

Effect of spin-orbit interaction on structure and two-proton radioactivity of ^{18}Mg

Authors: Yan-zhao, Prof. Wang, Li, Dr. Jie, Zhang, Dr. Wenhao, Dr. Yueqing Li, Prof. Gu Jianzhong, Prof. Wang Yan-zhao

Date: 2025-11-10T17:11:29+00:00

Abstract

The influence of spin-orbit coupling strength (W) on the structure and two-proton ($2p$) radioactivity of ^{18}Mg is investigated using the spherical Skyrme-Hartree-Fock-Bogoliubov (SHFB) approach with the SLy4 interaction and a mean-field cluster potential model. Our calculations reveal that increasing W enhances the splitting of the single-proton $1d$ orbitals. Concurrently, the $2s_{1/2}$ proton state evolves from a weakly bound state into a resonance in the continuum. As W increases, the occupation probability of the $2s_{1/2}$ proton state decreases, and its density distribution near the nuclear surface becomes less diffuse. Furthermore, both the spectroscopic factor S'_{2p} and the decay energy Q_{2p} for $2p$ radioactivity gradually decrease with increasing W , resulting in a longer half-life. When Q_{2p} is held constant, the half-life is significantly enhanced by S'_{2p} . At the mean time, it is found that the depth of the diproton cluster potential well increases with W , while the corresponding S'_{2p} becomes smaller, indicating that the diproton cluster is considerably looser than the α -cluster. Additionally, a clear linear correlation is observed between $\log_{10}S'_{2p}$ and Q_{2p} , as well as between $\log_{10}S'_{2p}$ and W . The logarithmic half-lives, both with and without inclusion of S'_{2p} , exhibit good linear relationships with W and $Q_{2p}^{-1/2}$, respectively. Finally, using the experimental Q_{2p} value of ^{18}Mg , the optimal W is determined to be $1.15W_0$ with $W_0=123 \text{ MeV fm}^5$.

Full Text

Preamble

Effect of Spin-Orbit Interaction on Structure and Two-Proton Radioactivity of ^{18}Mg

Yanzhao Wang (王艳召)^{1,2,3,4,5,*}, Jie Li (李婕)^{1,2,3,6}, Wenhao Zhang (张文昊)^{1,2,3}, Yueqing Li (李月晴)^{1,2,3}, and Jianzhong Gu (顾建中)^{5†}

¹School of Physics, North China Electric Power University, Baoding 071003, China

²Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

³Hebei Key Laboratory of Physics and Energy Technology, North China Electric Power University, Baoding 071000, China

⁴Hebei Research Center of the Basic Discipline Engineering Mechanics, Shijiazhuang Tiedao University, Shijiazhuang 050043, China

⁵School of Physics, Xi'an Jiaotong University, Xi'an 710049, China

⁶School of Physics, Xi'an Jiaotong University, Xi'an 710049, China

(Dated: November 10, 2025)

Abstract

The influence of spin-orbit coupling strength (W) on the structure and two-proton ($2p$) radioactivity of ^{18}Mg is investigated using the spherical Skyrme-Hartree-Fock-Bogoliubov (SHFB) approach with the SLy4 interaction and a mean-field cluster potential model. Our calculations reveal that increasing W enhances the splitting of the single-proton $1d$ orbitals. Concurrently, the $2s_{1/2}$ proton state evolves from a weakly bound state into a resonance in the continuum. As W increases, the occupation probability of the $2s_{1/2}$ proton state decreases, and its density distribution near the nuclear surface becomes less diffuse. Furthermore, both the spectroscopic factor S_{2p} and the decay energy Q_{2p} for $2p$ radioactivity gradually decrease with increasing W , resulting in a longer half-life. When Q_{2p} is held constant, the half-life is significantly enhanced by S_{2p} . At the same time, it is found that the depth of the diproton cluster potential well increases with W , while the corresponding S_{2p} becomes smaller, indicating that the diproton cluster is considerably looser than the α -cluster. Additionally, a clear linear correlation is observed between $\log_{10}S_{2p}$ and W . The logarithmic half-lives, both with and without inclusion of S_{2p} , exhibit good linear relationships with W and Q_{2p} , respectively. Finally, using the experimental Q_{2p} value of ^{18}Mg , the optimal W is determined to be $1.15W_0$ with $W_0 = 123 \text{ MeV} \cdot \text{fm}^5$.

Keywords: $2p$ radioactivity; spin-orbit coupling strength; SHFB theory; mean-field cluster potential approach

I. INTRODUCTION

Two-proton ($2p$) radioactivity is a rare decay mode observed in nuclei near the proton drip line, which was predicted by Zel'dovich and Goldansky in the

1960s [1-3]. Owing to the pairing correlation effect, direct (true) 2p radioactivity occurs in even- Z nuclei because one-proton emission is energetically forbidden while 2p radioactivity is allowed [4]. True 2p radioactivity was first observed from the ground state of ^{45}Fe at the beginning of the 21st century [5, 6]. Later, more events of 2p radioactivity from ground states were discovered in ^{48}Ni [7], ^{54}Zn [8], ^{67}Kr [9], and ^{19}Mg [10]. Moreover, 2p emission may occur from excited states populated in β decay, which is usually named β -delayed 2p radioactivity [11-20]. Additionally, some 2p emitters such as ^{14}O [21], $^{17,18}\text{Ne}$ [22-26], ^{22}Mg [27, 28], and $^{28,29}\text{S}$ [29, 30] were discovered from excited states fed by nuclear reactions. Furthermore, 2p emission from the 21^+ isomer state of ^{94}Ag was reported by the group of Mukha [31].

To understand the mechanism of 2p radioactivity, numerous microscopic and phenomenological models have been developed [32-63]. Generally, the 2p radioactivity mechanism includes three types: (i) Strongly correlated emission due to the attraction of the two protons, also called diproton emission, where the emission occurs through the penetration of a preformed ^2He cluster. The emitted protons have a small angular distribution and share similar energies [32-48]. (ii) Three-body simultaneous emission, where the simultaneously emitted protons distribute isotropically but usually have comparable energies [45-63]. (iii) Two-body sequential emission, where a parent nucleus first emits a proton and decays to an intermediate state, which then emits another proton to reach the final state [45-63].

