Spectroscopic factors of single proton capture from silicon to phosphorus isotopes

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Abstract

Proton capture reactions from silicon to phosphorus play a crucial role in hydrogen burning during nova explosions. However, experimental measurements of reactions involving radioactive isotopes or extremely short-lived nuclei are exceptionally challenging. Calculating the spectroscopic factors for these reactions can provide guidance for cross-section estimates and serve as an essential reference for experimental studies. In this work, we employ the nuclear shell model to compute the spectroscopic factors associated with the silicon isotopic chain, with a specific focus on proton capture processes that form phosphorus nuclei. The results for the reactions of $^{25-28}$Si are in good agreement with existing experimental data and previous theoretical calculations. Additionally, we predict the proton spectroscopic factor of the ^{25}P ground state is approximately 0.6.

Full Text

Preamble

Spectroscopic Factors for Single-Proton Capture from Silicon to Phosphorus Isotopes

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Proton capture reactions from silicon to phosphorus play a crucial role in hydrogen burning during nova explosions. However, experimental measurements involving radioactive isotopes or extremely short-lived nuclei are exceptionally challenging. Calculating spectroscopic factors for these reactions can provide guidance for cross-section estimates and serve as an essential reference for experimental studies. In this work, we employ the nuclear shell model to compute

the spectroscopic factors associated with the silicon isotopic chain, with a specific focus on proton capture processes that form phosphorus nuclei. The results for reactions involving $^{25-28}$Si show good agreement with existing experimental data and previous theoretical calculations. Additionally, we predict that the proton spectroscopic factor of the ^{25}P ground state is approximately 0.6.

Keywords: spectroscopic factors; direct proton capture; silicon isotopes

Introduction

Capture reactions are key determinants of reaction rates in explosive nucleosynthesis within high-temperature, high-density stellar environments [**?**, **?**]. They not only influence stellar energy output but also directly govern the evolution of elemental abundances, serving as a fundamental driving force behind complex nuclear reaction networks [**?**, **?**]. In the rapid proton capture process (rp-process), the (p, γ) reaction is particularly critical [**?**, **?**]. For proton-unstable nuclei, proton capture rates may approach or even exceed β^+-decay rates, thereby dominating explosive hydrogen combustion. However, determining the cross sections of these astrophysically important reactions remains a challenging issue in nuclear astrophysics. While direct measurements are the most reliable method [**?**, **?**], (p, γ) reaction cross sections in the astrophysically relevant energy range are on the order of microbarns to picobarns. For most isotopes of interest, radioactive ion beams available from current facilities have low intensities, making direct (p, γ) reaction measurements extremely difficult. Therefore, it is important to employ indirect methods [**?**], such as using proton-transfer reactions like (d, n) or (^{3}He, d) to identify which states play significant roles in direct proton capture, or to assess them theoretically by determining proton spectroscopic factors [**?**, **?**, **?**].

Nuclear spectroscopic factors, which link nuclear structure and nuclear reactions, are regarded as one of the most important inputs in nuclear astrophysics models [**?**, **?**]. Quantities such as the proton width and capture cross section are proportional to the spectroscopic factors, after accounting for the single-particle width and the relevant Coulomb/centrifugal penetration factors. Spectroscopic factors derived from direct proton-capture studies are generally considered more reliable than those inferred from proton-transfer reactions [**?**, **?**]. In nuclear astrophysics, proton capture reactions are particularly crucial during stellar explosive hydrogen burning, where proton capture reactions on silicon isotopes play an important role [**?**, **?**, **?**, **?**, **?**, **?**]. For example, the ^{26}Si(p, γ)^{27}P reaction is critical at this stage and represents one of the key pathways to ^{26}Al [**?**, **?**], while the ^{29}Si(p, γ)^{30}P rate modulates the ^{29}Si/^{30}P branching ratio and thus indirectly affects the final ^{29}Si abundance observed in classical novae [**?**, **?**, **?**].

In this work, we calculate the spectroscopic factors for direct proton capture reactions along the silicon isotopic chain using the nuclear shell model. The resulting proton spectroscopic factors for $^{26-29}$P show favorable consistency with both available experimental data and prior theoretical computations. We then

examine the spectroscopic factors in relation to the excitation energies of the residual nuclei. Additionally, we find that certain excited states make significant contributions to the spectroscopic factors and, under specific conditions, are more likely to undergo proton capture than the ground state. This finding provides critical parameters for refining reaction networks involving silicon isotopes in nuclear astrophysics and facilitates a more accurate understanding of elemental abundance evolution in stellar interiors.

