Prediction of $p\bar{\Omega}$ States and Femtoscopic Study

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Abstract

Inspired by recent research on the $p\Omega$ and $p\bar{\Lambda}$ systems, we investigate the $p\bar{\Omega}$ systems within the framework of the quark delocalization color screening model. Our result indicates that the nucleon-$\bar{\Omega}$ interaction is slightly stronger than the nucleon-Ω interaction, implying a higher likelihood for the $p\bar{\Omega}$ system to form bound states. Dynamic calculations show that the $p\bar{\Omega}$ systems with $J^P = 1^-$ and 2^- form bound states, whose binding energies are deeper than that of the $p\Omega$ system with $J^P = 2^+$. The scattering phase shifts and extracted scattering parameters also support the existence of $p\bar{\Omega}$ bound states. Additionally, we discuss the behavior of the femtoscopic correlation function for the $p\bar{\Omega}$ pairs for the first time. Building on the recent experimental progress on the $p\Omega$ correlation function, future femtoscopic investigations of the $p\bar{\Omega}$ system in heavy-ion collisions will be particularly valuable for constraining baryon-antibaryon interactions.

Full Text

Preamble

Prediction of $p\bar{\Omega}$ States and Femtoscopic Study

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Inspired by recent research on the $p\Omega$ and $p\bar{\Lambda}$ systems, we investigate the $p\bar{\Omega}$ systems within the framework of the quark delocalization color screening model. Our result indicates that the nucleon-$\bar{\Omega}$ interaction is slightly stronger than the nucleon-Ω interaction, implying a higher likelihood for the $p\bar{\Omega}$ system to form

bound states. Dynamic calculations show that the $p\bar{\Omega}$ systems with $J^P = 1^-$ and 2^- form bound states, whose binding energies are deeper than that of the $p\Omega$ system with $J^P = 2^+$. The scattering phase shifts and extracted scattering parameters also support the existence of $p\bar{\Omega}$ bound states. Additionally, we discuss the behavior of the femtoscopic correlation function for the $p\bar{\Omega}$ pairs for the first time. Building on the recent experimental progress on the $p\Omega$ correlation function, future femtoscopic investigations of the $p\bar{\Omega}$ system in heavy-ion collisions will be particularly valuable for constraining baryon-antibaryon interactions.

Keywords: $p\bar{\Omega}$ systems; femtoscopic correlation function; bound states; hadron-hadron interaction; scattering phase shifts

Introduction

The study of baryon-antibaryon bound states dates back to the proposal by Fermi and Yang [?] to form the pion from a nucleon-antinucleon pair. In the traditional one-boson-exchange theory of nucleon-nucleon interactions, it is shown that the nucleon-antinucleon system is more attractive than the nucleon-nucleon system due to the strong ω-exchange [?]. Therefore, possible bound states and resonances of nucleon-antinucleon systems have been proposed for many years, with an extensive and comprehensive review of possible $N\bar{N}$ bound states provided in Ref. [?].

More recently, the BESIII Collaboration reported the observation of a new $X(1880)$ state in the line shape of the $3(\pi^+\pi^-)$ invariant mass spectrum [?], which is considered as evidence for the existence of a proton-antiproton bound state. Many theoretical works have been sparked to study the $p\bar{p}$ system and the properties of $X(1880)$ [?]. In addition to the possible proton-antiproton state, there has also been great progress in recent work related to $p\bar{\Lambda}$ [?]. A narrow structure in the $p\bar{\Lambda}$ system near the mass threshold, named $X(2085)$, is observed in the process $e^+e^- \to pK^-\bar{\Lambda}$ with a statistical significance exceeding 20σ. Its spin and parity are slightly favored to be $J^P = 1^+$ through an amplitude analysis. Further theoretical results and discussions can be found in Refs. [?]. Building on the significant progress in the studies of nucleon-antinucleon and nucleon-$\bar{\Lambda}$ systems, it is natural to explore whether bound states or resonance states can be formed between nucleon and other hyperons, or between nucleon and other antihyperons.

In recent years, progress in understanding the strange dibaryon $p\Omega$ has renewed interest in dibaryon systems. The STAR Collaboration measured the $p\Omega$ correlation functions in Au+Au collisions at the Relativistic Heavy-Ion Collider (RHIC) [?] and reported a positive scattering length for the $p\Omega$ interaction, which supports the hypothesis of a $p\Omega$ bound state. In addition, the ALICE Collaboration reported measurements of the p-Ω correlation in pp collisions at $\sqrt{s} = 13$ TeV at the Large Hadron Collider (LHC) [?]. Beyond the $p\Omega$ system, femtoscopic techniques and correlation function studies have made significant progress, both experimentally [?] and theoretically [?].

The $S = -3$, $I = 1/2$, $J = 2$ $N\Omega$ state was first predicted by J. T. Goldman et al. as a narrow resonance in a relativistic quark model [?]. M. Oka also proposed the existence of a quasi-bound state with $I(J^P) = 1/2(2^+)$ using a constituent quark model [?]. A lattice QCD study by the HAL QCD Collaboration reported that the $p\Omega$ state is a bound state at a pion mass of 875 MeV [?]. Later, the bound nature was also confirmed with nearly physical quark masses ($m_\pi \simeq 146$ MeV and $m_K \simeq 525$ MeV) [?]. Using the interactions obtained from $(2+1)$-flavor lattice QCD simulations, K. Morita et al. studied the two-pair momentum correlation functions of the $p\Omega$ state in relativistic heavy-ion collisions to further investigate the existence of a $p\Omega$ bound state [?, ?]. This state has also been confirmed to be a bound state in the frameworks of the chromomagnetic model [?], QCD sum rules [?], and other quark models [?]. Additionally, studies on the production of the $p\Omega$ and $NN\Omega$ systems can be found in Refs. [?].

Furthermore, by analogy to the nucleon-nucleon and nucleon-antinucleon systems, one may expect attractive interactions in both the $p\Omega$ and $p\bar{\Omega}$ channels. If the $p\Omega$ state can be confirmed through further experimental measurements, we hope to observe an even stronger signal for the $p\bar{\Omega}$ state in experiments. The nucleon-antinucleon state would annihilate very quickly in the ground state due to the quark content of this system, making it challenging to provide a convincing theoretical confirmation of the nucleon-antinucleon bound state or resonance. In contrast, the $p\bar{\Omega}$ state cannot annihilate into the vacuum, as the nucleon consists of three $u(d)$ quarks and $\bar{\Omega}$ consists of three $\bar{s}$ quarks. In this context, the $p\bar{\Omega}$ state is expected to be relatively stable and may serve as an ideal system for studying baryon-antibaryon interactions. The copious production of anti-baryons in high-energy colliders offers excellent opportunities to study this type of spectrum theoretically. Clearly, the $p\bar{\Omega}$ system is both interesting and necessary, as it can provide valuable insights for experimental searches of baryon-antibaryon bound states.

