Study on Localization of High-Dose Regions Based on Dose Gradient Characteristics and the Least Squares Method

Authors: Jiang, Mr. Lin Jiang, Jiang, Mr. Lin Jiang

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Abstract

In nuclear decommissioning scenarios, determining the distribution of radiation doses in the spatial domain, particularly the location of high-dose areas, plays a crucial role in monitoring nuclear decommissioning facilities. The conventional approach involves performing interpolation between measurement points and identifying high-dose area locations based on interpolation results. However, the positioning deviation of results obtained by currently prevalent interpolation methods is relatively substantial. Therefore, this paper proposes a method for locating high-dose radiation areas based on dose gradient characteristics and the least squares method, aiming to enhance the accuracy of high-dose area localization. In this study, dose gradient characteristics during the radiation propagation process are utilized to determine the direction of high-dose areas, combined with the least squares optimization method to predict radiation source location, with high-dose area location results obtained through the radiation dose attenuation formula. Comparative experiments with currently prevalent interpolation methods demonstrate that the proposed method reduces the mean squared error (MSE) of dose calculations by 82.1% compared to the Kriging interpolation method and by 47.9% compared to the inverse distance weighting method, thereby significantly improving localization accuracy. This method is applicable to radiation scenarios comprising gamma ray (γ-ray) detection point data and can provide technical support for radiation risk assessment and emergency response to contamination, enhancing the nuclear safety monitoring system.

Full Text

Preamble

Study on Localization of High Radiation Dose Areas Based on Dose Gradient Characteristics and the Least Squares Method

Lin-Jiang Jiang, Jun Liu, Yue Sun, Xiao-Feng Yang
Chengdu University of Technology, Chengdu 610059, China

In nuclear decommissioning contexts, determining the spatial distribution of radiation doses—particularly localizing high-dose areas—plays a crucial role in monitoring decommissioning facilities. The conventional approach involves interpolating between measurement points and identifying high-dose regions based on these interpolation results. However, widely used interpolation methods currently exhibit significant localization errors. This paper proposes a method for locating radiation high-dose areas based on dose gradient characteristics and the least squares approach, aiming to improve localization accuracy. This study utilizes dose gradient characteristics during radiation propagation to determine the direction of high-dose areas, predicts source location using a least squares optimization method, and obtains high-dose area localization results through the radiation dose attenuation formula. Comparative experiments with current popular interpolation methods show that the proposed method reduces the mean square error (MSE) of dose calculation by 82.1% compared to Kriging interpolation and by 47.9% compared to the inverse distance weighting method, significantly enhancing localization accuracy. This method is applicable to radiation scenarios composed of gamma (γ-ray) detector data points and can provide technical support for radiation risk assessment and emergency pollution response, thereby improving the nuclear safety monitoring system.

Keywords: Dose gradient, Least squares method, Radiation scenario

Introduction

In nuclear decommissioning scenarios, collecting dose rate data from various locations within a site and spatially interpolating these measurement points allows for observation of the radiation dose distribution across the area. Particularly, the results of high-dose area localization can further assess the potential impact of the current site and provide core data support for developing subsequent disposal plans. Therefore, researching spatial methods suitable for locating high-dose regions in radiation environments has become an important area of study in the field of nuclear safety monitoring.

Current academic and engineering research on calculating radiation dose distribution mainly focuses on two types of interpolation methods: Kriging interpolation and inverse distance weighting (IDW). Li Hua and colleagues were the first to use the Kriging method to reconstruct and visualize radiation dose fields, achieving good reconstruction results, though with considerable accuracy deviations [**?**]. Jin Guodong and colleagues presented the principles of inverse distance weighted interpolation and Kriging interpolation methods, then compared the two, concluding that Kriging interpolation performs better than inverse distance weighting in most application scenarios [?]. Building on this, Hu Jifeng and colleagues introduced the Radial Basis Function (RBF) interpolation method. Compared to Kriging, RBF generally performs less well overall, but its reconstruction accuracy at the radiation source position exceeds that of Kriging [?]. Xie Xingwen and others incorporated network function interpolation into traditional dose rate interpolation methods, using sparsely sampled nodes to reconstruct three-dimensional radiation fields [?]. Chi Mingwen improved the Kriging method and conducted a comparative study with inverse distance squared surface interpolation, finding that inverse distance squared interpolation is efficient but less accurate, while the improved Kriging method provides better fitted data but with relatively lower efficiency; overall, Kriging interpolation is more suitable for practical engineering applications [?]. Zhang Biao and others found that radiation fields often contain mixtures of multiple radioactive nuclides, making inversion extremely difficult, and therefore proposed an optimized inverse distance weighting algorithm to reconstruct gamma radiation fields. The reconstruction results obtained using the optimized inverse distance weighting algorithm had a significantly lower mean absolute percentage error compared to results obtained using the Kriging method [?].