II. Theoretical Basis

Spectroscopic factors were first introduced into nuclear reaction theory approximately sixty years ago, marking a significant milestone in the advancement of nuclear physics [**?**, **?**]. This concept has since become a cornerstone in the study of nuclear structure and reactions, as it provides a quantitative measure of the overlap between nuclear states before and after a reaction [**?**]. By comparing spectroscopic factors derived from experimental data with those calculated using various nuclear models, researchers can probe the internal structure of atomic nuclei and assess the validity of theoretical frameworks. This comparative approach has proven instrumental in refining our understanding of nuclear forces and configurations. Consequently, the study of spectroscopic factors has remained a central theme in both theoretical and experimental nuclear physics, inspiring extensive research over the decades and resulting in a substantial body of literature on the topic [**?**, **?**, **?**, **?**].

The spectroscopic factor is closely related to the occupancy probability of a single-particle orbital. It reflects both the probability that a specific orbital is occupied in the target nuclear state and the overlap integral between this orbital and the single-particle state involved in the transfer reaction. Therefore, the higher the occupancy probability of an orbital, the larger its corresponding spectroscopic factor tends to be. Cole et al. calculated spectroscopic factors for single-nucleon transfer reactions in nuclei with mass numbers A = 23–31 based on the shell model [**?**]. Specifically, the nuclear wave functions are calculated within the full or truncated sd-shell configuration space, and an appropriate creation or annihilation operator is applied to the target wave function (an annihilation operator in the case of pick-up reactions). The spectroscopic factor is then obtained by squaring the overlap integral between the resulting wave function and that of the final state.

According to the nuclear shell model, a spectroscopic factor depends on the overlap integral between the final state and the state formed by coupling the target state with the transferred particle to a coupled-channel state, which is defined as [**?**, **?**, **?**, **?**, **?**]:

$$\langle \phi_{A+1} \| \hat{a}^{+} \| \phi_{A} \rangle$$

where ϕ_{A+1} is the wave function of the final-state nuclei, ϕ_A is the wave function

of the initial-state nuclei, $A+1$ is the mass number of the nucleus, and $\hat{a}^+$ is the creation operator. By precise definition, this usually includes the Clebsch-Gordan coefficient and is therefore written as [**?**]:

$$C^{2S} = \left\langle T_i T_z; \tfrac{1}{2}\tfrac{1}{2} \| T_f T_z - \tfrac{1}{2} \right\rangle^2 \left| \langle \phi_{A+1} \| \hat{a}^+ \| \phi_A \rangle \right|^2$$

where T_i denotes the isospin quantum number of the initial nucleus ^{A}Si, and T_z is its corresponding isospin z-component. The quantities $\frac{1}{2}$ represent the isospin and isospin z-component of the incoming proton, respectively. T_f denotes the isospin quantum number of the final nucleus, while $T_z - \frac{1}{2}$ corresponds to the isospin z-component of the final nucleus ^{A}P after the proton is captured.

In the process of proton emission [**?**], the decay can be expressed as ${}^{A+1}_{Z+1}X \rightarrow {}^{A}_{ZX}$ $+p$, where ${}^{A+1}_{Z+1}X$ denotes the parent nucleus, ${}^{A}_{ZX}$ represents the daughter nucleus, and p is the emitted proton. The Coulomb barrier involved in this process can typically reach values as high as 15 MeV. The total half-life of proton emission is determined by the decay width (or resonance width), which is related through [**?**, **?**]:

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\hbar \ln 2}{C^{2S}\Gamma_0}$$

where Γ is the decay width, and Γ_0 denotes the effective decay width corrected by the spectroscopic factor. Both quantities have the dimension of energy and are commonly expressed in MeV. The decay width is connected to the transition amplitude via $\Gamma = 2\pi |T_{A+1,Z+1;A,Z}|^2$. Within the distorted-wave Born approximation (DWBA), the transition amplitude takes the form:

$$T_{A+1,Z+1;A,Z} = \langle \psi_{A,p}, \Psi_{A,p} | V_{A,p} | \Psi_{A+1} \rangle$$

where $\psi_{A,p}$ denotes the incoming spherical wave that describes the relative motion between the proton and the daughter nucleus. The initial wave function of the proton–daughter system is given by $\Psi_{A,p} = \Phi_A \Phi_p$, with Φ_A and Φ_p being the intrinsic functions of the daughter nucleus and the proton, respectively. Detailed formulas can be found in the references [**?**].

III. Details of the Calculation

In this paper, calculations are performed in the full $1d_{3/2}1d_{5/2}2s_{1/2}$ (sd) shell model space using the W, CW, and CWH effective interactions, with the shell-model code NuShellX, which performs diagonalization [**?**]. NuShellX is a core-shell model code whose important capabilities include predicting transition rates, spectroscopic factors, cluster spectroscopic factors, etc. [**?**, **?**].