In our previous work, we studied the $p\Omega$ interactions and correlation functions based on the quark delocalization color screening model (QDCSM) [?]. According to our calculations, the depletion of the $p\Omega$ correlation functions caused by the $J^P = 2^+$ bound state, which is not observed in the ALICE Collaboration's measurements [?], can be explained by the contribution of the attractive $J^P = 1^+$ component in spin-averaging. The QDCSM is a constituent quark model [?, ?] that introduces two key ingredients: first, quark delocalization, which accounts for orbital excitation by allowing quarks to delocalize from one cluster to another; second, the color screening factor, which modifies the confinement interaction between quarks in different cluster orbits. In the study of nucleon-nucleon and nucleon-hyperon interactions and the properties of the deuteron, the mechanism of quark delocalization and color screening plays a crucial role in generating intermediate-range attraction [?]. This model has also been used to investigate various dibaryon candidates, such as d^* [?], $p\Omega$ [?, ?], and others [?]. It has been extended to study baryon-antibaryon systems, including $p\bar{p}$ and $p\bar{\Lambda}$ [?, ?]. Extending it to the $p\bar{\Omega}$ system is a natural progression. Therefore, we continue to investigate the $p\bar{\Omega}$ system within the framework of the QDCSM. In this work, the $p\bar{\Omega}$ system is studied from three aspects: energy spectrum, scattering processes, and correlation functions.

This paper is organized as follows. A brief introduction of the QDCSM is given in the next section. The correlation function and the inverse scattering method are introduced in Sec. IIB and Sec. IIC, respectively. Sec. III is devoted to the numerical results and discussions. The summary is shown in the last section.

II. Theoretical Formalism

A. Quark Delocalization Color Screening Model

The details of the QDCSM employed in the present work can be found in Refs. [?]. Here, we present the salient features of the model. The model Hamiltonian is given by:

$$H = \sum_{i=1}^{6}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{\text{c.m.}} + \sum_{j>i=1}^{3} V_{qq}(r_{ij}) + \sum_{j>i=4}^{6} V_{\bar{q}\bar{q}}(r_{ij}) + \sum_{i=1}^{3}\sum_{j=4}^{6} V_{q\bar{q}}(r_{ij}),$$

where m_i is the quark mass, p_i is the momentum of the quark, and $T_{\text{c.m.}}$ is the center-of-mass kinetic energy. The dynamics of the hexaquark system is driven by two-body potentials, including color confinement (V_{CON}), perturbative one-gluon exchange interaction (V_{OGE}), and dynamical chiral symmetry breaking (V_χ):

$$V_{qq}(r_{ij}) = V_{\text{CON}}(r_{ij}) + V_{\text{OGE}}(r_{ij}) + V_\chi(r_{ij}).$$

Here, a phenomenological color screening confinement potential (V_{CON}) is used as:

$$V_{\text{CON}}(r_{ij}) = -a_c \lambda_i^c \cdot \lambda_j^c [f(r_{ij}) + V_0],$$

$$f(r_{ij}) = \begin{cases} r_{ij}^2 & \text{if } i, j \text{ occur in the same cluster} \\ \frac{1-e^{-\mu_{q_iq^j} r_{ij}^2}}{\mu_{q_iq^j}} & \text{if } i, j \text{ occur in different clusters} \end{cases}$$

where a_c, V_0, and $\mu_{q_iq^j}$ are model parameters, and λ^c stands for the SU(3) color Gell-Mann matrices. Among them, the color screening parameter $\mu_{q_iq^j}$ is determined by fitting the deuteron properties, nucleon-nucleon scattering phase shifts, and hyperon-nucleon scattering phase shifts, respectively, with μ_{qq} =

0.45, $\mu_{qs} = 0.19$, and $\mu_{ss} = 0.08$ fm^{-2}, satisfying the relation $\mu_{qs}^2 = \mu_{qq}\mu_{ss}$ [?]. The one-gluon exchange potential (V_{OGE}) is written as:

$$V_{\text{OGE}}(r_{ij}) = \alpha_s^{q_i q j} \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2}\delta(r_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_{im}j} \right) \right],$$

where σ is the Pauli matrices and α_s is the quark-gluon coupling constant. In order to cover the wide energy range from light to strange quarks, an effective scale-dependent quark-gluon coupling $\alpha_s(\mu)$ was introduced [?]:

$$\alpha_s(\mu) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)}.$$

Owing to the dynamical breaking of chiral symmetry, SU(3) Goldstone boson exchange interactions arise between the constituent light quarks u, d, and s. Accordingly, the chiral interaction is expressed as:

$$V_\chi(r_{ij}) = V_\pi(r_{ij}) + V_K(r_{ij}) + V_\eta(r_{ij}).$$

Among them,

$$V_\pi(r_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\pi^2}{12m_{im}j} \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} \times \left[Y(m_\pi r_{ij}) - \frac{\Lambda_\pi}{m_\pi} Y(\Lambda_\pi r_{ij}) \right] (\sigma_i \cdot \sigma_j)(\lambda_i^a \cdot \lambda_j^a),$$

$$V_K(r_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_K^2}{12m_{im}j} \frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} \times \left[Y(m_{Kr_{ij}}) - \frac{\Lambda_K}{m_K} Y(\Lambda_{Kr_{ij}}) \right] (\sigma_i \cdot \sigma_j)(\lambda_i^a \cdot \lambda_j^a),$$

$$V_\eta(r_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\eta^2}{12m_{im}j} \frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} \times \left[Y(m_\eta r_{ij}) - \frac{\Lambda_\eta}{m_\eta} Y(\Lambda_\eta r_{ij}) \right] (\sigma_i \cdot \sigma_j)[\cos\theta_p(\lambda_i^8 \cdot \lambda_j^8) - \sin\theta_p(\lambda_i^0 \cdot \lambda_j^0)],$$

where $Y(x) = e^{-x}/x$ is the standard Yukawa function. The physical η meson is considered by introducing the angle θ_p instead of the octet one. The λ^a are the SU(3) flavor Gell-Mann matrices. The values of m_π, m_K, and m_η are the masses of the SU(3) Goldstone bosons, which adopt the experimental values [?]. The chiral coupling constant g_{ch} is determined from the πNN coupling constant through $g_{ch}^2/(4\pi) = (3/5)^2 g_{\pi NN}^2/(4\pi) = 0.54$. Assuming that flavor SU(3) is an exact symmetry, it will only be broken by the different mass of the strange quark. The other symbols in the above expressions have their usual meanings.