The aforementioned research methods demonstrate advantages in computational efficiency in specific scenarios and can quickly complete radiation dose distribution predictions. However, they generally exhibit limitations in scenario adaptability: methods such as Kriging and RBF interpolation perform well in dose inversion within the coverage area of measurement points but tend to produce significant deviations when extrapolating to areas outside the coverage range. Although the IDW interpolation method can achieve extrapolation predictions within a certain range under simple scenarios or with a single radiation source, in complex situations such as multiple overlapping radiation sources and obstacle shielding, not only does interpolation accuracy significantly decrease, but there may also be incorrect predictions of dose distribution. When the radiation source is located at the edge of the monitoring area or in areas difficult for detection equipment to reach, existing methods struggle to achieve radiation source localization and dose inversion. Therefore, this paper conducts further research on these issues.

This paper proposes a radiation dose distribution calculation method that integrates dose gradient analysis with least squares optimization. It aims to address the accuracy of radiation dose interpolation in complex scenarios and to achieve three-dimensional visualization of predicted high-dose distribution areas. The principle and implementation process of the method will be explained in Section 2. In Section 3, we set up comparative experiments between the method proposed in this paper and the Kriging and inverse distance weighting methods. The effectiveness of our method is verified through error comparison and image scene comparison, with results analyzed in Section 4. Finally, Section 5 summarizes the main conclusions of this paper.

II. Method Design

As shown in Fig. 1 [Figure 1: see original paper], this section focuses on gradient methods and least squares methods, explaining in detail their theoretical basis, computational logic, and specific design ideas through three steps: regional division, gradient method regional constraints, and least squares method simulation calculations.

A. Gradient Characteristics and Directional Constraints

The radiation dose distribution of a single radiation source in space has distinctive characteristics: it exhibits a monotonic decreasing gradient outward in all directions from the center of the source (as shown in Fig. 2 [Figure 2: see original paper]). This figure includes three measurement locations, labeled 1, 2, and 3, as well as the extended lines of rays between each pair of locations. By observing the distribution characteristics, key patterns can be extracted: Positions 1 and 2 fall within different gradient ranges, and the spatial distance between the two points is relatively short. The direction of radiation extending from low to high dose is closer to the center of the radiation source. Positions 2 and 3 belong to the same gradient range, with consistent dose attenuation characteristics. Although positions 1 and 3 are in different gradient ranges, the distance between them is relatively large, and the direction of radiation extending from low to high dose along the line connecting the two points deviates from the center of the radiation source.

Based on the above rules, the approximate direction of the radiation source can be determined by the magnitude and direction of the gradient. The core logic is: the more significant the gradient change, the region where radiation shifts from low to high dose in a more concentrated direction is closer to the radiation source. The calculation of a single gradient magnitude is centered on the "dose difference between two points" and the "spatial distance between two points," with the formula defined as follows:

$$(\Delta r)^2$$

Here, ΔD represents the absolute value of the dose difference between two points, and Δr is the distance between the two points along the branch line.