A few years ago, four-proton radioactivity was discovered from the ground state of ^{18}Mg [64]. In fact, it can be seen as two sequential 2p emissions through the intermediate nucleus ^{16}Ne . To understand its 2p decay mechanism, the structure of ^{18}Mg was investigated using a three-body Gamow coupled-channel method [65]. The results showed that the ground state of ^{18}Mg is significantly influenced by the continuum, resulting in a substantial s-wave component. Recently, we explored the tensor force effect on the 2p radioactivity of ^{18}Mg within the framework of spherical SHFB theory and the mean-field cluster potential approach [66, 67]. It was shown that a 2p halo-like structure appears in ^{18}Mg and its spectroscopic factor becomes larger with the increase of the 2p halo-like size caused by the tensor force. Furthermore, the decay mode prefers ^2He emission [66, 67]. Although the 2p emission of ^{18}Mg has been investigated by various models, its mechanism has not yet been fully understood [48, 65-67]. Therefore, investigating the 2p emission mechanism of ^{18}Mg remains a topic of significant interest.

The spin-orbit force is a key ingredient in the nucleon-nucleon interaction, which plays a crucial role in nuclear structure and radioactivity [68-71]. For nuclei around the drip line, the spin-orbit coupling strength W is not only a core parameter in nuclear models but also key to understanding exotic nuclear structures, decay mechanisms, and shell evolution [68-71]. However, its magnitude has not yet been precisely determined. Therefore, it is essential to investigate the influence of W on nuclear structure and decay processes. Very recently, we

explored the effect of W on the α -decay of ^{210}Po within spherical SHFB theory. It was found that the localization of the α -cluster and α -decay energy are enhanced significantly by the increase of W [72]. This motivates us to consider whether 2p radioactivity is also substantially influenced by W . Moreover, the mechanism by which the spin-orbit interaction affects 2p radioactivity has not been clearly understood, and the observables of 2p emission can be used to constrain the magnitude of W .

Driven by the above motivation, in this article we explore how the structure and 2p emission of ^{18}Mg are influenced by the spin-orbit interaction within the spherical SHFB theory and mean-field cluster potential approach. This article is organized as follows: In Sec. II, the theoretical framework is presented. Sec. III shows the calculated results and discussions. Finally, conclusions are drawn in the last section.

II. THEORETICAL FRAMEWORK

A. SHFB Theory

It is well known that SHFB theory is a powerful tool for describing nuclear structure [73-76]. It provides a unified and self-consistent description of both the mean field and the pairing field in terms of Bogoliubov quasi-particles. Numerous nuclear properties have been successfully described with the SHFB approach.

The Skyrme interaction $V_{\{\text{Skyrme}\}}$ is written as [73-76]:

$$V_{\text{Skyrme}} = t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2) + \frac{1}{2}t_1(1+x_1P_\sigma)[\mathbf{k}'^2\delta(\mathbf{r}_1-\mathbf{r}_2) + \delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}^2] + t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k} + \frac{1}{6}t_3(1+x_3P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}'\cdot\mathbf{k}$$

where t_i , x_i ($i = 1, 2, 3$) and W are interaction parameters, P_σ is the spin-exchange operator, and σ_j ($j = 1, 2$) are Pauli matrices. The operator $\mathbf{k} = (\nabla_1 - \nabla_2)/2i$ acts on the right and $\mathbf{k}' = (\nabla_1 - \nabla_2)/2i$ on the left.

In the particle-particle (pairing) channel, a density-dependent δ interaction is usually employed, with the form [74]:

$$V_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2}V_0 \left(1 - \left(\frac{\rho(\mathbf{r}_1)}{\rho_0} \right)^\gamma \right) \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

where V_0 is the pairing strength parameter, $\gamma = 1$ corresponds to the mixed pairing force, and ρ_0 is the saturation density. The V_0 value is determined by the empirical pairing energy gap [77-79].

In the framework of the SHFB approach, expressions for the effective mass, abnormal effective mass, particle-hole field, particle-particle field, and Coulomb field can be found in Refs. [73-76]. The spin-orbit coupling fields are expressed as:

$$\mathbf{B}_q = -\frac{1}{2}(t_{1x}1 + t_{2x}2)\mathbf{J} + \frac{1}{2}(t_1 - t_2)\mathbf{J}_q + W\nabla(\rho + \rho_q)$$

$$\tilde{\mathbf{B}}_q = \frac{1}{2}(1 + x')\mathbf{J}_q + W'\nabla\tilde{\rho}_q$$

In Eqs. (3-4), ρ and \mathbf{J} are the particle and spin-current vector densities, respectively. $\tilde{\mathbf{J}}$ is the corresponding abnormal spin-current vector density in the particle-particle channel. For the SLy4 interaction, \mathbf{J} and $\tilde{\mathbf{J}}$ equal zero. The subscript q stands for neutrons or protons.

Bulk and microscopic properties can be obtained by solving the following SHFB equation in coordinate representation:

$$\begin{pmatrix} h_q - \lambda_q & \Delta_q \\ \Delta_q^\dagger & -h_q + \lambda_q \end{pmatrix} \begin{pmatrix} U_{n,q} \\ V_{n,q} \end{pmatrix} = E_{n,q} \begin{pmatrix} U_{n,q} \\ V_{n,q} \end{pmatrix}$$

where h_q is the single-particle Hamiltonian, λ_q is the Fermi energy, Δ_q is the pairing potential, and $U_{n,q}$ and $V_{n,q}$ are the quasi-particle wave functions.

Furthermore, the 2p decay energy (Q_{2p}) is calculated by:

$$Q_{2p}(Z, A) = -S_{2p}(Z, A) = E(Z, A) - E(Z - 2, A - 2)$$

where $E(Z, A)$ and $E(Z - 2, A - 2)$ are the binding energies of the parent nucleus and its daughter nucleus, respectively, and S_{2p} is the 2p separation energy.