As for $V_{\bar{q}\bar{q}}(r_{ij})$ and $V_{q\bar{q}}(r_{ij})$ in Eq. (1), which represent the antiquark-antiquark ($\bar{q}\bar{q}$) and quark-antiquark ($q\bar{q}$) interactions, for the antiquark we replace λ_i^c in Eqs. (3) and (4) with $-\lambda_i^{c*}$, and replace λ_i^a in Eqs. (7)-(9) with λ_i^{a*}. In this way, the forms of $V_{\bar{q}\bar{q}}$ and $V_{q\bar{q}}$ can be derived. It is noted that there is no annihilation between quark and antiquark. The reason is that the $p\bar{\Omega}$ state cannot annihilate to the vacuum due to the different quark flavor contents of N and $\bar{\Omega}$. All the parameters used in this work and the calculated baryon masses are listed in Table 1 and Table 2 , respectively. In the quark model, the corresponding antibaryons have the same masses as their baryon partners.

In addition, quark delocalization was introduced to enlarge the model variational space to take into account the mutual distortion or the internal excitations of nucleons in the course of interaction. It is realized by specifying the single-particle orbital wave function of the QDCSM as a linear combination of left and right Gaussians, the single-particle orbital wave functions used in the ordinary quark cluster model:

$$\psi_\alpha(S_i,\epsilon)=\frac{\phi_\alpha(S_i)+\epsilon\phi_\alpha(-S_i)}{N(\epsilon)},$$

$$\psi_\beta(-S_i,\epsilon)=\frac{\phi_\beta(-S_i)+\epsilon\phi_\beta(S_i)}{N(\epsilon)},$$

$$N(S_i,\epsilon)=\sqrt{1+\epsilon^2+2\epsilon e^{-S_i^2/4b^2}}.$$

It is worth noting that the mixing parameter ϵ is not an adjusted one but determined variationally by the dynamics of the multiquark system itself. In this way, the multiquark system chooses its favorable configuration in the interacting process. This mechanism has been used to explain the crossover transition between the hadron phase and quark-gluon plasma phase [**?**].

B. Two-Particle Correlation Function

Experimentally, the correlation function $C(k)$ can be measured based on:

$$C(k)=\xi(k)\frac{N_{\text{same}}(k)}{N_{\text{mixed}}(k)},$$

where $N_{\text{same}}(k)$ and $N_{\text{mixed}}(k)$ represent the k distributions of hadron-hadron pairs produced in the same and in different collisions, respectively, and $\xi(k)$ denotes the corrections for experimental effects. In theoretical studies, the correlation function can be calculated using the Koonin-Pratt (KP) formula [**?**]:

$$C(k)=\frac{N_{12}(p_1,p_2)}{N_1(p_1)N_2(p_2)}=\int d^4x_1d^4x_2S_1(x_1,p_1)S_2(x_2,p_2)|\Psi(\mathbf{r},\mathbf{k})|^2,$$

where $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{k}$ is the relative momentum. The source function $S_{12}(\mathbf{r})$ is usually assumed to be a Gaussian form:

$$S_{12}(\mathbf{r}) = \frac{1}{(2\sqrt{\pi}R)^3} e^{-\mathbf{r}^{2/4R^2}},$$

where R is the size parameter of the source. Thus, two important factors of the correlation function are included in Eq. (13): the collision system, which is related to the source function $S_{12}(\mathbf{r})$, and the two-particle interaction, which is embedded in the relative wave function $\Psi(\mathbf{r}, \mathbf{k})$.

For a pair of non-identical particles, such as $p\bar{\Omega}$, assuming that only the S-wave part of the wave function is modified by the two-particle interaction, $\Psi(\mathbf{r}, \mathbf{k})$ can be given by:

$$\Psi_{p\bar{\Omega}}(\mathbf{r}, \mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{r}} - j_0(kr) + \psi_{p\bar{\Omega}}(\mathbf{r}, \mathbf{k}),$$

where the spherical Bessel function $j_0(kr)$ represents the S-wave part of the non-interacting wave function, and $\psi_{p\bar{\Omega}}$ stands for the scattering wave function affected by the two-particle interaction. Substituting the relative wave function $\Psi_{p\bar{\Omega}}(\mathbf{r}, \mathbf{k})$ into the KP formula yields the correlation function:

$$C_{p\bar{\Omega}}(k) = 1 + 4\pi \int_0^\infty r^2 dr S_{12}(r) \left[|\psi_{p\bar{\Omega}}(r, k)|^2 - |j_0(kr)|^2 \right].$$

The $\psi_{p\bar{\Omega}}(r, k)$ can be obtained by solving the Schrödinger equation, and a similar approach has been utilized in the femtoscopic correlation analysis tool using the Schrödinger equation [**?**]:

$$\nabla^2 \psi_{p\bar{\Omega}}(\mathbf{r}, \mathbf{k}) + 2\mu[V(\mathbf{r}) - E]\psi_{p\bar{\Omega}}(\mathbf{r}, \mathbf{k}) = 0,$$

where $\mu = m_p m_\Omega/(m_p + m_\Omega)$ is the reduced mass of the system.

Considering the case of the S-wave, the wave function can be separated into a radial term $R_k(r)$ and an angular term $Y_0^0(\theta, \phi)$ and expressed as:

$$\psi_{p\bar{\Omega}}(\mathbf{r}, \theta, \phi) = R_k(r) Y_0^0(\theta, \phi).$$

Considering the interaction between a proton and an $\bar{\Omega}$ baryon, which includes both the strong interaction and the repulsive Coulomb interaction, the potential can be written as:

$$V(r) = V_{\text{Strong}}(r) + V_{\text{Coulomb}}(r),$$

where $V_{\text{Coulomb}}(r) = +\alpha_{\text{em}}/r$, and α_{em} is the fine-structure constant. The method to obtain the strong interaction potential $V_{\text{Strong}}(r)$ will be introduced in the next section.

