

## Development and optimization of a virtual model for the joint calculation of efficiency calibration and trajectory length in TGS emission measurements

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### Abstract

In tomographic gamma scanning (TGS) emission measurements, conventional methods typically compute efficiency calibration and trajectory length separately, which leads to reduced overall computational efficiency. Our analysis reveals a spatial consistency between efficiency calibration and trajectory length. Building upon this physical correlation, this paper proposes a novel joint calculation model that simultaneously resolves both efficiency calibration and trajectory length. To further improve emission image quality and nuclide activity inversion accuracy, radiation sources within voxels are treated as uniformly distributed multi-point sources, thereby constructing a joint calculation model under the multi-point source and detector framework. For model verification, three calculation methods under single-point source emission mode (point-point model, average trajectory model, and the point source mode of this model) and three multi-point source emission modes (uniform 8-point, uniform 64-point, and uniform volume source) were established. The Maximum Likelihood Expectation Maximization algorithm (MLEM) was employed to reconstruct images of volume sources at three different locations within seven different waste drums. The results demonstrate that this model significantly improves computational efficiency, with multi-point source modes outperforming single-point source modes; among multi-point source modes, increasing the number of source points within voxels does not necessarily yield better results, with the uniform 64-point mode exhibiting the best overall performance and showing significant improvements in both emission image quality and nuclide activity accuracy compared to traditional point-to-point and average trajectory models.

## Full Text

# Development and Optimization of a Virtual Model for Coordinated Efficiency Calibration and Trajectory Length Estimation in TGS Emission Measurements

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## Abstract

In tomographic gamma scanning (TGS) emission measurements, conventional methods typically compute efficiency calibration and trajectory length separately, which reduces overall computational efficiency. Our analysis reveals a spatial consistency between these two parameters. Leveraging this physical correlation, we propose a novel joint calculation model that simultaneously resolves both efficiency calibration and trajectory length. To further improve emission image quality and nuclide activity inversion accuracy, radiation sources within voxels are treated as uniformly distributed multi-point sources, constructing a joint calculation model under the multi-point source and detector framework. For model verification, we established three calculation methods under single-point source emission mode (point-point model, average trajectory model, and the point source mode of this model) and three multi-point source emission modes (uniform 8-point, uniform 64-point, and uniform volume source). The Maximum Likelihood Expectation Maximization (MLEM) algorithm was used to reconstruct images of volume sources at three different locations in seven different waste drums. Results demonstrate that this model significantly improves computational efficiency, with multi-point source modes outperforming single-point source modes. Among multi-point source modes, increasing the number of source points does not necessarily yield better performance; the uniform 64-point mode exhibits the best comprehensive performance, showing significant improvements in both emission image quality and nuclide activity accuracy compared to traditional point-to-point and average trajectory models.

**Keywords:** tomographic gamma scanning (TGS), emission imaging, efficiency calibration, trajectory length

## 1 Introduction

Under global carbon-neutral targets, nuclear power has regained prominence as a low-carbon, dispatchable baseload source. The International Energy Agency (IEA) projects that installed nuclear capacity must roughly double by 2050, contributing approximately 10% to total global electricity generation [1]. Although nuclear energy contributes significantly to carbon reduction and energy security, its expansion inevitably generates radioactive waste during the entire fuel cycle. The safe and effective management of such waste has become a key focus for the international community [2-4]. Therefore, accurately assessing the radioactivity level of nuclear waste is an essential step in ensuring the sustainable development and safe management of nuclear energy [3, 4].

In the field of radioactive waste characterization, Tomographic Gamma Scanning (TGS) has emerged as an advanced nondestructive  $\gamma$ -ray assay technique for drum waste inspection (as shown in Fig. 1 [Figure 1: see original paper]). The detection principle of TGS is as follows: first, a transmission measurement is performed to obtain the three-dimensional (3D) spatial distribution of materials inside the drum. Then, emission measurements are conducted to determine the 3D activity distribution of radionuclides. Finally, the radioactive level of the waste is evaluated based on the reconstructed activity distribution [5-11]. In the overall TGS detection process, the emission measurement is the key factor that determines the accuracy of activity assessment. Therefore, this study focuses on the emission measurement process in TGS, where the precise computation of the efficiency calibration matrix and trajectory length matrix constitutes the core technical challenge.