B. Mean-Field Cluster Potential Approach with Skyrme Interaction

For 2p radioactivity, the angular momentum carried by the ${}^2\text{He}$ cluster is zero, so the centrifugal potential vanishes. Based on SHFB theory, the diproton cluster potential can be decomposed as [80, 81]:

$$V_{2p}(r) = V_N(r) + V_C(r)$$

where $V_N(r)$ and $V_C(r)$ are the nuclear and Coulomb potentials, respectively, between the diproton cluster and the core. These are expressed approximately by:

$$V_N(r) = \lambda[U_p(r) + B_p(r)]$$

$$V_C(r) = 2U_c(r)$$

where λ is a folding factor, $U_p(r)$, $B_p(r)$, and $U_c(r)$ refer to the single-proton potential without the spin-orbit term, the proton spin-orbit potential, and the single-proton Coulomb potential generated by the core, respectively. These expressions can be found in Refs. [73–76].

The folding factor λ can be determined by the Bohr-Sommerfeld quantization condition:

$$\int_{r_1}^{r_2} dr \sqrt{2\mu|Q_{2p} - V_{2p}(r)|} = (2n + 1) \frac{\pi}{2} = (G + 1) \frac{\pi}{2}$$

where r_1 (and r_2 later) is the classical inner (and outer) turning point obtained from $V_{2p}(r) = Q_{2p}$, μ is the reduced mass for the cluster-daughter nucleus system, and n is the node number of the radial wave function of the cluster motion within the potential. A quasibound state of the cluster orbiting the core is characterized by a global quantum number G , estimated by the Wildermuth rule:

$$G = 2n + l = \sum_{i=1}^2 (2n_i + l_i)$$

where g_i is the oscillator quantum number of a cluster nucleon orbiting the core.

The width of 2p radioactivity can be calculated by:

$$\Gamma_{2p} = S'_{2p} \frac{\hbar^2}{4\mu} \exp\left(-2 \int_{r_1}^{r_2} dr k(r)\right)$$

where S'_{2p} is the spectroscopic factor.

For ^{18}Mg , we assume the two emitted protons originate from the $2s_{1/2}$ orbitals. The S'_{2p} value is calculated by a BCS model [82]:

$$S'_{2p} = V_{2s_{1/2}}^4 U_{2s_{1/2}}^2$$

where $V_{2s_{1/2}}^2$ and $U_{2s_{1/2}}^2$ are the occupation and unoccupation probabilities of the $2s_{1/2}$ proton states for ^{18}Mg and ^{16}Ne , respectively.

In Eq. (12), F is the normalization factor determined by:

$$F = \exp\left(\int_{r_1}^{r_2} dr' k(r') - \frac{\pi}{2}\right)$$

with the wave number $k(r)$ calculated by:

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2} |Q_{2p} - V_{2p}(r)|}$$

Finally, the 2p radioactivity half-life is calculated by:

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_{2p}}$$

III. RESULTS AND DISCUSSIONS

In the SHFB calculations, the HFBRAD code with the SLy4 interaction is employed [74]. Specifically, the spherical box and mesh sizes are selected as 20 fm and 0.1 fm, respectively. The quasiparticle energies are cut off at 60 MeV. The maximum angular momenta of the quasi-neutron and quasi-proton are set to be 39 \hbar and 25 \hbar , respectively. All calculations in this article are converged with these conditions.

Within the mixed pairing force, the single-proton energy spectra near the Fermi energies of ^{18}Mg are calculated for W values in the range of 0–1.4 W_0 MeV \cdot fm⁵ with an interval of 0.2 W_0 MeV \cdot fm⁵, as plotted in Fig. 1 [Figure 1: see original paper]. The single-proton energies refer to those in the canonical basis. From Fig. 1, one can see that the splitting of the 1d orbital becomes increasingly pronounced with increasing W . Moreover, the 2s_{1/2} energy levels are below the Fermi energies for $W = 0W_0, 0.2W_0, 0.4W_0,$ and $0.6W_0$, indicating that the 2s_{1/2} states are weakly bound. However, with enhanced 1d orbital splitting, the 2s_{1/2} energy level exceeds the 1d_{5/2} orbital and is located in the resonant states of the continuum energy region when $W \geq 0.8W_0$. Regardless of whether the 2s_{1/2} state is weakly bound or not, a certain number of protons occupy it. For the 2s_{1/2} state, the centrifugal barrier vanishes, so valence protons can extend to large r regions, leading to a 2p halo or 2p halo-like structure. This situation is similar to the formation of neutron halos [83, 84]. Therefore, the contribution of the 2s_{1/2} state to the full density in the large r region is dominant, as seen clearly in Fig. 2 [Figure 2: see original paper], which shows the density distribution of each orbit near the Fermi energy as a function of r for $W = 0.6W_0, 1.0W_0,$ and $1.4W_0$. Our recent studies with the T31 and SLy5 interactions suggested that the deformation of ^{18}Mg is very weak [66, 67]. Furthermore, relevant studies indicate that the 2p halo-like structure rather than deformation is responsible for the 2p correlation and emission mechanism [85–89]. Thus, the microscopic single-proton energy spectra are sufficient to examine the 2p halo or 2p halo-like structure of ^{18}Mg .

With enhanced 1d orbital splitting, fewer protons occupy the 2s_{1/2} orbital. As shown in column 3 of Table I, the occupation probability of the 2s_{1/2} state

($V_{2s_{1/2}}^2$) decreases with increasing W . Consequently, the tail of the density distribution of the $2s_{1/2}$ orbit is less extended, as clearly seen in Fig. 2. Hence, the 2p halo or 2p halo-like sizes of ^{18}Mg are suppressed by increasing W . Similarly, the occupation probability of the $2s_{1/2}$ state of its daughter nucleus ^{16}Ne falls with rising W , meaning that the unoccupation probability $U_{2s_{1/2}}^2$ increases with W , as seen in column 4 of Table I. Using Eq. (13), we obtain the corresponding S'_{2p} values, listed in column 5 of Table I. Overall, the S'_{2p} value changes very little with W . Note that the diproton cluster preforms more easily when the density on the nuclear surface is low. The weakening 2p halo or 2p halo-like structure in Fig. 2 suggests that the density distribution on the nuclear surface is less extended with enhanced W . As a result, the S'_{2p} value may be strongly correlated with the 2p halo or 2p halo-like size. To study this correlation, our recent work used a virtual charge radius to represent the 2p halo-like size of ^{18}Mg because it is difficult to distinguish a halo from a gas-like structure for an unbound nucleus [66, 67]. We found a pronounced linear correlation between S'_{2p} and the 2p halo-like size [66, 67], so we will not discuss this correlation further in the present article.