Once the total interaction potential is determined, the radial Schrödinger equation can be solved:

$$\frac{d^2 u_k(r)}{dr^2} + \left[k^2 - 2\mu V(r)\right] u_k(r) = 0,$$

where $E = \hbar^2 k^2/(2\mu)$ and $u_k(r) = rR_k(r)$. On this basis, the correlation function $C_{p\bar{\Omega}}(k)$ for given spin-parity quantum numbers can be calculated through Eq. (16). The calculation of the correlation functions described above is based on obtaining the scattering wave functions by solving the Schrödinger equation in coordinate space [?, ?, ?, ?, ?]. Additionally, the scattering wave functions can also be obtained by solving the Lippmann-Schwinger (Bethe-Salpeter) equation in momentum space [?, ?, ?, ?]. Further details on correlation functions for various systems can be found in the references mentioned above.

Additionally, for the S-wave $p\bar{\Omega}$ dibaryon system, the possible spin-parity quantum numbers are $J^P = 1^-$ and 2^-, respectively. Since the experimentally measured correlation function is spin-averaged, the theoretically obtained correlation function should also consider the average over systems with different quantum numbers:

$$C_{p\bar{\Omega}}(k) = \frac{1}{2}\left[C_{p\bar{\Omega}}^{J=1}(k) + C_{p\bar{\Omega}}^{J=2}(k)\right].$$

C. Gel'fand-Levitan-Marchenko Method for Inverse Scattering Problem

To solve Eq. (20), a two-body interaction potential $V(r)$ is absolutely necessary. The QDCSM is actually a treatment for few-body problems, which means directly extracting a two-body interaction potential $V(r)$ from it will not be so natural. Hence, the QDCSM can be employed to investigate the scattering processes, from which the desired potential can be obtained, since the hadronization is fully incorporated in the model.

The approach we adopted to extract the two-body equivalent potential $V(r)$ is the GLM method, which is a very powerful tool in inverse scattering theory [?]. It can provide us a systematic approach to reconstruct an equivalent potential from the scattering data of a specific process, which makes it a very classical "inverse problem". Thus, this method will give us another path to understand the nature of two-body interaction.

The key equation of the GLM method used in the work is the Marchenko equation [?, ?], which can be written in the S-wave case in an integration equation form as:

$$K(r,r') + F(r,r') + \int_r^{\infty} K(r,s)F(s,r')ds = 0.$$

Here, the kernel function $K(r,r')$ is the solution of the equation to be determined, and $F(r,r')$ is the inverse Fourier transformation of the reflection coefficient as:

$$F(r,r') = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikr}\{1 - S(k)\}e^{ikr'} + \sum_{i=1}^{n} M_i e^{-\kappa_i r} e^{-\kappa_i r'},$$

where the partial-wave scattering matrix $S(k)$ is given by $S(k) = \exp(2i\delta(k))$, with $\delta(k)$ the scattering phase shift satisfying $k \cot\delta = -1/a_0 + \frac{1}{2}r_{\text{eff}}k^2$. Here, a_0 and r_{eff} represent the scattering length and the effective range, respectively. Additionally, n is the number of bound states, κ_i denotes the wavenumber of the i-th bound state, and M_i is the norming constant. Then, after solving the Marchenko equation and obtaining $K(r,r')$, the potential can be reconstructed as:

$$V(r) = -2\frac{d}{dr}K(r,r).$$

There is one point we want to emphasize here. Generally, when there exist bound states, this method cannot give us a fully determined potential but ends up with a set of phase-equivalent potentials [**?**]. However, if one fixes all the M_i in a unique way such as calculating from Jost solution, the obtained potential will be unique for further calculation [**?**, **?**]. By using this method, preparation for further calculation can be done. For a more comprehensive discussion on this method, one can refer to Refs. [**?**].

III. Results and Discussion

The S-wave $p\bar{\Omega}$ systems with isospin $I = 1/2$, spin parity $J^P = 1^-$ and 2^- are investigated on the basis of the QDCSM. In order to see whether or not there is any bound state, a dynamic calculation is performed as a first step. The resonating group method (RGM) is employed to solve the bound-state problem. In this approach, the total wave function of the six-quark (three quarks and three antiquarks) system is constructed as:

$$\Psi = \mathcal{A}\phi_p(\xi_1)\phi_{\bar{\Omega}}(\xi_2)\chi(\mathbf{R}),$$

where ϕ_p and $\phi_{\bar{\Omega}}$ are the internal wave functions of the proton and $\bar{\Omega}$ clusters, $\chi(\mathbf{R})$ represents the relative motion wave function, and $\mathcal{A}$ is the antisymmetrization operator accounting for quark exchange effects. The RGM equation is derived by projecting the Schrödinger equation onto the cluster basis, leading to a coupled integro-differential equation:

$$\int [H(\mathbf{R}, \mathbf{R}') - EN(\mathbf{R}, \mathbf{R}')]\chi(\mathbf{R}')d\mathbf{R}' = 0,$$

where H and N are the Hamiltonian and normalization kernels. The relative wave function $\chi(\mathbf{R})$ is expanded with Gaussian basis functions, converting the integral equation into a generalized eigenvalue problem. Solving this equation provides the binding energies and wave functions of the $p\bar{\Omega}$ states. This method has been widely and successfully applied to baryon-baryon interactions and multiquark systems [**?**].

The binding energies of the $p\bar{\Omega}$ systems with $J^P = 1^-$ and $J^P = 2^-$, denoted as E_B, are listed in Table 3 . Here, E_{Theo} represents the theoretical threshold, and E_{Theo} represents the eigenvalue of the corresponding system. The calculation for the $p\bar{\Omega}$ systems does not involve channel coupling, because we limit our study to color-singlet sub-clusters consisting of three u/d quarks and three $\bar{s}$ quarks.

From Table 3, we can find that both the $J^P = 1^-$ and $J^P = 2^-$ $p\bar{\Omega}$ form bound states with binding energies of about 10 MeV and 9 MeV, respectively. By contrast, in our previous work on the $p\Omega$ systems, the single-channel calculation shows that neither the $J^P = 1^+$ nor $J^P = 2^+$ $p\Omega$ is bound. After channel coupling, only the $p\Omega$ with $J^P = 2^+$ forms a bound state with binding energy about 6 MeV. According to the results obtained with the second parameter set, the binding energies of the $J^P = 1^-$ and $J^P = 2^-$ $p\bar{\Omega}$ states are about 18 MeV and 16 MeV, respectively. The corresponding binding energy of the $p\Omega$ state with $J^P = 2^+$ after channel coupling is about 14 MeV. These numerical results indicate that it is more possible for the $p\bar{\Omega}$ system rather than the $p\Omega$ system to form bound states in our calculations. Therefore, considering that the attractive $p\Omega$ interaction is implied in the experimental measurements of $p\Omega$ correlation functions [**?**], we look forward to the experimental progress on $p\bar{\Omega}$ correlation functions in the future.

