Rectangular Cross-Section FOV Rotational Computed Laminography and Its Analytical Reconstruction Method

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Abstract

Rotational computed laminography (CL) has broad application potential in three-dimensional imaging of plate-like objects, as it only needs x-ray to pass through the tested object in the thickness direction during the imaging process. In this study, a rectangular cross-section field of view rotational CL (RC-CL) was proposed to circuit board imaging. Compared with other rotational CL systems, its field of view is biggest and more suitable for rectangular circuit board. Meanwhile, because the imaging geometry of RC-CL is significantly different from cone beam CT, the Feldkamp-Davis-Kress (FDK) reconstruction algorithm cannot be used directly. On the other hand, transferring the projection data to fit into the CBCT geometry by two-dimensional interpolation will introduce interpolation error. Therefore, the FDK-type analytical reconstruction algorithm applicable to the RC-CL was derived. The effectiveness of the method was validated through numerical experiments and the influence of tilt angle on the reconstruction results was analyzed. Finally, the RC-CL was applied to the real defect detection research of circuit boards

Full Text

Preamble

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Rotational computed laminography (CL) has broad application potential for three-dimensional imaging of plate-like objects, as it only requires X-rays to pass through the tested object in the thickness direction during the imaging process. In this study, a rectangular cross-section field of view rotational CL (RC-CL) was proposed for circuit board imaging. Compared with other rotational CL systems, its field of view is largest and more suitable for rectangular circuit boards. Meanwhile, because the imaging geometry of RC-CL is significantly different from cone-beam CT, the Feldkamp-Davis-Kress (FDK) reconstruction algorithm cannot be used directly. On the other hand, transferring the projection data to fit into the CBCT geometry by two-dimensional interpolation introduces interpolation error. Therefore, an FDK-type analytical reconstruction algorithm applicable to RC-CL was derived. The effectiveness of the method was validated through numerical experiments, and the influence of tilt angle on the reconstruction results was analyzed. Finally, RC-CL was applied to real defect detection research on circuit boards.

Keywords: Computed tomography (CT), Computed laminography (CL), Field of view, FDK, Analytical reconstruction

Introduction

Computed tomography (CT) technology is widely used in industrial applications as a non-destructive testing technique [?]. However, when imaging plate-like objects such as fossils, paintings, composite panels in the aerospace industry, and printed circuit boards (PCB), the commonly used circular cone-beam CT (CBCT) struggles to obtain high-precision three-dimensional (3D) images due to limitations of imaging space and radiation source energy [?]. At the same time, computed laminography (CL) technology only requires rays to pass through the object in the thickness direction, giving it great potential for imaging plate-like objects [?]. In the early days, CL could only record images on the focal plane of an object. With the development of computers, digital detectors, and CL reconstruction algorithms, CL can now obtain 3D images of objects like CT [?].

According to differences in scan trajectory, CL can be divided into translational CL [?, ?], rotational CL [?], and swing CL [?], among which rotational CL is widely used due to its strong adaptability, rich projection information, and same resolution in the xy direction [?]. In terms of scanning geometry, rotational CL is analogous to CBCT: the detector and X-ray source rotate 360° around the rotation axis (i.e., z-axis) to collect projection information. However, the angles between the central ray and the rotation axis (i.e., tilt angle in Fig. 1) differ between the two systems. In CBCT, the central ray is perpendicular to the rotation axis ($= 90°$), while the tilt angle in rotational CL is less than 90° ($< 90°$), as shown in Fig. 1(b). This characteristic enables X-rays to pass through the plate-like object only in the thickness direction during the 360° scanning process [?].

In both CBCT and CL, flat-panel detectors are widely used. However, the detector in CBCT is vertically set and facing the source during rotation, which can make full use of the detector [?, ?]. Different from CT, there are various settings of flat-panel detector in rotational CL. As shown in Fig. 2(a), the first setting has the detector parallel to the rotation axis, similar to CBCT. The second and third settings are currently commonly used. In the second setting, the detector is perpendicular to the central ray during rotation, as shown in Fig. 2(b). In the third setting shown in Fig. 2(c), the detector is perpendicular to the rotation axis and rotates in-plane so that its v-axis always points to the rotation axis.

The fourth setting is our proposed configuration. Similar to the third setting, its detector is perpendicular to the rotation axis. However, the detector has only translational motion and its orientation remains unchanged during rotation. Different detector settings mean different scanning geometries, which directly impact image reconstruction [?].

Image reconstruction is an important part of CL imaging [?, ?]. Existing CL reconstruction methods can be divided into three categories: analytical methods [?, ?], iterative methods [?, ?], and deep learning methods [?, ?]. Although some studies [?, ?, ?] have shown that deep learning methods have excellent performance in computational efficiency and accuracy, there are still many challenges (e.g., lack of training data), and further optimization is needed before they can be extensively accepted. Meanwhile, analytical and iterative methods are widely used in practical applications. Iterative methods have good noise resistance and the ability to process incomplete projection data. However, they require large amounts of computation, making real-time reconstruction difficult. In contrast, analytical algorithms have lower computational complexity and no parameters needed, making them widely used in commercial fields. However, analytical methods are specifically bound to imaging geometry, and different geometries require different analytical algorithms [?].