Nevertheless, when W is weak the S'_{2p} value becomes smaller. Therefore, the S'_{2p} value decreases with increasing W . Using Eq. (6), we obtain different Q_{2p} values for different W values, shown in column 6 of Table I, which demonstrates that Q_{2p} values gradually decrease with increasing W . Previous studies have suggested a dependence of S'_{2p} on Q_{2p} [4, 48, 50, 51, 90]. Therefore, we plot the logarithm S'_{2p} versus Q_{2p} and versus W in Figs. 3(a) and 3(b) [Figure 3: see original paper], respectively. From Fig. 3(a), a good linear correlation between $\log_{10} S'_{2p}$ and Q_{2p} with Pearson coefficient $R = 0.9975$ is found. Similarly, a less pronounced linear correlation between $\log_{10} S'_{2p}$ and W with $R = -0.8899$ is seen in Fig. 3(b).

To examine the W effect on diproton cluster potentials V_{2p} , we calculate V_{2p} for different W values using the mean-field cluster potential approach. The V_{2p} values with $W = 0.6W_0$, $1.0W_0$, and $1.4W_0$ are shown in Fig. 4(a) [Figure 4: see original paper]. To investigate the potential in the nuclear surface region influenced by W , the V_{2p} curves in the surface region with $W = 0.6W_0$, $1.0W_0$, and $1.4W_0$ are shown in Fig. 4(b). From Figs. 4(a) and 4(b), one can see that the potential well becomes deeper with increasing W , while the potential barriers in the surface region grow gradually with rising W . Studies on α -decay showed that the preformation probability of α -cluster is enhanced with increasing potential well depth [72, 91]. Nevertheless, for the S'_{2p} of the diproton cluster of ^{18}Mg , it becomes smaller when the potential well becomes deeper. This implies that the diproton cluster differs substantially from the α -cluster and should be much looser than the α -cluster.

Next, we calculate the 2p radioactivity half-lives with and without S'_{2p} using the mean-field cluster potential approach by inputting the Q_{2p} values for different W values. The $T_{1/2}$ and $T'_{1/2}$ values for different W values are listed in

column 7 and the last column of Table I, respectively. For cases with $W \leq 0.6W_0$, the Q_{2p} value exceeds the height of the diproton potential barrier, thus the half-life cannot be calculated within the tunneling model and is not listed. The logarithmic half-lives with and without S'_{2p} are shown in Figs. 4(c) and 4(d), respectively. From these figures, we see that $\log_{10}T_{1/2}$ and $\log_{10}T'_{1/2}$ show good linear relationships with W and Q_{2p} , respectively. As expected from the tunneling picture, decreasing Q_{2p} and S'_{2p} values resulting from increasing W lead to longer half-lives, as seen in Table I and Figs. 4(c) and 4(d). Moreover, the logarithmic half-lives are enhanced by S'_{2p} . For the case of $0.8W_0$, the half-life increases by a factor of 2.375. When $W = 1.4W_0$, the half-life is extended by 11.496 times.

The Q_{2p} value of ^{18}Mg was measured as 3.440 MeV [64]. Using this experimental Q_{2p} value, the W value can be determined. We found that when $W = 1.15W_0$, the calculated Q_{2p} value exactly equals the experimental one. With $W = 1.15W_0$, the corresponding t_0 , $V_{2s_{1/2}}^2$, $U_{2s_{1/2}}^2$, S'_{2p} , Q_{2p} , $T_{1/2}$, and $T'_{1/2}$ values are calculated and listed in the last line of Table I, from which it is easy to obtain that $T'_{1/2}$ is 5.181 times as large as $T_{1/2}$.

IV. CONCLUSIONS

In this article, the impact of spin-orbit coupling strength W on the structure and 2p radioactivity of ^{18}Mg has been investigated within spherical SHFB theory and the mean-field cluster potential approach using the SLy4 interaction. Specifically, the single-proton spectra, density distributions, spectroscopic factors, diproton cluster potentials, and 2p radioactivity half-lives have been calculated for W values in the range of 0– $1.4W_0$.

The following conclusions can be drawn:

- (i) The splitting of the 1d proton orbital becomes increasingly evident with increasing W . Moreover, the $2s_{1/2}$ proton state is weakly bound for $W = 0W_0$, $0.2W_0$, $0.4W_0$, and $0.6W_0$. However, the $2s_{1/2}$ state becomes a resonant state in the continuum energy region when $W \geq 0.8W_0$.
- (ii) With enhanced 1d orbital splitting, the occupation probabilities of the $2s_{1/2}$ state decrease gradually. As a result, the tail of the density distribution of the $2s_{1/2}$ orbital is less extended. Correspondingly, the spectroscopic factor S'_{2p} becomes smaller.
- (iii) The diproton cluster potential well becomes deeper with increasing W . Moreover, the deeper the diproton potential well, the smaller the S'_{2p} . This implies that the diproton cluster is much looser than the α -cluster.
- (iv) The declining S'_{2p} and Q_{2p} values caused by increasing W lead to longer 2p radioactivity half-lives. Moreover, the half-life is enhanced by S'_{2p} when Q_{2p} is held constant.

- (v) A pronounced linear correlation between $\log_{10}S'_{2p}$ and Q_{2p} and that between $\log_{10}S'_{2p}$ and W are found. Additionally, $\log_{10}T'_{1/2}$ and $\log_{10}T'_{1/2}$ show good linear relationships with W and Q_{2p} , respectively.
- (vi) Using the experimental Q_{2p} value of ^{18}Mg , the optimal W value is constrained to be $1.15W_0$.

ACKNOWLEDGEMENTS

We thank Professor Ligang Cao, Professor Jianmin Dong, and Professor Yibin Qian for helpful suggestions and comments. This work was supported by the S&T Program of Hebei (Grant No. 236Z4601G); Scientific Research Foundation for the Introducing Returned Overseas Chinese Scholars of Hebei Province (Grant No. C20230360); and Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics, Chinese Academy of Sciences (Grant No. IMPKFKT2021002).