In order to further study the interaction between nucleon and $\bar{\Omega}$, we calculated the scattering phase shifts of the $p\bar{\Omega}$ systems. The calculation is based on the well-developed Kohn-Hulthen-Kato (KHK) variational method; the details of this method can be found in Refs. [**?**, **?**]. The low-energy scattering phase shifts of the $p\bar{\Omega}$ systems with two parameter sets are shown in Fig. 1 [Figure 1: see original paper]. For the $p\bar{\Omega}$ systems with both $J^P = 1^-$ and $J^P = 2^-$, the scattering phase shifts approach 180° when $E_{\text{c.m.}} \to 0$ MeV and rapidly decrease when $E_{\text{c.m.}}$ increases, which indicates the existence of a bound state in these systems. This conclusion is consistent with the bound state calculation discussed earlier.

Then, we can extract the scattering length a_0 and the effective range r_{eff} of the $p\bar{\Omega}$ systems from the low-energy phase shifts obtained above by using the expansion:

$$k \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^4),$$

where k is the momentum of the relative motion with $k = \sqrt{2\mu E_{\text{c.m.}}}$, μ is the reduced mass of two baryons, and $E_{\text{c.m.}}$ is the incident energy; δ is the low-energy scattering phase shift. And the binding energy E'_B can be calculated according to the following relation:

$$E'_B = \frac{\hbar^2 \alpha^2}{2\mu},$$

where α is the wave number which can be obtained from the relation [?]:

$$\alpha = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} \alpha^2.$$

Note that this is another way to calculate the binding energy; therefore it is labeled E'_B. The scattering parameters of the $p\bar{\Omega}$ systems, along with the binding energies obtained using the scattering parameters, are listed in Table 4 .

From Table 4, our results show that the scattering lengths are positive for the $p\bar{\Omega}$ systems with $J^P = 1^-$ and 2^-, which also confirms the existence of bound states. Besides, the binding energies of the two systems obtained by Eq. (28) are broadly consistent with the numerical results shown in Table 3, which are obtained by the dynamic calculation. Additionally, in the method of obtaining the binding energies using scattering parameters, the binding energies of the two $p\bar{\Omega}$ systems are also deeper than that of the $p\Omega$ system with $J^P = 2^+$.

Moreover, by solving the inverse scattering problem, we can further study the behavior of the $p\bar{\Omega}$ correlation functions on the basis of the $p\bar{\Omega}$ scattering process and the KP formula in Eq. (13). Before that, we can study the general properties of the $p\bar{\Omega}$ correlation functions through an effective square well potential model. The correlation functions corresponding to different degrees of square well potentials are presented in Fig. 2 [Figure 2: see original paper]. The solid red lines represent the correlation functions influenced only by the repulsive Coulomb interaction. The dashed blue lines represent the correlation functions influenced only by square well potentials. We introduce three square well potentials with width $r_0 = 2$ fm: weak attraction in panel (a) corresponding to $V_0 = -10$ MeV, moderate attraction in panel (b) corresponding to $V_0 = -28$ MeV, and relatively strong attraction in panel (c) corresponding to $V_0 = -40$ MeV. The dotted black lines represent the correlation functions influenced by both Coulomb interaction and square well potentials through Eq. (19). Additionally, we adopt a source size parameter $R = 0.95$ fm in Eq. (14), which is the same value used in our previous $p\Omega$ correlation analysis [?]. This value was originally extracted by the ALICE Collaboration [?, ?]. It should be emphasized that this source size is determined by the specific collision system and experimental conditions in heavy-ion collisions.

In Fig. 2 [Figure 2: see original paper], panels (a) and (b), the correlation function affected only by the square potential is above unity in the low-energy region, which is due to the attractive interaction. The difference is that the weak attraction is not enough to form a bound state; therefore the correlation function is always above unity, while the moderate attraction forms a shallow bound state. The existence of the bound state leads to the depletion of the correlation function, so there exists a part below unity. After taking into account both the Coulomb interaction and square well potentials, which mainly dominate the low-energy region ($0 < k < 25$ MeV), the correlation function forms a peak-like structure. In panel (c), the correlation function remains below unity for a relatively strong attraction. A discussion about this phenomenon can be found in Refs. [?, ?]. After considering the Coulomb interaction, one can see that the correlation function in the low-energy region nearly coincides with the result obtained by considering only the Coulomb interaction. As the relative momentum k increases, the correlation function closely matches the result obtained by considering only a relatively strong attraction.

After replacing the square well potentials with the effective potentials obtained by solving the inverse scattering problem using the GLM method, which is briefly introduced in Sec. IIC, we can study the correlation function of the $p\bar{\Omega}$ systems. As the two parameter sets give similar results, the discussion of the correlation function is presented based on the first parameter set for conciseness. Both the effective potentials of the $p\bar{\Omega}$ systems with $J^P = 1^-$ and 2^- are obtained. The total $p\bar{\Omega}$ correlation function is the superposition of the correlation functions corresponding to the two quantum numbers according to Eq. (21). Since the $p\bar{\Omega}$ system forms bound states for both quantum numbers and the interactions are similar, we omit the comparison of the correlation functions for the two quantum numbers here. The total correlation functions calculated for different values of source size parameter R are shown in Fig. 3 [Figure 3: see original paper]. In addition, recent studies on the emission source properties can be found in Refs. [?].

In Fig. 3 [Figure 3: see original paper], panels (a)-(e), the dashed gray lines and the dotted orange lines represent the $p\bar{\Omega}$ correlation functions considering only the Coulomb interaction and both the Coulomb interaction and strong interaction, respectively. In panel (f), the gray band stands for the correlation functions influenced only by the Coulomb interaction with size parameter R ranging from 1.0 to 2.5 fm, while the other lines summarize the correlation functions shown in panels (a)-(e). According to our results, the change of the size parameter R can greatly influence the $p\bar{\Omega}$ correlation functions. An obvious feature is that as R increases, the peak-like structure caused by the different dominant regions of the Coulomb interaction and $p\bar{\Omega}$ strong interaction gradually becomes less obvious and eventually disappears. Since two bound states are obtained in our calculation, it is very important to verify this conclusion in the correlation functions. It can be seen that as the correlation function influenced only by the Coulomb interaction gradually approaches unity, the depletion caused by the bound states leads to the correlation function being below that of the Coulomb-only case.