Different reconstruction methods have different application scenarios [?]. Although analytical methods produce worse artifacts in reconstructed images compared with iterative methods, they are efficient and suitable for scenarios requiring efficiency, such as online detection of circuit board defects. In the analytical algorithm study of rotational CL, Yang et al. proposed a filtered backprojection reconstruction formula suitable for rotational CL in 2010 [?]. However, this method only focuses on the backprojection process and does not discuss the filtering process. Sun et al. proposed a reconstruction algorithm based on projection transformation (PT-FDK) [?]. In this method, the CL scanning data and parameters are converted into those of CT that conform to the FDK conditions [?]. Then, the filtered backprojection operation is carried out on the converted CL data. By this means, the CL projection data can be reconstructed through the standard FDK algorithm. Compared with Yang' s work, this method converts projection data to standard geometry and adopts standard FDK, thus having high applicability. However, this method requires a large amount of computation, and the interpolation error could be significant enough to degrade image reconstruction quality.

In this study, for fast and high-precision imaging of circuit boards, we first proposed a rotational CL detector setting and compared its field of view (FOV) with other detector settings. Then, an FDK-type analytical reconstruction algorithm for our proposed detector setting was derived and verified through numerical experiments. Finally, the proposed rotational CL scheme was validated on a real system for PCB inspection.

II. FOV Analysis with Different Detector Settings

As shown in Fig. 3, during rotational CL imaging, the imaging range under a projection angle is the quadrangular pyramid region SP P P P formed by the X-ray source S and the four vertices of the detector. The intersection of the imaging ranges under all projection angles is the field of view (FOV) of the CL imaging system. The projection information of voxel points within the FOV can be recorded by the detector at all projection angles. In CL imaging, to ensure reconstruction quality, it is necessary that all voxels of interest are located within the FOV. Therefore, a larger FOV allows larger objects to be scanned.

Under a projection angle , let R , R , R , and R be the intersection points of the rays SP , SP , SP , and SP with the z = z plane, respectively, and the quadrilateral region R R R R is the imaging range of CL on the z = z plane. Correspondingly, the intersection of the quadrilateral region R R R R of CL under all projection angles is the FOV of CL on the z = z plane. Generally speaking, the FOV of CL on the z = z plane varies with different coordinates z . However, because the circuit board has small size in the thickness direction (i.e., the z direction), it is most important to evaluate the FOV of the CL system during imaging of circuit boards by directly analyzing its FOV on the z = 0 plane.

According to the formula derivation (see Appendix for the detailed derivation process), as shown in Fig. 4, under the first three settings, the shape of the imaging region R R R R of CL on the z = 0 plane does not change with the projection angle and just rigidly rotates around the origin O during imaging. Therefore, their FOV shapes on the z = 0 plane are circles, and the radius of these circles can be determined by finding the minimum distance from origin O to the four sides (i.e., lines R R , R R , R R , and R R) of the quadrangle region R R R R . Meanwhile, in the fourth setting, the quadrangle R R R R not only has a constant shape but also does not rotate around the origin O. Therefore, its FOV is the quadrangle R R R R , which is a rectangle.

Let H *{O-R R }, H* {O-R R }, H *{O-R R }, and H* {O-R R } be the distances in the i-th (i=1, 2, 3, 4) setting from origin O to lines R R , R R , R R , and R R , respectively. Because H *{O-R R } = H* {O-R R } (the explanation is given in Appendix A), the circle radius of the first three settings can be expressed as:

$$R^{(1)} = \min\left\{H^{(1)}_{O-R_1R_2}, H^{(1)}_{O-R_4R_1}\right\}$$

$$R^{(2)} = \min\left\{H^{(2)}_{O-R_1R_2}, H^{(2)}_{O-R_4R_1}\right\}$$

$$R^{(3)} = \min\left\{H^{(3)}_{O-R_1R_2}, H^{(3)}_{O-R_4R_1}\right\}$$

In practical applications, the tilt angle of CL is less than 60°, i.e., 0° <   60°. At this time, we can obtain:

$$H^{(1)}_{O-R_1R_2} = \frac{L_u|SO|\sin^2\alpha}{\sqrt{L_u^2\cos^2\alpha + 4|SD|^2}} < H^{(2)}_{O-R_1R_2} = \frac{L_u|SO|\sin\alpha}{\sqrt{L_u^2\sin^2\alpha + 4|SD|^2}}$$

$$H^{(1)}_{O-R_4R_1} = \frac{L_v|SO|\sin\alpha + 2|SD|\cos\alpha\sin\alpha}{|L_v - 2|SD|\cos\alpha|} < H^{(2)}_{O-R_4R_1} = \frac{L_v|SO|\sin\alpha + 2|SD|\cos\alpha}{L_v\sin\alpha + 2|SD|\cos\alpha}$$

Therefore, $R^{(1)} < R^{(2)} < R^{(3)}$. Further, the areas of circular FOVs are:

$$S^{(1)} = \pi\left(R^{(1)}\right)^2 < S^{(2)} = \pi\left(R^{(2)}\right)^2 < S^{(3)} = \pi\left(R^{(3)}\right)^2$$

Meanwhile, because the FOV shape of the fourth setting is rectangular, its area can be calculated as:

$$S^{(4)} = \frac{2L_v|SO|}{|SD|} \times \frac{2L_u|SO|}{|SD|} = \frac{4|SO|^2}{|SD|^2}L_uL_v > S^{(3)}$$

To sum up, when 0° <   60°, $S^{(1)} < S^{(2)} < S^{(3)} < S^{(4)}$. That is to say, under the same imaging conditions, the fourth setting has the largest FOV, followed by the third and second settings, with the first setting being the worst.