The W interaction refers to the two-body matrix elements derived from a linear fit [**?**]. Under the assumption of mass-independent two-body matrix elements,

Chung and Wildenthal introduced two different sets of two-body matrix elements: the CW interaction for the A = 17–28 region and the CWH interaction for the A = 28–39 region [**?**, **?**]. The spin and parity data required in the calculation process come from Ref. [**?**]. Since the mass numbers of the silicon and phosphorus isotopes fall near the applicable ranges of these effective interactions, their use is reasonable. Furthermore, as these interactions have been validated in previous studies and show good agreement with experimental data [**?**, **?**, **?**], their adoption in this work is reliable.

In the theoretical description of proton emission from a nucleus, the total spectroscopic factor is generally the sum of contributions from all accessible single-particle orbitals, such as $d_{5/2}$ and $d_{3/2}$. Each orbital contributes a partial spectroscopic factor, which can be interpreted as the probability of emitting a proton from that specific orbital. Accordingly, the partial half-life for each orbital can be calculated to reflect the tunneling probability and barrier penetrability associated with that single-particle configuration. The overall proton-emission half-life of the nucleus is then determined by the combination of these partial half-lives, allowing the decay branching structure to be accurately taken into account and enabling direct comparison between theoretical predictions and experimental measurements. Although the calculation procedure is relatively intricate, it can be systematically implemented in existing computational frameworks to handle contributions from different orbitals. In this work, we calculate the total proton-emission half-life $T_{1/2}$ and the corresponding proton width Γ_0. Consequently, the contributions from individual orbitals are directly connected to the overall decay properties, allowing for a consistent comparison between theoretical calculations and experimental results.

[TABLE I]

IV. Results and Discussion

We calculated the spectroscopic factors for the silicon isotope chain, selected nuclear states with spectroscopic factors greater than 0.1 for analysis, and systematically compared the results with theoretically predicted values and experimentally measured data reported in existing literature, as detailed in Table I. The analysis revealed a high degree of consistency among the results obtained through different interactions within the same shell model. The calculated spectroscopic factors, which range from 0 to 1, closely matched the literature values, indicating the accuracy of the shell model calculations. For the ground state ($E_x = 0$ MeV), the proton spectroscopic factors of $^{25-31}$P are between 0.44 and 0.63. Furthermore, we found that the proton spectroscopic factor of the ^{25}P ground state is relatively large, approximately 0.6, with a value of 0.63 under the W interaction and 0.60 under the CW interactions.

[Figure 1: see original paper] presents the shell model calculation results for various nuclear states under the W and CW/CWH interactions. These results are compared with all available theoretical values and experimental measurements

reported in the literature listed in Table I, and the relative deviations between the calculated values and the corresponding averages are analyzed. Through the plotting of error bars, it can be clearly observed that the overall error range is small, and the shell model calculation results maintain good consistency with both theoretical and experimental data points. Specifically, for the nuclear states of ^{29}P($1/2^+$), ^{29}P($5/2^+$), and ^{31}P($1/2^+$), the calculation results from this study exhibit a higher degree of agreement with experimental measurement values. However, for the ^{29}P($3/2^+$) nuclear state, there is a relatively large deviation between the calculated values and the experimental values. Overall, our calculation results demonstrate high reliability, which validates the effectiveness of the shell model method.

We selectively extracted the data for the isotopes ^{26}P to ^{29}P, which have attracted considerable interest, and plotted the spectroscopic factors as a function of excitation energy, as shown in [Figure 2: see original paper]. In most of the analyzed (p, γ) reactions, the proton spectroscopic factor of the ground state of P is typically more than twice that of the excited states. However, several notable exceptions emerge in which the proton spectroscopic factors of certain excited states are comparable to, or even exceed, those of the corresponding ground states. For instance, in the ^{26}Si(p, γ)^{27}P reaction (Figure 2: see original paper), the proton spectroscopic factor of the ^{27}P($3/2^+$) excited state at $E_x = 0.895$ MeV is 0.41, which is very close to the ground state value of 0.46. In the ^{28}Si(p, γ)^{29}P reaction (Figure 2: see original paper), the proton spectroscopic factor of the ^{29}P($3/2^+$) excited state at $E_x = 1.394$ MeV is 0.66, exceeding that of the ground state (0.45). Furthermore, as shown in Table I, the proton spectroscopic factor of the ^{30}P(2^+) excited state at $E_x = 1.491$ MeV in the ^{29}Si(p, γ)^{30}P reaction is 0.35, which is comparable to that of the ground state (0.44). A similar situation is observed in the ^{30}Si(p, γ)^{31}P reaction, where the proton spectroscopic factor of the ^{31}P($3/2^+$) excited state at $E_x = 1.21$ MeV is 0.54, close to the ground state ^{31}P($1/2^+$) value of 0.51. These results further highlight the importance of considering not only ground states but also excited states in nuclear reaction models. Taking into account the tabulated data as a whole, we also observe that nuclei with higher binding energies generally exhibit larger spectroscopic factors in their excited states, as exemplified by the $^{29-31}$P isotopes. Overall, the present results show good agreement with existing experimental or theoretical data, demonstrating the reliability of the employed computational approach.