While femtoscopic measurements in heavy-ion collisions provide important access to the $p\bar{\Omega}$ interaction, such bound states may also be produced in other high-energy environments capable of forming multi-strange baryon-antibaryon pairs, such as pp and e^+e^- collisions, as well as high-energy fixed-target experiments, although the production probability may vary across systems. If a $p\bar{\Omega}$ bound state is formed, it can decay through the weak decay of the $\bar{\Omega}$, leading to final states such as $p\bar{\Omega} \to p\bar{\Lambda}K$ and $p\bar{\Omega} \to p\bar{\Xi}\pi$, or through quark rearrangement into a three-meson final state, $p\bar{\Omega} \to KKK$, which may provide complementary detection signatures beyond femtoscopy in future searches.

In recent years, experimental data on correlation functions have increased rapidly [**?**], providing unprecedented insights into hadron-hadron interactions across a variety of systems. Together with femtoscopic techniques, these studies open up new possibilities for extracting low-energy scattering parameters that are difficult to access otherwise. On the theoretical side, continuous progress in lattice QCD, effective field theory, and quark-model-based approaches has greatly enriched our understanding and offered valuable guidance for interpreting the experimental observations [**?**]. The synergy between experimental measurements and theoretical developments will not only deepen our knowledge of the strong interaction, but also pave the way for future explorations of exotic hadronic states and nuclear physics [**?**]. In this context, it is well established that nucleon-nucleon and hyperon-nucleon interactions provide the basis for the formation of nuclei and hypernuclei [**?**], while antihyperon-antinucleon interactions give rise to anti-hypernuclei [**?**, **?**]. An open question is whether antihyperon-nucleon interactions could also give rise to the formation of novel nuclear systems. As a continuation of the present study, we will further explore such systems based on the antihyperon-nucleon interactions obtained in this work.

IV. Summary

In this work, we investigate the S-wave $p\bar{\Omega}$ systems with isospin $I = 1/2$, spin parity $J^P = 1^-$ and 2^- in the framework of the QDCSM. The results show that the $p\bar{\Omega}$ systems with both $J^P = 1^-$ and 2^- form bound states, and the attraction between nucleon and $\bar{\Omega}$ is slightly larger than that between nucleon and Ω, which suggests that the $p\bar{\Omega}$ system has a higher likelihood of forming bound states than the $p\Omega$ system. The calculation of the low-energy scattering phase shifts and scattering parameters of the $p\bar{\Omega}$ systems also supports the existence of the $p\bar{\Omega}$ bound states with $J^P = 1^-$ and 2^-. Besides, considering that the nucleon is composed of three light quarks $u(d)$ and $\bar{\Omega}$ of three strange quarks $\bar{s}$, the $p\bar{\Omega}$ state cannot annihilate to the vacuum. In this context, the $p\bar{\Omega}$

state is a special state, which can provide useful information for the experimental search of baryon-antibaryon bound states.

By using the GLM method, we solve the inverse scattering problem and obtain the effective $p\bar{\Omega}$ potentials. On this basis, the $p\bar{\Omega}$ correlation functions are calculated, taking into account both the Coulomb interaction and spin averaging. We present correlation functions corresponding to different source size parameters R, which can be used for future comparison with experimental measurements.

Understanding hadron-hadron interactions is one of the important issues in the study of hadron physics. The study of the interaction between baryons and antibaryons in this work is also an effective place to test this mechanism. The femtoscopic correlation function has become one of the important ways to explore hadron-hadron interactions, and further theoretical and experimental investigations are essential for a deeper understanding of such baryon-antibaryon systems.

References

[1] E. Fermi and C. N. Yang, Phys. Rev. 76, 1739 (1949).
[2] K. Erkelenz, Phys. Rept. 13, 191 (1974).
[3] J. M. Richard, Nucl. Phys. B Proc. Suppl. 86, 361 (2000).
[4] M. Ablikim et al. [BESIII], Phys. Rev. Lett. 132, 151901 (2024).
[5] S. G. Salnikov and A. I. Milstein, Nucl. Phys. B 1002, 116539 (2024).
[6] Y. Xiao, J. X. Lu and L. S. Geng, Phys. Rev. C 110, 6 (2024).
[7] M. Karliner and J. L. Rosner, Phys. Rev. D 110, 094058 (2024).
[8] P. Y. Niu, Z. Y. Zhang, Y. Y. Li, Q. Wang and Q. Zhao, Phys. Rev. D 110, 094020 (2024).
[9] P. G. Ortega, D. R. Entem, F. Fernandez and J. Segovia, Phys. Lett. B 862, 139281 (2025).
[10] Z. S. Jia, Z. H. Zhang, F. K. Guo and G. Li, Phys. Rev. D 111, 054014 (2025).
[11] Q. H. Yang, L. Y. Dai and U. G. Meißner, [arXiv:2412.07599 [hep-ph]].
[12] M. Ablikim et al. [BESIII], Phys. Rev. Lett. 131, 151901 (2023).
[13] T. G. Li, S. C. Zhang, G. Y. Wang and Q. F. Lü, Phys. Rev. D 110, 114020 (2024).
[14] J. Haidenbauer and U. G. Meißner, Eur. Phys. J. A 60, 119 (2024).
[15] X. H. Zhang, S. Q. Zhang and C. F. Qiao, Eur. Phys. J. C 85, 693 (2025).
[16] J. Adam et al. [STAR], Phys. Lett. B 790, 490 (2019).
[17] A. Collaboration et al. [ALICE], Nature 588, 232 (2020) [erratum: Nature 590, E13 (2021)].
[18] L. Adamczyk et al. [STAR], Nature 527, 345 (2015).
[19] S. Acharya et al. [ALICE], Phys. Rev. Lett. 124, 092301 (2020).
[20] L. Fabbietti, V. Mantovani Sarti and O. Vazquez Doce, Ann. Rev. Nucl. Part. Sci. 71, 377 (2021).
[21] S. Acharya et al. [ALICE], Phys. Rev. Lett. 127, 172301 (2021).
[22] S. Acharya et al. [ALICE], Phys. Lett. B 833, 137272 (2022).