Regarding efficiency calibration, most existing studies simplify the radioactive source as an ideal point source to reduce computational complexity. Typical approaches include the spatially symmetric equivalent matrix element method [12, 13], polar coordinate partitioning method, and function-fitting techniques [14-18]. However, the point-source assumption neglects the actual spatial distribution of radioactivity within a voxel. Cheng Yu' s study [14] demonstrated that, within the collimator' s field of view, a volume source contributes more detectable  $\gamma$ -rays than a point source, leading to significant discrepancies in efficiency calibration results. Thus, incorporating the geometric distribution of the source into the efficiency calibration process is crucial for accurately evaluating its influence on emission image quality [14-16].

For the trajectory length model, the Estep team at Los Alamos National Laboratory (LANL) simplified the TGS physical model into a "point-source and point-detector" configuration for trajectory length calculation [5]. This model greatly simplifies computation and has since become the mainstream approach. However, by treating both the source and detector as point-like, the model neglects their actual geometrical dimensions, introducing significant deviations in computed trajectory length and subsequently degrading image reconstruction quality.

To correct these geometric simplifications, subsequent research introduced more refined geometric representations. Based on the structure of the TGS system, a “point-source and detector” configuration was proposed, which considers the real dimensions of the detector rather than treating it as a point. The China Institute of Atomic Energy first incorporated the concept of detector solid angle into the average trajectory length model [19]. Subsequently, various equivalent trajectory length calculation methods were developed under this configuration, including: Monte Carlo methods (e.g., the volume flux method developed by Chengdu University of Technology [20] and the Monte Carlo-based equivalent trajectory length model established by our group for transmission measurements [21]); mean-value methods (e.g., Han Miaomiao of Harbin Engineering University proposed a method based on the mean value of line connections between the point source and finite points on the detector surface [22]); and computer graphics clipping methods (e.g., Zhang Quanhu of the China Institute of Atomic Energy used the Cyrus-Beck algorithm [19] and Yan Yucheng of Chengdu University of Technology applied the Cohen-Sutherland algorithm [23]).

However, most of these methods still assume the source to be a point source without considering the actual spatial distribution of radioactivity. Consequently, when voxels are relatively large, significant deviations in equivalent trajectory length remain. Although assuming a uniformly distributed volume source can reduce bias, applying mean-value or clipping methods requires extensive computations of numerous sub-voxel points, making the process extremely cumbersome. In contrast, Monte Carlo methods can directly simulate particle transport in 3D space via probabilistic sampling, making them more suitable for handling multi-point distributions within a voxel.

Moreover, in existing emission measurement studies based on the “point-source-detector” configuration, efficiency calibration and trajectory length are typically computed independently, resulting in low computational efficiency, especially when the voxel source is non-point-like. Therefore, integrating the efficiency calibration and trajectory length computation processes while accurately reflecting the geometric distribution of the source is crucial for improving emission image quality. It is noteworthy that the spatial scope for Monte Carlo-based efficiency calibration is consistent with that for trajectory length calculation. Specifically, the range of particle paths contributing to the full-energy peak count is identical to the computational range of trajectory length. The Geant4 Monte Carlo toolkit provides direct access to these particle trajectory step lengths, offering a foundation for joint computation of efficiency calibration and trajectory length while properly accounting for source geometry.

Based on the actual TGS emission measurement scenario, this study addresses the cumbersome independent calculation of efficiency calibration and trajectory length in traditional workflows. Using a virtual vacuum waste drum model that preserves only geometric features, we propose for the first time a joint computation model of efficiency calibration and trajectory length under a “single-point-source-detector” configuration. Furthermore, to account for source distribution

characteristics, the radioactivity within each voxel was uniformly distributed to the centers of its subregions, leading to an innovative “multi-point-source-detector” joint computation model that more realistically represents source distribution. This model aims to improve computational efficiency while effectively enhancing the reconstruction quality of emission images.