REFERENCES

- [1] Y. B. Zel' dovich, *Sov. Phys.-JETP* 11, 812 (1960).
- [2] V. I. Goldansky, On neutron-deficient isotopes of light nuclei and the phenomena of proton and two-proton radioactivity, *Nucl. Phys.* 19, 482 (1960). [https://doi.org/10.1016/0029-5582\(60\)90258-3](https://doi.org/10.1016/0029-5582(60)90258-3).
- [3] V. I. Goldansky, Two-proton radioactivity, *Nucl. Phys.* 27, 648 (1961). [https://doi.org/10.1016/0029-5582\(61\)90309-1](https://doi.org/10.1016/0029-5582(61)90309-1).
- [4] M. Pfützner, M. Karny, L. V. Grigorenko, and K. Riisager, Radioactive decays at limits of nuclear stability, *Rev. Mod. Phys.* 84, 567 (2012). <https://doi.org/10.1103/RevModPhys.84.567>.
- [5] M. Pfützner, E. Badura, C. Bingham et al., First evidence for the two-proton decay of ^{45}Fe , *Eur. Phys. J. A* 14, 279 (2002). <https://doi.org/10.1140/epja/i2002-10033-9>.
- [6] J. Giovinazzo, B. Blank, M. Chartier et al., Two-proton radioactivity of ^{45}Fe , *Phys. Rev. Lett.* 89, 102501 (2002). <https://doi.org/10.1103/PhysRevLett.89.102501>.
- [7] C. Dossat, A. Bey, B. Blank et al., Two-proton radioactivity studies with ^{45}Fe and ^{48}Ni , *Phys. Rev. C* 72, 054315 (2005). <https://doi.org/10.1103/PhysRevC.72.054315>.
- [8] B. Blank, A. Bey, G. Cachel et al., First observation of ^{54}Zn and its decay by two-proton emission, *Phys. Rev. Lett.* 94, 232501 (2005). <https://doi.org/10.1103/PhysRevLett.94.232501>.
- [9] T. Goigoux, P. Ascher, B. Blank et al., Two-proton radioactivity of ^{67}Kr , *Phys. Rev. Lett.* 117, 162501 (2016). <https://doi.org/10.1103/PhysRevLett.117.162501>.
- [10] I. Mukha, K. Sümmerer, L. Acosta et al., Observation of two-proton radioactivity of ^{19}Mg by tracking the decay products, *Phys. Rev. Lett.* 99, 182501 (2007). <https://doi.org/10.1103/PhysRevLett.99.182501>.

- [11] M. D. Cable, J. Honkanen, R. F. Parry et al., Discovery of β -delayed two-proton radioactivity: ^{22}Al , Phys. Rev. Lett. 50, 404 (1983). <https://doi.org/10.1103/PhysRevLett.50.404>.
- [12] B. Blank, F. Boué, S. Andriamonje et al., Spectroscopic studies of the β p and $\beta\beta$ decay of ^{23}Si , Z. Phys. A 357, 247 (1997). <https://doi.org/10.1007/s002180050241>.
- [13] J. Honkanen, M. D. Cable, R. F. Parry et al., Beta-delayed two-proton decay of ^{26}P , Phys. Lett. B 133, 146 (1983). [https://doi.org/10.1016/0370-2693\(83\)90547-6](https://doi.org/10.1016/0370-2693(83)90547-6).
- [14] V. Borrel et al., Beta-delayed proton decay of the $T_z = -5/2$ isotope ^{31}Ar , Nucl. Phys. A 473, 331 (1987). [https://doi.org/10.1016/0375-9474\(87\)90148-5](https://doi.org/10.1016/0375-9474(87)90148-5).
- [15] J. E. Reiff, M. A. C. Hotchkis, D. M. Moltz et al., A fast in-beam recoil catcher wheel and the observation of β -delayed two-proton emission from ^{31}Ar , Nucl. Instrum. Methods A 276, 228 (1989). [https://doi.org/10.1016/0168-9002\(89\)90637-2](https://doi.org/10.1016/0168-9002(89)90637-2).
- [16] J. Äystö, D. M. Moltz, X. J. Xu et al., Observation of the first $TZ = -5/2$ nuclide, ^{35}Ca , via its β -delayed two-proton emission, Phys. Rev. Lett. 55, 1384 (1985). <https://doi.org/10.1103/PhysRevLett.55.1384>.
- [17] D. M. Moltz, J. C. Batchelder, T. F. Lang et al., Beta-delayed two-proton decay of ^{39}Ti , Z. Phys. A 342, 273 (1992). <https://doi.org/10.1007/BF01291509>.
- [18] V. Borrel, R. Anne, D. Bazin et al., The decay modes of proton drip-line nuclei with A between 42 and 47, Z. Phys. A 344, 135 (1992). <https://doi.org/10.1007/BF01291696>.
- [19] J. Giovannozzi et al., First direct observation of two protons in the decay of ^{45}Fe with a TPC, Phys. Rev. Lett. 99, 102501 (2007). <https://doi.org/10.1103/PhysRevLett.99.102501>.
- [20] C. Dossat, N. Adimi, F. Aksouh et al., The decay of proton-rich nuclei in the mass, Nucl. Phys. A 792, 18 (2007). <https://doi.org/10.1016/j.nuclphysa.2007.05.004>.
- [21] C. R. Bain, P. J. Woods, R. Coszach et al., Two proton emission induced via a resonance reaction, Phys. Lett. B 373, 35 (1996). [https://doi.org/10.1016/0370-2693\(96\)00109-8](https://doi.org/10.1016/0370-2693(96)00109-8).
- [22] M. J. Chromik, B. A. Brown, M. Fauerbach et al., Excitation and decay of the first excited state of ^{17}Ne , Phys. Rev. C 55, 1676 (1997). <https://doi.org/10.1103/PhysRevC.55.1676>.
- [23] M. J. Chromik, P. G. Thirolf, M. Thoennessen et al., Evidence for 2p-radioactivity in ^{17}Ne , Phys. Rev. C 66, 024313 (2002). <https://doi.org/10.1063/1.57331>.
- [24] T. Zerguerras, B. Blank, Y. Blumenfeld et al., Study of light proton-rich nuclei by complete kinematics measurements, Eur. Phys. J. A 20, 389 (2004). <https://doi.org/10.1140/epja/i2003-10176-1>.
- [25] J. Gómez del Campo, A. Galindo-Uribarri, J. R. Beene et al., Decay of a Resonance in ^{18}Ne by the simultaneous emission of two protons, Phys. Rev. Lett. 86, 43 (2001). <https://doi.org/10.1103/PhysRevLett.86.43>.
- [26] G. Raciti, G. Cardella, M. De Napoli et al., Experimental Evidence of ^2He Decay from ^{18}Ne excited states, Phys. Rev. Lett. 100, 192503 (2008). <https://doi.org/10.1103/PhysRevLett.100.192503>.
- [27] Y. G. Ma, D. Q. Fang, X. Y. Sun et al., Different mechanism of two-proton