However, the gradient directions of radiation from low to high doses do not necessarily point directly at the center of the radiation source, and the occurrence of 1,3-line extension cable deviations is more common. Therefore, in most cases, the prediction area can only be narrowed down by calculating multiple gradient rays. The specific formula is as follows:

$$T(D_i) = \max\left\{\frac{|D_i - D_j|}{(P_i - P_j)^2} \mid i \neq j\right\}$$

Among them, $T(D_i)$ is the set of gradient calculations, D_i is the dose rate (μGy/h) at the i-th measurement point, D_j is the dose rate at the j-th measurement point ($j \neq i$), and P_i and P_j are the spatial coordinates of the i-th and j-th measurement points, respectively. The gradient calculation result of the current measurement point i with all other measurement points is taken as the maximum value to serve as the core gradient feature of this point, and the direction vector is calculated using the following formula:

$$\vec{\alpha}_i = \vec{P}_i - \vec{P}_j$$

Here, $\vec{\alpha}_i$ is the ray direction vector corresponding to the maximum gradient at the i-th measurement point (direction from the low dose point to the high dose point), and $\vec{P}_i$ and $\vec{P}_j$ are the spatial position vectors of the i-th and j-th measurement points, respectively. The extension direction of this vector is the candidate orientation of the radiation source.

As shown in Fig. 3 [Figure 3: see original paper], gradient methods can efficiently compute the gradient direction in scenarios with a single radiation source. However, in the multi-radiation source scenario shown in Fig. 4 [Figure 4: see original paper], multiple sources create overlapping high-gradient regions, causing confusion in the ray direction and interfering with the determination of the radiation source orientation.

Therefore, before performing gradient evaluation, the data needs to be processed. On one hand, high-dose points are segmented, and on the other hand, points in low-dose regions are excluded as initial points for gradient rays. Using dose thresholds and spatial distance thresholds as criteria, the measurement points are divided into multiple independent high-dose regions, defined by the following formula:

$$A_i \ni P_j \begin{cases} D_j \geq D_{\text{Threshold}} \\ r_j \leq r_{\text{Threshold}} \end{cases}$$

Here, A_i represents the i-th high-dose region, P_j is the position of the j-th measurement point, $D_{\text{Threshold}}$ is the preset dose determination threshold, and only high-dose points are retained for subsequent calculations. $r_{\text{Threshold}}$ is the preset distance threshold that controls the spatial extent of a single region. When the j-th measurement point satisfies both "dose $\geq D_{\text{Threshold}}$" and "distance to region $A_i \leq r_{\text{Threshold}}$", the point is assigned to region A_i.

To ensure the accuracy of regional division, it is necessary to update the minimum distance from each measurement point to its corresponding region in real time. The formula is as follows:

$$r_j = \min\{(P_j - P_i)^2 \mid P_i \in A_i\}$$

Here, r_j is the minimum distance from the j-th measurement point to its corresponding region A_i, and P_i is any measurement point within region A_i.

Through the above preprocessing, the multi-source mixed regions in Fig. 4 can be divided into two separate high-dose areas, red and blue, as shown in Fig. 5 [Figure 5: see original paper], eliminating the gradient interference from low to medium doses. Subsequently, it is only necessary to calculate the gradient ray direction within each independent area to constrain the radiation source localization region.

B. The Least Squares Method and Implementation

The core idea of the least squares method is to quantify the difference between the "model-predicted values" and the "actual observed values" by defining a "sum of squared errors function," and then to find the model parameters that minimize the error. Specifically, this is done by defining an error function that measures the degree of difference between the observed value y and the function $f(x)$ at the corresponding x, typically using the sum of squared errors as this error function. That is, for a given set of n data points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, if the fitting function is $y = f(x)$, then the sum of squared errors S is:

$$\sum (y_i - f(x_i))^2$$

The ultimate goal is to find a set of parameters that minimizes the sum of squared errors S. In this process, this method is used to optimize the simulated position of the radiation source. By constructing an error function between the simulated dose values and the actual values, the simulation position that corresponds to the smallest error is identified as the optimal simulated position of the radiation source.

Under realistic conditions, radioactive materials are generally stored in containers and pipelines or attached to other objects. Therefore, we introduced scene modeling data as the basis for the attachment of radioactive materials, limiting the simulation range of radiation source locations to the mesh-modeled scenes. In the gradient method of part A, we have obtained a series of gradient rays. After extending these rays from low dose to high dose to the scene-modeled locations, we determined the specific areas of the radiation sources.