[23] S. Acharya et al. [ALICE], Phys. Rev. D 106, 052010 (2022).
[24] B. E. Aboona et al. [STAR], Phys. Lett. B 864, 139412 (2025).
[25] A. Ohnishi, Y. Hirata, Y. Nara, S. Shinmura and Y. Akaishi, Nucl. Phys. A 670, 297 (2000).
[26] K. Morita, T. Furumoto and A. Ohnishi, Phys. Rev. C 91, 024916 (2015).
[27] A. Ohnishi, K. Morita, K. Miyahara and T. Hyodo, Nucl. Phys. A 954, 294 (2016).
[28] T. Hatsuda, K. Morita, A. Ohnishi and K. Sasaki, Nucl. Phys. A 967, 856 (2017).
[29] J. Haidenbauer, Nucl. Phys. A 981, 1 (2019).
[30] Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi and W. Weise, Phys. Rev. Lett. 124, 132501 (2020).
[31] J. Haidenbauer, G. Krein and T. C. Peixoto, Eur. Phys. J. A 56, 184 (2020).
[32] A. Ohnishi, Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita and K. Ogata, Few Body Syst. 62, 42 (2021).
[33] K. Ogata, T. Fukui, Y. Kamiya and A. Ohnishi, Phys. Rev. C 103, 065205 (2021).
[34] S. Mrówczyński and P. Słoń, Phys. Rev. C 104, 024909 (2021).
[35] Ł. K. Graczykowski and M. A. Janik, Phys. Rev. C 104, 054909 (2021).
[36] Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi and T. Hatsuda, Phys. Rev. C 105, 014915 (2022).
[37] J. Haidenbauer and U. G. Meißner, Phys. Lett. B 829, 137074 (2022).
[38] Z. W. Liu, K. W. Li and L. S. Geng, Chin. Phys. C 47, 024108 (2023).
[39] Z. W. Liu, J. X. Lu and L. S. Geng, Phys. Rev. D 107, 074019 (2023).
[40] Z. W. Liu, J. X. Lu, M. Z. Liu and L. S. Geng, Phys. Rev. D 108, L031503 (2023).
[41] R. Molina, Z. W. Liu, L. S. Geng and E. Oset, Eur. Phys. J. C 84, 328 (2024).
[42] I. Vidana, A. Feijoo, M. Albaladejo, J. Nieves and E. Oset, Phys. Lett. B 846, 138201 (2023).
[43] V. M. Sarti, A. Feijoo, I. Vidaña, A. Ramos, F. Giacosa, T. Hyodo and Y. Kamiya, Phys. Rev. D 110, L011505 (2024).
[44] R. Molina, C. W. Xiao, W. H. Liang and E. Oset, Phys. Rev. D 109, 054002 (2024).
[45] M. Albaladejo, A. Feijoo, I. Vidaña, J. Nieves and E. Oset, Eur. Phys. J. A 61, 187 (2025).
[46] A. Feijoo, L. R. Dai, L. M. Abreu and E. Oset, Phys. Rev. D 109, 016014 (2024).
[47] H. P. Li, J. Y. Yi, C. W. Xiao, D. L. Yao, W. H. Liang and E. Oset, Chin. Phys. C 48, 053107 (2024).
[48] A. Feijoo, M. Korwieser and L. Fabbietti, Phys. Rev. D 111, 014009 (2025).
[49] N. Ikeno, G. Toledo and E. Oset, Phys. Lett. B 847, 138281 (2023).
[50] M. Albaladejo, J. Nieves and E. Ruiz-Arriola, Phys. Rev. D 108, 014020 (2023).
[51] J. M. Torres-Rincon, À. Ramos and L. Tolos, Phys. Rev. D 108, 096008

(2023).
[52] L. M. Abreu, P. Gubler, K. P. Khemchandani, A. Martinez Torres and A. Hosaka, Phys. Lett. B 860, 139175 (2025).
[53] H. P. Li, C. W. Xiao, W. H. Liang, J. J. Wu, E. Wang and E. Oset, Phys. Rev. D 110, 114018 (2024).
[54] M. Albaladejo, A. Feijoo, J. Nieves, E. Oset and I. Vidaña, Phys. Rev. D 110, 114052 (2024).
[55] F. Etminan, Phys. Rev. C 111, 014912 (2025).
[56] A. Jinno, Y. Kamiya, T. Hyodo and A. Ohnishi, J. Subatomic Part. Cosmol. 1, 100005 (2024).
[57] F. Etminan, Phys. Lett. B 866, 139564 (2025).
[58] J. T. Goldman, K. Maltman, G. J. Stephenson, K. E. Schmidt and F. Wang, Phys. Rev. Lett. 59, 627 (1987).
[59] M. Oka, Phys. Rev. D 38, 298 (1988).
[60] F. Etminan et al. [HAL QCD], Nucl. Phys. A 928, 89 (2014).
[61] T. Iritani et al. [HAL QCD], Phys. Lett. B 792, 284 (2019).
[62] K. Morita, A. Ohnishi, F. Etminan and T. Hatsuda, Phys. Rev. C 94, 031901 (2016).
[63] K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya and A. Ohnishi, Phys. Rev. C 101, 015201 (2020).
[64] B. Silvestre-Brac and J. Leandri, Phys. Rev. D 45, 4221 (1992).
[65] X. H. Chen, Q. N. Wang, W. Chen and H. X. Chen, Phys. Rev. D 103, 094011 (2021).
[66] L. R. Dai, D. Zhang, C. R. Li and L. Tong, Chin. Phys. Lett. 24, 389 (2007).
[67] H. Huang, J. Ping and F. Wang, Phys. Rev. C 92, 065202 (2015).
[68] Q. B. Li and P. N. Shen, Eur. Phys. J. A 8, 417 (2000).
[69] S. Zhang and Y. G. Ma, Phys. Lett. B 811, 135867 (2020).
[70] J. Pu, K. J. Sun, C. W. Ma and L. W. Chen, Phys. Rev. C 110, 024908 (2024).
[71] L. Zhang, S. Zhang and Y. G. Ma, Eur. Phys. J. C 82, 416 (2022).
[72] Y. Yan, Q. Huang, Y. Yang, H. Huang and J. Ping, Sci. China Phys. Mech. Astron. 68, 232012 (2025).
[73] F. Wang, G. h. Wu, L. j. Teng and J. T. Goldman, Phys. Rev. Lett. 69, 2901 (1992).
[74] G. H. Wu, L. J. Teng, J. L. Ping, F. Wang and J. T. Goldman, Phys. Rev. C 53, 1161 (1996).
[75] J. L. Ping, F. Wang and J. T. Goldman, Nucl. Phys. A 657, 95 (1999).
[76] G. h. Wu, J. L. Ping, L. j. Teng, F. Wang and J. T. Goldman, Nucl. Phys. A 673, 279 (2000).
[77] H. R. Pang, J. L. Ping, F. Wang and J. T. Goldman, Phys. Rev. C 65, 014003 (2002).
[78] J. L. Ping, H. X. Huang, H. R. Pang, F. Wang and C. W. Wong, Phys. Rev. C 79, 024001 (2009).
[79] Y. Yan, Y. Wu, X. Hu, H. Huang and J. Ping, Phys. Rev. D 105, 014027 (2022).