- emission from proton-rich nuclei ^{23}Al and ^{22}Mg , *Phys. Lett. B* 743, 306 (2015). <https://doi.org/10.1016/j.physletb.2015.02.066>.
- [28] D. Q. Fang, Y. G. Ma, X. Y. Sun et al., Proton-proton correlations in distinguishing the two-proton emission mechanism of ^{23}Al and ^{22}Mg , *Phys. Rev. C* 94, 044621 (2016). <https://doi.org/10.1103/PhysRevC.94.044621>.
- [29] C. J. Lin, X. X. Xu, H. M. Jia et al., Experimental study of two-proton correlated emission from ^{29}S excited states, *Phys. Rev. C* 80, 014310 (2009). <https://doi.org/10.1103/PhysRevC.80.014310>.
- [30] X. X. Xu, C. J. Lin, H. M. Jia et al., Excluding a generic spin-2 higgs impostor, *Phys. Lett. B* 727, 126 (2013). <https://doi.org/10.1016/j.physletb.2013.10.064>.
- [31] I. Mukha, E. Roeckl, L. Batist et al., Proton-proton correlations observed in two-proton radioactivity of ^{94}Ag , *Nature* 439, 298 (2006). <https://doi.org/10.1038/nature04453>.
- [32] J. Jänecke, The emission of protons from light neutron-deficient nuclei, *Nucl. Phys.* 61, 326 (1965). [https://doi.org/10.1016/0029-5582\(65\)90907-7](https://doi.org/10.1016/0029-5582(65)90907-7).
- [33] B. A. Brown, Diproton decay of nuclei on the proton drip line, *Phys. Rev. C* 43, R1513 (1991). <https://doi.org/10.1103/PhysRevC.44.924>.
- [34] W. Nazarewicz, J. Dobaczewski, T. R. Werner et al., Structure of proton drip-line nuclei around doubly magic ^{48}Ni , *Phys. Rev. C* 53, 740 (1996). <https://doi.org/10.1103/PhysRevC.53.740>.
- [35] W. E. Ormand, Mapping the proton dripline up to $A = 70$, *Phys. Rev. C* 55, 2407 (1997). <https://doi.org/10.1103/PhysRevC.55.2407>.
- [36] F. C. Barker, ^{12}O ground-state decay by emission, *Phys. Rev. C* 63, (2001). <https://doi.org/10.1103/PhysRevC.63.047303>.
- [37] B. A. Brown, F. C. Barker, Diproton decay of ^{45}Fe , *Phys. Rev. C* 67, 041304(R) (2003). <https://doi.org/10.1103/PhysRevC.67.041304>.
- [38] D. S. Delion, R. J. Liotta and R. Wyss, Simple approach to two-proton emission, *Phys. Rev. C* 87, 034328 (2013). <https://doi.org/10.1103/PhysRevC.87.034328>.
- [39] J. Rotureau, J. Okolowicz and M. Ploszajczak, Theory of the two-proton radioactivity in the continuum shell model, *Nucl. Phys. A* 767, 13 (2006). <https://doi.org/10.1016/j.nuclphysa.2005.12.005>.
- [40] M. Goncalves, N. Teruya and O. Tavares et al., Two-proton emission half-lives in the effective liquid drop model, *Phys. Lett. B* 774, 14 (2017). <https://doi.org/10.1016/j.physletb.2017.09.032>.
- [41] J. P. Cui, Y. H. Gao, Y. Z. Wang and J. Z. Gu, Two-proton radioactivity within a generalized liquid drop model, *Phys. Rev. C* 101, 014301 (2020); *Phys. Rev. C* 104, 029902(E) (2021). <https://doi.org/10.1103/PhysRevC.104.029902>.
- [42] Y. Z. Wang, J. P. Cui, Y. H. Gao, J. Z. Gu, Two-proton radioactivity of exotic nuclei beyond proton drip-line, *Commun. Theor. Phys.* 73, 075301 (2021). <https://doi.org/10.1088/1572-9494/abfa00>.
- [43] F. Z. Xing, J. P. Cui, Y. Z. Wang, J. Z. Gu, Two-proton radioactivity of ground and excited states within a unified fission model, *Chin. Phys. C* 45, 124105 (2021). <https://doi.org/10.1088/1674-1137/ac2425>.
- [44] H. M. Liu, Y. T. Zou, P. Xiao et al., New Geiger-Nuttall law for two-proton radioactivity, *Chin. Phys. C* 45, 024108 (2021). <https://doi.org/10.1088/1674-1137/abd01e>.