To more intuitively compare the FOVs, we compared the FOV of different detector settings using numerical tests. In the numerical test, four rotational CL systems with different detector settings as shown in Fig. 2 were simulated using the ASTRA toolbox [**?**]. These systems have the same imaging parameters except for the detector setting. Table 1 lists the detailed imaging parameters. In the simulation, if the projections of a reconstruction point locate inside the detector at all projection angles, this point belongs to the FOV. The more such points, the larger the FOV. To present this more intuitively, Fig. 5 shows two mutually perpendicular sections and their area values for the four FOVs.

Fig. 5(a) shows the coronal plane (i.e., yz cross-section, x = 0 voxel), and Fig. 5(b) shows the transverse plane (i.e., xy cross-section, z = 0 voxel). The volumes of the FOVs are also given. It can be seen that, in terms of FOV distribution, the shapes of the four FOVs are irregular in coronal planes. Meanwhile, as the theoretical formulas show, the xy cross-sections of the first three settings are circular, while because the detector has the same size in two directions, the FOV of the fourth setting is a special rectangle: a square. The volume of the FOV in the first setting is the smallest, followed by the third and second settings, and the fourth is largest. Although the volumes of the FOV in the second and fourth settings are similar, the second is more slender along the z-direction and not suitable for imaging plate-like objects.

From the above analysis, it can be concluded that the proposed setting has the largest FOV under the same imaging parameters. Besides, its xy cross-section shape is largest and rectangular, which is beneficial for CL imaging of plate-like objects such as circuit boards, as most of these objects are rectangular. Finally, its detector is horizontal, which requires smaller installation space. Because this setting has a rectangular FOV shape, it was named "rectangular cross-section FOV rotational CL (RC-CL)" in this study.

III. Analytical Reconstruction Algorithm for RC-CL

Because the imaging geometry of RC-CL is different from CBCT, the classical FDK algorithm cannot be directly used. Although one can transfer the projection data of RC-CL to fit into the CBCT geometry by 2D interpolation so that FDK can be applied to reconstruction similar to PT-FDK, there are two unfavorable factors to consider: (1) transferring projections of RC-CL to CBCT could require a much larger virtual detector because RC-CL projections correspond to large cone angles in CBCT; (2) 2D interpolation error in this situation will significantly reduce image reconstruction quality. Therefore, an analytical reconstruction method specifically for RC-CL is necessary for efficient and high-quality reconstruction.

The 3D schematic diagram of an RC-CL system is illustrated in Fig. 6(a), where a global coordinate system O-xyz is defined with z being the rotation axis and the origin O being the intersection of the axis z and the central ray connecting the source (S) and the center of the detector (D). The zenith angle is referred to as the CL tilt angle. Plane E is the plane where the detector is located and O is the intersection of plane E and the rotation axis. S is the projection of S on plane E. Fig. 6(b) is the 2D schematic diagram from the top view on plane E. There is a native coordinate system D-uv on the detector. During rotation, the directions of axes u and v in D-uv remain parallel to the axes x and y, respectively.

A. Formulation of Analytical Reconstruction on a Virtual 2D Problem

To derive the reconstruction formula for RC-CL, we follow the idea of the FDK method and start from a 2D filtered backprojection (FBP) reconstruction on plane E. First, a rotational coordinate system D-u v is configured on the detector with axis v pointing to the rotation axis during rotation to form a 2D virtual problem. For convenience, we define the angle between axis v and v to be our projection angle . The relation between D-uv and D-u v at projection angle   is:

$$\begin{cases} u' = u\cos\beta - v\sin\beta \\ v' = u\sin\beta + v\cos\beta \end{cases}$$

As shown in Fig. 7, on the detector plane E, if we regard point S as the X-ray source in the 2D problem, projection data along v at a certain v = v * gives a standard view of a fan-beam CT. Hence, we can apply an FBP reconstruction algorithm in this situation:

$$f(x,y) = \frac{1}{2}\int_0^{2\pi} \frac{|S'D| - v'_*}{|S'R'|^2} \int_{-\infty}^{\infty} \frac{|S'D| - v'_*}{(|S'D| - v'_*)^2 + u'^2} p_\beta(u', v'_*) h(u' - u'_*) du' d\beta$$

where (x,y) are the coordinates of a reconstruction point R, D is the point on the v axis where v = v , *|S R | is the projected distance of |S R| on |S D |,* *u* and v * are the projection positions of point R on the detector, |S D| and |S O | are the distances from source S to the detector center D and origin O , respectively, p__ (u , v *) represents projection data under coordinate system D-u v , and h(u) is a ramp filter.

In Eq. 6, the FBP filtering is performed along the u axis. However, the projection data in RC-CL is recorded along u and v axes. Therefore, a reconstruction formula for RC-CL will be derived based on Eq. 6.