[80] Y. Yan, X. Hu, Y. Wu, H. Huang, J. Ping and Y. Yang, Eur. Phys. J. C 83, 524 (2023).
[81] Y. Yan, Y. Wu, H. Huang, J. Ping and X. Zhu, Eur. Phys. J. C 83, 610 (2023).
[82] Y. Yan, X. Hu, H. Huang and J. Ping, Phys. Rev. D 108, 094045 (2023).
[83] Y. Yan, H. Huang, X. Zhu and J. Ping, Phys. Rev. D 109, 034036 (2024).
[84] H. Huang, H. Pang and J. Ping, Mod. Phys. Lett. A 26, 1231 (2011).
[85] H. Huang, J. Ping and F. Wang, Mod. Phys. Lett. A 27, 1250039 (2012).
[86] M. Chen, H. Huang, J. Ping and F. Wang, Phys. Rev. C 83, 015202 (2011).
[87] J. Vijande, F. Fernandez and A. Valcarce, J. Phys. G 31, 481 (2005).
[88] S. Navas et al. [Particle Data Group], Phys. Rev. D 110, 030001 (2024).
[89] M. Xu, M. Yu and L. Liu, Phys. Rev. Lett. 100, 092301 (2008).
[90] S. E. Koonin, Phys. Lett. B 70, 43 (1977).
[91] S. Pratt, T. Csorgo and J. Zimanyi, Phys. Rev. C 42, 2646 (1990).
[92] W. Bauer, C. K. Gelbke and S. Pratt, Ann. Rev. Nucl. Part. Sci. 42, 77 (1992).
[93] D. L. Mihaylov, V. Mantovani Sarti, O. W. Arnold, L. Fabbietti, B. Hohlweger and A. M. Mathis, Eur. Phys. J. C 78, 394 (2018).
[94] K. Chadan and P. C. Sabatier, Inverse Problems in Quantum Scattering Theory, 2nd ed. (Springer, New York, 1989).
[95] V. A. Marchenko, Dokl. Akad. Nauk SSSR, 104, 695 (1955).
[96] Z. S. Agranovich and V. A. Marchenko, The Inverse Problem of the Scattering Theory (Gordon and Breach, New York, 1963).
[97] S. A. Sofianos, A. Papastylianos, H. Fiedeldey and E. O. Alt, Phys. Rev. C 42, R506 (1990).
[98] S. E. Massen, S. A. Sofianos, S. A. Rakityansky and S. Oryu, Nucl. Phys. A 654, 597 (1999).
[99] R. G. Newton, Scattering Theory of Waves and Particles, 2nd ed. (Dover, New York, 2002).
[100] L. Jade, M. Sander and H. V. von Geramb, Lect. Notes Phys. 488, 124 (1997).
[101] N. A. Khokhlov and L. I. Studenikina, Phys. Rev. C 104, 014001 (2021).
[102] N. A. Khokhlov, Phys. Rev. C 107, 044001 (2023).
[103] J. A. Wheeler, Phys. Rev. 52, 1083 (1937).
[104] M. Kamimura, Prog. Theor. Phys. Suppl. 62, 236 (1977).
[105] Y. Yan, Q. Huang, X. Zhu, H. Huang and J. Ping, Phys. Rev. D 110, 014021 (2024).
[106] V. A. Babenko and N. M. Petrov, Phys. Atom. Nucl. 66, 1319 (2003).
[107] S. Acharya et al. [ALICE], Phys. Lett. B 811, 135849 (2020).
[108] S. Acharya et al. [ALICE], Eur. Phys. J. C 85, 198 (2025).
[109] D. Mihaylov and J. González González, Eur. Phys. J. C 83, 590 (2023).
[110] D. F. Wang, M. Y. Chen, Y. G. Ma, Q. Y. Shou, S. Zhang and L. Zheng, Nucl. Sci. Tech. 36, 154 (2025).
[111] O. Vázquez Doce, D. Mihaylov and L. Fabbietti, Eur. Phys. J. A 61, 53 (2025).
[112] P. Braun-Munzinger and B. Dönigus, Nucl. Phys. A 987, 144 (2019).

[113] J. H. Chen, J. Chen, F. K. Guo, Y. G. Ma, C. P. Shen, Q. Y. Shou, Q. Shou, Q. Wang, J. J. Wu and B. S. Zou, Nucl. Sci. Tech. 36, 55 (2025).
[114] D. Johnson, I. Polyakov, T. Skwarnicki and M. Wang, Ann. Rev. Nucl. Part. Sci. 74, 583 (2024).
[115] T. Shao et al. [A1], Phys. Rev. Lett. 134, 162501 (2025).
[116] J. Haidenbauer, U. G. Meißner and A. Nogga, Eur. Phys. J. A 56, 91 (2020).
[117] B. Aboona et al. [STAR], Phys. Rev. Lett. 130, 212301 (2023).
[118] J. Chen, X. Dong, Y. G. Ma and Z. Xu, Sci. Bull. 68, 3252 (2023).
[119] S. Acharya et al. [ALICE], Phys. Rev. Lett. 131, 102302 (2023).
[120] Y. G. Ma, Nuc. Sci. Tech. 34, 97 (2023).
[121] J. H. Chen, L. S. Geng, E. Hiyama, Z. W. Liu and J. Pochodzalla, Chin. Phys. Lett. 42, 100101 (2025).
[122] B. I. Abelev et al. [STAR], Science 328, 58 (2010).
[123] M. Abdulhamid et al. [STAR], Nature 632, 1026 (2024).

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