- [45] D. Q. Fang, Y. G. Ma, Progress of experimental studies on two-proton emission, *Chin. Sci. Bull.* 65, 4018 (2020) (in Chinese). <https://doi.org/10.1360/TB-2020-0423>.
- [46] L. Zhou, S. M. Wang, D. Q. Fang, Y. G. Ma, Recent progress in two-proton radioactivity, *Nucl. Sci. Tech.* 33, 105 (2022). <https://doi.org/10.1007/s41365-022-01091-1>.
- [47] Z. Zhang, C. Yuan, C. Qi et al., Extended R-matrix description of two-proton radioactivity, *Phys. Lett. B* 838, 137740 (2023). <https://doi.org/10.1016/j.physletb.2023.137740>.
- [48] M. Pfützner, I. Mukha, S. M. Wang, Two-proton emission and related phenomena, *Pro. Part. Nucl. Phys.* 132, 104050 (2023). <https://doi.org/10.1016/j.pnpnp.2023.104050>.
- [49] E. Olsen, M. Pfützner, N. Birge et al., Landscape of two-proton radioactivity, *Phys. Rev. Lett.* 110, 222501 (2013); *Phys. Rev. Lett.* 111, 139903(E) (2013). <https://doi.org/10.1103/PhysRevLett.110.222501>.
- [50] B. Blank and M. Ploszajczak, Two-proton radioactivity, *Rep. Prog. Phys.* 71, 046301 (2008). <https://doi.org/10.1088/0034-4885/71/4/046301>.
- [51] B. Blank and M. J. G. Borge, Nuclear structure at the proton drip line: Advances with nuclear decay studies, *Prog. Part. Nucl. Phys.* 60, 403 (2008). <https://doi.org/10.1016/j.pnpnp.2007.12.001>.
- [52] V. Galitsky and V. Cheltsov, Two-proton radioactivity theory, *Nucl. Phys.* 56, 86 (1964). [https://doi.org/10.1016/0029-5582\(64\)90455-9](https://doi.org/10.1016/0029-5582(64)90455-9).
- [53] B. V. Danilin and M. V. Zhukov, *Phys. At. Nucl.* 56, 460 (1993).
- [54] V. Vasilevsky, A. V. Nesterov, F. Arickx, and J. Broeckhove, Algebraic model for scattering in three-s-cluster systems. II. Resonances in the three-cluster continuum of ${}^6\text{He}$ and ${}^6\text{Be}$, *Phys. Rev. C* 63, 034607 (2001). <https://doi.org/10.1103/PhysRevC.63.034607>.
- [55] L. V. Grigorenko, R. C. Johnson, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, Theory of two-proton radioactivity with application to ${}^{19}\text{Mg}$ and ${}^{48}\text{Ni}$, *Phys. Rev. Lett.* 85, 22 (2000). <https://doi.org/10.1103/PhysRevLett.85.22>.
- [56] L. V. Grigorenko, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, Two-proton widths of ${}^{12}\text{O}$, ${}^{16}\text{Ne}$, and three-body mechanism of thomas-ehrman shift, *Phys. Rev. Lett.* 88, 042502 (2002). <https://doi.org/10.1103/PhysRevLett.88.042502>.
- [57] P. Descouvemont, E. Tursunov, and D. Baye, Three-body continuum states on a Lagrange mesh, *Nucl. Phys. A* 765, 370 (2006). <https://doi.org/10.1016/j.nuclphysa.2005.11.010>.
- [58] S. M. Wang, N. Michel, W. Nazarewicz, and F. R. Xu, Structure and decays of nuclear three-body systems: the Gamow coupled-channel method in Jacobi coordinates, *Phys. Rev. C* 96, 044307 (2017). <https://doi.org/10.1103/PhysRevC.96.044307>.
- [59] S. M. Wang, W. Nazarewicz, R. J. Charity, and L. G. Sobotka, Structures and decay properties of extremely proton-rich nuclei ${}^{11,12}\text{O}$, *Phys. Rev. C* 99, 054302 (2019). <https://doi.org/10.1103/PhysRevC.99.054302>.
- [60] E. Garrido, A. S. Jensen, and D. V. Fedorov, Necessary conditions for accurate computations of three-body partial decay widths, *Phys. Rev. C* 78, 034004 (2008). <https://doi.org/10.1103/PhysRevC.78.034004>.

- [61] R. Álvarez-Rodríguez, A. S. Jensen, E. Garrido, D. V. Fedorov and H. O. U. Fynbo, Momentum distributions of α -particles from decaying low-lying ^{12}C -resonances, *Phys. Rev. C* **77**, 064305 (2008). <https://doi.org/10.1103/PhysRevC.77.064305>.
- [62] R. Álvarez-Rodríguez, A. S. Jensen, E. Garrido, and D. V. Fedorov, Structure and three-body decay of ^9Be resonances, *Phys. Rev. C* **82**, 034001 (2010). <https://doi.org/10.1103/PhysRevC.82.034001>.
- [63] B. Blank, P. Ascher, L. Audirac et al., Two-proton radioactivity as a tool of nuclear structure studies, *Acta Phys. Pol. B* **42**, 545 (2011). <https://doi.org/10.5506/APhysPolB.42.545>.
- [64] Y. Jin, C. Y. Niu, K. W. Brown et al., First observation of the four-proton unbound nucleus ^{18}Mg , *Phys. Rev. Lett.* **127**, 262502 (2021). <https://doi.org/10.1103/PhysRevLett.127.262502>.
- [65] L. Zhou, D.-Q. Fang, S.-M. Wang, and H. Hua, Structure and 2p decay mechanism of ^{18}Mg , *Nucl. Sci. Tech.* **35**, 107 (2024). <https://doi.org/10.1007/s41365-024-01479-1>.
- [66] Yan-Zhao Wang, Feng-Zhu Xing, Jian-Po Cui, Yong-Hao Gao, Jian-Zhong Gu, Roles of tensor force and pairing correlation in two-proton radioactivity of halo nuclei, *Chin. Phys. C* **47**, 084101 (2023). <https://doi.org/10.1088/1674-1137/acd680>.
- [67] Yanzhao Wang, Fengzhu Xing, Wenhao Zhang, Jianpo Cui, and Jianzhong Gu, Tensor force effect on two-proton radioactivity of ^{18}Mg and ^{20}Si , *Phys. Rev. C* **110**, 064305 (2024). <https://doi.org/10.1103/PhysRevC.110.064305>.
- [68] A. N. Bezbakh, G. G. Adamian, and N. V. Antonenko, Role of spin-orbit strength in the prediction of closed shells in superheavy nuclei, *Phys. Rev. C* **105**, 054305 (2022). <https://doi.org/10.1103/PhysRevC.105.054305>.
- [69] H. Muther, Spin-orbit term in the nuclear shell model, *Phys. Rev. C* **103**, (2021). <https://doi.org/10.1103/PhysRevC.103.024306>.
- [70] G. C6, M. Anguiano, V. De Donno, and A. M. Lallena, Matter distribution and spin-orbit force in spherical nuclei, *Phys. Rev. C* **97**, 034313 (2018). <https://doi.org/10.1103/PhysRevC.97.034313>.
- [71] S. S. Zhang, E. G. Zhao, and S. G. Zhou, Theoretical study of the two-proton halo candidate ^{17}Ne including contributions from a resonant continuum and pairing correlations, *Eur. Phys. J. A* **49**, 77 (2013). <https://doi.org/10.1140/epja/i2013-13077-1>.
- [72] W. H. Zhang, Y. H. Gao, J. P. Cui, Y. Z. Wang, and J. Z. Gu, Roles of spin-orbit interaction and pairing correlation in α decay of ^{210}Po , *Phys. Rev. C* **112**, 024319 (2025). <https://doi.org/10.1103/dw7k-fj9j>.
- [73] P. Bonche, H. Flocard, P. H. Heenen, Solution of the Skyrme HF+BCS equation on a 3D mesh, *Comput. Phys. Commun.* **171**, 49 (2005). <https://doi.org/10.1016/j.cpc.2005.05.001>.
- [74] K. Bennaceur and J. Dobaczewski, Coordinate-space solution of the Skyrme-Hartree-Fock-Bogolyubov equations within spherical symmetry. The program HFBRAD (v1.00), *Comput. Phys. Commun.* **168**, 96 (2005). <https://doi.org/10.1016/j.cpc.2005.02.002>.
- [75] J. Dobaczewski, W. Satula, B. G. Carlsson et al., Solution of the Skyrme-