To simplify the derivation, let us define:

$$p_{\text{filtered}}^{(\beta,u'_*,v'_*)}(u') \triangleq \int_{-\infty}^{\infty} \frac{|S'D| - v'_*}{(|S'D| - v'_*)^2 + u'^2} p_\beta(u', v'_*) h(u'_* - u') du'$$

It can be further expressed as:

$$p_{\text{filtered}}^{(\beta,u'_*,v'_*)}(u', v') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{|S'D| - v'_*}{(|S'D| - v'_*)^2 + u'^2} \delta(v' - v'_*) dv' p_\beta(u', v') h(u'_* - u') du'$$

Substituting Eq. 5 into Eq. 8 gives:

$$g_{\text{filtered}}^{(\beta,u_*,v_*)}(u,v)=p_{\text{filtered}}^{(\beta,u'_*,v'_*)}(u',v')=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{|S'D|-u_*\sin\beta-v_*\cos\beta}{(|S'D|-u_*\sin\beta-v_*\cos\beta)^2+(u\cos\beta-v\sin\beta)^2}\delta(u\sin\beta+v\cos\beta$$

where g_ (u, v) is the projection data on physical detector grids, recorded in coordinate system D-uv, and u_* and v_* are the corresponding coordinates in D-uv of u * and v *.

In Eq. 9, according to the scaling property of the Dirac delta function, we can obtain:

$$\delta(u\sin\beta+v\cos\beta-u_*\sin\beta-v_*\cos\beta)=\frac{1}{|\cos\beta|}\delta\left(v-\frac{u_*\sin\beta+v_*\cos\beta-u\sin\beta}{\cos\beta}\right)$$

By substituting Eq. 10 into Eq. 9, we obtain:

$$g_{\text{filtered}}^{(\beta,u_*,v_*)}(u,v)=\frac{1}{|\cos\beta|}\int_{-\infty}^{\infty}\frac{|S'D|-u_*\sin\beta-v_*\cos\beta}{(|S'D|-u_*\sin\beta-v_*\cos\beta)^2+(u\cos\beta-v\sin\beta)^2}g_\beta(u,v)h(u_*\cos\beta-v_*\sin\beta-u\cos$$

From $v+\frac{u\sin\beta-u_*\sin\beta-v_*\cos\beta}{\cos\beta}=0$, we obtain:

$$g_{\text{filtered}}^{(\beta,u_*,v_*)}(u)=\frac{1}{|\cos\beta|}\int_{-\infty}^{\infty}\frac{|S'D|-u_*\sin\beta-v_*\cos\beta}{\sqrt{(|S'D|-u_*\sin\beta-v_*\cos\beta)^2+\left(u-u_*\sin^2\beta-v_*\cos\beta\sin\beta\right)^2}}g_\beta\left(u,\frac{u_*\sin\beta+v_*}{\cos}\right.$$

According to the Fourier transform property $\mathcal{F}\{f(at)\}=\frac{1}{|a|}F\left(\frac{w}{a}\right)$, we obtain:

$$h\left(\frac{u_*-u\cos\beta}{\cos\beta}\right)=(\cos\beta)^2h(u_*-u)$$

By substituting Eq. 14 into Eq. 13, we obtain:

$$g_{\text{filtered}}^{(\beta,u_*,v_*)}(u)=|\cos\beta|\int_{-\infty}^{\infty}\frac{|S'D|-u_*\sin\beta-v_*\cos\beta}{\sqrt{(|S'D|-u_*\sin\beta-v_*\cos\beta)^2+\left(u-u_*\sin^2\beta-v_*\cos\beta\sin\beta\right)^2}}g_\beta\left(u,\frac{u_*\sin\beta+v_*\cos}{\cos}\right.$$

By combining Eq. 15 and Eq. 6, we obtain an FBP-type reconstruction formula for RC-CL:

$$f(x,y)=\frac{1}{2}\int_0^{2\pi}\frac{|S'D|-v'_*}{|S'R'|^2}d\beta\int_{-\infty}^{\infty}\frac{|S'D|-u_*\sin\beta-v_*\cos\beta}{\sqrt{(|S'D|-u_*\sin\beta-v_*\cos\beta)^2+\left(u-u_*\sin^2\beta-v_*\cos\beta\sin\beta\right)^2}}g_\beta\left(u,\frac{u_*\,\mathrm{s}}{}\right.$$

B. Extension to a 3D Scenario

In the 3D situation, the z dimension must be considered during reconstruction. Same as the derivation of the FDK algorithm for CBCT, when extending the FBP algorithm from 2D to 3D in RC-CL, two items in FBP need to be modified.

The first is the weighting factor prior to the filtering operation (recorded as). According to Eq. 16, the expression of   in FBP is:

$$\eta_1 = \frac{|S'D'|}{\sqrt{|S'D'|^2 + u'^2}} = \frac{|S'D| - u_* \sin\beta - v_* \cos\beta}{\sqrt{(|S'D| - u_* \sin\beta - v_* \cos\beta)^2 + \left(u - u_* \sin^2\beta - v_* \cos\beta \sin\beta\right)^2}}$$

Physically speaking, in the 2D case,   represents the cosine of the fan angle (i.e.,   in Fig. 7) of reconstruction point R. In the 3D case, as shown in Fig. 8, the fan angle of reconstruction point R is P SD . Meanwhile, the influence of the cone angle (i.e., PSP in Fig. 8) needs to be considered. Therefore, the expression of   is:

$$\eta_1 = \cos\angle D_s SP_s \times \cos\angle PSP_s = \cos\angle O_s SP_s \times \cos\angle PSP_s$$