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## 2.1 Transmission Measurement

In emission measurement, the count rate of a specific gamma ray can be described by Equation (1).

Among them, self-absorption attenuation correction for efficiency calibration is required, as shown in Equation (2). The solution for the absorption attenuation of gamma rays by the medium are presented in Equations (3) and (4) [5, 19, 24]. (cid:88)  $F_{ij} \cdot S_j$  In the equation,  $D_i$  represents the count rate of gamma rays emitted by all voxels in the sample and detected at the  $i$ -th measurement position;  $F_{ij}$  denotes the efficiency matrix element corrected for self-absorption attenuation, which is referred to as the attenuation-corrected efficiency matrix element;  $A_{ij}$  stands for the activity of the radioactive source in the  $j$ -th voxel.

$F_{ij} = E_{ij} \cdot A_{ij}$  In the equation,  $E_{ij}$  is the detection efficiency of the detector at the  $i$ -th scanning measurement position for the radioactive source in the  $j$ -th voxel;  $A_{ij}$  represents the attenuation factor of gamma rays emitted from the  $j$ -th voxel and detected by the detector at the  $i$ -th scanning measurement position, where the attenuation is caused by the medium. (cid:32)  $A_{ij} = \exp$  (cid:33)  $T_{ijk} \cdot \mu_k$  (cid:88) In the equation,  $T_{ijk}$  represents the linear attenuation thickness, which is the thickness of gamma rays emitted from the  $j$ -th voxel and detected by the detector at the  $i$ -th scanning measurement position being attenuated by the  $k$ -th voxel along the trajectory before reaching the detector;  $\mu_k$  denotes the linear attenuation coefficient of the  $k$ -th voxel.

According to the MC theory,  $i_j$  is expressed as Equation (4)[21]:

$T_{ijk} =$  (cid:80)  $N_j \sum_{l=1}^i T_{ijkl}$  (cid:80)  $N_j \sum_{l=1}^i i_l$  In the equation,  $i_l$  denotes the detection efficiency at a point  $l$  on the end-face of the detector when the detector is at position  $i$ ;  $T_{ijkl}$  represents the trajectory length of gamma particles from voxel  $j$  passing through voxel  $k$  before reaching point  $l$  on the detector's end-face;  $N_j$  is the total number of particles emitted from voxel  $j$ .

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## 2.2 TGS Model for Joint Calculation of Efficiency Calibration and Equivalent Trajectory Length

To invert the activity distribution inside the waste drum, it is necessary to pre-solve the efficiency calibration matrix corrected for absorption and attenuation, as shown in Equation (1). By combining Equations (2) and (3), this

problem can be transformed into solving the efficiency calibration matrix  $E$  and trajectory length matrix  $T$ . Most existing studies calculate these two matrices independently, resulting in cumbersome processes. Notably, when the Monte Carlo method is used to calculate detector efficiency matrix elements, the range of particle paths contributing to the full-energy peak count is identical to the computational range of trajectory length, and the MC program Geant4 can conveniently obtain ray trajectory step lengths. Therefore, both the efficiency calibration matrix elements and trajectory length matrix elements in this study were calculated using the Monte Carlo method. The equivalent trajectory length is calculated using Equation (4), which can be understood as the average trajectory length of all  $\gamma$ -rays from voxel  $j$  that pass through voxel  $k$  and are detected by the detector end-face at position  $i$ . Based on this, a virtual vacuum nuclear waste drum model retaining only geometric features was constructed using Geant4, and a joint calculation model for efficiency calibration and trajectory length was proposed for the first time to simplify the solution process and simultaneously obtain the efficiency calibration matrix and equivalent trajectory length matrix. This model includes three modules: TGS system geometric modeling, particle source emission, and calculation of efficiency matrix  $E$  and trajectory length matrix  $T$ , providing a new approach for the synchronous solution of efficiency calibration and equivalent trajectory length.