Within each area, source data is simulated:

$$D_{1\times i} = |D_1 D_2 \cdots D_i|$$

$$L_{i\times j} = \begin{pmatrix} (P_D(1) - P_{D1})^2 & \cdots & (P_D(i) - P_{D1})^2 \\ \vdots & \ddots & \vdots \\ (P_D(1) - P_{Dj})^2 & \cdots & (P_D(i) - P_{Dj})^2 \end{pmatrix}$$

Here, $D_{1\times i}$ represents the simulated source dose matrix for each region, $P_D(i)-P_{Dj}$ represents the distance from the simulated source position to the measurement point, and $L_{i\times j}$ represents the squared distance matrix from all simulated sources to all measurement points.

Classify and process the measurement point data, setting high, medium, and low dose ranges. In each range, use 10% of the data as interpolated unknown points, and the remaining data as the set of interpolated known points.

$$D_{\text{Known } j\times 1} = (\vdots)$$
$$D_{\text{Unknown } k\times 1} = (\vdots)$$

Establish a matrix equation based on the radiation attenuation function and solve it using simulated source dose data:

$$D_{1\times i} \cdot L_{i\times j} = D_{\text{Known } j\times 1}$$

Obtain the simulated point dose data for each region in the matrix $D_{1\times i}$, and then perform multi-source joint superposition calculations of the dose at the interpolated unknown point positions.

$$D_{\text{Total}}(k) = \sum D_n \cdot L_n \cdot e^{-\mu_n \cdot d_n(k)}$$

Here, $D_{\text{Total}}(k)$ represents the dose rate at the k-th interpolation unknown point calculated from the simulated data, N represents the number of simulated sources, D_n is the reference dose rate of the n-th simulated source, L_n is the distance from the position of the n-th simulated source to the k-th interpolation unknown point, μ_n is the linear attenuation coefficient of the medium, which is related to the type of radiation and the material of the medium, and d_n is the thickness of the medium that the radiation passes through.

Finally, the error is calculated using the least squares method:

$$\sum (D_{\text{Total}}(k) - D_{\text{Unknown}}(k))^2$$

Here, S represents the current error calculation value, M is the number of measurement points, and $D_{\text{Unknown}}(k)$ represents the value of the k-th interpolated unknown point.

Within the range of the gradient-limited area, find the next scene grid unit, update the simulation point, and recompute. After traversing all situations, find the minimum value from the final array S:

$$S_{\text{Best}} = \min(S)$$

Among them, each regional simulated source position corresponding to S_{Best} is the optimal predicted position of the resulting radiation source.

The best simulated sources obtained from each region are combined and superimposed, with their dose rate attenuation strictly following the inverse square law and medium absorption corrections, to calculate the radiation dose rate at an unknown point, as shown in the following formula:

$$\sum \cdot e^{-\mu \cdot d}$$

Here, D_x represents the dose rate at any unknown point being measured, D_i represents the dose rate of the i-th simulated radiation source point, n represents the number of simulated radiation sources, r_i represents the distance from the simulated source point to the actual source, r_x represents the distance from the position of the unknown point D_x to the actual source, μ represents the "linear attenuation coefficient" of the medium, and d represents the thickness of the medium traversed.

III. Experimental Design

The experimental data are sourced from actual measurements at a nuclear waste treatment site, including dose data at measurement points and site point cloud reconstruction data. The experimental environment was built using self-developed 3D radiation field imaging analysis software. Among them, the dose rate at measurement points ranges from 0.02 to 0.9 µGy/h. The experiment grouped the data according to the rule of "dividing one dose interval for every 0.1 µGy/h." Within each interval, 10% of the measurement points were selected as interpolation unknown points for comparison and verification, while the remaining 90% were used as the training set for subsequent model calculations and performance validation.

To verify the effectiveness of the proposed method, this paper conducts an analysis through comparative experiments. The comparison objects include the Kriging interpolation method and the inverse distance weighting method. The evaluation criterion is designed using the cross-validation method, with mean absolute error (MAE) and mean squared error (MSE) as the core evaluation indicators. The performance of the Kriging method, the inverse distance weighting method, and the method designed in this paper is quantitatively compared. The calculations are repeated five times, and the results are retained to four decimal places to avoid the impact of sampling randomness. The MSE is calculated based on the average MAE of the five repetitions as the final evaluation criterion. At the same time, to explore the error distribution of each method in different dose intervals, the unknown interpolation points are ranked from high to low according to their true values. Then, the computed value of each unknown interpolation point is compared with the actual value, and finally, the absolute errors of the three methods are compared.