According to the cosine theorem, Eq. 18 can be written as:

$$\eta_1 = \cos\angle O_s SP_s \times \cos\angle PSP_s = \frac{|SP_s|^2 + |SO_s|^2 - |O_s P_s|^2}{2 \cdot |SP_s| \cdot |SO_s|} \times \frac{|SP_s|}{|SP|} = \frac{|SP_s|^2 + |SO_s|^2 - |O_s P_s|^2}{2 \cdot |SP| \cdot |SO_s|}$$

With global coordinates in O-xyz: S(0, 0, -|SO|cos), O (|SO|sin sin , -|SO|sin cos , -|SO|cos), P(-|OD|sin sin + u, |OD|sin cos - v, |OD|cos), and P (-|OD|sin sin + u, |OD|sin cos - v, -|OD|cos), Eq. 19 can be expressed as:

$$\eta_1 = \frac{|SD| \sin\alpha - v \cos\beta - u \sin\beta}{\sqrt{|SD|^2 - 2|SD| \sin\alpha (u \sin\beta + v \cos\beta) + u^2 + v^2}}$$

By substituting Eq. 12 into Eq. 20, we obtain:

$$\eta_1 = \frac{|SD| \sin\alpha - u_* \sin\beta - v_* \cos\beta}{\sqrt{|SD|^2 - 2|SD| \sin\alpha (u_* \sin\beta + v_* \cos\beta) + u^2 + \left(\frac{u_* \sin\beta + v_* \cos\beta - u \sin\beta}{\cos\beta}\right)^2}}$$

The second item is the weighting factor for backprojection (recorded as). According to Eq. 16, the expression of   in FBP is:

$$\eta_2 = \left(\frac{|S'D| - v'_*}{|S'R'|}\right)^2$$

According to the triangle similarity theorem, we can obtain:

$$\frac{|SR_1|}{|QQ_s|} = \frac{|R_1R_2|}{|SD| \cdot \cos(\alpha)} = \frac{z + |SO| \cdot \cos(\alpha)}{|SD| \cdot \cos(\alpha)}$$

Physically speaking, is determined by the source-to-detector distance |S D| - v * and the projected distance |S R | between the source and the reconstruction point on the central ray. Therefore, as shown in Fig. 8, its expression in the 3D case is:

$$\eta_2 = \left(\frac{|SR_1|}{|SP|}\right)^2 = \left(\frac{|SD| \cdot \cos(\alpha)}{z + |SO| \cdot \cos(\alpha)}\right)^2$$

where z is the coordinate of reconstruction point R. Therefore, Eq. 23 can be written as:

$$\eta_2 = \left(\frac{|SR_1|}{|SP|}\right)^2 = \left(\frac{|SD| \cdot \cos(\alpha)}{z + |SO| \cdot \cos(\alpha)}\right)^2$$

Replacing and with Eq. 21 and Eq. 25, the FDK-type reconstruction formula in RC-CL can be obtained:

$$f(x, y, z) = \frac{1}{2}\int_0^{2\pi} \left(\frac{|SD| \cdot \cos(\alpha)}{z + |SO| \cdot \cos(\alpha)}\right)^2 d\beta \int_{-\infty}^{\infty} \frac{g_\beta\left(u, \frac{u_* \sin\beta + v_* \cos\beta - u \sin\beta}{\cos\beta}\right) |\cos\beta}{\sqrt{|SD| \sin\alpha - u_* \sin\beta - v_* \cos\beta}\sqrt{|SD|^2 - 2|SD| \sin\alpha(u_* \sin\beta}$$

The implementation steps for the proposed algorithm can be summarized as follows:

1. **Preweighting**: Multiply the two-dimensional projection data by a weighting factor computed by:

   $$\text{factor} = \frac{|\cos\beta|}{\sqrt{|SD| \sin\alpha - u_* \sin\beta - v_* \cos\beta}\sqrt{|SD|^2 - 2|SD| \sin\alpha(u_* \sin\beta + v_* \cos\beta) + u^2 + \left(\frac{u_* \sin\beta + v_* \cos\beta}{\cos\beta}\right.}}$$

2. **Filtration**: In numerical implementation, to lower discretization error and avoid cos = 0 at = /2 or = 3 /2, we divide [0, 2) into four parts: [- /4, /4), [/4, 3 /4), [3 /4, 5 /4), and [5 /4, 7 /4) to perform filtration, as shown in Fig. 9.

Specially, when [- /4, /4), |cos | |sin |. The relation $v = \frac{u_* \sin\beta + v_* \cos\beta - u \sin\beta}{\cos\beta}$ (i.e., Eq. 12) is adopted to replace v in Eq. 11. The formula derivations of Eq. 13–Eq. 26 are based on this situation. Accordingly, the value of the u coordinate can be manually specified, and the filtration is performed along the u-axis.

Meanwhile, when [/4, 3 /4) [5 /4, 7 /4), |sin | |cos |. The relation $u = \frac{u_* \sin\beta + v_* \cos\beta - v \cos\beta}{\sin\beta}$ can be adopted to replace u in Eq. 11. For brevity, we do not give the detailed formula derivation here, but the reader can easily derive it according to Eq. 13–Eq. 26. Now, the value of the v coordinate can be manually specified, and the filtration is performed along the v-axis.

