Other empirical terms, including constant and lunar solid tide factor: 2 parameters.

The range residuals plotted per station are shown in Fig. S1.

Fig. S1. LLR residuals at the European stations.

Table S1 presents the root mean square (RMS) temperature values by station and year from 2012 to 2022, along with the overall RMS across all years.

Fig. S2 displays the annual range residual statistics for each station.

S.2 Polar Motion on the Ground

For interested readers, **Fig. S3** provides a clearer and more intuitive visualization of the polar motion. Fig. S3 is a map near Earth' s north pole, where we provide the direction to 0° longitude (Greenwich) and 90°W longitude (Chicago). The ITRF north pole is in the upper-left direction outside this map, where pole offsets x_p and y_p are both zero.

Fig. S4 shows the prediction errors obtained by comparing actual polar motion data with predictions from the mean pole model defined in the IERS Conventions (2010).

S.3 Partial Derivatives

Table S2 displays the mean absolute values of the partial derivatives of the ranging measurements with respect to the ERP components, where the UT1 component is listed in both the ERA and the UT1 columns, with units of meters per arcsecond and meters per second, respectively.

Table 1. Settings of the simulation and augmentation cases

Phases/Stages	Case ID	UT1 bias	Range residuals	Number of solutions
Simulation	S0	0	Real data	8,191
	SU1, SU2	$±$100 s	Real data	2 × 8,191
	SG1, SG2	0	GRSM: $±$50 mm	2 × 8,191
	SM1, SM2	0	MATM: $±$50 mm	2 × 8,191
	SW1, SW2	0	WETL: $±$50 mm	2 × 8,191

Phases/Stages	Case ID	UT1 bias	Range residuals	Number of solutions
Augmentation stage ‘1+2’	AG1-AG4	0	GRSM: Real data; Others: $±$36 mm	4 × 8,191
	AM1-AM4	0	MATM: Real data; Others: $±$48 mm	4 × 8,191
	AW1-AW4	0	WETL: Real data; Others: $±$22 mm	4 × 8,191
Augmentation stage ‘2+1’	AGM1, AGM2	0	GRSM-MATM: Real data; WETL: $±$22 mm	2 × 126
	AGW1, AGW2	0	GRSM-WETL: Real data; MATM: $±$48 mm	2 × 55

Table 2. Variation in the solutions of the simulation cases

Perturbations	Solution variation/mas		
	RMSE x_p	RMSE y_p	MaxAbs Pole Position
Control Case (S0)	(trivial zero)	(trivial zero)	(trivial zero)
UT1 $±$100 s (SU1, SU2)	3.2	4.8	18
GRSM $±$50 mm (SG1, SG2)	18	24	180
MATM $±$50 mm (SM1, SM2)	20	45	175
WETL $±$50 mm (SW1, SW2)	18	23	165

Table 3. Solution errors in the augmentation cases

Case ID	Solution error/mas			
	MAE x_p	MAE y_p	MaxAbs Pole Position	% Better than Prediction
'1+2' summary				
AG1-AG4	53	26	752	88.5%
AM1-AM4	42	31	698	88.5%
AW1-AW4	18	15	321	88.5%
'2+1' summary				
AGM1, AGM2	99	34	899	91.2%
AGW1, AGW2	34	28	456	91.2%

Table 4. Comparison of PM accuracy of the presented method with that of other existing methods

Parameter	RMSE x_p/mas	RMSE y_p/mas	Time resolution
LLR TCV (This paper)			
Aug '1+2'	53	26	~15 min
Aug '2+1'	99	34	~15 min
LLR long-term			
Singh et al.	0.050−0.080	0.050−0.080	1 day
VLBI			
Schuh and Behrend	0.05	0.05	1 day

Fig. 1. LLR stations in Europe. (Map showing locations of GRSM, MATM, and WETL stations with baselines)

Fig. 2. Relation between the condition number and the lunar declination. (Plot showing inverse relationship on logarithmic scale)

Fig. 3. Perturbation effects of UT1 (A) and range at GRSM (B), MATM (C), and WETL (D); the corresponding case IDs are SU1, SG1, SM1, and SW1, respectively. (Scatter plots of solution errors)

Fig. 4. Solution errors in the augmentation phase. (A) '1+2' augmentation, (B) '2+1' augmentation. (Scatter plots with different scales)

Fig. S1. LLR residuals at the European stations. (Time series plot of residuals)

Fig. S2. RMS of range residuals of the stations in the selected time span. (Annual RMS values by station)

Fig. S3. Predicted and reference pole offset from 2010 to 2022. Note that the y_p axis is inverted according to geographical convention. The reference pole (black solid curve) drifts around within a certain area, while the predicted pole (dashed line) moves linearly. Deviations between the reference and predicted poles are <300 mas within the selected time span. In the lower-right corner, a 1-m rule is shown to scale (1 m 32.3 mas).

Fig. S4. Prediction error of pole motion coordinates. (Time series of prediction errors for x_p and y_p)

Table S1. RMS of range residuals by station and year (2012-2022)

Stations	RMS of range residuals/mm
GRSM	[values by year]
MATM	[values by year]
WETL	[values by year]

Table S2. Mean of the absolute values of the ranging partial derivatives to all ERP components

Stations	$\left\lvert\frac{\partial\tau}{\partial x_p}\right\rvert$ (m arcsec^{-1})	$\left\lvert\frac{\partial\tau}{\partial y_p}\right\rvert$ (m arcsec^{-1})	$\left\lvert\frac{\partial\tau}{\partial ERA}\right\rvert$ (m arcsec^{-1})	$\left\lvert\frac{\partial\tau}{\partial UT1}\right\rvert$ (m s^{-1})
GRSM	[value]	[value]	[value]	[value]
MATM	[value]	[value]	[value]	[value]
WETL	[value]	[value]	[value]	[value]

Note: Figure translations are in progress. See original paper for figures.

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