To intuitively display the experimental scene, measurement data, and positioning results, image rendering is performed using 3D visualization technology. The specific parameter settings are as follows: the scene adopts a meshing method to construct a 3D model of the experimental site, with a mesh radius set to 0.005 m to balance point cloud density and rendering efficiency, ensuring clear presentation of the site structure details; measurement points are marked in the form of point clouds, with a point size set to 1.0, making it easy to distinguish measurement points from the scene background and highlight the locations of the original data; within the spatial range covered by the measurement points and scene data, interpolation calculation points are generated at 0.1 m intervals. Continuous visualization of radiation dose distribution is achieved through volume rendering, allowing intuitive presentation of continuous dose gradients; the transparency of the scene model and measurement points is set to 1.0 to ensure the original scene and data are clearly visible, while the transparency of the interpolation results is set to 0.5 to avoid obscuring the original data while highlighting the dose distribution trend.

A. Kriging Interpolation Method

Kriging interpolation is based on spatial autocorrelation and achieves the optimal unbiased estimation of unknown points through linear weighting of known sample points. Its core calculation formula is as follows:

$$Z(x_0) = \sum \lambda_i \cdot Z(x_i)$$

Here, $Z(x_0)$ represents the estimated value at the unknown point x_0, λ_i represents the weight of the i-th known point, $Z(x_i)$ represents the attribute value of the known point, and n represents the number of known sample points involved in the calculation.

By combining the spatial attenuation law of radiation dose and quantifying spatial correlation, this experiment matches the weight λ_i to the exponential variogram model, with the specific form as follows:

$$\lambda_i = \text{Nugget} + \text{Sill} \times \left(1 - e^{(-d/\text{Range})}\right)$$

The nugget is set to 0 by default, and the sill, which is the "stable maximum value" of the variogram, is set to 1.0. The distance d represents the distance from an unknown point x_0 to a known point x_i. The range is the "maximum distance" of spatial correlation. Based on the range between known and unknown points, the distance range of the test data in this study is within 8 meters, so the range is set to 8.0 here.

B. Inverse Distance Weighting Method

The core logic of inverse distance weighting is that the closer a known sample point is to the unknown point, the greater its weight on the estimated value of the unknown point, which conforms to the attenuation characteristic of radiation dose being "larger when near and smaller when far." The dose estimation formula is as follows:

$$Z(x_0) = \frac{\sum_{i=1}^{n} \frac{Z(x_i)}{d_{i0}^p}}{\sum_{i=1}^{n} \frac{1}{d_{i0}^p}}$$

Here, $Z(x_0)$ represents the estimated value at the unknown point x_0, $Z(x_i)$ represents the actual attribute value of the i-th known sample point, d_{i0} represents the direct distance between the known point x_i and the unknown point x_0 ($d_{i0} \neq 0$ to avoid division by zero), p represents the distance decay coefficient, and according to the rule of radiation decaying with the square of the distance, p is set to 2 here. n represents the number of known sample points involved in the calculation.

C. Gradient and Least Squares Method

Regional Division Part: In this experiment, the strategy first calculates the maximum dose rate of all known interpolation points and ranks them. Starting from the first known interpolation point as the core, it extends outward by 1.0 m (0.1 times the adaptive scenario range) to form the initial regional boundary. Then, new known interpolation points are evaluated: if a new known interpolation point falls within the existing regional boundary, a boundary extending 1.0 m from the new point is combined with the original region to form a new regional range, ensuring coverage of all associated high-dose points; if a new known interpolation point falls outside all existing regional boundaries and its dose rate is greater than 0.25 times the maximum dose rate (with a dose attenuation factor of 0.25 at 1.0 m), a new independent calculation region is created centered on this point according to the initial regional rules.