Spectroscopic factors play a crucial role in both nuclear structure and nuclear reactions. Accurate spectroscopic factors facilitate more precise calculations of capture cross sections, which are essential for understanding the probability of nuclear reactions and for improving both theoretical predictions and the interpretation of experimental results. Similarly, reliable spectroscopic factors allow for more accurate evaluations of proton widths, which are critical for understanding the stability of excited nuclear states and their decay processes [**?**, **?**, **?**]. Given that nuclear reaction data often require comparison with theoretical predictions, the spectroscopic factors obtained in this study can provide

a valuable reference for the further refinement of relevant theoretical models, thereby offering potentially valuable guidance and support for future experimental investigations.

In addition, the half-life and proton width of the proton emission from ^{25}P to ^{24}Si were calculated using the DWBA method [**?**]. As shown in [TABLE II], the proton-decay properties of ^{25}P and ^{29}P exhibit distinct behaviors. For ^{25}P, the proton-emission energy is 0.941 MeV, corresponding to a half-life of 5.08×10^{-20} s and a relatively small proton width of only 0.009 MeV. In contrast, ^{29}P displays a pronounced multi-channel decay pattern with three accessible final states at 0.936, 2.323, and 4.249 MeV. A clear correlation between decay energy and decay probability is observed: as the proton-emission energy increases, the half-life decreases drastically from 7.81×10^{-20} s to 4.01×10^{-21} s, and further to 3.40×10^{-21} s. This trend is consistent with the corresponding increase in proton width from 0.006 MeV to 0.114 MeV and 0.135 MeV, indicating enhanced barrier penetrability in the higher-energy channels. Consequently, the effective lifetime of ^{29}P is mainly governed by the highest-energy decay mode and is significantly shorter than that of ^{25}P. These results reveal a general rule that, in nuclei with multiple emission pathways, higher decay energies typically correspond to larger proton widths and faster decay rates. With increasing proton-emission energy, the rapid reduction of the half-life and the significant enhancement of the proton width manifest the characteristic quantum-tunneling behavior, further confirming the shell model' s capability in accurately describing the proton-emission process in light nuclei such as phosphorus.

[TABLE II]

V. Summary

In this study, we systematically calculated the proton spectroscopic factors for the phosphorus isotopic chain and compared them with existing experimental and theoretical values. Our analysis revealed a high degree of consistency across results obtained using various nuclear interactions within the same shell model framework. The calculated spectroscopic factors, which are constrained to the range 0–1, exhibit strong agreement with experimental data and published theoretical values, thereby validating the reliability and accuracy of our shell model approach. A detailed examination of the data showed that, for the ground state (with excitation energy $E_x = 0$ MeV), the proton spectroscopic factors of isotopes ^{25}P through ^{31}P generally lie in the range of 0.44 to 0.63. Furthermore, we observed that excited states make non-negligible contributions to the total spectroscopic strength, and the tabulated data reveal that the spectroscopic factors of stable nuclei are larger than those of unstable nuclei. This indicates that including these excited states is essential for a comprehensive understanding of nuclear structure.

In addition to the isotopic analysis, we present a theoretical prediction for the proton spectroscopic factor of the ^{25}P ground state, which is associated with

the nuclear reaction ^{24}Si(p, γ)^{25}P, estimating its value to be approximately 0.6. These calculated spectroscopic factors not only enhance our understanding of the nuclear structure of silicon isotopes but also serve as a valuable reference for modeling astrophysical phenomena and nuclear synthesis processes, particularly in proton-rich environments.

Finally, we calculated the proton emission half-lives and proton widths of ^{25}P and ^{29}P using the calculated spectroscopic factors in combination with the DWBA method. The results show that ^{25}P has a relatively low proton emission energy, a longer half-life, and a smaller proton width, whereas for ^{29}P, among multiple decay channels, the higher-energy channels correspond to shorter half-lives and larger proton widths. As the decay energy increases, the proton width becomes larger and the decay rate faster, reflecting the quantum tunneling effect. Overall, these results illustrate the applicability of the shell model in describing proton emission processes in light nuclei.

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Note: Figure translations are in progress. See original paper for figures.