During filtration, although we can assign the u coordinate as an integer when filtering along the u-axis and the v coordinate as an integer when filtering along the v-axis, the corresponding v and u coordinates need to be calculated by $v = \frac{u_* \sin\beta + v_* \cos\beta - u \sin\beta}{\cos\beta}$ and $u = \frac{u_* \sin\beta + v_* \cos\beta - v \cos\beta}{\sin\beta}$, which are usually not integers. Therefore, interpolation is needed to get the projection values at these positions. However, different from the 2D interpolation in projection data transferring algorithms, we only need 1D interpolation, which is easily done as shown in Fig. 9.

3. **Weighted Backprojection**: The 3D backprojection weighted by $\left(\frac{|SD|\cdot\cos(\alpha)}{|SO|\cdot\cos(\alpha)+z}\right)^2$ is similar to other FDK-type reconstructions.

IV. Experimental Analysis

A. Simulation Study

To verify the proposed reconstruction method (referred to below as CL-FDK), we simulated an RC-CL system. In the simulation, the radiation source is regarded as a point source. Meanwhile, ray- and voxel-driven models were chosen as forward- and back-projectors, respectively. A PCB phantom as shown in Fig. 10 was used. The phantom contains three copper circuit layers that are interconnected. The mass attenuation coefficients used reference the table of X-ray mass attenuation coefficients from the National Institute of Standards and Technology (NIST) [**?**, **?**], with a range of [0.05, 0.46]. The detailed imaging parameters were: the tilt angle was 45°, the distances from source to origin and detector center were 45.79 mm and 194.58 mm, respectively. The detector was simulated with a 768×768 array and a 0.17×0.17 mm² pixel size, and 256 projection images were acquired. The reconstruction image grids are 300×300×80 with a 0.07×0.07×0.07 mm³ voxel size.

To quantitatively evaluate the quality of the reconstructed PCB images, three metrics were used to measure the similarity between the reconstructed image and the reference image: root mean square error (RMSE), mean structural similarity index (MSSIM), and peak signal-to-noise ratio (PSNR). Smaller RMSE indicates better reconstruction quality, while larger MSSIM and PSNR indicate better reconstruction quality.

For comparison, reconstruction results by the PT-FDK method [**?**] and the simultaneous iterative reconstruction technique (SIRT) are also presented, with the SIRT iteration count set to 200. The reconstructed results of the three algorithms are shown in Fig. 11 and Fig. 12. It can be seen that all three methods can reconstruct the main structural features in the phantom. The reconstructed 2D cross-section images of slice #40 (i.e., z = 40 voxel) in Fig. 12 clearly show that artifacts are unavoidable because of the incomplete data situation in CL scanning. All reconstructed images are darker compared with the original image. The difference between reference and reconstructed slice images shows that the SIRT result has relatively the least artifact, while the PT-FDK result has the most significant error. Horizontal profiles along lines a and b in Fig. 12 are plotted in Fig. 13, which confirms that the difference between the intensity of SIRT reconstruction and the phantom is the smallest, and the error in the CL-FDK result is smaller than PT-FDK.

Fig. 14 shows the values of the three metrics for different reconstruction methods. It can be seen that the SIRT reconstruction method is the best, followed by CL-FDK, and PT-FDK is the worst. As a filtered backprojection algorithm, the PT-FDK algorithm has lower accuracy than CL-FDK mainly because PT-FDK requires out-of-plane interpolation of the projection image during the transformation process, and the additional interpolation operation not only increases the computational load but also brings interpolation errors.

B. Influence of Tilt Angle

In CL imaging, the tilt angle is an important parameter. To study its influence on RC-CL, we experimented by setting the tilt angle to 25°, 35°, 45°, 55°, and 65°, respectively, with other parameters kept constant. Fig. 15 shows the reconstruction results at slice #30 and their difference from the reference. As shown, CL-FDK can reconstruct the main internal features of the phantom at different tilt angles. However, the smaller the , the more severe the artifacts. The superimposed structure from other layers is less strong when increasing the tilt angle.

Fig. 16 shows the variation in quantitative metrics with respect to tilt angle. It can be seen that the RMSE decreases with increasing tilt angle. Its value at 25° is 1.69 times larger than at 65°. On the other hand, the MSSIM increases with increasing tilt angle.

C. Real Experimental Study

In this section, we performed an experimental CL scan of a PCB sample. The experiment was carried out using an RC-CL system as shown in Fig. 17(a). The radiation source used in the experiment is a microfocus X-ray source. Its main characteristics include: X-ray tube voltage operational range 60 to 110 kVp, X-ray tube current operational range 10 to 800 µA, and X-ray focal spot size (nominal value) of 4 µm.

In this study, the X-ray tube was set at 80 kVp and 20 µA. The PCB sample scanned is a computer motherboard with an L-shaped geometry as shown in Fig. 17(b). Since the bottom of the PCB sample is not flat, it was placed on an aluminum base during imaging.

In the experiment, the tilt angle was set to 45° and 512 projections uniformly distributed over 2 were acquired. Each projection has 2048×2048 detector bins and each bin size is 0.14×0.14 mm². The distance between source and detector is 263.101 mm, and the distance between source and origin is 28.681 mm.