Gradient Calculation Section: Within each independent region, the candidate range of radiation sources is screened based on gradient directions. Using the maximum dose rate of interpolated known points within the current region as a reference, a screening threshold is set at 0.8 times the maximum dose rate of the current region (the amplification threshold is set for high dose intervals in this study). Only interpolated known points with dose rates greater than this threshold are retained for gradient calculation, thereby eliminating interference from low-dose points. For each filtered interpolated known point, the gradient value with respect to other interpolated known points in the region is calculated, and the direction with the largest gradient is selected as the core gradient direction. This direction is extended from low dose to high dose until it reaches the boundary of the experimental scenario. The "starting point coordinates" and "boundary endpoint coordinates" of all core gradient rays are collected, and the maximum and minimum values along the X, Y, and Z axes are extracted. The rectangular space formed by these six extreme values then represents the candidate range for radiation sources in the region. Subsequent searches for the radiation source location are carried out only within this range, thereby reducing computational effort.

Radiation Source Localization: Within the candidate range of each area, the simulated radiation source position is iteratively optimized using the least squares method to achieve precise localization. To balance positioning accuracy and computational efficiency, a three-dimensional grid is generated at 0.1 m intervals, with each grid point serving as a candidate simulated radiation source. Combinations of candidate simulated sources from all areas are then considered, and the least squares method is used to calculate the sum of squared errors between the simulated radiation doses and the actual doses at interpolated unknown points. The position parameters of candidate simulated sources in each area are iteratively updated, repeating the error calculation process. Finally, the simulated source position with the minimum sum of squared errors is selected as the final localization result of the radiation source for that area. The dose rate at any interpolated unknown point to the simulated radiation source is calculated using the radiation dose rate attenuation model (Equation 15). In this experiment, since the material and thickness data of the scene medium are unknown, medium attenuation corrections are not considered, and the linear absorption attenuation coefficient μ is set to 0 (the calculation results may be overestimated); the simulated radiation source is treated as a real source, with r_i set to 1.0 and r_x being the distance from the interpolated unknown point to the simulated radiation source.

IV. Experimental Results

The Kriging method calculation results are shown in Table 1 and Fig. 6 [Figure 6: see original paper], with the calculated mean square error being 0.0715 $(\mu Gy/h)^2$.

Table 1 . Numerical Experimental Results of the Kriging Method.

Number	Average calculated value(μGy/h)	Actual value(μGy/h)	Mean Absolute Error(μGy/h)	MSE$(\mu Gy/h)^2$
				0.0715

The computed results using the inverse distance weighting method are shown in Table 3 and Fig. 7 [Figure 7: see original paper], and the calculated mean square error is 0.0206 $(\mu Gy/h)^2$.

The calculation results of the method designed in this paper are shown in Table ?? and Fig. 8 [Figure 8: see original paper], and the calculated mean square error result is 0.0128 (μGy/h)^2.

The absolute errors of the three methods at different dosage ranges are shown in Fig. 9 [Figure 9: see original paper].

Table 2 . Numerical Experimental Results of the Kriging Method.

Number	Average calculated value(μGy/h)	Actual value(μGy/h)	Mean Absolute Error(μGy/h)	MSE(μGy/h)^2
				0.0207

Table 3. Numerical Experimental Results of the Kriging Method.

Number	Average calculated value(μGy/h)	Actual value(μGy/h)	Mean Absolute Error(μGy/h)	MSE(μGy/h)^2
				0.0128

Based on the MSE quantification results, there are significant differences in the accuracy performance of the three methods. The MSE of the method designed in this study is approximately 82.1% lower than that of the Kriging method and about 47.9% lower than that of the inverse distance weighting method. In terms of performance across different dose ranges, the method designed in this study shows higher accuracy than both the Kriging and inverse distance weighting methods in the low-dose range; in the medium-dose range, it outperforms the Kriging method and has similar accuracy to the inverse distance weighting method; in the high-dose range, although the calculated results are slightly higher, the accuracy still surpasses that of the other two methods. This indicates that the method proposed in this study effectively improves the accuracy of dose estimation, with the inverse distance weighting method being second, and the Kriging method having the lowest accuracy.

Next, the measured point dose data will be superimposed in 3D with the simulated dose distribution data using experimental software for visualization, applying a color mapping rule where the dose ranges from high to low are represented by red → yellow → green → blue. Additionally, all visualization results will be ensured to use the same viewing angle and observation area to eliminate the interference of perspective differences on the analysis results.