The model was constructed as follows: This model is based on a single HPGe detection system and performs geometric modeling of a standard 200L waste drum. The density of the entire drum was set to zero, and it could be axially divided into  $Z$  layers. Each layer was further subdivided into  $V \times V$  rectangular voxels, resulting in a total of  $Z \times V \times V$  voxels. Each layer can be configured with 1 to  $I$  ( $1 \leq I \leq V$ ) measurement positions, and the entire system has  $Z \times I$  detection positions.

The particle source adopts a single-point emission mode at the voxel center (as shown in Figure 2 Figure 2: see original paper), with particles uniformly distributed at the center of each voxel through uniform sampling. It is worth noting that this design includes a technical highlight: for the rotating working condition of the waste drum, the system can real-time match the rotational attitude of the drum, drive the particle source position to dynamically adjust synchronously with the spatial coordinates of the voxel, and maintain precise alignment with the voxel center at all times. This design can effectively avoid particle distribution deviation caused by drum rotation, ensure that the uniformity of particles in the detection area is not affected by rotation during the entire detection process, and provide stable support for the accuracy of subsequent matrix calculations.

The specific process is as follows: During each particle emission, uniform sampling was used to select the particle-emitting voxel, and the center coordinates of this voxel were obtained using G4VPhysicalVolume as the initial particle position. The affine transformation class in Geant4, which can realize coordinate transformation through rotation and translation matrices, is utilized. Before

particle emission, the initial position coordinates can be affine-transformed with the rotation matrix of the waste drum to obtain the rotated particle emission coordinates, as expressed in Equation (5):

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### 2.3 Joint Calculation Model of Efficiency Calibration and Equivalent Trajectory Length Under Multi-Point Source Emission

In the equation,  $P$  denotes the transformed coordinate vector,  $M$  represents the  $3 \times 3$  rotation transformation matrix,  $P$  is the coordinate vector before transformation, and  $T$  stands for the translation transformation vector.

In this model, the solution process for the efficiency matrix  $E$  and equivalent trajectory length matrix  $T$  is as follows (Figure 3 [Figure 3: see original paper]):

1. When a single photon is emitted, the G4Track is used to trace the transport process of each photon. When the photon has a ParentID of 0 and a Step of 0, the geometry it is located in the particle source voxel  $j$ , and the trajectory length  $x_{jj}$  within this voxel is obtained.
2. The photon transport status was continuously tracked. When the ParentID is 0, Step is greater than 0, and the photon is inside the waste drum, the geometric space where the particle resides is the photon trajectory voxel  $k$ , and the trajectory length  $x_{jk}$  within this voxel can be obtained.
3. The photon entering the detector geometry was monitored. When a photon enters the  $i$ -th detector geometry, the energy deposited in this step is acquired using the `GetTotalEnergyDeposit()` member function of the `G4Step` class. The energy deposition variables corresponding to the  $i$ -th detector are statistically added up to the energy deposited each time.
4. At the end of each event, the trajectory length identification is recorded, and it is checked whether the total deposited energy  $e_{ij}$  in detector  $i$  equals the initial energy  $e_0$ . If so, the photon was considered to be detected. The track length  $x_{ijk}$  through the  $k$ -th voxel is retained, and the value of the detector count matrix element  $D_{ij}$  is incremented by one.

After completing the transport and reaction processes of all  $N$  sampled photons, the full-energy peak detection efficiency matrix element  $E_{ij}$  is calculated by dividing the number of photons with full energy deposition  $(\text{cid:80})N_{j \ l=1}^i$  recorded by the detector at position  $i$  for the emission source voxel  $j$  (where  $D_{ij} = (\text{cid:80})N_{j \ l=1}^i$ ) by the total number of emitted photons  $N_j$  in voxel  $j$ . Using Eq. (4), the equivalent trajectory length matrix element  $T_{ijk}$  is obtained through a ratio. The numerator is the total valid trajectory length,  $(\text{cid:80})N_{j \ l=1}^i x_{ijkl}$ , of photons from voxel  $j$  that pass through voxel  $k$  and are effectively detected. The denominator is the expected number of photons with full-energy

deposition,  $(\text{cid:80})N_{j \ l=1 \ i}$ , recorded for the emission source voxel  $j$ . (a) Radioactive distribution of single-point source at the voxel center. (b) Radioactive distribution of 8-point sources within a voxel.