- Hartree-Fock-Bogolyubov equations in the Cartesian deformed harmonic-oscillator basis. (VI) HFODD (v2.40h): A new version of the program, *Comput. Phys. Commun.* 180, 2361 (2009). <https://doi.org/10.1016/j.cpc.2009.08.009>.
- [76] M. V. Stoitsov, N. Schunck, M. Kortelainen et al., Axially deformed solution of the Skyrme-Hartree-Fock-Bogoliubov equations using the transformed harmonic oscillator basis (II) HFBTHO v2.00d: A new version of the program, *Comput. Phys. Commun.* 184, 1592 (2013). <https://doi.org/10.1016/j.cpc.2013.01.013>.
- [77] D. Madland, J. Nix, New model of the average neutron and proton pairing gaps, *Nucl. Phys. A* 476, 1 (1988). [https://doi.org/10.1016/0375-9474\(88\)90370-3](https://doi.org/10.1016/0375-9474(88)90370-3).
- [78] P. Möller, J. Nix, Nuclear pairing models, *Nucl. Phys. A* 536, 20 (1992). [https://doi.org/10.1016/0375-9474\(92\)90244-E](https://doi.org/10.1016/0375-9474(92)90244-E).
- [79] T. Duguet, P. Bonche, P.-H. Heenen, J. Mayer, Pairing correlations. II. Microscopic analysis of odd even mass staggering in nuclei, *Phys. Rev. C* 65, 014311 (2001). <https://doi.org/10.1103/PhysRevC.65.014311>.
- [80] F. R. Xu, J. C. Pei, Mean-field cluster potentials for various cluster decays, *Phys. Lett. B* 642, 322 (2006). [10.1016/j.physletb.2006.09.048](https://doi.org/10.1016/j.physletb.2006.09.048)
- [81] J. C. Pei, F. R. Xu, Z. J. Lin, and E. G. Zhao, α -decay calculations of heavy and superheavy nuclei using effective mean-field potentials, *Phys. Rev. C* 76, 044326 (2007). <https://doi.org/10.1103/PhysRevC.76.044326>.
- [82] C. Basu, *Pramana-J. Phys. Spectroscopic factors two-proton radioactive nuclei*, 63, 1047 (2004). <https://doi.org/10.1007/BF02704343>.
- [83] J.-L. An, K.-Y. Zhang, Q. Lu et al., A unified description of the halo nucleus ^{37}Mg from microscopic structure to reaction observables, *Phys. Lett. B* 849, 138422 (2024). <https://doi.org/10.1016/j.physletb.2023.138422>.
- [84] S. S. Zhang, S. Y. Zhong, B. Shao and M. S. Smith, Self-consistent description of the halo nature of ^{31}Ne with continuum and pairing correlations, *J. Phys. G: Nucl. Part. Phys.* 49, 025102 (2022). <https://doi.org/10.1088/1361-6471/ac430e>.
- [85] G. Saxena, M. Kumawat, M. Kaushik, S. K. Jain, Mamta Aggarwal, Two-proton radioactivity with 2p halo in light mass nuclei $A=18-34$, *Phys. Lett. B* 775, 126 (2017). <https://doi.org/10.1016/j.physletb.2017.10.055>.
- [86] X. X. Xu, C. J. Lin, H. M. Jia et al., Correlations of two protons emitted from excited states of ^{28}S and ^{27}P , *Phys. Lett. B* 727, 126 (2013). <https://doi.org/10.1016/j.physletb.2013.10.029>.
- [87] C. J. Lin, X. X. Xu, H. M. Jia et al., Experimental study of two-proton correlated emission from ^{29}S excited states, *Phys. Rev. C* 80, 014310 (2009). <https://doi.org/10.1103/PhysRevC.80.014310>.
- [88] X. X. Xu, C. J. Lin, H. M. Jia et al., Investigation of two-proton emission from excited states of the odd-Z nucleus ^{27}P by complete-kinematics measurements, *Phys. Rev. C* 81, 054317 (2010). <https://doi.org/10.1103/PhysRevC.81.054317>.
- [89] C. J. Lin, X. X. Xu, J. S. Wang et al., Proton and two-proton emissions from proton-rich nuclei with $10 \leq Z \leq 20$, *Nucl. Phys. Rev.* 33, 160 (2016). <https://doi.org/10.11804/NuclPhysRev.33.02.160>.
- [90] O. A. P. Tavares and E. L. Medeiros, A calculation model to half-life

estimate of two-proton radioactive decay process, Eur. Phys. J. A 54, 65 (2018). <https://doi.org/10.1140/epja/i2018-12495-4>.

[91] J.-P. Ebran, E. Khan, T. Nikšić, and D. Vretenar, How atomic nuclei cluster, Nature 487, 341 (2012). <https://doi.org/10.1038/nature11246>.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv – Machine translation. Verify with original.