The initial scene is shown in Fig. 10 [Figure 10: see original paper], with the image containing measurement points marked in color and the site background model. The red high-dose measurement points are clustered around the cylindrical barrel-like objects in the scene, the yellow-green medium-dose points spread around the high-dose points, and the blue low-dose points are distributed along the periphery of the site, clearly reflecting the spatial dose gradient characteristics of the measurement data.

After dividing the high-dose regions, the gradient rays from low dose to high dose are drawn as shown in Fig. 11 [Figure 11: see original paper]: gradient rays consist of continuous red dots, starting from the high-dose measurement points and extending along the direction of maximum gradient to the boundary of the site. The concentrated directionality of the rays provides an intuitive basis for determining the candidate range of radiation sources.

Within the candidate range defined by the gradient rays, the simulated radiation source points obtained through iterative optimization using the least squares method are shown in Fig. 12 [Figure 12: see original paper]: the white marked point in the figure represents the optimal simulated radiation source location, which happens to be inside a cylindrical barrel. This barrel is a known container for radioactive materials in the experimental site. The simulation results highly correspond to the actual scenario, validating the rationality of this positioning method.

The final dose distribution visualization results are shown in Fig. 13 [Figure 13: see original paper]: the high-dose red region not only fully encompasses the high-dose measurement points but also extends into the interior of the cylindrical container, consistent with the actual location of the radioactive material; the yellow-green medium-dose region only covers the area corresponding to the yellow measurement points, and the blue low-dose region is highly consistent with the low-dose measurement points, with dose boundaries closely matching the actual gradient pattern.

The visualization results of the dose distribution using ordinary Kriging interpolation are shown in Fig. 14 [Figure 14: see original paper]: the high-dose red areas ($ $0.8 µGy/h) do not align with the high-dose measurement points, instead concentrating in the blank areas between medium-high dose measurement points; the yellow-green medium-dose areas (0.4–0.6 µGy/h) are excessively spread, encompassing many green and blue low-dose measurement points, which does not correspond to the actual dose gradient pattern, reflecting the failure of this method' s spatial correlation assumption in multi-source scenarios.

The dose distribution results of the inverse distance weighting method are shown in Fig. 15 [Figure 15: see original paper]: the high-dose red region accurately covers the high-dose measurement points without obvious deviation; the yellow-green medium-dose region includes only a few blue low-dose measurement points, and the dose boundaries are relatively clear; however, the high-dose region spreads outward from the measurement points, covering only the upper surface and top of the cylindrical bucket.

In summary, evaluating from the perspectives of the matching degree at different dose measurement points and the predictive performance in high-dose regions, the gradient and least squares method designed in this study yielded the best results, followed by the inverse distance weighting method, with the Kriging method performing the worst. This result further validates the effectiveness of the method proposed in this study, and the visualization of the radiation dose distribution better reflects the characteristics of the actual radiation field.

V. Conclusion

To address the issues of low accuracy and large prediction deviations in traditional interpolation algorithms for radiation dose distribution calculations, this paper proposes a radiation dose distribution calculation method that integrates dose gradient analysis with least squares optimization. Experimental results show that the MSE of the proposed method is 82.1% lower than that of the ordinary Kriging method and 47.9% lower than that of the inverse distance weighting method, fully demonstrating the advantage of this method in dose interpolation accuracy. Furthermore, three-dimensional visualization results indicate that the simulated radiation source points generated by the proposed method can be precisely located inside the storage barrels of radioactive materials, consistent with the physical laws of actual radiation fields, whereas the Kriging method produces offset positioning and the inverse distance weighting method only covers the barrel surfaces, both failing to reflect the actual spatial positions of radiation sources. This demonstrates the superiority of the proposed method in radiation source prediction accuracy.

Considering both error accuracy and practical features, the above analysis confirms the feasibility and practicality of the proposed method in complex radiation scenarios.

Although the method presented in this paper demonstrates advantages in accuracy and practicality, there are still two areas that need optimization: compared with traditional interpolation algorithms, the method designed in this paper involves more computational steps (excluding error calculation, the computation time is 200 ms); when the number of measurement points in high-dose regions is small (< 2), insufficient input data can easily lead to large deviations in radiation source prediction. These issues need to be further optimized in the future.

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Note: Figure translations are in progress. See original paper for figures.