Fig. 3. Schematic diagram of different radioactive distribution settings within the voxel of this model.

In traditional emission imaging, the calculation of efficiency calibration and trajectory length usually assumes that the radioactive source is concentrated at the center of the voxel (as shown in Figure 2(a)). However, radioactive sources in actual waste drums are mostly distributed as volume sources, and it is too rough to perform relevant calculations based solely on the assumption of a point source at the voxel center.

To address this, this section optimizes the particle emission module based on the TGS model for the joint calculation of efficiency calibration and equivalent trajectory length. The radioactive distribution is regarded as covering a wider area inside the voxel, and two new emission modes are introduced accordingly: the uniform volume source emission mode within the voxel and the uniform multi-point source emission mode within the voxel. Compared with the traditional point-source emission mode, these two new modes only change the sampling rules for particle emission positions, whereas other key calculation links (including efficiency calibration calculation, trajectory length calculation, and the total number of sampled particles in each voxel) remain unchanged. Specifically, the calculation logic for efficiency calibration and trajectory length is still consistent with the “single-point source emission at the voxel center” mode in Section 2.2, with a single voxel as the basic calculation unit. The main difference lies in the design of the uniform multi-point source emission mode within the voxel: this mode further subdivides a single voxel into multiple sub-regions of equal volume, takes the center of each sub-region as the point-source emission position, and assumes that radioactivity is uniformly distributed at these center points. In essence, the two new emission modes only adjust the distribution of radioactive sources within the voxel, allowing more particles to reach the detector within the collimator’s field of view, thus conforming better to the volume source distribution characteristics in actual scenarios. Figure 2(b) shows the specific radioactive distribution when a single voxel is divided into 8 equal-volume regions.

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### 3.1 TGS Geometry and System Setup

To verify the processing effect of the proposed model on emission images, the experimental setup was as follows:

As shown in Figure 4 [Figure 4: see original paper], each layer of the standard 200L waste drum was divided into  $10 \times 10$  voxels, with 5 horizontal scanning positions configured. The experiment selected half of the drum area ( $Z = 8$ ),

and the waste drum rotates around its axis at intervals of 15 degrees ( ), with a total of 24 rotations. Combined with the above parameters, a total of 768,000 matrix elements were formed. The detection equipment used an HPGe detector with a crystal specification of 6.41 cm in diameter and 7.55 cm in length[25].

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### 3.2 Sampling Position Settings for Radioactive Source Emission Modes

The emission modes include two types: single-point and multi-point sources. In the particle source module of the multi-point source emission mode, when the emission source is distributed as a volume source, uniform sampling within the voxel is adopted; when the emission source is distributed as a uniform limited point source, a general method for generating three-dimensional source points based on power-of-8 division is used. This method can flexibly achieve regular divisions into 8, 64, and even higher power-of-8 points. In this study, four distribution modes are configured as shown in Figure 5 [Figure 5: see original paper]: voxel center point source, voxel uniform 8-point source, voxel uniform 64-point source, and uniform volume source, which correspond to the joint point-detector (JPD) model, joint 8-point-detector (J8PD) model, joint 64-point-detector (J64PD) model, and joint volume-detector (JVD) model, respectively.

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### 3.3 Medium and Nuclide Positions Inside the Drum

To quantitatively evaluate the results of the emission measurements, this study sets 7 filling methods for the medium inside the drum and three volume source positions. The linear attenuation coefficient  $\mu$  of the medium was calculated using the Monte Carlo method.