Due to the large size of the PCB, we only selected several representative areas in the sample for imaging during the experiment. Fig. 18(a) shows the reconstructed results of ball grid array (BGA) solder joints. Multiple bubble defects featured by black holes can be seen in the reconstructed images, for example, the ones pointed by the red arrows in the figure. Fig. 18(b) shows the reconstructed results of quad flat no-leads (QFN) package solder joints. The square area in the image represents the QFN solder joints, and the irregular circular area inside represents the internal bubble defects. Fig. 18(b) also shows the grey-value profile along the yellow line in three QFNs; it can be seen that the change pattern of gray values is highly correlated with the location of defects. According to these results, it can be concluded that the proposed CL-FDK algorithm can well reconstruct the main internal features of the tested objects and be applied in real systems.

V. Conclusion

This work proposed a new rotational CL imaging system with a horizontal and fixed-orientation detector, and the analytical reconstruction algorithm suitable for it was derived. Research results show that the proposed imaging system has the largest FOV under the same conditions. On the other hand, the proposed reconstruction algorithm has superior performance over the commonly used projection re-sorting reconstruction algorithm. Based on this, the influence of tilt angle on reconstruction results was analyzed, and a larger tilt angle is suggested for better performance. Finally, the proposed imaging system and its reconstruction algorithm were validated on a system imaging circuit boards for defect detection.

Although the proposed method provides a new approach for 3D imaging of plate-type objects, due to the intrinsic shortcoming of rotational CL (i.e., lack of projection information at certain angles), the reconstructed image contains interlayer aliasing artifacts, which are difficult to eliminate through traditional methods. In this situation, deep learning methods can be a good choice, and we plan to conduct further research on this topic.

Meanwhile, it should be pointed out that although motion artifact and scattering artifact are two common artifacts in CT/CL imaging, they are not the main errors in this study. For motion artifact, the step-and-shoot mode is used to record the projection images, and the motions of detector and source are well controlled, so the motion artifact in the reconstructed image is negligible. However, if using continuous mode to record projection images, for example in online detection of circuit boards, the influence of motion artifact cannot be ignored. On the other hand, for scattering artifact, because the research object of this study is circuit board, which has high contrast ratio, the effect of scatter on the reconstruction is small, and we have not conducted research on this question. However, we also found that the CT value of air region is not zero during CL image reconstruction, which indicates the existence of scattering artifact in CL imaging. We think that if using CL for low contrast ratio objects, a detailed analysis of scattering artifact is essential.

VI. Appendix: Calculation of FOV in Rotational CL

Taking the first setting as an example, its calculation process of the FOV is introduced in detail. During the imaging process, the coordinates of the X-ray source S and the detector center D of CL can be expressed as:

$$\begin{cases} S_x = |SO|\sin(\alpha)\sin(\beta) \\ S_y = -|SO|\sin(\alpha)\cos(\beta) \\ S_z = -|SO|\cos(\beta) \end{cases}$$

$$\begin{cases} D_x = -|OD|\sin(\alpha)\sin(\beta) \\ D_y = |OD|\sin(\alpha)\cos(\beta) \\ D_z = |OD|\cos(\beta) \end{cases}$$

where is the tilt angle, is the projection angle, |SO| is the distance between S and O, and |OD| is the distance between D and O.

Meanwhile, the coordinates of four vertices P , P , P , and P of the detector can be represented as:

$$\begin{cases} P_{1x} = D_x - 0.5L_u\cos(\beta) \\ P_{1y} = D_y - 0.5L_u\sin(\beta) \\ P_{1z} = D_z + 0.5L_v \end{cases}$$

$$\begin{cases} P_{2x} = D_x - 0.5L_u\cos(\beta) \\ P_{2y} = D_y - 0.5L_u\sin(\beta) \\ P_{2z} = D_z - 0.5L_v \end{cases}$$

$$\begin{cases} P_{3x} = D_x + 0.5L_u\cos(\beta) \\ P_{3y} = D_y + 0.5L_u\sin(\beta) \\ P_{3z} = D_z - 0.5L_v \end{cases}$$

$$\begin{cases} P_{4x} = D_x + 0.5L_u\cos(\beta) \\ P_{4y} = D_y + 0.5L_u\sin(\beta) \\ P_{4z} = D_z + 0.5L_v \end{cases}$$

where L_u and L_v are the length and width of the detector, as shown in Fig. 3. Based on the coordinates of P , P , P , P , and S, the coordinates of R , R , R , and R on the z = 0 plane can be calculated:

$$\begin{cases} R_{1x} = -\frac{|SO|(L_u\cos\alpha\cos\beta - L_v\sin\alpha\sin\beta)}{L_v+2|SD|\cos\alpha} \\ R_{1y} = -\frac{|SO|(L_u\cos\alpha\sin\beta + L_v\sin\alpha\cos\beta)}{L_v+2|SD|\cos\alpha} \\ R_{1z} = 0 \end{cases}$$

$$\begin{cases} R_{2x} = \frac{|SO|(L_u\cos\alpha\cos\beta + L_v\sin\alpha\sin\beta)}{L_v-2|SD|\cos\alpha} \\ R_{2y} = \frac{|SO|(L_u\cos\alpha\sin\beta - L_v\sin\alpha\cos\beta)}{L_v-2|SD|\cos\alpha} \\ R_{2z} = 0 \end{cases}$$

$$\begin{cases} R_{3x} = -\frac{|SO|(L_u\cos\alpha\cos\beta - L_v\sin\alpha\sin\beta)}{L_v-2|SD|\cos\alpha} \\ R_{3y} = -\frac{|SO|(L_u\cos\alpha\sin\beta + L_v\sin\alpha\cos\beta)}{L_v-2|SD|\cos\alpha} \\ R_{3z} = 0 \end{cases}$$

$$\begin{cases} R_{4x} = \frac{|SO|(L_u\cos\alpha\cos\beta + L_v\sin\alpha\sin\beta)}{L_v+2|SD|\cos\alpha} \\ R_{4y} = \frac{|SO|(L_u\cos\alpha\sin\beta - L_v\sin\alpha\cos\beta)}{L_v+2|SD|\cos\alpha} \\ R_{4z} = 0 \end{cases}$$