As shown in Figure 6 Figure 6: see original paper, three volume source samples were positioned in different layers with different radii, all emitting

#### 107 Bq/s each.

As shown in Figure 6(b), each layer inside the waste drum was filled with three identical cubes at the same positions, among which 4 voxels are filled with water, 3 voxels with nylon 11, and 1 voxel with aluminum. The remaining matrix positions were filled with the following materials respectively: wood (0.443 g/cm<sup>3</sup>), polyethylene (0.94 g/cm<sup>3</sup>), nylon 11 (1.425 g/cm<sup>3</sup>), concrete (2.3 g/cm<sup>3</sup>), aluminum (2.699 g/cm<sup>3</sup>), calcium oxide (3.3 g/cm<sup>3</sup>), and aluminum oxide (3.97 g/cm<sup>3</sup>).

The average densities of the resulting waste drums were 0.7402 g/cm<sup>3</sup>, 1.0782 g/cm<sup>3</sup>, 1.4080 g/cm<sup>3</sup>, 2.0030 g/cm<sup>3</sup>, 2.2743 g/cm<sup>3</sup>, 2.6830 g/cm<sup>3</sup>, and 3.1386

g/cm<sup>3</sup> respectively.

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## 4 Results and Discussion

To intuitively evaluate the computational model proposed in this study, the MLEM algorithm was used for image reconstruction. Six emission modes were established for systematic comparative analysis, including the efficiency calibration-trajectory length joint calculation model under the point-source emission mode (MLEM-JPD), three efficiency calibration-trajectory length joint calculation models under multi-point source emission modes, namely, the 8-point source (MLEM-J8PD), 64-point source (MLEM-J64PD), and voxel-uniform volume source modes (MLEM-JVD), and two comparative models: the point-point model (MLEM-PP) and the average trajectory length model.

As shown in Figs.7-9, they respectively present the imaging results of each volume source under different average densities of waste drums for the three emission modes: MLEM-PP, MLEM-JPD, and MLEM-J8PD. Since the material settings of each layer are consistent, the imaging of radioactive sources is hardly affected by different layers. Among them, MLEM-JPD is similar to MLEM-AT, and MLEM-J8PD is similar to MLEM-J64PD and MLEM-JVD; therefore, they are not displayed separately. It can be seen from Figs.7-9 that the emission measurement images under the multi-point source emission mode have the best quality, and the positioning and recognition of each volume source are superior to those of MLEM-PP and MLEM-JPD. Specifically, MLEM-PP could no longer locate volume source B when the average density reached 2.0030 g/cm<sup>3</sup>, whereas MLEM-JPD could barely locate the source at volume source B when the average density was 2.6830 g/cm<sup>3</sup>, with poor imaging quality.

In this case, when there is a large density difference between the drum's matrix density and other filling media, the image noise in high-density voxel regions will increase significantly, leading to a decline in image quality. Thus, the quality of the emission images is closely related to the medium distribution around the volume source and the average density inside the drum.

Fig.10 shows the NMSE values and Neighboring Ratios (NR) of different models under varying medium densities.

As shown in Fig.10(a), the MLEM-PP emission mode exhibits the worst overall NMSE performance, particularly in the high-density range. Moreover, single-point source emission modes are generally inferior to multi-point source modes.

Fig.10(a)-10(d) further indicate that MLEM-PP performs the worst in terms of NR values, with the most significant discrepancy observed at volume source B under high-density conditions. Based on the complete dataset analysis, MLEM-AT and MLEM-JPD demonstrate minimal differences in image quality, fundamentally because both models are built upon the point source-detector con-

figuration assumption. However, MLEM-JPD offers significant advantages in computational workflow: for the large number of matrix elements in the experimental setup, this method enables simultaneous computation of both the efficiency calibration matrix and trajectory length matrix, whereas MLEM-AT requires separate independent calculations for these two matrices. Additionally, the three models under multi-point source emission modes can accurately locate radioactive sources. Compared with MLEM-PP, at a density of 2.2743 g/cm<sup>3</sup>, MLEM-JVD reduces NMSE by 47.3%, 47.2%, and 48%, respectively, while increasing NR at volume source B by 84.7%, 84.9%, and 87.6%, respectively. Compared with MLEM-AT, at a density of 2.6830 g/cm<sup>3</sup>, MLEM-JVD reduces NMSE by 12%, 14%, and 14.9%, respectively, and increases NR at volume source B by 26.9%, 30%, and 35.8%, respectively.