According to Eq. A3, we can obtain:

$$\begin{cases} |OR_1| = |OR_2| = |OR_3| = |OR_4| = \frac{2L_v|SO|\cos\alpha}{\sqrt{L_u^2\cos^2\alpha + L_v^2\sin^2\alpha}} \\ |R_1R_2| = |R_3R_4| = \frac{2L_u|SO|\cos\alpha}{\sqrt{L_u^2\cos^2\alpha + L_v^2\sin^2\alpha}} \\ |R_2R_3| = |R_4R_1| = \frac{2L_v|SO|\cos\alpha}{\sqrt{L_u^2\cos^2\alpha + L_v^2\sin^2\alpha - 4|SD|^2\cos^2\alpha}} \end{cases}$$

From Eq. A4, it can be noted that the x and y coordinates of R , R , R , and R are related to the projection angle , but the distance from each point to the origin O is independent of , and the distance between points is also independent of . Therefore, the imaging ranges on the z = 0 plane under different projection angles can be obtained by rigidly rotating the quadrilateral region R R R R around O. Finding the intersection of the quadrilateral R R R R at all projection angles is equivalent to finding the minimum inscribed circle of the quadrilateral R R R R with the origin O as the center, that is, finding the minimum value of the distance from the origin O to the four sides of the quadrilateral R R R R .

According to the calculation:

$$\begin{cases} H^{(1)}_{O-R_1R_2} = \frac{L_u|SO|\sin^2\alpha}{\sqrt{L_u^2\cos^2\alpha+4|SD|^2}} \\ H^{(1)}_{O-R_2R_3} = \frac{L_v|SO|\sin\alpha}{|L_v-2|SD|\cos\alpha|} \\ H^{(1)}_{O-R_3R_4} = \frac{L_u|SO|\sin^2\alpha}{\sqrt{L_u^2\cos^2\alpha+4|SD|^2}} \\ H^{(1)}_{O-R_4R_1} = \frac{L_v|SO|\sin\alpha}{L_v+2|SD|\cos\alpha} \end{cases}$$

From Eq. A5, we can know $H^{(1)}_{O-R_1R_2} = H^{(1)}_{O-R_3R_4}$. Because $H^{(1)}_{O-R_1R_2} < H^{(1)}_{O-R_4R_1}$ and $H^{(1)}_{O-R_2R_3}$ is related to the tilt angle , detector size, and |SD|, the radius of the inscribed circle can be expressed as $R^{(1)} = \min\left\{H^{(1)}_{O-R_1R_2}, H^{(1)}_{O-R_4R_1}\right\}$.

Similarly, the distance formulas in the second setting are:

$$\begin{cases} H^{(2)}_{O-R_1R_2} = \frac{L_u|SO|\sin\alpha}{\sqrt{L_u^2\sin^2\alpha+4|SD|^2}} \\ H^{(2)}_{O-R_2R_3} = \frac{L_v|SO|\sin\alpha}{|L_v\sin\alpha-2|SD|\cos\alpha|} \\ H^{(2)}_{O-R_3R_4} = \frac{L_u|SO|\sin\alpha}{\sqrt{L_u^2\sin^2\alpha+4|SD|^2}} \\ H^{(2)}_{O-R_4R_1} = \frac{L_v|SO|\sin\alpha}{L_v\sin\alpha+2|SD|\cos\alpha} \end{cases}$$

The radius of the inscribed circle can be expressed as $R^{(2)} = \min\left\{H^{(2)}_{O-R_1R_2}, H^{(2)}_{O-R_4R_1}\right\}$.

The distance formulas in the third setting are:

$$\begin{cases} H^{(3)}_{O-R_1R_2} = \frac{L_u|SO|}{\sqrt{L_u^2+4|SD|^2}} \\ H^{(3)}_{O-R_2R_3} = \frac{L_v|SO|}{|L_v-2|SD|\cos\alpha|} \\ H^{(3)}_{O-R_3R_4} = \frac{L_u|SO|}{\sqrt{L_u^2+4|SD|^2}} \\ H^{(3)}_{O-R_4R_1} = \frac{L_v|SO|}{L_v+2|SD|\cos\alpha} \end{cases}$$

The radius of the inscribed circle can be expressed as $R^{(3)} = \min\left\{H^{(3)}_{O-R_1R_2}, H^{(3)}_{O-R_4R_1}\right\}$.

In the fourth setting, the distance formulas are the same as Eq. A7, and its FOV area is:

$$S^{(4)} = \frac{2L_u|SO|}{|SD|} \times \frac{2L_v|SO|}{|SD|} = \frac{4L_uL_v|SO|^2}{|SD|^2}$$

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