Fig.11 presents the relative errors of the reconstructed activities for the direct radioactivity (DR) and summed radioactivity (SR) of the three volume sources in the waste drum under the seven medium conditions across the six models. Except for some density points of volume source A, where DR is slightly better than SR, the overall reconstruction accuracy of SR for the three volume sources is generally superior to that of DR.

A comparison of the 3-point source methods reveals that the activity reconstruction accuracy of MLEM-JPD is significantly better than that of MLEM-PP, with the greatest difference occurring at volume source B. At this point, the relative error difference in the Specific Reconstruction (SR) compared to MLEM-PP at the highest average density point is reduced by 53.1%. Compared to MLEM-AT, MLEM-JPD shows only slight differences at high-density points, with overall minimal discrepancy.

Further analysis of Fig.10 and Fig.11 shows that MLEM-J64PD is the optimal emission mode, with little difference in NMSE and NR compared to MLEM-JVD, and both can accurately locate radioactive sources. MLEM-J64PD achieves the smallest overall activity reconstruction errors at volume sources A and C, with a maximum difference of only 1.35% from MLEM-JVD at volume source B. This indicates that more points do not necessarily yield better results under multi-point source emission modes. Compared with MLEM-AT, MLEM-J64PD achieves the optimal improvement in SR, with a relative error reduction of 7.42% at volume source C under an average density of 2.6830 g/cm<sup>3</sup>.

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## 5 Conclusion

Based on a virtual vacuum nuclear waste drum that retains only geometric features, this study establishes, for the first time, a joint calculation model of efficiency calibration and trajectory length based on a point-source-detector configuration. Measurement data were acquired under seven medium-filled waste drum conditions, and the MLEM algorithm was used for nuclide activity inver-

sion. Under the single-point source emission mode, MLEM-JPD exhibits better imaging quality and more accurate radionuclide activity quantification than MLEM-PP. This method shows little difference compared to MLEM-AT. However, corresponding to the large number of matrix elements in the experimental setup, it avoids the cumbersome process of separately calculating efficiency calibration and trajectory length.

Furthermore, targeting the characteristic that radioactive sources in waste drums are mostly distributed as volume sources, this study innovatively proposes a joint calculation model of efficiency calibration and trajectory length under a “multi-point source-detector” configuration. Three different multi-point source division methods were established: 8-point, 64-point, and uniform distributions within voxels. Experimental comparison results indicate that more division points do not necessarily lead to better performance under the multi-point source emission mode, among which MLEM-J64PD achieves the optimal comprehensive evaluation. Compared with MLEM-AT, the optimal improvement is observed at an average density of  $2.6830 \text{ g/cm}^3$ : the relative error of the summed radioactive activity at volume source C is reduced by 7.42%, NMSE is decreased by 14%, and the maximum difference in NR value is an increase of 30% at volume source B. This model effectively improves the quality of the emission images while enhancing computational efficiency.

Thus, this study avoids the cumbersomeness of separately calculating efficiency calibration and trajectory length, and simultaneously considers the spatial distribution of radioactive sources within voxels to improve image inversion quality, providing a reference scheme for TGS emission imaging. This research was conducted using a single HPGc detector system, which has a long measurement time. Additionally, the voxel division scale adopted is relatively large, failing to explore the variation in model accuracy under finer voxel divisions. In future work, the model can be extended to an array scintillator detection system, and the model accuracy can be investigated using finer voxel division methods.

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*Note: Figure translations are in progress. See original paper for figures.*

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